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Chiral gauged WZNW models and heterotic string backgrounds

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Abstract

We construct new heterotic string backgrounds which are analogous to superstring solutions corresponding to coset models but are not simply the 'embeddings'of the latter. They are described by the (1,0) supersymmetric extension of the G/H chiral gauged WZNW models. The 'chiral gauged' WZNW action differs from the standard gauged WZNW action by the absence of the $A\bar{A}$-term (and thus is not gauge invariant in the usual sense) but can still be expressed as a combination of WZNW actions and is conformal invariant. We explain a close relation between gauged and chiral gauged WZNW models and prove that in the case of the abelian H the G/H chiral gauged theory is equivalent to a particular $(G \times H)/H$ gauged WZNW theory. In contrast to the gauged WZNW model, the chiral gauged one admits a (1,0) supersymmetric extension which is consistent at the quantum level. Integrating out the $2d$ gauge field we determine the exact (in α') form of the couplings of the corresponding heterotic sigma model. While in the bosonic (superstring) cases all the fields depend (do not depend) non-trivially on α' here the metric receives only one $O(\alpha')$ correction while the antisymmetric tensor and the dilaton remain semiclassical. As a simplest example, we discuss the basic $D = 3$ solution which is the heterotic string counterpart of the 'black string' $SL(2,R) \times R/R$ background.

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1. Introduction

The Wess-Zumino-Novikov-Witten (WZNW) model [1][2] is a prototypical example of a local field theory which is conformally invariant, i.e. which provides a realisation of the conformal operator algebra [3]. The G/H gauged WZNW model (or GWZNW) can be represented as a particular combination of the WZNW models for a group G and a subgroup H. This explains its conformal invariance as well as its relation [4][5] to a coset conformal field theory [6]. Starting with GWZNW it is possible to construct various string solutions to the leading order in α' (see, for example, [7][8][9][10]), or to all orders in α' (see, for example, [11][12][13]).

One may question if there are other non-trivial combinations of WZNW models which also correspond to local conformal field theories and thus generate new string backgrounds. A closely related to GWZNW model is the so called chiral gauged WZNW model [14] (CWZNW) in which one does not include the 'counterterm' $\text{Tr}\,(A\bar{A})$ in the action [9] ($A_m = (A, \bar{A})$ is a $2d$ vector field with values in the algebra of H) and thus one has only 'chiral' gauge invariance (with gauge parameters constrained to be holomorphic or antiholomorphic). As we shall see below, the action of CWZNW is, in fact, the only local modification of the GWZNW action which can also be represented in terms of a combination of independent WZNW actions and thus is certainly conformally invariant at the quantum level. As a consequence, the σ-model for the coordinates of the group space (G) obtained from a CWZNW model by integrating out the $2d$ gauge field (see [15][16] for particular examples and [17] for a general case) will also be conformally invariant.

An interpretation that can be given to a 'product' of WZNW models depends crucially on which combinations of fields are treated as 'fundamental' and which – as 'auxiliary' or Lagrange multiplier-type variables. Since the actions of both GWZNW and CWZNW are local, being expressed in terms of the group variable g and the $2d$ vector field A_m (with the latter having no kinetic term), it is natural to treat A_m as an auxiliary field which should be integrated out (without introducing a source term for it) in the path integral. It is within such an approach that the GWZNW model is related to the coset model. Below we shall study a possibility to give a similar interpretation to CWZNW.[1] We shall find that if all the fields are treated on an equal footing, i.e. if one admits a possibility of making field redefinitions that mix g with A_m, then the CWNW models are essentially equivalent to a subclass of GWZNW models: $(G/H)_{CWZNW}$ can be identified (modulo field redefinitions) with the $(G/H \times H)_{GWZNW}$. When H is abelian it is possible to establish a more direct relation without the necessity to redefine A_m: $(G/H)_{CWZNW} = [(G \times H)/H]_{GWZNW}$, where in the GWZNW case the subgroup H is embedded into $G \times H$ in a specific way and is gauged axially. The latter equivalence provides a general explanation for the observations in [15][16] (to the leading order in $1/k$) and [17] (exactly in $1/k$) that the $SL(2,\mathbb{R})/\mathbb{R}$ CWZNW model is a particular limit of the $SL(2,\mathbb{R}) \times \mathbb{R}/\mathbb{R}$ GWZNW (or 'black string' [18][13]) model.

Our interest in the CWZNW models, besides their importance as bosonic models with exact conformal invariance (and in connection with $O(d,d)$ duality which seems to relate different 'mixtures' of WZNW models, see e.g. [19][20][21]) was originally motivated by a desire to understand if they can be used for a construction of heterotic string backgrounds which are not simply the embeddings of $(1,1)$ supersymmetric coset solutions. As we shall show, this is indeed the case: CWZNW model has a consistent $(1,0)$ supersymmetric generalisation and may thus serve as a basis for a non-trivial heterotic string world sheet theory.

The question about heterotic string solutions related to coset models was recently discussed in [22] where it was pointed out that the direct (1,0) truncation of the (1,1) supersymmetric gauged WZNW model does not correspond to a consistent heterotic string background since the resulting $2d$ theory is anomalous (the fermions couple chirally to the $2d$ gauge field which is integrated over in the path integral). It was suggested to

[1] We shall mostly consider CWZNW models in the cases when the 'left' and 'right' subgroups are the same. We disagree with the claim [14] that the CWZNW model with $H_L = H_R = H$ is equivalent to G/H or GWZNW model.

cancel the $2d$ gauge anomaly by introducing an additional world sheet coupling term which corresponds to a non-vanishing target space gauge field background [22]. As a result, the theory becomes effectively (1,1) supersymmetric and can be interpreted as a coset model superstring solution embedded into the heterotic string theory. Because of (1,1) supersymmetry the background fields are then not modified by α' corrections [23][12][24] [25].

The key observation made below is that it is possible to construct closely related but non-trivial (1,0) supersymmetric model by giving up the $2d$ gauge invariance already at the classical level and replacing the gauged WZNW model by the chiral gauged WZNW one, i.e. by using the (1,0) supersymmetric CWZNW as a starting point for a construction of a heterotic σ-model which has an exact conformal field theory interpretation. Practically, this means setting to zero the coefficient of the $A\bar{A}$-type term in the GWZNW action or, equivalently, adding the anomalous gauge degree of freedom to the set of dynamical variables and thus remaining on the target space of dimension equal to dim G (and not to dim G/H). Then no inconsistency appears at the quantum level. In contrast to the GWZNW case, the world-sheet theory here has only (1,0) (and not (1,1)) supersymmetry, the target space dimension of the corresponding heterotic σ-model is $D = \dim G$ and the background metric receives $1/k$ (or α') correction. The first possibility for a non-trivial solution is thus in $D = 3$, corresponding, e.g., to $G = SL(2,\mathbb{R})$ and $H = \mathbb{R}$. From the point of view of the heterotic string theory, this $D = 3$ model is the simplest yet basic example, similar to what the $SL(2,\mathbb{R})/\mathbb{R}$ $D = 2$ 'black hole' is for the bosonic or (1,1) supersymmetric string theory.

We shall start in Section 2 with a description of the path integral formulation of the GWZNW and CWZNW models clarifying their close connection. We shall establish their formal equivalence relation (eq.(2.22)) and illustrate it by computing the central charges. In Section 3 we shall compare the expressions for the Hamiltonians of the corresponding conformal field theories demonstrating that the CWZNW Hamiltonian is different from the combination of the standard Hamiltonians of the 'left' (G/H_L) and 'right' (G/H_R) coset models. In Section 4 we shall consider the case when the subgroup H is abelian and identify G/H CWZNW model with a particular axially gauged $(G\times H)/H$ WZNW model.

The (1,1) and (1,0) supersymmetric versions of CWZNW theory will be discussed in Sections 5 and 6. We shall first review the manifestly supersymmetric path integral quantisation of the (1,1) GWZNW and then apply a similar approach to the CWZNW case (discussing also the component formulation). In Section 6 we shall treat the case of (1,0) supersymmetric CWZNW model, establishing, in particular, the expression for its effective action which, in contrast to the (1,1) supersymmetric case, will contain a quantum correction term. Finally, in Section 7 we shall apply the effective action approach [26][27][25][17] to derive the exact (in $1/k$) form of the background fields of the corresponding heterotic string solutions and present the first non-trivial $D = 3$ example which is similar to the $SL(2,\mathbb{R})\times\mathbb{R}/\mathbb{R}$ GWZNW model. Section 8 will contain some concluding remarks.

2. Gauged and 'chiral gauged' WZNW models

2.1. Path integral

Consider the classical action of the form

$$I_a(g,A) = I_0(g,A) + \frac{a}{\pi}\int d^2z \ \mathrm{Tr}\ (A\bar{A}) \ , \tag{2.1}$$

where $A = A_z, \ \bar{A} = A_{\bar{z}}, \ a$ is a constant parameter and $I_0(g,A)$ is the gauged WZNW action [1][28]

$$\begin{aligned} I_{GWZNW} =& I_0(g,A) = I(g) + \frac{1}{\pi}\int d^2z \ \mathrm{Tr}\ \left(A\,\bar{\partial}gg^{-1} - \bar{A}\,g^{-1}\partial g + g^{-1}Ag\bar{A} - A\bar{A}\right) \ , \\ & I(g) = \frac{1}{2\pi}\int d^2z \ \mathrm{Tr}\ (\partial g^{-1}\bar{\partial} g) + \frac{i}{12\pi}\int \ \mathrm{Tr}\ (g^{-1}dg)^3 \ . \end{aligned} \tag{2.2}$$

The action (2.2) is invariant under the standard vector gauge transformations ($A, \bar{A}$ take values in the algebra $\mathcal{L}(H)$ of the subgroup H)

$$g \to u^{-1}gu \ , \quad A \to u^{-1}(A-\partial)u \ , \quad \bar{A} \to u^{-1}(\bar{A}-\bar{\partial})u \ , \qquad u = u(z,\bar{z}) \in H \ . \tag{2.3}$$

For $a = 1$ the action (2.1) is the CWZNW action of [9][14]

$$I_{CWZNW} = I_1(g, A) = I(g) + \frac{1}{\pi} \int d^2z \ \mathrm{Tr} \ \left(A \, \bar{\partial} g g^{-1} - \bar{A} \, g^{-1} \partial g + g^{-1} A g \bar{A} \right) \ . \tag{2.4}$$

This action is invariant under the following gauge-type transformations

$$\begin{aligned} & g \to u^{-1} g \bar{u} \ , \quad A \to u^{-1}(A - \partial) u \ , \quad \bar{A} \to \bar{u}^{-1}(\bar{A} - \bar{\partial}) \bar{u} \ , \\ & u = u(z) \in H \ , \quad \bar{u} = \bar{u}(\bar{z}) \in H \ . \end{aligned} \tag{2.5}$$

In general one can consider the 'left' and 'right' subgroups of G to be different, i.e. $u \in H_R$, $\bar{u} \in H_L$, $A \in \mathcal{L}(H_R)$ and $\bar{A} \in \mathcal{L}(H_L)$ with $H_R \neq H_L$.[2] Since the chiral gauge transformations do not actually eliminate dynamical degrees of freedom (since u and $\bar{u}$ are holomorphic and antiholomorphic functions) it is more appropriate to consider them as global symmetry transformations of the action (2.1) with $a = 1$.

Parametrising A and $\bar{A}$ in terms of h and $\bar{h}$ which take values in H

$$A = \partial h h^{-1} \quad , \quad \bar{A} = \bar{\partial} \bar{h} \bar{h}^{-1} \quad , \tag{2.6}$$

one can use the Polyakov-Wiegmann identity [29] to represent the action (2.1) in terms of one WZNW action corresponding to the group G and two WZNW actions corresponding to the subgroup H,

$$\begin{aligned} & I_a(g, A) = I(\tilde{g}) - I(\tilde{h}) + a \ [\ I(\tilde{h}) - I(h^{-1}) - I(\bar{h}) \ ] \ , \\ & \tilde{g} = h^{-1} g \bar{h} \ , \qquad \tilde{h} = h^{-1} \bar{h} \ . \end{aligned} \tag{2.7}$$

Clearly, the action I_a (2.1) (or equivalently (2.7)) is *classically* conformally invariant for any value of a. However, it is only for $a = 0$ or $a = 1$ that I_a reduces to a sum of WZNW

[2] In our notation the 'left' and 'right' in the group action sense and the world sheet $(z, \bar{z})$ sense are 'cross-related' in the action (but not in the Hamiltonian), i.e. H_L acts from the left not on g but on g^{-1}.

actions for *independent* fields and only in these two cases it is obvious that conformal invariance is preserved at the quantum level. [3] Namely, we find

$$a = 0 : \qquad I_0 = I_{GWZNW} = I(\tilde{g}) - I(\tilde{h}) \ , \tag{2.8}$$

and for

$$a = 1 : \qquad I_1 = I_{CWZNW} = I(\tilde{g}) - I(h^{-1}) - I(\bar{h}) \ . \tag{2.9}$$

For the same values of a an extra local or 'semi-local' symmetry appears in (2.7) which corresponds to the gauge symmetries (2.3), (2.5) :

$$\begin{aligned} a = 0 : \quad & g \to u^{-1} g u \ , \quad h \to u^{-1} h \ , \quad \bar{h} \to u^{-1} \bar{h} \ , \quad u = u(z, \bar{z}) \in H \\ a = 1 : \quad & g \to u^{-1} g \bar{u} \ , \quad h \to u^{-1} h \ , \quad \bar{h} \to \bar{u}^{-1} \bar{h} \ , \quad u(z), \bar{u}(\bar{z}) \in H \ . \end{aligned} \tag{2.10}$$

For all other values of a the above symmetries degenerate to global ones with constant transformation parameters. It is clear that (2.9) (but not (2.8)) admits a straightforward generalisation to the case when h and $\bar{h}$ belong to diffent subgroups H_R and H_L of G.

The corresponding path integral has the form

$$\begin{aligned} Z_p &= \int [dg][dA][d\bar{A}] \exp[-k I_a(g, A)] \\ &= J_0 \int [dg][dh][d\bar{h}] \exp\{-k I(\tilde{g}) + (k + 2\mathrm{g_H}) I(\tilde{h}) - p[I(\tilde{h}) - I(h^{-1}) - I(\bar{h})]\} \ , \end{aligned} \tag{2.11}$$

where $p \equiv ak - 2q\mathrm{g_H}$ and $\mathrm{g_H}$ is the dual Coxeter number for the subgroup H. We have used the fact that there exists a freedom of introducing a local counterterm $\sim q \ \mathrm{Tr}\ (A\bar{A})$ in the definition of the Jacobian

$$\begin{aligned} J &= \det D(A) \det \bar{D}(\bar{A}) = J_0 \exp\{2\mathrm{g_H}[\ I(h^{-1}\bar{h}) + \frac{q}{\pi} \int d^2 z \ \mathrm{Tr}\ (A\bar{A})\ ]\} \ , \\ J_0 &= [\ \det \partial\bar{\partial}\ ]^{d_H} \ , \end{aligned} \tag{2.12}$$

---

[3] Let us note that in the acse of the abelian H the action (2.1) can be interpreted as a gauge-fixed form of the action of the $G \times H/H$ GWZNW nodel with a being related to the parameters of embedding of H into $G \times H$ (see eq. (4.5)). Being understood in this way, the action (2.1) corresponds to a conformal theory for all a. In particular, the σ-model obtained by integrating out $A, \bar{A}$ should also be conformal invariant for any a. An example of such theory is provided by the $SL(2, \mathbb{R}) \times \mathbb{R}/\mathbb{R}$ GWZNW model (the conformal invariance of the associated σ-model was checked at the two-loop level in [30]).

of the transformation from $A, \bar{A}$ to $h, \bar{h}$. In the GWZNW case q is chosen to be zero in order to preserve the vector gauge symmetry [5]. In the CWZNW case the natural choice is the 'left-right decoupled' one $q = -1$, i.e. $J \sim \exp\{2g_H[I(h^{-1}) + I(\bar{h})]\}$ [14]. It is only for the two values of p

$$
\begin{aligned}
p = 0: \qquad & Z_0 = Z_{GWZNW} = J_0 \int [d\tilde{g}][d\tilde{h}] \exp[-S_{GWZNW}(\tilde{g}, \tilde{h})] \ , \\
& S_{GWZNW} = kI(\tilde{g}) - (k + 2g_H) I(\tilde{h}) \ ,
\end{aligned}
\tag{2.13}
$$

and

$$
\begin{aligned}
p = k + 2g_H: \qquad & Z_1 = Z_{CWZNW} = J_0 \int [d\tilde{g}][dh][d\bar{h}] \exp[-S_{CWZNW}(\tilde{g}, h, \bar{h})] \ , \\
& S_{CWZNW} = kI(\tilde{g}) - (k + 2g_H)[I(h^{-1}) + I(\bar{h})] \ ,
\end{aligned}
\tag{2.14}
$$

that the resulting quantum theory reduces to a combination of independent WZNW theories and therefore is guaranteed to be conformally invariant. In fact, a formal proof of conformal invariance for the theories (2.13) and (2.14) reduces to that for the WZNW model and an observation that conformal invariance (or UV finiteness) property is essentially preserved under field redefinitions.

The quantum effective actions corresponding to GWZNW (2.2) and CWZNW (2.4) models are obtained (up to a non-local field redefinition [25] which we shall ignore) by replacing k and $-k$ by $k + g_G$ and $-k + g_H$ in (2.13) and (2.14) (or by multiplying the G and H terms in (2.8) and (2.9) by $k + g_G$ and $k + g_H$) [26][27][25][17]

$$
\begin{aligned}
\Gamma_{GWZNW}(g, A) &= (k + g_G) I(h^{-1} g \bar{h}) - (k + g_H) I(h^{-1} \bar{h}) \\
&= (k + g_G) I_{GWZNW}(g, A) + (g_G - g_H) \Omega(A) \ ,
\end{aligned}
\tag{2.15}
$$

and

$$
\begin{aligned}
\Gamma_{CWZNW}(g, A) &= (k + g_G) I(h^{-1} g \bar{h}) - (k + g_H)[I(h^{-1}) + I(\bar{h})] \\
&= (k + g_G) I_{CWZNW}(g, A) + (g_G - g_H)[\omega(A) + \bar{\omega}(\bar{A})] \ ,
\end{aligned}
\tag{2.16}
$$

where $\Omega(A)$ is a non-local gauge invariant functional of A and $\bar{A}$,

$$
\Omega(A) \equiv I(h^{-1}\bar{h}) = \omega(A) + \bar{\omega}(\bar{A}) + \frac{1}{\pi} \int d^2 z \ \mathrm{Tr}\ (A\bar{A}) \ , \tag{2.17}
$$

and the functionals ω and $\bar\omega$ are given by

$$\omega(A) \equiv I(h^{-1}) = -\frac{1}{\pi}\int d^2z \ \mathrm{Tr}\ \{\frac{1}{2}A\frac{\bar\partial}{\partial}A + \frac{1}{3}A[\frac{1}{\partial}A, \frac{\bar\partial}{\partial}A] + O(A^4)\} \ ,$$
$$\bar\omega(\bar A) \equiv I(\bar h) = -\frac{1}{\pi}\int d^2z \ \mathrm{Tr}\ \{\frac{1}{2}\bar A\frac{\partial}{\bar\partial}\bar A - \frac{1}{3}\bar A[\frac{1}{\bar\partial}\bar A, \frac{\partial}{\bar\partial}\bar A] + O(\bar A^4)\} \ . \qquad (2.18)$$

As at the classical level, the effective action of the CWZNW is obtained from the effective action of GWZNW by dropping out the $A\bar A$-terms (which, of course, were crucial for gauge invariance of (2.2) and (2.15)).

Let us note that in the general case of $H_L \neq H_R$ when h and $\bar h$ belong to H_R and H_L the action S_{CWZNW} in (2.14) and J_0 in (2.12) are replaced by

$$S_{CWZNW} = kI(\tilde g) - (k + 2\mathrm{g}_{\mathrm{H_R}})I(h^{-1}) - (k + 2\mathrm{g}_{\mathrm{H_L}})I(\bar h) \ , \qquad (2.19)$$

and

$$J_{0R}J_{0L} = (\ \det\ \partial)^{d_{H_R}}(\ \det\ \bar\partial)^{d_{H_L}} \ . \qquad (2.20)$$

Eq. (2.20) implies that if dim $H_R \neq$ dim H_L the numbers of the 'left' and 'right' bosonic degrees of freedom do not match and the theory is thus really 'chiral' in the $2d$ sense, i.e. it has a Lorentz anomaly on a curved $2d$ background. Using it to construct a consistent string theory, one needs to compensate the anomaly by introducing extra chiral degrees of freedom.

2.2. Relation between 'chiral gauged' and gauged WZNW models

The models (2.8),(2.9) or (2.13),(2.14) are particular representatives of a general class of models which can be called 'twisted' products of WZNW models,

$$S = \sum_{i=1}^{N} \kappa_i I_{G_i}(\tilde g_i) \ , \qquad (2.21)$$

where I_{G_i} stands for a WZNW action for a group G_i; the arguments are related to some 'original' variables g_i by field redefinitions respecting global symmetries (e.g. $\tilde g_i$ are given by particular products of some g_i). Such models are conformally invariant and unless there are some ‘accidental’ gauge symmetries (as in the case of GWZNW) they can be represented as σ-models on the target space (of dimension $\sum_{i=1}^N \dim G_i$) equivalent to the direct product of the group spaces. For example, in the CWZNW case (2.14) we can parametrise $g, h, \bar{h}$ in terms of local group coordinates $X_g, X_h, X_{\bar{h}}$; then the coordinates corresponding to $\tilde{g}$ in (2.7) ($\tilde{g} = \exp(T \cdot X_{\tilde{g}})$) are given by a local transformation of $X_g, X_h, X_{\bar{h}}$. As a result, the action (2.14) will take the form of a σ-model on the group space $G \times H_L \times H_R$ represented in terms of ‘transformed’ coordinates.

All becomes less trivial once we decide to treat some subsets of the fields among g_i as more ‘fundamental’ than others (by constructing observables in terms of them only, i.e. by introducing sources in the path integral for them only). Such a split may be motivated by locality considerations: one is prompted to treat h and $\bar{h}$ in (2.8),(2.9) as auxiliary fields (which should be integrated out first) by the observation that being expressed in terms of the corresponding currents $A, \bar{A}$ in (2.6) the classical actions (2.2), (2.4) are local (while they are non-local being expressed in terms of the current corresponding to g). Another possible reason may be the elimination of negative norm fields associated with the negative sign terms in (2.21) (as in the case of the GWZNW models).

Once one first integrates over a subset of ‘auxiliary’ fields ($2d$ gauge fields in the case of GWZNW and CWZNW models) one induces (on a curved $2d$ background) the dilaton term in the effective action. The dilaton term is necessary in order to preserve the Weyl invariance of the resulting lower dimensional σ-model [7]. Thus the appearance of the dilaton can be considered as an artifact of concentrating on an ‘intermediate’ (or ‘reduced’) theory with a smaller number of fields than the original one.

If one does not make a separation into ‘fundamental’ and ‘auxiliary’ fields one may be able to establish various formal equivalences between the models in (2.7). For example, as we will show below, the G/H chiral gauged WZNW model can be represented as the gauged WZNW model for the coset $(G_k/H_k) \times H_{-k-2\mathrm{g_H}}$ in the sense that the corresponding quantum actions in (2.13) and (2.14) are related by a field redefinition. This is not too surprising since at the level of the path integral (2.13), (2.14) the two theories are represented by combinations of WZNW models that can be directly related.

To demonstrate the formal equivalence

$$
\begin{aligned}
(G_k/H_k)_{CWZNW} &= [(G_k/H_k)\times H_{-k-2\mathrm{g_H}}]_{GWZNW} \\
&= [G_k \times H_{-k-2\mathrm{g_H}} \times H_{-k-2\mathrm{g_H}}]_{WZNW} \ ,
\end{aligned}
\tag{2.22}
$$

let us start with the quantum effective CWZNW action (2.16) and represent it as a sum of the quantum actions for the G_k/H_k GWZNW and $H_{-k-2\mathrm{g_H}}$ WZNW by making a field redefinition

$$
\begin{aligned}
\Gamma_{CWZNW}(G/H) &= (k+\mathrm{g_G})I(h^{-1}g\bar h) - (k+\mathrm{g_H})[I(h^{-1}) + I(\bar h)] \\
&= [\ (k+\mathrm{g_G})I(u^{-1}f\bar u) - (k+\mathrm{g_H})I(u^{-1}\bar u)\ ] - (k+\mathrm{g_H})I(v) \\
&= \Gamma_{GWZNW}(G_k/H_k) + \Gamma_{WZNW}(H_{-k-2\mathrm{g_H}})\ ,
\end{aligned}
\tag{2.23}
$$

where $f\in G$, $u,v\in H$ and

$$
g=\bar u^{-1}f\bar u v^{-1}\ ,\quad h=\bar u^{-1}u\ ,\quad \bar h = v\ . \tag{2.24}
$$

The same equivalence relation is true also between the classical actions in (2.2) and (2.4) and the 'quantum' actions in (2.13) and (2.14). The fields of CWZNW are obviously invariant under the gauge transformations of the fields $(f,u,\bar u)$ of G/H GWZNW. Since transformation (2.24) mixes different fields the 'intermediate' σ-models obtained by integrating out the gauge fields, in the GWZNW and CWZNW models will, in general, be different.[4]

In the case when H is abelian it is possible to establish a more explicit equivalence between the G/H GWZNW and axially gauged $(G\times H)/H$ WZNW models with a specific embedding of H (see Sect.4). In this case the gauge fields are not transformed and thus the corresponding σ-models (with G as a configuration space) are equivalent.

[4] For example, the $SL(2,\mathbb{R})/\mathbb{R}\times\mathbb{R}$ GWZNW theory reduces to the 'uncharged black string' $D=3$ σ-model [18] while $SL(2,\mathbb{R})/\mathbb{R}$ CWZNW theory gives a 'charged black string' $D=3$ σ-model [15][17].

In contrast to G/G GWZNW model which is topological and therefore trivial from the σ-model pont of view, the G/G CWZNW model is non-trivial and equivalent to $G_{-k-2\mathrm{g_G}}$ WZNW model. In order to prove that one uses the expression for the σ-model obtained from a general CWZNW model after integrating out the gauge fields [17]. One can show that if $H = G$ it takes the form of the σ-model appropriate for the $G_{-k-2\mathrm{g_G}}$ WZNW theory. This conclusion is of course in agreement with (2.22).

The equivalence (2.22) implies that if G and H are compact the resulting CWZNW model will contain dim H negative norm fields (or 'times') and therefore the physically interesting case is only that of an abelian H with dim $H = 1$ (the models considered in [15][16][17] belonged to this class).

2.3. Central charge

The formal relation (2.22) between the path integrals of GWZNW and CWZNW models implies the equality of the corresponding central charges. Adding the quantum shifts to the levels, $kI(g) \to (k+\mathrm{g_G})I(g),\ kI(h) \to (k+\mathrm{g_H})I(h)$ and accounting for the J_0 contribution in (2.12) it is straightforward to compute the values of the central charges for GWZNW (2.13) [5] and CWZNW (2.14) models ($d_G = \dim G,\ d_H = \dim H$):

$$C_{GWZNW}(G/H) = \frac{kd_G}{k+\mathrm{g_G}} + \frac{(-k-2\mathrm{g_H})d_H}{(-k-2\mathrm{g_H})+\mathrm{g_H}} - 2d_H = \frac{kd_G}{k+\mathrm{g_G}} - \frac{kd_H}{k+\mathrm{g_H}} \equiv C_{G/H}\ , \tag{2.25}$$

and

$$C_{CWZNW}(G/H) = \frac{kd_G}{k+\mathrm{g_G}} + 2\frac{(-k-2\mathrm{g_H})d_H}{(-k-2\mathrm{g_H})+\mathrm{g_H}} - 2d_H = C_{G/H} + \frac{(-k-2\mathrm{g_H})d_H}{(-k-2\mathrm{g_H})+\mathrm{g_H}}\ . \tag{2.26}$$

As a result, the G/H CWZNW model has the same central charge as the $(G_k/H_k) \times H_{-k-2\mathrm{g_H}}$ GWZNW model (and, in particular, it cannot be equivalent to G/H GWZNW model, cf. [14]). If H is abelian, $\mathrm{g_H} = 0$ and (2.26) is equal to the central charge of the G_k WZNW model.

In the case when h and $\bar{h}$ belong to different subgroups H_R and H_L of G one finds from (2.14), (2.20), (2.19) the following expression for the central charge of CWZNW [5]

$$C_{CWZNW} = \frac{kd_G}{k+\mathrm{g_G}} + \frac{(-k-2\mathrm{g_{H_L}})d_{H_L}}{(-k-2\mathrm{g_{H_L}})+\mathrm{g_{H_L}}} + \frac{(-k-2\mathrm{g_{H_R}})d_{H_R}}{(-k-2\mathrm{g_{H_R}})+\mathrm{g_{H_R}}} - d_{H_L} - d_{H_R} \ . \quad (2.27)$$

This expression is equal to the central charge of the $G_k \times (H_L)_{-k-2\mathrm{g_{H_L}}} \times (H_R)_{-k-2\mathrm{g_{H_R}}} \times \mathbb{R}^{-d_{H_L}-d_{H_R}}$ WZNW model (or of the $(G/H_L)_k \times (H_R)_{-k-2\mathrm{g_{H_R}}} \times \mathbb{R}^{d_{H_L}-d_{H_R}}$ GWZNW model or of the $(G/H_R)_k \times (H_L)_{-k-2\mathrm{g_{H_L}}} \times \mathbb{R}^{d_{H_R}-d_{H_L}}$ GWZNW model) but is *different* from the central charge of the direct product of the 'left' and 'right' parts of the G/H_R and G/H_L GWZNW models (cf.[14]). In the latter case the central charge is

$$C = \frac{1}{2}C_{G/H_L} + \frac{1}{2}C_{G/H_R} = \frac{kd_G}{k+\mathrm{g_G}} - \frac{1}{2}\frac{kd_{H_L}}{k+\mathrm{g_{H_L}}} - \frac{1}{2}\frac{kd_{H_R}}{k+\mathrm{g_{H_R}}} \ . \quad (2.28)$$

Note that extra 'one-halves' in front of the H_L and H_R contributions do not actually appear in CWZNW (the $\mathrm{g_H}$ contributions from the Jacobians appear in (2.14) with the same coefficient as in GWZNW). As we shall see in the next section, the Hamiltonian of the GWZNW model is also *different* from the naive combination of the left part of the Hamiltonian of the G/H_L with the right part of the Hamiltonian of the G/H_R GWZNW models.

3. Hamiltonians

The form of the stress tensor for the GWZNW and CWZNW models can be easily read off from (2.13), (2.14) by replacing each WZNW action by the bilinear products of the corresponding currents (since the WZ-term does not depend on the world-sheet metric). The zero mode of the T_{00}-component of the classical stress tensor is the Hamiltonian $\mathcal{H} = L_0 + \bar{L}_0$ associated with the classical action (2.8) or (2.9). The zero mode of the

[5] This is the expression for the Weyl anomaly coefficient, i.e. for the 'left' plus 'right' central charge $C = \frac{1}{2}(C_l + C_r)$. As we have mentioned above, the theory contains also the Lorentz anomaly, proportional to $\frac{1}{2}(C_l - C_r) = d_{H_R} - d_{H_L}$.

quantum stress tensor is the Hamiltonian associated with the quantum effective action (2.15) or (2.16) with shifted k's. The 'J^2' structure of the Hamiltonian follows from the similar structure of the action in the $1d$ dimensional reduction limit (in which the WZ-term in the action does not contribute). For example,

$$S = k_G I_G - k_{H_L} I_{H_L} - k_{H_R} I_{H_R} \ \rightarrow \ \mathcal{H} = \frac{1}{k_G} J_G^2 - \frac{1}{k_{H_L}} J_{H_L}^2 - \frac{1}{k_{H_R}} \bar{J}_{H_R}^2 \ . \tag{3.1}$$

It is instructive to derive the general expression for the Hamiltonian corresponding to the theory (2.1) supplemented by extra A^2 and $\bar{A}^2$ terms. Such terms are not Lorentz invariant but they can be considered as originating (in the $d=1$ reduction limit relevant for the derivation of the zero mode Hamiltonian) from the Lorentz invariant non-local terms $A(\bar{\partial}/\partial)A + \ldots$ and $\bar{A}(\partial/\bar{\partial})\bar{A} + \ldots$ which appear in the quantum effective action of the GWZNW and CWZNW models (see (2.15), (2.16), (7.3)).

Let us start with the following action

$$I = I_0(g, A) + \frac{1}{\pi} \int d^2 z \ \mathrm{Tr}\ (aA\bar{A} + \frac{1}{2}\beta A^2 + \frac{1}{2}\bar{b}\bar{A}^2) \ , \tag{3.2}$$

where $I_0(g, A)$ is the GWZNW action (2.2). When $a = 1, n = m = 0$ this is the classical CWZNW action (2.4). The case of

$$a = -b \ , \quad\ b = \bar{b} = -\frac{\mathrm{g_G} - \mathrm{g_H}}{k + \mathrm{g_G}} \ , \tag{3.3}$$

corresponds to the quantum GWZNW effective action [26][27], whereas the case of

$$a = 1, \quad b = \bar{b} = -\frac{\mathrm{g_G} - \mathrm{g_H}}{k + \mathrm{g_G}} \ , \tag{3.4}$$

corresponds to the quantum CWZNW effective action [17]. Let us find the classical Hamiltonian $\mathcal{H}$ of the model (3.2) treating $A, \bar{A}$ as non-dynamical fields that should be eliminated at the end. One should express $\mathcal{H}$ in terms of currents that should satisfy proper Poisson bracket current algebra relations as in the WZNW model (see e.g. [31]). These are the left $(g^{-1}\partial g)$ and right $(-\bar{\partial} g g^{-1})$ currents expressed in terms of momenta (which will now contain $A, \bar{A}$ so that the currents will also depend on $A, \bar{A}$). Let $J_G, \bar{J}_G$ and $J_H, \bar{J}_H$ denote the currents corresponding to G and H. In what follows we shall switch to the component notation: $J_G = (J_G^A),\ J_H = (J_H^a),\ A^2 = A^a A_a = -\ \mathrm{Tr}\ (A^2)$, etc. Then $\mathcal{H}$ will be given by (we rescale $\mathcal{H}$ by the factor of 2 and omit obvious factors of $1/(k + \mathrm{g_G})$ and $1/\pi$)

$$\mathcal{H} = \frac{1}{2}J_G^2 + \frac{1}{2}\bar{J}_G^2 - 2J_H\bar{A} - 2\bar{J}_H A + 2(a-1)A\bar{A} + (b+1)A^2 + (\bar{b}+1)\bar{A}^2 \ . \tag{3.5}$$

For $a = b = \bar{b} = 0$ one obtains the Hamiltonian for the classical WZNW action (2.2) [31]

$$\mathcal{H}_0 = \frac{1}{2}J_G^2 + \frac{1}{2}\bar{J}_G^2 - 2J_H\bar{A} - 2\bar{J}_H A + (A - \bar{A})^2 \ . \tag{3.6}$$

In contrast to (3.6) where the A^2-term is singular (as a consequence of the gauge invariance of the GWZNW action (2.2)) it is straightforward to eliminate $A, \bar{A}$ from (3.5). The singular case (3.6) can then be defined as the limit $a, b, \bar{b} \to 0$ (in this way one is able to avoid the complications (due to constraints) dealt with in [31]). We find (using that $\bar{J}_G^2 = J_G^2$)

$$\begin{aligned} &\mathcal{H} = J_G^2 + \frac{1}{\Delta}[\ -2(a-1)J_H\bar{J}_H + (b+1)J_H^2 + (\bar{b}+1)\bar{J}_H^2\ ] \ , \\ &\Delta \equiv (a-1)^2 - (b+1)(\bar{b}+1) \ . \end{aligned} \tag{3.7}$$

The 'determinant' Δ is singular both in the classical ($a = b = \bar{b} = 0$) and quantum ($-a = b = \bar{b}$) GWZNW cases. To reproduce the well-known expression for the GWZNW Hamiltonian from (3.7) let us consider the following limit: $b = \bar{b},\ a = -b + \epsilon,\ \Delta = -2(b+1)\epsilon + O(\epsilon^2),\ \epsilon \to 0$. Then

$$\mathcal{H}_{GWZNW} = J_G^2 + \frac{1}{b+1}J_H\bar{J}_H - \frac{1}{2\epsilon}(J_H + \bar{J}_H)^2 \ . \tag{3.8}$$

To get a non-singular Hamiltonian one is to restrict it to a gauge invariant subspace on which $J_H + \bar{J}_H = 0$. Then (3.8) reduces to the standard expression[6]

$$\mathcal{H}_{GWZNW} = J_G^2 - \frac{1}{b+1}J_H^2 = J_G^2 - \frac{k + \mathrm{g_G}}{k + \mathrm{g_H}}J_H^2 \ . \tag{3.9}$$

[6] The singular term in (3.8) can be thought of as giving rise to a δ-function in the path integral approach.

The CWZNW Hamiltonian ($a=1,\ b=\bar{b}$, $\Delta=-(b+1)^2$) is given by

$$
\begin{aligned}
\mathcal{H}_{CWZNW} &= J_G^2 - \frac{k+\mathrm{g_G}}{k+\mathrm{g_H}}(J_H^2+\bar{J}_H^2) \\
&= \frac{1}{2}[\ J_G^2 - 2\ \frac{k+\mathrm{g_G}}{k+\mathrm{g_H}} J_H^2\ ] + \frac{1}{2}[\ \bar{J}_G^2 - 2\ \frac{k+\mathrm{g_G}}{k+\mathrm{g_H}} \bar{J}_H^2\ ]\ .
\end{aligned}
\tag{3.10}
$$

In contrast to the GWZNW case (3.9), here there is no need to restrict to a sector in which $J_H+\bar{J}_H=0$. One can show [25][17] that the σ-model expressions for the metric and dilaton obtained using the quantum effective actions (2.15), (2.16) coincide with the corresponding expressions obtained in the operator approach using the Hamiltonians (3.8), (3.10).

As we have noted already, the structure of $\mathcal{H}_{CWZNW}$ directly reflects the structure of the action in (2.9) and (2.14). According to the relation (2.22) it is possible to identify the CWZNW theory with the GWZNW theory for the product $(G/H)\times H$ under a proper definition of the currents. Namely, it should be possible to introduce the new currents J'_G, $\bar{J}'_G$, J'_H, $\bar{J}'_H$,$\tilde{J}_H$, $\bar{\tilde{J}}_H$ such that on the subspace of states satisfying $J'_H+\bar{J}'_H=0$ eq. (3.10) takes the form

$$
\mathcal{H}_{CWZNW} = [\ J_G'^2 - \frac{k+\mathrm{g_G}}{k+\mathrm{g_H}} J_H'^2\ ] - \frac{k+\mathrm{g_G}}{k+\mathrm{g_H}} \tilde{J}_H^2\ , \tag{3.11}
$$

where we have used the fact that $\tilde{J}_H^2 = \bar{\tilde{J}}^2{}_H$. Here the first and second terms represent the G_k/H_k and $H_{-k-2\mathrm{g_H}}$ factors in (2.22).

The Hamiltonian (3.10) has an obvious generalisation to the case of the different 'left' and 'right' subgroups H_L and H_R

$$
\mathcal{H}_{CWZNW} = \frac{1}{2}[\ J_G^2 - 2\ \frac{k+\mathrm{g_G}}{k+\mathrm{g_{H_L}}} J_{H_L}^2\ ] + \frac{1}{2}[\ \bar{J}_G^2 - 2\ \frac{k+\mathrm{g_G}}{k+\mathrm{g_{H_R}}} \bar{J}_{H_R}^2\ ]\ . \tag{3.12}
$$

This expression is *different* from the straightforward combination of the 'left' part of the Hamiltonian of the G/H_L GWZNW with the 'right' part of the Hamiltonian of the G/H_R GWZNW

$$
\begin{aligned}
\mathcal{H} &= [\mathcal{H}_{GWZNW}(G/H_L)]_l + [\mathcal{H}_{GWZNW}(G/H_R)]_r \\
&= \frac{1}{2}[\ J_G^2 - \frac{k+\mathrm{g_G}}{k+\mathrm{g_{H_L}}} J_{H_L}^2\ ] + \frac{1}{2}[\ \bar{J}_G^2 - \frac{k+\mathrm{g_G}}{k+\mathrm{g_{H_R}}} \bar{J}_{H_R}^2\ ]\ .
\end{aligned}
\tag{3.13}
$$

The difference is due to the crucial coefficients 2 in front of the subgroup current terms in the CWZNW case. We already noted a similar difference in the values of the central charges.

In the case when H_L is different from H_R it is not clear how to interpret CWZNW in terms of a particular GWZNW model. At the same time, the path integral analysis of Sect.2.1 implies that this model can be represented as the $G_k \times (H_L)_{-k-2\mathrm{g}_{\mathrm{H_L}}} \times (H_R)_{-k-2\mathrm{g}_{\mathrm{H_R}}}$ WZNW model. That means that while for generic values of $a, b, \bar{b}$ the Hamiltonian (3.5) should not correspond to a conformal theory, the CGWZW Hamiltonian (3.12) should. One may question how the structure of (3.11) is consistent with the known fact that the only (relevant in the present case) solutions of the affine - Virasoro master equation [32][33] are represented by coset (GWZNW) models. The answer is again implied by the structure of the path integral (2.14) (with h and $\bar{h}$ now belonging to H_R and H_L): it should be possible to define the holomorphic and antiholomorphic currents $J'_G,\ J'_{H_L},\ J'_{H_R}$ and $\bar{J}'_G,\ \bar{J}'_{H_L},\ \bar{J}'_{H_R}$ such that in terms of them (3.12) takes the form of the Hamiltonian of the three WZNW models

$$
\begin{aligned}
\mathcal{H}_{CWZNW} &= \mathcal{H}_l + \mathcal{H}_r = {J'}^2_G - \frac{k+\mathrm{g_G}}{k+\mathrm{g_{H_L}}}{J'}^2_{H_L} - \frac{k+\mathrm{g_G}}{k+\mathrm{g_{H_R}}}{J'}^2_{H_R} \\
&= \frac{1}{2}[\ {J'}^2_G - \frac{k+\mathrm{g_G}}{k+\mathrm{g_{H_L}}}{J'}^2_{H_L} - \frac{k+\mathrm{g_G}}{k+\mathrm{g_{H_R}}}{J'}^2_{H_R}\ ] \\
&+ \frac{1}{2}[\ \bar{J}'^2_G - \frac{k+\mathrm{g_G}}{k+\mathrm{g_{H_L}}}\bar{J}'^2_{H_L} - \frac{k+\mathrm{g_G}}{k+\mathrm{g_{H_R}}}\bar{J}'^2_{H_R}\ ]\ .
\end{aligned}
\tag{3.14}
$$

The new currents should directly correspond to the redefined variables $\tilde{g} = h^{-1} g \bar{h},\ h_R = h^{-1},\ h_L = \bar{h}$ in terms of which the path integral (2.14) factorises.

4. The case of the Abelian subgroup H

In this section we shall prove that the chiral gauged WZNW model with abelian $H_L = H_R = H$ is, in fact, equivalent to a specific (axially) gauged WZNW model $(G \times H)/H$ with a particular embedding of the subgroup H into $G \times H$. The basic idea is to 'cancel' the $A\bar{A}$-term in the GWZNW action (2.2) in order to make it look like the CWZNW action

(2.4). This is achieved by introducing extra H-degrees of freedom coupled to $A\bar{A}$ and then gauging them away. To make this idea work one is to consider GWZNW with an *axially* gauged abelian subgroup. In this case the action (cf. (2.2), (2.8))

$$
\begin{aligned}
I^{axial}_{GWZNW}(g,A) &= I(g) + \frac{1}{\pi}\int d^2z \ \mathrm{Tr}\ \left(A\,\bar{\partial}g g^{-1} - \bar{A}\,g^{-1}\partial g + g^{-1}Ag\bar{A} + A\bar{A}\right) \\
&= I(h^{-1}g\bar{h}) - I(h\bar{h}) \quad ,
\end{aligned}
\tag{4.1}
$$

computed for $g = I$ does not vanish but is proportional to the integral of $\mathrm{Tr}\ (A\bar{A})$. Let us consider the $(G\times H)/H$ axially gauged WZNW theory and represent the elements of $G\times H$ as block-diagonal matrices

$$
\tilde{g} = \mathrm{diag}(g, h_0)\ , \quad h_0 = \mathrm{diag}(p_1,\ldots,p_{r_H})\ , \quad p_a = \exp(y_a T_a)\ , \tag{4.2}
$$

where g belongs to G and h_0 is from H. An element of the abelian subgroup H (which we can take, without lack of generality, to be generated by the maximal abelian subalgebra of the algebra of G) of $G\times H$ can be represented in the form

$$
\tilde{h} = \mathrm{diag}(h,h')\ , \quad h = \exp(\sum_{a=1}^{r_H} x_a T_a)\ , \quad h' = \mathrm{diag}(f_1,\ldots,f_{r_H})\ , \quad f_a = \exp(n_a x_a T_a)\ , \tag{4.3}
$$

where T_a are the generators of the abelian group H and n_a parametrise the embedding of H into $G\times H$. Our aim will be to prove that there exist an embedding such that the quantum effective action of $(G\times H)/H$ GWZNW model in the axial gauging (the analog of (2.15)) is equal to the quantum effective action (2.16) of G/H CWZNW model, i.e. $(\mathrm{g_H} = 0)$

$$
\begin{aligned}
\Gamma^{axial}_{GWZNW}(G\times H/H) &= \Gamma_{WZNW}(\tilde{h}^{-1}\tilde{g}\bar{\tilde{h}}) - \Gamma_{WZNW}(\tilde{h}\bar{\tilde{h}}) \\
&= \Gamma_{WZNW}(h^{-1}g\bar{h}) - \Gamma_{WZNW}(h^{-1}) - \Gamma_{WZNW}(\bar{h}) \\
&= \Gamma_{CWZNW}(G/H)\ .
\end{aligned}
\tag{4.4}
$$

In the large k limit this relation implies the equivalence of the corresponding classical actions (4.1) and (2.9) (of course for abelian H only). The GWZNW action is invariant under the gauge transformations: $\tilde g \to \tilde u \tilde g \tilde u$, $\tilde h \to \tilde u \tilde h$, $\bar{\tilde h} \to \tilde u^{-1} \bar{\tilde h}$, where $\tilde u = \tilde u(z, \bar z)$. Therefore we can prove (4.4) in any gauge. It is convenient to fix the gauge so that $\tilde g = \mathrm{diag}(g, 1)$. Then computing the GWZNW action (4.4) using (4.3) we find

$$\begin{aligned}
&\Gamma_{GWZNW}(G \times H/H) = [\ (k + \mathrm{g_G}) I(h^{-1} g \bar h) + \sum_{a=1}^{r_H} k'_a I({f_a}^{-1} \bar f_a)\ ] \\
&- [\ k I(h \bar h) + \sum_{a=1}^{r_H} k'_a I(f_a \bar f_a)\ ] \qquad\qquad (4.5) \\
&= [\ (k + \mathrm{g_G}) I(h^{-1} g \bar h) - k I(h^{-1}) - k I(\bar h)\ ] - \frac{1}{\pi} \sum_{a=1}^{r_H} (2k'_a n_a^2 + k) \int d^2 z A^a \bar A^a \ .
\end{aligned}$$

Here k and k'_a are the coefficients of the WZNW actions corresponding to the G and H factors in the product $G \times H$, i.e. the central extensions in the current algebras defined in G and for each factor in H. Also, $A^a = \partial x^a$, $\bar A^a = \bar\partial \bar x^a$, $\mathrm{Tr}\ (T_a T_b) = -\delta_{ab}$ and we have used that in the abelian case the WZNW action contains just the quadratic term, i.e. $I(h) = I(h^{-1})$. Choosing the embedding parameters n_a that satisfy

$$2k'_a n_a^2 + k = 0 \ , \qquad \forall\ a = 1, \ldots, r_H \ , \tag{4.6}$$

we get the desired equality (4.4). Eq. (4.6) implies that if k is positive, all k'_a will be negative, i.e. the coordinates y_a corresponding to the H-factor in $G \times H$ will have the negative signs of their kinetic terms in the $(G \times H)/H$ GWZNW action. This can be considered to be a consequence of the negative sign in front of the H-terms in the CWZNW action in (4.4).

Since to prove (4.4) we did not make any field redefinitions, the established equivalence implies the equivalence of the corresponding 'reduced' σ-models obtained by eliminating the gauge fields from the GWZNW and CWZNW actions. In particular, the $D = 3$ models corresponding to the $[SL(2, \mathbb{R}) \times \mathbb{R}]/\mathbb{R}$ GWZNW model (with the particular embedding of $H = \mathbb{R}$) and $SL(2, \mathbb{R})/\mathbb{R}$ CWZNW model are, in fact, equivalent. This provides the general explanation for the observations made in [15][16] (to the leading order in $1/k$ expansion) and in [17] (exactly in the $1/k$ expansion).[7]

It should be noted that if we have started with the effective quantum action for $(G\times H)/H$ GWZNW model with a general embedding of an abelian H and with the *vector* gauging, the result would be equivalent to $(G/H)\times H$ which, however, bears no resemblance to the G/H CWZNW model. The reason is that under the vector gauge transformation $\tilde{g} \to \tilde{u}^{-1}\tilde{g}\tilde{u}$, the y_a's are invariant and therefore there must exist a field transformation which maps the action for the $(G\times H)/H$ GWZNW model to the corresponding action for the $G/H \times H$ one (this was worked out explicitly for the $SL(2,\mathbb{R}) \times \mathbb{R}^{d-2}/\mathbb{R}$ model in [9][13]).

5. (1,1) supersymmetric chiral gauged WZNW theory

5.1. (1,1) supersymmetric GWZNW model

Before a discussion of the (1,1) supersymmetric generalisation of the chiral gauged WZNW model (2.4) it is useful first to recall the supersymmetric version of the gauged WZNW case. We shall follow the manifestly supersymmetric (superfield) approach of [25] (see also [22]) and later compare this with the component formulation [34][35] in connection with the $N=1$ superconformal coset models [36][37].

The (1,1) supersymmetric generalisation of the gauged WZNW action (2.2) is given by

$$\begin{aligned} \hat{I}(\hat{g},\hat{A}) &= \hat{I}(\hat{g}) + \frac{1}{\pi}\int d^2z d^2\theta \ \mathrm{Tr}\ \left(\hat{A}\,\bar{D}\hat{g}\hat{g}^{-1} - \hat{\bar{A}}\,\hat{g}^{-1}D\hat{g} + \hat{g}^{-1}\hat{A}\hat{g}\hat{\bar{A}} - \hat{A}\hat{\bar{A}}\right) \\ &= \hat{I}(\tilde{\hat{g}}) - \hat{I}(\tilde{\hat{h}}) \ , \end{aligned} \qquad (5.1)$$

[7] An apparent disagreement in the exact values of the antisymmetric tensor couplings in the two models observed in [17] can be interpreted as an artifact of the procedure of determining B_{MN} from the local part of the effective action. For the $SL(2,\mathbb{R}) \times \mathbb{R}/\mathbb{R}$ GWZNW model the antisymmetric tensor coupling is given just by the semiclassical expression [27]. The procedure of extracting it is not, however, completely unambiguous (for a detailed discussion of this issue see [30]).

where the gauge superfields $\hat{A}, \hat{\bar{A}}$ take values in the algebra of the subgroup H and

$$\hat{A} = D\hat{h}\hat{h}^{-1} \ , \qquad \hat{\bar{A}} = \bar{D}\hat{\bar{h}}\hat{\bar{h}}^{-1} \ ,$$
$$\tilde{\hat{g}} \equiv \hat{h}^{-1}\hat{g}\hat{\bar{h}} \ , \qquad \tilde{\hat{h}} \equiv \hat{h}^{-1}\hat{\bar{h}} \ . \tag{5.2}$$

The quantisation of the theory can be reduced to that of the two ungauged supersymmetric WZNW theories corresponding to the group and the subgroup

$$Z^{(1,1)}_{GWZNW} = \int [d\hat{g}][d\hat{A}][d\hat{\bar{A}}] \ \exp\{-k\hat{I}(\hat{g},\hat{A})\} = \int [d\tilde{\hat{g}}][d\tilde{\hat{h}}] \ \mathcal{J} \ \exp\{-kI(\tilde{\hat{g}}) + kI(\tilde{\hat{h}})\} \ . \tag{5.3}$$

Here $\mathcal{J}$ stands for the product of Jacobians of the change of superfield variables from $\hat{A}$ to $\hat{h}$ and from $\hat{\bar{A}}$ to $\hat{\bar{h}}$ (and includes also a gauge fixing factor). While in the bosonic case the corresponding product (regularised in the left-right symmetric way) is non-trivial and leads to the shift of the coefficient of the H-term in the action, in the (1,1) superfield case each of the Jacobians is proportional to a field-independent factor.[8] It is important to note that in the present case one should not include an extra local counterterm $A\bar{A}$ (needed to preserve gauge invariance in the bosonic theory) since here the Jacobians are trivial.

The theory can thus be represented as a 'product' of the two (1,1) supersymmetric WZNW theories for the groups G and H with the levels k and $-k$. Since in the (1,1) case there is no shift of k at the quantum level [38][34] the corresponding effective action is the same as the classical one (up to a field renormalisation) [25]. To see this in detail at the component level, let us start with (5.3), express it in the component notation and make the chiral rotations to decouple fermions from bosons (or, equivalently, integrate fermions out). Then

$$Z^{(1,1)}_{GWZNW} = N' \int [d\tilde{g}][d\tilde{h}] \ \exp[-(k - \mathrm{g_G})I(\tilde{g}) + (k + \mathrm{g_H})I(\tilde{h})] \ . \tag{5.4}$$

[8] This happens because the non-trivial contribution of the bosonic determinant is cancelled out by the contribution of the fermionic one (this cancellation is similar to that of the bosonic and fermionic contributions to the coefficient k of the effective action in the ungauged supersymmetric WZNW theory). In fact, as in the bosonic case, the Jacobian of the change of variables $\hat{A} \to \hat{h}$ can be expressed in terms of the path integral with the action $\int d^2z d^2\theta U(DV + [\hat{A}, V])$, where U and V are superfields of opposite statistics. Rewriting this action in component fields and integrating them out, it is easy to see that this Jacobian is $\hat{A}$-independent.

Ignoring the free-theory factors, we can represent the resulting theory as the product of the bosonic WZNW theories for the groups G and H with levels $(k-\mathrm{g_G})$ and $(k-\mathrm{g_H})$ (we separate the shift $\mathrm{g_H}$ corresponding to the bosonic change of variable).

The above approach can be compared with the one based on starting with the component formulation of the (1,1) supersymmetric gauged WZNW theory. One considers for both the 'left' and the 'right' movers the $N=1$ superconformal field theory based on the coset [36][37]

$$\frac{G_{\hat{k}} \times SO(\mathrm{dim}\ (G/H))_1}{H_{\hat{k}+\mathrm{g_G}-\mathrm{g_H}}} \ , \tag{5.5}$$

where the shifted level $\hat{k}=k-\mathrm{g_G}$ is a result of the redefinition of the bosonic currents necessary to decouple the fermions [36][37].

In [34][35] a Lagrangian formulation of the above models was given. One introduces free fermions $\psi,\bar{\psi}$ with values in the tangent space to G/H and couples them minimally to the gauge fields $A,\bar{A}$. Then the corresponding path integral is given by

$$\begin{aligned} Z^{(1,1)}_{GWZNW} &= \int [dg][dA][d\bar{A}][d\psi][d\bar{\psi}]\ \exp\{-\hat{k}I_0(g,A)-\hat{k}I_0(\psi,A)\} \ , \\ I_0(\psi,A) &= \frac{i}{2\pi}\int d^2z\ \mathrm{Tr}\ (\bar{\psi}D\bar{\psi}+\psi\bar{D}\psi) \ , \end{aligned} \tag{5.6}$$

where the covariant derivatives are defined as

$$D=\partial-[A,\ \ ] \ , \quad \bar{D}=\bar{\partial}-[\bar{A},\ \ ] \ . \tag{5.7}$$

The explicit form of the infinitesimal supersymmetric transformations is

$$\begin{aligned} &\delta g = i\epsilon\bar{\psi}g+i\bar{\epsilon}g\psi \ , \qquad && \delta\psi=-\bar{\epsilon}(g^{-1}Dg+i\psi^2)\big|_{G/H} \ , \\ &\delta A=\delta\bar{A}=0 \ , && \delta\bar{\psi}=-\epsilon(\bar{D}gg^{-1}-i\bar{\psi}^2)\big|_{G/H} \ , \end{aligned} \tag{5.8}$$

where the supersymmetry parameters are chiral, i.e. $\epsilon=\epsilon(\bar{z})$, $\bar{\epsilon}=\bar{\epsilon}(z)$. Integrating over the fermions in (5.6) and going through the same steps as in the bosonic case we finish with

$$\begin{aligned} Z^{(1,1)}_{GWZNW} =& N'\int[d\tilde{g}][d\tilde{h}]\ \exp\{-\hat{k}I(\tilde{g})+[\hat{k}+(\mathrm{g_G}-\mathrm{g_H})+2\mathrm{g_H}]I(\tilde{h})\} \ , \\ &\tilde{g}=h^{-1}g\bar{h} \ , \quad \tilde{h}=h^{-1}\bar{h} \ , \end{aligned} \tag{5.9}$$

where the contribution of the fermions is proportional to $\mathrm{g_G} - \mathrm{g_H}$. This expression becomes equivalent to (5.4) once the relation $\hat{k} = k - \mathrm{g_G}$ is being used. It is clear that after we perform the bosonic quantum shiftings in the G and H terms in (5.4) the quantum effective action one obtains is the same as the semi-classical one (cf. (2.8)) [25], and the σ-model receives no $1/k$ corrections [23][12][24][25].

5.2. (1,1) supersymmetric CWZNW model

Let us now repeat the above analysis in the case of the (1,1) supersymmetric extension of the chiral gauged WZNW action which is obtained by dropping out the $\hat{A}\hat{\bar{A}}$ term in (5.1)

$$\begin{aligned}\hat{I}^{(1,1)}_{CWZNW}(\hat{g},\hat{A}) &= \hat{I}(\hat{g}) + \frac{1}{\pi}\int d^2z d^2\theta \ \mathrm{Tr}\ \left(\hat{A}\,\bar{D}\hat{g}\hat{g}^{-1} - \hat{\bar{A}}\,\hat{g}^{-1}D\hat{g} + \hat{g}^{-1}\hat{A}\hat{g}\hat{\bar{A}}\right) \\ &= \hat{I}(\tilde{\hat{g}}) - \hat{I}(\hat{h}^{-1}) - \hat{I}(\hat{\bar{h}})\ , \qquad \tilde{\hat{g}} \equiv \hat{h}^{-1}\hat{g}\hat{\bar{h}}\ .\end{aligned} \tag{5.10}$$

Instead of (5.3) we now get

$$\begin{aligned}Z^{(1,1)}_{CWZNW} &= \int [d\hat{g}][d\hat{A}][d\hat{\bar{A}}]\ \exp\{-kI^{(1,1)}_{CWZNW}(\hat{g},\hat{A})\} \\ &= \int [d\tilde{\hat{g}}][d\hat{h}][d\hat{\bar{h}}]\ \mathcal{J}\ \exp\{-kI(\tilde{\hat{g}}) + k\hat{I}(\hat{h}^{-1}) + k\hat{I}(\hat{\bar{h}})\}\ ,\end{aligned} \tag{5.11}$$

where the Jacobian is the same as in (5.3), i.e. is trivial. As a result, the theory can be represented as a product of the three (1,1) supersymmetric WZNW theories for the groups G, H and H with the levels $k, -k$ and $-k$. Using the (1,1) supersymmetric analog of (2.22), i.e.

$$(G/H)^{(1,1)}\ CWZNW = \frac{G_{\hat{k}} \times SO(\mathrm{dim}\ (G/H))_1}{H_{\hat{k}+\mathrm{g_G}-\mathrm{g_H}}} \times H_{-\hat{k}_H - 2\mathrm{g_H}} \times SO(\mathrm{dim}\ H)_1\ , \tag{5.12}$$

where $\hat{k}_H = k - \mathrm{g_H}$, it can also be interpreted as a 'product' of a supersymmetric gauged WZNW model and a supersymmetric WZNW model. Because of the particular structure of the levels of the various factors it is expected, as in the case of the (1,1) supesrymmetric WZNW model [12], that the effective quantum action will receive no $1/k$ corrections. As in the case of (1,1) supersymmetric gauged WZNW case, one should be able to reproduce the result (5.11) using the component approach. Since the bosonic CWZNW action is not gauge invariant, the fermionic partners of the bosons g here take values in the algebra of G itself and not in the tangent space to G/H as in the GWZNW case (5.6), i.e. $\psi, \bar{\psi} \in \mathcal{L}(G)$. Then the path integral corresponding to the (1,1) supersymmetric CWZNW model is

$$\begin{aligned} Z^{(1,1)}_{CWZNW} &= \int [dg][dA][d\bar{A}][d\psi][d\bar{\psi}] \ \exp\{-\hat{k} I_1(g,A) - \hat{k} I_0(\psi, A)\} \ , \\ I_0(\psi, A) &= \frac{i}{2\pi} \int d^2z \ \mathrm{Tr} \ (\bar{\psi} D \bar{\psi} + \psi \bar{D} \psi) \ , \end{aligned} \tag{5.13}$$

where the covariant derivatives were defined in (5.7). The explicit form of the infinitesimal supersymmetric transformations is

$$\begin{aligned} \delta g &= i\epsilon \bar{\psi} g + i \bar{\epsilon} g \psi \ , \qquad & \delta \psi &= -\bar{\epsilon} \big[g^{-1}(\partial g - A g) + \partial \bar{h} \bar{h}^{-1} + i \psi^2 \big] \ , \\ \delta A &= \delta \bar{A} = 0 \ , & \delta \bar{\psi} &= -\epsilon \big[(\bar{\partial} g + g \bar{A}) g^{-1} - \bar{\partial} h h^{-1} - i \bar{\psi}^2 \big] \ , \end{aligned} \tag{5.14}$$

where the supersymmetry parameters are chiral, i.e. $\epsilon = \epsilon(\bar{z})$, $\bar{\epsilon} = \bar{\epsilon}(z)$. Expressed in terms of the three superfields $\hat{g}$, $\hat{h}$, $\hat{\bar{h}}$ the CWZNW action is manifestly (1,1) supersymmetric. The supersymmetry is preserved if one integrates over $\hat{h}$, $\hat{\bar{h}}$ obtaining the effective action (or, up to non-local terms, a σ-model) for $\hat{g}$. Integrating out only the fermionic partners of h and $\bar{h}$ (or of A and $\bar{A}$) while keeping the bosonic fields $h, \bar{h}$ one gets the action in (5.13). The fields ψ are the fermionic components of the 'rotated' superfield $\tilde{\hat{g}}$ in (5.10), (5.11), i.e. they are the combinations of the original fermionic partners of $g, h, \bar{h}$. As we see from (5.14) the action (5.13) lacks a linearly realised supersymmetry. The formal supersymmetry transformation laws are non-local being expressed in terms of $A, \bar{A}$ (but local in terms of $h, \bar{h}$). They become local once one integrates out $A, \bar{A}$.

As in the GWZNW case (5.9) the combined contribution of the integrals over the fermionic partners of g and $A, \bar{A}$ is proportional to $\mathrm{g_G} - \mathrm{g_H}$ (the latter has the opposite sign since it may be thought of as originating from the measure). Not including (as in the bosonic CWZNW case) the local $A\bar{A}$ - counterterm in the results for both the bosonic Jacobian and the fermionic determinants we find (cf. (5.9))

$$Z^{(1,1)}_{CWZNW} = N' \int [d\tilde{g}][dh][d\bar{h}] \ \exp\{-\hat{k} I(\tilde{g}) + [\hat{k} + (\mathrm{g_G} - \mathrm{g_H}) + 2\mathrm{g_H}][I(h^{-1}) + I(\bar{h})]\} \ . \tag{5.15}$$

After one uses the relation $\hat{k} = k - g_G$ this expression becomes equivalent to the component form of (5.11) with the fermions integrated out (cf. (5.4))

$$Z^{(1,1)}_{CWZNW} = N' \int [d\tilde{g}][dh][d\bar{h}] \; \exp\{-(k - g_G) I(\tilde{g}) + (k + g_H)[I(h^{-1}) + I(\bar{h})]\} \; . \quad (5.16)$$

It is now straightforward to write down the expression for the effective action in the chiral gauged (1,1) supersymmetric WZNW theory. Using either the representation in terms of the ungauged supersymmetric WZNW theories (5.11) or the equivalent formulation in terms of the ungauged bosonic WZNW theories (5.16) we get the following expression for (the bosonic part of) the effective action (we omit non-local terms originating from field renormalisations [25])

$$\Gamma^{(1,1)}_{CWZNW}(g, A) = kI(h^{-1} g \bar{h}) - kI(h^{-1}) - kI(\bar{h}) \; . \quad (5.17)$$

As in the ungauged supersymmetric WZNW theory, but in contrast to the result in the bosonic CWZNW theory, here there are no shifts in the overall coefficients of the G- and H- terms in Γ_{CWZNW}. The local part of the effective action of the (1,1) supersymmetric CWZNW model is equal to the *classical* action of the bosonic CWZNW theory (2.4),

$$\Gamma^{(1,1)}_{CWZNW}(g, A) = k\big[I(g) + \frac{1}{\pi} \int d^2 z \; \mathrm{Tr} \; \big(A \, \bar{\partial} g g^{-1} - \bar{A} \, g^{-1} \partial g + g^{-1} A g \bar{A}\big)\big] \; , \quad (5.18)$$

i.e. in contrast to the bosonic case (2.4) it does not contain the quantum correction term proportional to $b = -\frac{g_G - g_H}{k + g_G}$. As a consequence, the exact form of the bosonic part of the corresponding σ-model is equivalent to the 'semiclassical' form of the σ-model in the bosonic theory. This result is similar to the one found for the (1,1) CWZNW theory, in agreement with the equivalence (2.22) between CWZNW and $(G/H) \times H$ GWZNW models which was established in the bosonic case and with similar relation (5.12) in the supersymmetric one. The quantum Hamiltonian corresponding to the (1,1) supersymmetric CWZNW model has the same form as the classical bosonic one, i.e. (3.10) in the $k \to \infty$ limit (we consider the case when the left and right subgroups are the same)

$$\mathcal{H}^{(1,1)}_{CWZNW} = \; J^2_G - J^2_H - \bar{J}^2_H = \frac{1}{2}[\; J^2_G - 2 J^2_H \;] + \frac{1}{2}[\; \bar{J}^2_G - 2 \bar{J}^2_H \;]. \quad (5.19)$$

Note again that this Hamiltonian is *not* equal to the sum of the Hamiltonians of the left and right G/H coset models because of the extra coefficients 2 in front of the subgroup current terms.

6. (1,0) supersymmetric chiral gauged WZNW model

Let us now turn to a less trivial (1,0) supersymmetric theory, which is to be related to the heterotic string σ-model [39]. We are going to repeat the previous analysis, replacing (1,1) superfields by (1,0) ones. Part of the discussion will be similar to that in [22]. It was concluded in [22] that since the direct (1,0) truncation of the (1,1) supersymmetric GWZNW theory is anomalous, it does not describe a consistent heterotic string background. Here instead we shall start with the (1,0) supersymmetric extension of bosonic (or truncation of (1,1) supersymmetric) chiral gauged WZNW model. Since we shall give up the gauge invariance and will not reduce to G/H already at the classical level, no contradiction will be found at the quantum level. As we shall see below in Section 7, the resulting model provides a non-trivial example of a heterotic string solution that is not effectively (1,1) supersymmetric (as were solutions in [22]) but still has a well defined conformal field theory counterpart.

6.1. Component approach

To illustrate the point that the 'anomalous' contribution of Weyl fermions does not represent a problem in the CWZNW case it is instructive to start with a heuristic discussion in the component approach. Using the component form of the action of the (1,1) supersymmetric CWZNW and dropping out the right component ($\bar{\psi}$) the fermionic fields we shall add instead some 'right' internal fermions $\bar{\psi}^I$ which are not coupled to $A, \bar{A}$ (but may be coupled to a background target space gauge field which in the present case we shall set equal to zero) and do not transform under supersymmetry. Then we get

$$I^{(1,0)}_{CWZNW}(g, A, \psi, \bar{\psi}^I) = I_{CWZNW}(g, A) + \frac{i}{2\pi} \int d^2z \ \mathrm{Tr}\ (\psi \bar{D} \psi) + I_{int}(\bar{\psi}^I) \ . \qquad (6.1)$$

If one changes the bosonic variables from $A, \bar{A}$ to $h, \bar{h}$ (without introducing the local $A\bar{A}$-counterterm) and integrates over the fermions $\psi, \bar{\psi}^I$ one finds the following expression for the quantum partition function

$$
\begin{aligned}
Z^{(1,0)}_{CWZNW} &= \int [dg][dh][d\bar{h}] \exp\{-\hat{k}I(h^{-1}g\bar{h}) + (\hat{k}+2\mathrm{g_H})[I(h^{-1}) + I(\bar{h})] + (\mathrm{g_G}-\mathrm{g_H})I(\bar{h})\} \\
&= \int [d\tilde{g}][dh][d\bar{h}]\ \exp\{-\hat{k}I(\tilde{g}) + [\hat{k}+2\mathrm{g_H} + (\mathrm{g_G}-\mathrm{g_H})]I(\bar{h}) + (\hat{k}+2\mathrm{g_H})I(h^{-1})\}.
\end{aligned}
\tag{6.2}
$$

Here the $(\mathrm{g_G}-\mathrm{g_H})$-term is the fermionic contribution. This expression is to be compared with (5.15), (5.16) found in the (1,1) supersymmetric case. Using $\hat{k} = k - \mathrm{g_G}$ as in the (1,1) supersymmetric case[9] we find

$$
Z^{(1,0)}_{CWZNW} = \int [d\tilde{g}][dh][d\bar{h}]\ \exp\{-(k-\mathrm{g_G})I(\tilde{g}) + (k-\mathrm{g_G}+2\mathrm{g_H})I(h^{-1}) + (k+\mathrm{g_H})I(\bar{h})\}\ . \tag{6.3}
$$

To get the effective action corresponding to (6.3) one is to make further (bosonic) shifts of the levels. Up to a non-local field redefinition we get (cf.(2.16))

$$
\begin{aligned}
\Gamma^{(1,0)}_{CWZNW}(g,A) &= kI(\tilde{g}) - (k-\mathrm{g_G}+\mathrm{g_H})I(h^{-1}) - kI(\bar{h}) \\
&= kI_{CWZNW}(g,A) + (\mathrm{g_G}-\mathrm{g_H})\omega(A)\ ,
\end{aligned}
\tag{6.4}
$$

where ω was defined in (2.18). We conclude that in contrast to the (1,1) supersymmetric model, in the (1,0) supersymmetric case there is a *non-trivial* quantum correction in the effective action (or in the quantum Hamiltonian of the corresponding conformal theory). The expression (6.4) can be obtained from its bosonic counterpart (2.16) by dropping out the $\bar{A}$-part of the quantum correction term and replacing the shifted k by the unshifted one in the classical term.

According to (6.3) the bosonic sector of the (1,0) theory is represented by a combination of the three WZNW theories for G, H and H. Comparing (6.2) with (5.11) we can interpret this theory as a 'product' of the supersymmetric $(G/H)_k = G_{k-\mathrm{g_G}}/H_{k-\mathrm{g_H}}$ GWZNW model and the bosonic $H_{-k-\mathrm{g_H}}$ WZNW model (cf.(2.16)). Since the resulting

[9] The shift of k that occurs in the (1,1) WZNW model must still be present in the (1,0) case since going from superfields to components we still get the coupling of the Weyl fermions to the g-dependent current and, as a result, the anomalous contribution that shifts the coefficient k of the bosonic term in the action.

theory is not (1,1) supersymmetric, it is not surprising that here we will have non-trivial $1/k$ corrections in the effective action (and, as a result, in the corresponding target space background fields).[10] We shall return to the discussion of the effective action (6.4) in Section 7 below.

6.2. Superfield approach

Let us now support the above component analysis by using directly the (1,0) superfield formulation. To obtain a manifestly (1,0) supersymmetric CWZNW action one starts with the (1,1) supersymmetric action (5.10) and truncates the (1,1) superfields to (1,0) superfields (in the heterotic string context we should also add the internal (1,0) superfields Ψ^I which will not, in contrast to the case considered in [22], be important in the present discussion). The resulting action is

$$\hat{I}^{(1,0)}_{CWZNW}(\hat{g},\hat{A},\Psi) = \hat{I}^{(1,0)}(\hat{g}) + \frac{1}{\pi}\int d^2z d\theta \ \mathrm{Tr}\ \left(\hat{A}\,\bar{\partial}\hat{g}\hat{g}^{-1} - \hat{\bar{A}}\,\hat{g}^{-1}D\hat{g} + \hat{g}^{-1}\hat{A}\hat{g}\hat{\bar{A}}\right) + I_{int}\ , \quad (6.5)$$

where $\hat{I}^{(1,0)}$ denotes the (1,0) supersymmetric WZNW action [40]

$$\hat{I}^{(1,0)}(\hat{g}) \equiv \frac{1}{2\pi}\int d^2z d\theta \{\ \mathrm{Tr}\ (D\hat{g}^{-1}\bar{\partial}\hat{g}) - i\int dt[\hat{g}^{-1}D\hat{g}, \hat{g}^{-1}\partial_t\hat{g}]\hat{g}^{-1}\bar{\partial}\hat{g}\ \}\ , \quad (6.6)$$

and

$$I_{int} = \int d^2z d\theta \Psi^I D\Psi^I\ , \quad \hat{g} = \exp(T_A X^A)\ , \quad X^A = x^A + \theta\psi^A_+\ , \quad \Psi^I = \psi^I_- + \theta f^I\ ,$$

$$\hat{A} = \chi_+ + \theta A = D\hat{h}\hat{h}^{-1}\ , \quad \hat{\bar{A}} = \bar{A} + \theta\chi_- = \bar{\partial}\hat{\bar{h}}\hat{\bar{h}}^{-1}\ . \quad (6.7)$$

We get the following path integral

$$Z^{(1,0)}_{CWZNW} = \int [d\hat{g}][d\hat{h}][d\hat{\bar{h}}]\ \mathcal{J}'\ \exp\{-kI^{(1,0)}(\hat{h}^{-1}\hat{g}\hat{\bar{h}}) + k[I^{(1,0)}(\hat{h}^{-1}) + I^{(1,0)}(\hat{\bar{h}})]\}\ , \quad (6.8)$$

---

[10] Note that the quantum correction in (6.4) is absent when $G = H$. In this case the theory effectively reduces to the (1,0) supersymmetric WZNW theory.

where $\mathcal{J}'$ is the product of Jacobians of the two changes of variables $\hat{A} \to \hat{h}$ and $\hat{\bar{A}} \to \hat{\bar{h}}$. As discussed in [22], the first Jacobian is essentially the same as in the bosonic case while the second one is still trivial as in the (1,1) supersymmetric case. Therefore,

$$Z^{(1,0)}_{CWZNW} = \int [d\hat{g}][d\hat{h}][d\hat{\bar{h}}] \quad \exp\{-k\hat{I}^{(1,0)}(\hat{h}^{-1}\hat{g}\hat{\bar{h}})$$

$$+ k[\hat{I}^{(1,0)}(\hat{h}^{-1}) + \hat{I}^{(1,0)}(\hat{\bar{h}})] + 2\mathrm{g_H}\hat{I}^{(1,0)}(\hat{h}^{-1})\} \quad . \tag{6.9}$$

There is also an extra anomaly term originating from non-invariance of the path integral measure under the (1,0) superfield rotation of $\hat{g}$ [22] so that the final result is

$$Z^{(1,0)}_{CWZNW} = \int [d\tilde{\hat{g}}][d\hat{h}][d\hat{\bar{h}}] \quad \exp\{-k\hat{I}^{(1,0)}(\tilde{\hat{g}})$$

$$+ k[\hat{I}^{(1,0)}(\hat{h}^{-1}) + \hat{I}^{(1,0)}(\hat{\bar{h}})] - (\mathrm{g_G} - \mathrm{g_H})\hat{I}^{(1,0)}(\hat{h}^{-1})\} \quad . \tag{6.10}$$

The corresponding superfield effective action

$$\Gamma^{(1,0)}_{CWZNW} = k\hat{I}^{(1,0)}_{CWZNW}(\hat{g}, \hat{A}) + (\mathrm{g_G} - \mathrm{g_H})\hat{I}^{(1,0)}(\hat{h}^{-1}) \tag{6.11}$$

is thus perfectly consistent with the expression (6.4) found above in the component approach.

In the case of CWZNW model with two different subgroups H_L and H_R its (1,0) supersymmetric extension is different from the (0,1) one.[11] The corresponding (bosonic parts of) Hamiltonians of the (1,1), (1,0) and (0,1) supersymmetric CWZNW models can be found using the general expressions (3.1),(3.4) (cf.(3.11),(5.16))

$$\mathcal{H}^{(1,1)}_{CWZNW} = \frac{1}{2}[J^2_G - 2J^2_{H_L}] + \frac{1}{2}[\bar{J}^2_G - 2\bar{J}^2_{H_R}] = \frac{1}{2}[J^2_G - J^2_{H_L} - \bar{J}^2_{H_R}] \ , \tag{6.12}$$

$$\mathcal{H}^{(1,0)}_{CWZNW} = \frac{1}{2}[J^2_G - 2\frac{k}{k - (\mathrm{g_G} - \mathrm{g_{H_L}})}J^2_{H_L}] + \frac{1}{2}[\bar{J}^2_G - 2\bar{J}^2_{H_R}]$$

[11] Using such models for the construction of the heterotic string solutions we need to compensate for a mismatch between the numbers of left and right bosonic degrees of freedom (see the remark at the end of Section 2.1).

$$= J_G^2 - \frac{k}{k-(\mathrm{g_G}-\mathrm{g_{H_L}})} J_{H_L}^2 - \bar{J}_{H_R}^2 \ , \qquad (6.13)$$

$$\mathcal{H}^{(0,1)}_{CWZNW} = \frac{1}{2}[J_G^2 - 2J_{H_L}^2] + \frac{1}{2}[\bar{J}_G^2 - 2\frac{k}{k-(\mathrm{g_G}-\mathrm{g_{H_R}})}\bar{J}_{H_R}^2]$$

$$= J_G^2 - J_{H_L}^2 - \frac{k}{k-(\mathrm{g_G}-\mathrm{g_{H_R}})}\bar{J}_{H_R}^2 \ . \qquad (6.14)$$

Note that the Hamiltonians for the 'heterotic' cases (6.13),(6.14) are equal to the combinations of the left and right parts of the Hamiltonians of the bosonic and supersymmetric CWZNW models and *not* of the Hamiltonians of the bosonic and supersymmetric coset G/H models. The important difference is also that here the configuration space is not reduced to G/H but remains the group space itself. In contrast to a naive 'left plus right' combination of the bosonic and supersymmetric coset models the above models are well defined.

7. Heterotic string solutions corresponding to the (1,0) supersymmetric chiral gauged WZNW model

Let us now use the (1,0) supersymmetric CWZNW model as a basis for a construction of the heterotic string solutions. As in the bosonic or (1,1) supersymmetric GWZNW case the idea is to treat the $2d$ gauge field as an 'auxiliary' variable and thus to eliminate it from the (effective) action obtaining the σ-model for the 'observable' coordinates x^M of the configuration space (G/H in the GWZNW case and group space G in the CWZNW case). In the case of bosonic CWZNW model this was already discussed on particular examples in [15][16] and in general in [17]. If we start with the (1,0) supersymmetric CWZNW theory the resulting σ-model is bound to be conformal and (1,0) supersymmetric and therefore to represent a solution of the heterotic string theory.

7.1. General expressions for the background fields

As follows from the comparison of the corresponding effective actions (2.16), (5.18) and (6.4) the semiclassical (or leading order in $1/k$) expressions for the background fields are the same in the bosonic, (1,1) and (1,0) supersymmetric CWZNW cases. Like the bosonic one, the heterotic solution is modified by the $1/k$ (or α') corrections. The generic expression for the effective action can be represented in the form (cf. (2.1), (2.2), (2.15), (2.16), (3.2), (6.4))

$$\Gamma(g,A,\bar{A}) = \kappa \,[\, I(g) + \frac{1}{\pi}\int d^2z \ \text{Tr}\ \big(A\,\bar{\partial}gg^{-1} - \bar{A}\,g^{-1}\partial g + g^{-1}Ag\bar{A} - A\bar{A}\big) \\ + \frac{a}{\pi}\int d^2z \ \text{Tr}\ (A\bar{A}) - b\omega(A) - \bar{b}\bar{\omega}(\bar{A})\ ]\ , \tag{7.1}$$

where the values of the constants $a, b, \bar{b}$ corresponding to the bosonic GWZNW and CWZNW models were given in (3.3), (3.4). The heterotic or (1,0) CWZNW case is intermediate between the bosonic and (1,1) supersymmetric CWZNW ones (in the latter case $\kappa = k,\ a = 1,\ b = \bar{b} = 0$)

$$\kappa = k\ , \quad a = 1\ , \quad b = -\frac{1}{k}(\mathrm{g_G} - \mathrm{g_H})\ , \quad \bar{b} = 0\ . \tag{7.2}$$

The derivation of the σ-model corresponding to the general effective action (7.1) which encompasses all known GWZNW and CWZNW cases is given in [30]. Here for simplicity we restrict the discussion to the heterotic case (6.4) or (7.1) with (7.2). Dropping out the higher order non-local $O(A^3)$ terms in $\omega(A)$ (since they do not affect the derivation of the local part of the σ-model action [25]) we obtain[12]

$$\Gamma^{(1,0)}(g,A,\bar{A}) = \ k\ [I(g) + \frac{1}{\pi}\int d^2z \ \text{Tr}\ \big(A\,\bar{\partial}gg^{-1} - \bar{A}\,g^{-1}\partial g + g^{-1}Ag\bar{A} + \frac{1}{2}bA\frac{\bar{\partial}}{\partial}A)]\ . \tag{7.3}$$

[12] This is equivalent to approaching the point particle limit of the theory, in which all non-local terms drop out, and then restoring formally the Lorentz invariance [27].

Introducing the notation ($T_A = (T_a, T_i)$ are the generators of G; T_a are the generators of H; $A = 1, \ldots, d_G$; $a = 1, \ldots, d_H$; η_{AB} is negative definite in the compact case)

$$
\begin{aligned}
& A = A^a T_a \ , \quad C_{AB} \equiv \text{ Tr } (T_A g T_B g^{-1}) \ , \quad \text{Tr } (T_A T_B) = \eta_{AB} \ , \\
& J_a = \text{ Tr } (T_a g^{-1} \partial g) = L_{aM}(x) \partial x^M \ , \quad \bar{J}_a = - \text{ Tr } (T_a \bar{\partial} g g^{-1}) = R_{aM}(x) \bar{\partial} x^M \ , \\
& \tilde{J}_a = \text{ Tr } (T_a g^{-1} \bar{\partial} g) = L_{aM}(x) \bar{\partial} x^M \ , \quad \tilde{\bar{J}}_a = - \text{ Tr } (T_a \partial g g^{-1}) = R_{aM}(x) \partial x^M \ , \\
& R^A_M = -C^A{}_B L^B_M \ , \quad C^{AD} C_{BD} = \delta^A_B \ ,
\end{aligned}
\tag{7.4}
$$

we find the following solution for $A^a, \bar{A}^a$

$$
A = (C^T)^{-1} J \ , \qquad \bar{A} = C^{-1} \bar{J} - b (C^T C)^{-1} \tilde{J} + \ldots \ , \qquad C = (C_{ab}) \ , \tag{7.5}
$$

where dots stand for non-local terms. Inserting (7.5) back into the action (7.3) we get for the local part of the effective action (7.3)

$$
\Gamma^{(1,0)}_{loc}(g, A, \bar{A}) = \ k \ \{ I(g) + \frac{1}{\pi} \int d^2 z \ \text{Tr} \left[-J C^{-1} \bar{J} + \frac{1}{2} b J (C^T C)^{-1} \tilde{J} \right] \} \ . \tag{7.6}
$$

Identifying this action with (the bosonic part of) the heterotic σ-model action we obtain the exact expressions for the target space metric and the antisymmetric tensor coupling

$$
S(x) = \Gamma^{(1,0)}_{loc}(g) = \frac{k}{2\pi} \int d^2 z \ \mathcal{G}_{MN}(x) \partial x^M \bar{\partial} x^N \ , \tag{7.7}
$$

where

$$
\begin{aligned}
& G_{MN} \equiv \mathcal{G}_{(MN)} = G_{0MN} - 2 M^{-1}{}_{ab} L^a_{(M} R^b_{N)} + b (M^T M)^{-1}{}_{ab} L^a_M L^b_N \ , \\
& B_{MN} \equiv \mathcal{G}_{[MN]} = B_{0MN} - 2 M^{-1}{}_{ab} L^a_{[M} R^b_{N]} \ , \qquad M_{ab} \equiv C_{ab} \ ,
\end{aligned}
\tag{7.8}
$$

where G_{0MN} and B_{0MN} are the original WZNW (group space) couplings,

$$
G_{0MN} = -\eta_{AB} L^A_M L^B_N \ , \qquad 3 \partial_{[K} B_{0MN]} = L^A_K L^B_M L^C_N f_{ABC} \ . \tag{7.9}
$$

We conclude that the exact expression for the metric contains only the leading (two-loop) correction (recall that $b = -(\mathrm{g_G} - \mathrm{g_H})/\kappa$) while the antisymmetric tensor is the same as in the semiclassical approximation. It is easy to see that the determinant of the matrix in the quadratic $(A,\bar{A})$-term in (7.3) does not depend on b and thus the expression for the dilaton also remains semiclassical

$$\phi = \phi_0 - \frac{1}{2} \ln \det M \ . \tag{7.10}$$

Note that the 'measure factor' $\sqrt{G} \exp(-2\phi)$ still does not receive quantum corrections since one can prove (cf. [25][17]) that $G = \det G_{MN}$ does not depend non-trivially on b (there is only an overall b-dependent factor). It is also clear that when $G = H$ we get just the WZNW model with the opposite sign in front of the first term in the action: $G_{MN} = -G_{0MN}$, $B_{MN} = B_{0MN}$ (the dilaton in (7.10) is then constant since according to (7.4) $|\det C_{AB}| = 1$).

These expressions are to be compared with the (1,1) supersymmetric (superstring) CWZNW case where all the fields are given by semiclassical expressions as well as with the bosonic CWZNW case [17] where all the fields in general receive quantum corrections to all orders in the $1/k$ expansion.[13]

What is the geometrical interpretation of the resulting spaces? These are some 'deformations' of group spaces with the matrix C playing the role of a deformation 'parameter'. The deformation is related to the presence of a non-trivial dilaton which, in turn, is necessary in order to satisfy the σ-model conformal invariance conditions once the model is perturbed from the original (group space) conformal point. It should be stressed that the 'J^2-perturbation' of the WZNW model in (7.6) is not in general of an integrably marginal type; the 'perturbed' model is conformal only for the specific function $C_{ab}(g)$ that appears in the CWZNW model.

[13] For completeness, let us also recall that in the GWZNW case the semiclassical ($b \to 0$) expressions for the corresponding background fields are formally the same (before projection on G/H) as in (7.8), (7.10) with $M_{ab} = C_{ab} - \eta_{ab}$. However, in this case the residual gauge invariance of the model demands gauge fixing, i.e. reducing the configuration space to G/H.

7.2. Basic $D=3$ example: heterotic $SL(2,\mathbb{R})/\mathbb{R}$ CWZNW model

Since the σ-model configuration space in the CWZNW case has dimension $D = \dim G$ the first non-trivial example of the heterotic solution based on CWZNW model is found for $G = SU(2)$ or $SL(2,\mathbb{R})$. The form of the σ-model corresponding to the bosonic $SL(2,\mathbb{R})/\mathbb{R}$ CWZNW theory was determined in [15][17] and it was noted that this model is closely related to a specific limit of the $[SL(2,\mathbb{R})\times\mathbb{R}]/\mathbb{R}$ GWZNW theory [18][13] (see Section 4). Let us first recall the exact expressions for the background fields in the bosonic model. In the bosonic case the exact metric, antisymmetric tensor and dilaton are given by [17]

$$\begin{aligned} ds^2 &= -\frac{z-1}{z+b}\ dt^2 - \frac{z+1}{z-b}\ dx^2 + \frac{dz^2}{4(z^2-1)}\ , \\ B_{tx} &= -(1-b)\frac{z}{z^2-b^2}\ , \\ \phi &= \phi_0 - \frac{1}{4}\ \ln\ (z^2-b^2)\ . \end{aligned} \tag{7.11}$$

where $b = -(g_G - g_H)/\kappa = 2/(k-2)$ (and $\alpha' = 1/(k-2)$). In the heterotic case ($b = 2/k$, $\alpha' = 1/k$) we get the semiclassical expressions for the antisymmetric tensor and the dilaton, i.e.

$$B_{tx} = -\frac{1}{z}\ , \qquad \phi = \phi_0 - \frac{1}{2}\ \ln\ z\ , \tag{7.12}$$

while the metric one finds from (7.8)

$$\begin{aligned} ds^2 = &-\frac{z-1}{z}\ dt^2 - \frac{z+1}{z}\ dx^2 + \frac{dz^2}{4(z^2-1)} \\ &- \frac{b}{2z^2}\big[(z-1)\ dt + (z+1)\ dx\big]^2 \end{aligned} \tag{7.13}$$

contains a non-trivial $O(\alpha')$ correction term.[14]

It is useful to give the explicit derivation of the expressions for the bosonic and heterotic $SL(2,\mathbb{R})/\mathbb{R}$ backgrounds using the following parametrisation of the $SL(2,\mathbb{R})$ group

[14] The variables t, x in (7.11) have been rescalled with respect to the original 'classical' variables of the $SL(2,\mathbb{R})$ group element, i.e. $(t,x) \to \big(2/(b+1)\big)^{1/2}(t,x)$. The corresponding rescaling in (7.13) is $(t,x) \to \sqrt{2}(t,x)$.

element[15]

$$g = \mathrm{e}^{\frac{i}{2}\theta_L\sigma_2}\mathrm{e}^{\frac{1}{2}r\sigma_1}\mathrm{e}^{\frac{i}{2}\theta_R\sigma_2} \ , \qquad \theta_L = \theta + \tilde{\theta} \ , \quad \theta_R = \tilde{\theta} - \theta \ . \tag{7.14}$$

Taking $A, \bar{A}$ to be in the $U(1)$ subgroup generated by $\frac{1}{2}\sigma_2$, the classical CWZW action (2.4) can be represented in the form

$$S_1(g, A) = kI_1(g, A) = \frac{k}{2\pi}\int d^2z[\ \frac{1}{2}(\partial r\bar{\partial}r - \partial\theta_L\bar{\partial}\theta_L - \partial\theta_R\bar{\partial}\theta_R - 2C\ \bar{\partial}\theta_L\partial\theta_R)$$

$$-A(\bar{\partial}\theta_R + C\ \bar{\partial}\theta_L) + \bar{A}(\partial\theta_L + C\ \partial\theta_R) + CA\bar{A}\ \ ] \ , \quad C = C(r) \equiv \cosh r \ . \tag{7.15}$$

The effective actions corresponding to the bosonic and (1,0) supersymmetric CWZNW theories (2.16) and (6.4) (or (7.1) with (3.4) and (7.2)) in the present model are particular cases of (here the subgroup is abelian so the quantum terms are bilinear in $A, \bar{A}$)[16]

$$\Gamma(g, A) = \frac{\kappa}{2\pi}\int d^2z\ [\ \frac{1}{2}(\partial r\bar{\partial}r - \partial\theta_L\bar{\partial}\theta_L - \partial\theta_R\bar{\partial}\theta_R - 2C\ \bar{\partial}\theta_L\partial\theta_R)$$

$$-A(\bar{\partial}\theta_R + C\ \bar{\partial}\theta_L) + \bar{A}(\partial\theta_L + C\ \partial\theta_R) + CA\bar{A} + \frac{1}{2}bA\frac{\bar{\partial}}{\partial}A + \frac{1}{2}\bar{b}\bar{A}\frac{\partial}{\bar{\partial}}\bar{A}\ ] \ . \tag{7.16}$$

Eliminating $A, \bar{A}$ from (7.16) and dropping out the non-local terms we get the following σ-model action

$$S(r, \theta_L, \theta_R) = \frac{\kappa}{4\pi}\int d^2z[\partial r\bar{\partial}r + (1+\bar{b})(C^2+b)V^{-1}\partial\theta_L\bar{\partial}\theta_L + (1+b)(C^2+\bar{b})V^{-1}\partial\theta_R\bar{\partial}\theta_R$$

$$+\ (1+\bar{b})(1+b)CV^{-1}(\partial\theta_L\bar{\partial}\theta_R + \partial\theta_R\bar{\partial}\theta_L) + (1-b\bar{b})CV^{-1}(\partial\theta_L\bar{\partial}\theta_R - \partial\theta_R\bar{\partial}\theta_L)] \ , \tag{7.17}$$

where the function V and the dilaton are given by

$$V \equiv C^2 - b\bar{b} \ , \qquad \phi = \phi_0 - \frac{1}{4}\ \ln V \ . \tag{7.18}$$

[15] This parametrisation was used in [11] and also in [26]. The discussion that follows is very close to that in [26] where the case of the $SL(2, \mathbb{R})/U(1)$ GWZNW model was considered. Note that in our present notation the sign of A was changed.

[16] Comparing with Section 7.1 note that here we use a different normalisation of the generators: the generator of the subgroup is $T = \frac{1}{2}\sigma_2$ so that Tr $A^2 = \frac{1}{2}A^2$.

In terms of the coordinates $r, \theta, \tilde{\theta}$

$$S(r,\theta,\tilde{\theta}) = \frac{\kappa}{4\pi} \int d^2z [\partial r \bar{\partial} r + G_{\theta\theta} \partial\theta\bar{\partial}\theta + G_{\tilde{\theta}\tilde{\theta}} \partial\tilde{\theta}\bar{\partial}\tilde{\theta}$$

$$+ G_{\theta\tilde{\theta}} (\partial\theta\bar{\partial}\tilde{\theta} + \partial\tilde{\theta}\bar{\partial}\theta) + B_{\theta\tilde{\theta}} (\partial\theta\bar{\partial}\tilde{\theta} - \partial\tilde{\theta}\bar{\partial}\theta)] \ , \tag{7.19}$$

$$G_{\theta\theta} = (C-1)[(1+\bar{b})(C-b) + (1+b)(C-\bar{b})]V^{-1} \ , \tag{7.20}$$

$$G_{\tilde{\theta}\tilde{\theta}} = (C+1)[(1+\bar{b})(C+b) + (1+b)(C+\bar{b})]V^{-1} \ , \tag{7.21}$$

$$G_{\theta\tilde{\theta}} = (\bar{b} - b)(C^2 - 1)V^{-1} \ , \quad B_{\theta\tilde{\theta}} = 2(1 - b\bar{b})CV^{-1} \ . \tag{7.22}$$

If we identify α' with $1/\kappa$ as in (7.11) (so that G_{MN} and B_{MN} are to be multiplied by 1/4) then in the bosonic case ($b = \bar{b} = 2/(k-2) = 2\alpha'$) we get

$$4ds^2 = dr^2 + 2(1+b)\Big[\frac{C-1}{C+b} d\theta^2 + \frac{C+1}{C-b} d\tilde{\theta}^2\Big] \ , \quad B_{\theta\tilde{\theta}} = \frac{(1-b^2)C}{2(C^2-b^2)} \ , \tag{7.23}$$

while in the heterotic one ($b = 2/k = 2\alpha', \ \bar{b} = 0$)

$$4ds^2 = dr^2 + (1+bC^{-2})d\theta_L^2 + (1+b)d\theta_R^2 + 2(1+b)C^{-1} d\theta_L d\theta_R$$

$$= dr^2 + 2\frac{C-1}{C}[1 + b\frac{C-1}{2C}]d\theta^2 + 2\frac{C+1}{C}[1 + b\frac{C+1}{2C}]d\tilde{\theta}^2 - 2b\frac{C^2-1}{C^2} d\theta d\tilde{\theta} \ , \tag{7.24}$$

$$B_{\theta\tilde{\theta}} = \frac{1}{2C} \ . \tag{7.25}$$

The backgrounds (7.23) and (7.24),(7.25) coincide in the semiclassical ($b = 0$) limit and are related to (7.11) and (7.13) by the coordinate transformations

$$z = C = \cosh r \ , \quad t = i[\frac{1}{2}(1+b)]^{1/2}\theta \ , \quad x = i[\frac{1}{2}(1+b)]^{1/2}\tilde{\theta} \ , \tag{7.26}$$

and

$$z = C = \cosh r \ , \quad t = \frac{i}{\sqrt{2}}\theta \ , \quad x = \frac{i}{\sqrt{2}}\tilde{\theta} \ . \tag{7.27}$$

The metric (7.24) has rather peculiar 'heterotic' (left-right asymmetric) form. For comparison, let us recall the exact form [13][27] of the 'charged black string' background [18] (corresponding to $[SL(2,\mathbb{R})\times\mathbb{R}]/\mathbb{R}$ gauged WZNW theory) represented in the same coordinates $r,\theta,\tilde\theta$

$$4ds^2=dr^2+2(1+p+b)\frac{C-1}{C+p+b}d\theta^2+2(1-p+b)\frac{C+1}{C+p-b}d\tilde\theta^2\ , \tag{7.28}$$

$$B_{\theta\tilde\theta}=\frac{(1-p_0^2)C}{2(C+p_0)}\ ,\quad \phi=\phi_0-\frac{1}{4}\ln\left[(C+p-b)(C+p+b)\right]\ ,\quad p=p_0+b\ ,\ \ p_0=1+\sigma^2\ , \tag{7.29}$$

where σ is the parameter that governs the embedding of the subgroup (for $\sigma=0$ (7.28),(7.29) reduces to the exact $D=2$ black hole background [11]).[17] Note also that the $SL(2,\mathbb{R})$ group space background is (see (7.15))

$$4ds^2=dr^2+2(C-1)d\theta^2-2(C+1)d\tilde\theta^2\ ,\quad B_{\theta\tilde\theta}=\frac{1}{2}C\ ,\ \phi=\phi_0\ . \tag{7.30}$$

8. Concluding remarks

There exist five distinct classes of conformal σ-models associated with gauged or chiral gauged WZNW models: (1) models corresponding to bosonic G/H gauged WZNW theories; (2) models corresponding to bosonic G/H chiral gauged WZNW theories; (3) models corresponding to (1,0) supersymmetric G/H chiral gauged WZNW theories; (4) models corresponding to (1,1) supersymmetric G/H gauged WZNW theories; (5) models corresponding to (1,1) supersymmetric G/H chiral gauged WZNW theories. For all these models the exact dependence of the couplings (background fields) on α' is known and is different in different classes. The background fields in the first two classes depend non-trivially on α' (contain terms of all orders in expansion in α'). There is only one $O(\alpha')$ term in the metric in the third class while the dilaton and the antisymmetric tensor are α'-independent (i.e. semiclassical). The fields in the last two classes do not depend on α'. The backgrounds of the first and fourth classes coincide in the $\alpha'\to 0$ limit (the same is

[17] To derive (7.28),(7.29) one is to add to the $SL(2,\mathbb{R})/\mathbb{R}$ GWZNW action an extra scalar scalar term $(\partial y-\sigma A)(\bar\partial y+\sigma\bar A)$ and then fix $y=0$ as a gauge. The resulting effective action will be (7.16) with $CA\bar A$-term replaced by $(C+p)A\bar A$ (cf. (4.5)).

true for the backgrounds of the second, third and fifth classes). The σ-models associated with gauged (chiral gauged) WZNW theories have the configuration space of dimension equal to dim G/H (dim G). In the case of the abelian subgroup H the models of the second (fifth) class are equivalent to a particular subset of models (axially gauged ($G \times H/H$ with a special embedding of H) in the first (fourth) class. The heterotic string solutions are represented by the models of the third class (and also by the models of the fourth class 'embedded' into the heterotic string theory by introducing an extra gauge field background [22]).

The conformal invariance of the simplest ($D = 2$) $SL(2,\mathbb{R})/U(1)$ model of the first class was checked explicitly to α'^3 [41](and α'^4 [23]) order. In [30] we have checked that the simplest ($D = 3$) background (7.11) of the second class solves the σ-model conformal invariance conditions in the α' and α'^2 approximation. Though there are no doubts that the heterotic background (7.12),(7.13) of the third class solves the corresponding heterotic σ-model conformal invariance conditions (or, equivalently, the heterotic string effective equations) it may be of interest to check this directly in the α'^2 approximation. There exists a scheme [42] in which the G, B, ϕ–dependent part of the α'^2 term in the heterotic string effective action is given by one half of the α'^2 term in the bosonic string effective action plus the non-covariant contribution $\alpha' H(\omega R - \frac{2}{3}\omega\omega\omega)$ coming from the Lorentz Chern-Simons modification of $H = dB$. In contrast to the bosonic case, there is no (modulo a field redefinition) explicit $O(\alpha'^3)$ term in the heterotic string effective action. Though the metric (7.13) contains only the two-loop correction, this does not of course imply that the string equations will be automatically satisfied to all higher orders.

The backgrounds corresponding to the chiral gauged G/H WZNW theories discussed in this paper may be of interest from the point of view of a possible cosmological or black hole – type interpretation. There are two cases when the resulting space-time metric has the physical signature. If the group G is compact then according to the equivalence relation (2.22)we can get just one time-like coordinate if a compact subgroup H is one-dimensional, i.e. is $U(1)$. If G is non-compact, but H is compact, one can consider the non-compact coset G_{-k}/H_{-k} and require that the corresponding σ-model has just one time-like coordinate. Since H appears in (2.22)with level $k-2\mathrm{g_H}$ we will get no additional time-like coordinates as long as the condition $k>2\mathrm{g_H}$ is satisfied.[18]

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[18] This counting argument apparently does not depend on whether the 'auxiliary' fields are integrated out or not. All single time coordinate models characterised by G/H cosets based on simple as well as direct product non-compact groups are classified in [43][9]. The complete list of chiral gauged WZNW theories with one time-like coordinate is: $SU(p,q)/SU(p)\times SU(q)$, $SO(p,2)/SO(p)$, $Sp(2p,\mathrm{I\!R})/SU(p)$, $SO^*(2p)/SU(p)$, $E_6/SO(10)$, E_7/E_6. One can also take direct products of the above models with any WZNW or GWZNW theory with no time-like coordinates. The lowest-dimensional examples have $D=6$, i.e. $SO(2,2)/SO(2)$ CGWZN and $SO^*(4)/SU(2)$ CWZNW (in both cases it is assumed that the overall coefficient in the action is negative, i.e. $-k$).

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