

# Leptogenesis without CP Violation at Low Energies

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## Abstract

In this letter we give a class of examples where the decays of the heavy Majorana neutrinos may violate CP even if there is no CP violation at low energies, i. e., leptogenesis can take place without Majorana or Dirac type CP phases at low energies.

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There is strong experimental evidence for neutrino oscillations [1], [2] thus implying non zero neutrino masses and mixing in the leptonic sector, together with the possibility of CP violation. The most straightforward way of extending the Standard Model (SM) in order to incorporate neutrino masses is to add one neutrino field per generation, singlet under  $SU(3)_c \times SU(2) \times U(1)$  gauge symmetry, in analogy with the quark sector. The fact that neutrinos are neutral particles allows for the introduction of a Majorana mass term for the right-handed gauge singlets together with the usual Dirac mass term, provided that lepton number conservation is not imposed. This leads to the seesaw mechanism [3] which accounts in an elegant and simple way for the smallness of neutrino masses. Leptonic CP violation may play a crucial rôle in the generation of the observed baryon number asymmetry of the universe (BAU) via leptogenesis. In this framework a CP asymmetry would be generated through out-of-equilibrium L-violating decays of heavy Majorana neutrinos [4] leading to a lepton asymmetry  $L \neq 0$  while  $B = 0$  is still maintained. Subsequently, sphaleron processes [5] which are  $(B + L)$  violating and  $(B - L)$  conserving restore  $(B + L) = 0$  thus creating a nonvanishing  $B$ . Several groups have analysed the requirements on models leading to a viable leptogenesis [6]. At low energies the decoupling limit is an excellent approximation and there are three CP violating phases in the corresponding mixing matrix, one of Dirac type which could be observed in neutrino oscillations [7] and two of Majorana type which can be interpreted in terms of unitary triangles [8]. The question of whether or not it is possible to establish a connexion between leptogenesis and CP violation at low energies is very interesting and has been addressed by several authors [9], [10]. It has been shown that although in general this connexion cannot be established there are several frameworks where the sign and size of the observed baryon asymmetry obtained through leptogenesis can be related to CP violation at low energies.

It is well known that in the case of three generations with no lefthanded Majorana mass term there are six CP violating phases in the leptonic sector [11]. It is possible to choose a Weak Basis (WB) where all of these phases only appear in the Dirac type neutrino mass matrix. These phases may be parametrized in such a way that the three low energy CP violating phases are a function of all of them whilst leptogenesis can be written in terms of only three phases [10].

In this work we want to emphasize that leptogenesis can take place even if there is no CP violation at low energies. The prospects of finding CP violating effects at low energies, for instance in future neutrino factories, are extremely exciting yet it is important to notice that leptogenesis remains in principle a viable scenario even if no CP violation is seen at low energies.

**Framework:** After spontaneous symmetry breaking the leptonic mass term for a minimal extension of the SM which consists of adding to the standard spectrum one right-handed neutrino per generation can be written as:

$$\mathcal{L}_m = -[\overline{\nu}_L^0 m \nu_R^0 + \frac{1}{2} \nu_R^{0T} C M \nu_R^0 + \overline{l}_L^0 m_l l_R^0] + h.c. =$$

$$= -\left[\frac{1}{2}n_L^T C \mathcal{M}^* n_L + \overline{l}_L^0 m_l l_R^0\right] + h.c. \quad (1)$$

where  $m$ ,  $M$  and  $m_l$  denote the neutrino Dirac mass matrix, the right-handed neutrino Majorana mass matrix and the charged lepton mass matrix, respectively, and  $n_L = (\nu_L^0, (\nu_R^0)^c)$ . In this minimal extension of the SM a term of the form  $\frac{1}{2}\nu_L^{0T} C m_L \nu_L^0$  does not appear in the Lagrangean and the matrix  $\mathcal{M}$  is given by:

$$\mathcal{M} = \begin{pmatrix} 0 & m \\ m^T & M \end{pmatrix} \quad (2)$$

with a zero entry on the (11) block. The right-handed Majorana mass term is  $SU(2) \times U(1)$  invariant, consequently it can have a value much above the scale  $v$  of the electroweak symmetry breaking thus leading to the seesaw mechanism. The neutrino mass matrix  $\mathcal{M}$  is diagonalized by the transformation:

$$V^T \mathcal{M}^* V = \mathcal{D} \quad (3)$$

where  $\mathcal{D} = \text{diag.}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, M_{\nu_1}, M_{\nu_2}, M_{\nu_3})$ , with  $m_{\nu_i}$  and  $M_{\nu_i}$  denoting the physical masses of the light and heavy Majorana neutrinos, respectively. It is convenient to write  $V$  and  $\mathcal{D}$  in the following form:

$$V = \begin{pmatrix} K & R \\ S & T \end{pmatrix}; \quad \mathcal{D} = \begin{pmatrix} d & 0 \\ 0 & D \end{pmatrix}. \quad (4)$$

From Eq. (3) one obtains to an excellent approximation:

$$-K^\dagger m \frac{1}{M} m^T K^* = d \quad (5)$$

Together with the following exact relation:

$$R = m T^* D^{-1} \quad (6)$$

In the WB where the right-handed Majorana neutrino mass is diagonal it also follows to an excellent approximation that:

$$R = m D^{-1} \quad (7)$$

Eq.(5) is the usual seesaw formula with  $K$  a unitary matrix. The neutrino weak-eigenstates are related to the mass eigenstates by:

$$\nu_{iL}^0 = V_{i\alpha} \nu_{\alpha L} = (K, R) \begin{pmatrix} \nu_{iL} \\ N_{iL} \end{pmatrix} \quad \begin{pmatrix} i = 1, 2, 3 \\ \alpha = 1, 2, \dots, 6 \end{pmatrix} \quad (8)$$

and thus the leptonic charged current interactions are given by:

$$-\frac{g}{\sqrt{2}} \left( \overline{l}_{iL} \gamma_\mu K_{ij} \nu_{jL} + \overline{l}_{iL} \gamma_\mu R_{ij} N_{jL} \right) W^\mu + h.c. \quad (9)$$

From Eqs.(8), (9) it follows that  $K$  and  $R$  give the charged current couplings of charged leptons to the light neutrinos  $\nu_j$  and to the heavy neutrinos  $N_j$ , respectively. The unitary matrix  $K$ , which contains all the information about CP violation at low energies, can be parametrized as:

$$K = P_\xi \hat{U}_\rho P_\alpha \quad \longrightarrow \quad \hat{U}_\rho P_\alpha \quad (10)$$

with  $P_\xi = \text{diag.}(\exp(i\xi_1), \exp(i\xi_2), \exp(i\xi_3))$ , and  $P_\alpha = \text{diag.}(1, \exp(i\alpha_1) \exp(i\alpha_2))$  leaving  $\hat{U}_\rho$  with only one phase as in the case of the Cabibbo-Kobayashi-Maskawa matrix. Since  $P_\xi$  can be rotated away by a redefinition of the charged leptonic fields  $K$  is left with three CP violating phases, one of Dirac type  $\rho$  and two of Majorana character  $\alpha_1$  and  $\alpha_2$ .

The computation of the lepton number asymmetry, in this extension of the SM, resulting from the decay of a heavy Majorana neutrino  $N^j$  into charged leptons  $l_i^\pm$  ( $i = e, \mu, \tau$ ) leads to [12] :

$$\begin{aligned} A^j &= \frac{g^2}{M_W^2} \sum_{k \neq j} \left[ \text{Im} \left( (m^\dagger m)_{jk} (m^\dagger m)_{jk} \right) \frac{1}{16\pi} \left( I(x_k) + \frac{\sqrt{x_k}}{1-x_k} \right) \right] \frac{1}{(m^\dagger m)_{jj}}, \\ &= \frac{g^2}{M_W^2} \sum_{k \neq j} \left[ (M_k)^2 \text{Im} \left( (R^\dagger R)_{jk} (R^\dagger R)_{jk} \right) \frac{1}{16\pi} \left( I(x_k) + \frac{\sqrt{x_k}}{1-x_k} \right) \right] \frac{1}{(R^\dagger R)_{jj}} \end{aligned} \quad (11)$$

with the lepton number asymmetry from the  $j$  heavy Majorana particle,  $A^j$ , defined in terms of the family number asymmetry  $\Delta A^j_i = N^j_i - \bar{N}^j_i$  by :

$$A^j = \frac{\sum_i \Delta A^j_i}{\sum_i (N^j_i + \bar{N}^j_i)} \quad (12)$$

the sum in  $i$  runs over the three flavours  $i = e, \mu, \tau$ ,  $M_k$  are the heavy neutrino masses, the variable  $x_k$  is defined as  $x_k = \frac{M_k^2}{M_j^2}$  and  $I(x_k) = \sqrt{x_k} \left( 1 + (1+x_k) \log\left(\frac{x_k}{1+x_k}\right) \right)$ . From Eq. (11) it can be seen that the lepton number asymmetry is only sensitive to the CP violating phases appearing in  $m^\dagger m$  in the WB where  $M$  and  $m_l$  are diagonal (or equivalently in  $R^\dagger R$ ).

**Leptogenesis with no CP violation at low energies:** Let us go to the WB where  $M$  and  $m_l$  are diagonal, real and positive, ( $M \equiv D$ ) and choose a matrix  $m$  of the form [13]:

$$m = i\hat{U}_\rho P_\alpha \sqrt{d} O^c \sqrt{D} \quad (13)$$

where  $\sqrt{d}$  and  $\sqrt{D}$  are diagonal real matrices such that  $\sqrt{d}\sqrt{d} = d$ ,  $\sqrt{D}\sqrt{D} = D$  and  $O^c$  is an orthogonal complex matrix, i.e.,  $O^c O^{cT} = 1$  but  $O^c O^{c\dagger} \neq 1$ . In this WB all CP violating phases appear in  $m$ . From Eq. (13) together with Eq. (5) we obtain the matrix  $K$  given by Eq. (10) and in general it will violate CP. Particularizing for  $\alpha_1 = \alpha_2 = 0$  together with  $\rho = 0$  there is no CP violation at low energies. Yet leptogenesis is sensitive to the combination  $m^\dagger m$  which is given by:

$$h \equiv m^\dagger m = \sqrt{D} O^{c\dagger} d O^c \sqrt{D} \quad (14)$$

consequently, provided that the combination  $O^{c\dagger}dO^c$  is CP violating, we may have leptogenesis even without CP violation at low energies either of Dirac or Majorana type.

It is possible to write WB invariant conditions which have to vanish in order for CP invariance to hold. The nonvanishing of any of these invariants signals CP violation [14]. In Ref [10] the following WB invariants sensitive to CP violating phases which appear in leptogenesis were derived:

$$\begin{aligned}
I_1 &\equiv \text{ImTr}[hHM^*h^*M] = \\
&= M_1M_2(M_2^2 - M_1^2)\text{Im}(h_{12}^2) + M_1M_3(M_3^2 - M_1^2)\text{Im}(h_{13}^2) + \\
&\quad + M_2M_3(M_3^2 - M_2^2)\text{Im}(h_{23}^2); \\
I_2 &\equiv \text{ImTr}[hH^2M^*h^*M] = \\
&= M_1M_2(M_2^4 - M_1^4)\text{Im}(h_{12}^2) + M_1M_3(M_3^4 - M_1^4)\text{Im}(h_{13}^2) + \\
&\quad + M_2M_3(M_3^4 - M_2^4)\text{Im}(h_{23}^2); \\
I_3 &\equiv \text{ImTr}[hH^2M^*h^*MH] \\
&= M_1^3M_2^3(M_2^2 - M_1^2)\text{Im}(h_{12}^2) + M_1^3M_3^3(M_3^2 - M_1^2)\text{Im}(h_{13}^2) + \\
&\quad + M_2^3M_3^3(M_3^2 - M_2^2)\text{Im}(h_{23}^2) \tag{15}
\end{aligned}$$

The second equality for each  $I_i$  corresponds to the evaluation of these WB invariants in the WB where the right-handed neutrino mass is diagonal, with  $M_i$  the corresponding diagonal elements. The matrix  $H$  is defined by  $H \equiv M^\dagger M$ .

Choosing the matrix  $O^c$  of the form:

$$O^c = A_{12} \cdot A_{23} \cdot A_{13} \tag{16}$$

with

$$\begin{aligned}
A_{12} &= \begin{pmatrix} \cosh \theta_{12} & i \sinh \theta_{12} & 0 \\ -i \sinh \theta_{12} & \cosh \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}; A_{13} = \begin{pmatrix} \cosh \theta_{13} & 0 & i \sinh \theta_{13} \\ 0 & 1 & 0 \\ -i \sinh \theta_{13} & 0 & \cosh \theta_{13} \end{pmatrix}; \\
A_{23} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cosh \theta_{23} & i \sinh \theta_{23} \\ 0 & -i \sinh \theta_{23} & \cosh \theta_{23} \end{pmatrix} \tag{17}
\end{aligned}$$

we obtain (simplifying the notation with cosh and sinh replaced by ch and sh):

$$\begin{aligned}
\text{Re}h_{12} &= (\text{sh}^2\theta_{12}\text{sh}\theta_{13}\text{sh}\theta_{23}\text{ch}\theta_{23}d_1 + \text{sh}\theta_{13}\text{sh}\theta_{23}\text{ch}^2\theta_{12}\text{ch}\theta_{23}d_2 + \\
&\quad + \text{sh}\theta_{13}\text{sh}\theta_{23}\text{ch}\theta_{23}d_3)\sqrt{M_1}\sqrt{M_2}; \\
\text{Im}h_{12} &= i\text{sh}\theta_{12}\text{ch}\theta_{12}\text{ch}\theta_{13}\text{ch}\theta_{23}(d_1 + d_2)\sqrt{M_1}\sqrt{M_2}; \\
\text{Re}h_{13} &= -\text{sh}\theta_{12}\text{sh}\theta_{23}\text{ch}\theta_{12}(d_1 + d_2)\sqrt{M_1}\sqrt{M_3}; \\
\text{Im}h_{13} &= i[(\text{sh}\theta_{13}\text{ch}^2\theta_{12}\text{ch}\theta_{13} + \text{sh}^2\theta_{12}\text{sh}\theta_{13}\text{sh}^2\theta_{23}\text{ch}\theta_{13})d_1 + \\
&\quad + (\text{sh}^2\theta_{12}\text{sh}\theta_{13}\text{ch}\theta_{13} + \text{sh}\theta_{13}\text{sh}^2\theta_{23}\text{ch}^2\theta_{12}\text{ch}\theta_{13})d_2 +
\end{aligned}$$

$$\begin{aligned}
& + \operatorname{sh}\theta_{13}\operatorname{ch}\theta_{13}\operatorname{ch}^2\theta_{23}d_3]\sqrt{M_1}\sqrt{M_3}; \\
\operatorname{Re}h_{23} & = \operatorname{sh}\theta_{12}\operatorname{sh}\theta_{13}\operatorname{ch}\theta_{12}\operatorname{ch}\theta_{23}(d_1 + d_2)\sqrt{M_2}\sqrt{M_3}; \\
\operatorname{Im}h_{23} & = i(\operatorname{sh}^2\theta_{12}\operatorname{sh}\theta_{23}\operatorname{ch}\theta_{13}\operatorname{ch}\theta_{23}d_1 + \operatorname{sh}\theta_{23}\operatorname{ch}^2\theta_{12}\operatorname{ch}\theta_{13}\operatorname{ch}\theta_{23}d_2 \\
& + \operatorname{sh}\theta_{23}\operatorname{ch}\theta_{13}\operatorname{ch}\theta_{23}d_3)\sqrt{M_2}\sqrt{M_3}
\end{aligned} \tag{18}$$

The  $d_i$  are the diagonal elements of the matrix  $d$  (light neutrino masses). In general  $\operatorname{Im}h_{ij}^2, i \neq j$  do not vanish and there is leptogenesis. On the other hand if any of the  $\theta_{ij}$  is zero these imaginary parts vanish due to the fact that all products  $\operatorname{Re}h_{ij}\operatorname{Im}h_{ij}$  contain the factor  $\operatorname{sh}\theta_{12}\operatorname{sh}\theta_{13}\operatorname{sh}\theta_{23}$ . Equations (15) are no longer useful to discuss CP violation in the limit  $M_1 = M_2 = M_3$  since in this case they vanish trivially, although this degeneracy does not necessarily imply CP conservation at high energies [10].

**Final Additional Comments:** In this framework low energy physics only enters in Eq. (14) through the masses of the light neutrinos which are already constrained by experiment. In fact, Eq. (14) has no explicit dependence on mixing and CP violation at low energies since  $K$  cancels out. With the present experimental knowledge there is freedom in the choice of the masses of the heavy neutrinos. Furthermore low energy physics is insensitive to the matrix  $O^c$ . As a result one can only establish a connection between leptogenesis and CP violation at low energies in models where additional constraints are imposed so that, for instance, the matrix  $D$  is no longer independent of  $K$ .

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## References

- [1] Y. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Lett. B **433** (1998) 9 [arXiv:hep-ex/9803006]; Phys. Lett. B **436** (1998) 33 [arXiv:hep-ex/9805006]; Phys. Rev. Lett. **81** (1998) 1158 [Erratum-ibid. **81** (1998) 4279] [arXiv:hep-ex/9805021]; Phys. Rev. Lett. **82** (1999) 1810 [arXiv:hep-ex/9812009]; Phys. Rev. Lett. **82** (1999) 2430 [arXiv:hep-ex/9812011]; Phys. Rev. Lett. **82** (1999) 2644 [arXiv:hep-ex/9812014]; Phys. Lett. B **467** (1999) 185 [arXiv:hep-ex/9908049]; Phys. Rev. Lett. **85** (2000) 3999 [arXiv:hep-ex/0009001]; Phys. Rev. Lett. **86** (2001) 5651 [arXiv:hep-ex/0103032]; Phys. Rev. Lett. **86** (2001) 5656 [arXiv:hep-ex/0103033].
- [2] Q. R. Ahmad *et al.* [SNO Collaboration], Phys. Rev. Lett. **87** (2001) 071301 [arXiv:nucl-ex/0106015]; Phys. Rev. Lett. **89** (2002) 011301 [arXiv:nucl-ex/0204008]; Phys. Rev. Lett. **89** (2002) 011302 [arXiv:nucl-ex/0204009].
- [3] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, eds P. van Nieuwenhuizen and D. Z. Freedman (North Holland, Amsterdam, 1979), p.315; T.

Yanagida, in Proc. of the Workshop on the Unified Theory and Baryon Number in the Universe, KEK report 79-18 (1979), p.95.

- [4] M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45.
- [5] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155** (1985) 36.
- [6] For recent reviews and references see e.g.: A. Pilaftsis, Int. J. Mod. Phys. A **14** (1999) 1811 [arXiv:hep-ph/9812256]; W. Buchmüller and M. Plümacher, Int. J. Mod. Phys. A **15** (2000) 5047 [arXiv:hep-ph/0007176].
- [7] A. De Rujula, M. B. Gavela and P. Hernandez, Nucl. Phys. B **547** (1999) 21 [arXiv:hep-ph/9811390]; M. C. Gonzalez-Garcia, Y. Grossman, A. Gusso and Y. Nir, Phys. Rev. D **64** (2001) 096006 [arXiv:hep-ph/0105159].
- [8] J. A. Aguilar-Saavedra and G. C. Branco, Phys. Rev. D **62** (2000) 096009 [arXiv:hep-ph/0007025]; Y. Farzan and A. Y. Smirnov, Phys. Rev. D **65** (2002) 113001 [arXiv:hep-ph/0201105].
- [9] R. N. Mohapatra and X. Zhang, Phys. Rev. D **46** (1992) 5331; C. W. Chiang, Phys. Rev. D **63** (2001) 076009 [arXiv:hep-ph/0011195]; A. S. Joshipura, E. A. Paschos and W. Rodejohann, JHEP **0108** (2001) 029 [arXiv:hep-ph/0105175]; W. Buchmüller and D. Wyler, Phys. Lett. B **521** (2001) 291 [arXiv:hep-ph/0108216]; M. S. Berger and K. Siyeon, Phys. Rev. D **65** (2002) 053019 [arXiv:hep-ph/0110001]; G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim and M. N. Rebelo, arXiv:hep-ph/0202030; H. B. Nielsen and Y. Takanishi, Nucl. Phys. B **636** (2002) 305 [arXiv:hep-ph/0204027]; Z. z. Xing, arXiv:hep-ph/0206245; J. Ellis, M. Raidal and T. Yanagida, arXiv:hep-ph/0206300; S. Davidson and A. Ibarra, arXiv:hep-ph/0206304; W. Rodejohann, arXiv:hep-ph/0207053.
- [10] G. C. Branco, T. Morozumi, B. M. Nobre and M. N. Rebelo, Nucl. Phys. B **617** (2001) 475 [arXiv:hep-ph/0107164].
- [11] T. Endoh, T. Morozumi, T. Onogi and A. Purwanto, Phys. Rev. D **64** (2001) 013006 [Erratum-ibid. D **64** (2001) 059904] [arXiv:hep-ph/0012345].
- [12] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B **384** (1996) 169; M. Flanz, E. A. Paschos and U. Sarkar, Phys. Lett. B **345** (1995) 248 [Erratum-ibid. B **382** (1995) 447]; M. Plümacher, Z. Phys. C **74** (1997) 549.
- [13] This parametrization was already used by: J. A. Casas and A. Ibarra, Nucl. Phys. B **618** (2001) 171 [arXiv:hep-ph/0103065].
- [14] G. C. Branco, L. Lavoura and M. N. Rebelo, Phys. Lett. B **180** (1986) 264.