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**STRING LOOP CORRECTIONS TO GAUGE AND YUKAWA COUPLINGS<sup>a</sup>**

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We report on the recent progress in computing the effective supergravity action from superstring scattering amplitudes beyond the tree approximation. We discuss the moduli-dependent string loop corrections to gauge, gravitational and Yukawa couplings.

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## 1. Introduction

The basic property of string theory, which makes it so attractive from the point of view of particle physics, is that the physical couplings and masses are in principle calculable. They are determined by the vacuum expectation values (VEVs) of massless scalar fields, like the dilaton and moduli. The latter are scalar fields with flat potential, whose VEVs correspond to compactification parameters and determine the size and shape of the internal space. Any serious attempt to compute the low-energy parameters from string theory must address two basic questions: 1) How do masses and couplings depend on the VEVs of dilaton, moduli, Higgs scalars *etc.*? and 2) What fixes these VEVs? In the past several years, there has been some steady progress towards answering the second question, although the general perception is that the problem of scalar VEVs still escapes a satisfactory solution. On the other hand, there has been a lot of progress towards answering the first question, which we will generically call the problem of moduli dependence of physical parameters.

A very efficient method of studying the moduli dependence of low-energy parameters, developed in the last couple of years, relies on the computation of the effective supergravity action describing the physics of massless string excitations. The moduli dependence of the effective action can be determined by evaluating the appropriate superstring scattering amplitudes.<sup>1-5</sup> In this process, supergravitational interactions are determined directly from superstring theory. The moduli-dependent loop corrections obtained in this way give rise to the so-called threshold corrections to superstring unification parameters,<sup>6</sup> and determine the boundary conditions of gauge and Yukawa couplings at the string unification scale  $M_{SU}$ . They are also relevant for non-perturbative phenomena in string theory, which could generate dynamically potential for moduli providing a mechanism that fixes their VEVs. They may also be relevant for the mechanism of supersymmetry breaking, either via gaugino condensation,<sup>7</sup> or with a large compactification radius.<sup>8</sup>

The massless spectrum of any four-dimensional heterotic superstring model contains the supergravity multiplet, gauge multiplets, and a large number of chiral multiplets. In addition, there is a dilaton which belongs to a very distinct supersymmetry multiplet, together with the two-index antisymmetric tensor – the Kalb-Ramond field. The dilaton VEV plays the role of the string loop expansion parameter. Since the Kalb-Ramond field is equivalent to a pseudoscalar axion, one usually represents the dilaton and its supersymmetric partners by one chiral multiplet  $S$ . The most general  $N = 1$  supergravity action, describing local interactions involving up to two derivatives, is characterized by three functions of chiral superfields:<sup>9</sup> the real Kähler potential  $K$  which determines the kinetic terms, the analytic superpotential  $W$  related to the Yukawa couplings, and the analytic gauge function  $f$  associated with the gauge couplings.

At the tree level, the general structure of the  $f$ -function and the Kähler

potential is common to all compactifications:

$$f^{(0)} = kS, \quad K^{(0)} = -\ln(S + \bar{S}) + G^{(0)}(Z, \bar{Z}), \quad (1)$$

where  $k$  is the integer level of the Kač-Moody algebra that generates the gauge group, and  $Z$  denote chiral superfields other than  $S$ . Thus, the coupling constant  $g_a$  of a gauge group factor  $G_a$  is given at the tree-level by  $g_a^2 = g^2/k$ , where  $g$  is the four-dimensional string coupling. The function  $G^{(0)}(Z, \bar{Z})$  as well as the superpotential  $W(Z)$  depend on the details of compactification and can be determined by using a number of different methods. At low energies, the most relevant interactions among matter and gauge fields correspond to effective operators of dimension four. Consequently, one may expand  $G^{(0)}$  and  $W$  up to quadratic and cubic order in matter fields, respectively, with moduli-dependent coefficients which one is interested to compute. These are known in many examples, in particular for the case of (2,2) compactifications which exhibit the property of special geometry implying that both functions are given in terms of a single prepotential.<sup>1,10</sup>

## 2. Infrared divergences and threshold corrections

The tree-level effective action describes interactions of massless string excitations at energies below the string scale. These include contact interactions due to the propagation of heavy particles in one-(massive)-particle reducible diagrams. The masses of heavy particles depend on the moduli, *e.g.* the radii of compactified dimensions, therefore the induced massless particle interactions are also moduli-dependent. In order to compute the loop corrections to the effective action, one should integrate all diagrams involving heavy particles propagating inside loops. In string theory, it is very difficult to separate heavy from massless particles in higher genus diagrams, therefore the computation of the effective action becomes more subtle than in field theory. In fact integration over massless particles gives rise to infrared divergences for on-shell amplitudes, associated with the running of low energy couplings.<sup>6,11</sup> In the analogous field-theoretical computations, such logarithmic divergences are usually regulated by going off-shell, to momentum  $\mu^2 \neq 0$ . It is very important to realize that in string theory, as well as in quantum field theory, the momentum-dependence of coupling constants is a purely infrared effect, and therefore the corresponding  $\beta$ -function coefficients of the  $\mu^2 \rightarrow 0$  divergence depend on the massless particle content only.

Consider for instance the one-loop case. A generic on-shell amplitude  $\mathcal{A}$  corresponding to some physical coupling of the low-energy theory is written as an integral over the complex Teichmüller parameter  $\tau = \tau_1 + i\tau_2$  of the world-sheet torus inside its fundamental domain  $\Gamma \equiv \{|\tau_1| \leq \frac{1}{2}, |\tau| \geq 1\}$ :

$$\mathcal{A} = \int_{\Gamma} \frac{d^2\tau}{\tau_2} A(\tau, \bar{\tau}). \quad (2)$$

The presence of massless particles propagating in the loop implies that the integrand  $A$  goes to a constant  $A_0$  as  $\tau_2 \rightarrow \infty$ , and the the integral over the Teichmüller

parameter diverges in the infrared. When the logarithmic divergence is regularized and compared to the field-theoretical  $\overline{DR}$  scheme, it is converted to  $\ln(M_{SU}^2/\mu^2)$ , where  $\mu$  is the infrared cutoff and  $M_{SU} \simeq 5 \times g \times 10^{17}$  GeV is the string unification scale.<sup>6</sup> The remaining finite part of the integral yields the moduli-dependent string threshold corrections:

$$\mathcal{A} = A_0 \ln \frac{M_{SU}^2}{\mu^2} + \int_{\Gamma} \frac{d^2\tau}{\tau_2} [A(\tau, \bar{\tau}) - A_0]. \quad (3)$$

When computing the moduli dependence of threshold corrections we use two main ingredients: 1)  $N = 1$  space-time supersymmetry to relate physical couplings to amplitudes involving pseudoscalars which receive contributions only from odd spin structures, and thus are much easier to compute. 2) The invariance of the superstring and its low energy effective theory under the discrete duality group. This generalizes the simple  $R \rightarrow 1/R$  duality, where  $R$  is the radius of a circle on which one internal dimension is compactified to the case of more general compactifications. For instance, in the case of a plane one can define two complex parameters, each of them generating an  $SL(2, Z)$  group of duality transformations  $T \rightarrow 1/T$  and  $T \rightarrow T + i$ .

### 3. Gauge couplings

The moduli dependence of threshold corrections to gauge couplings can be determined in a way that circumvents the problem of infrared divergences. It is based on the observation that the  $\beta$ -function which is the coefficient of the infrared divergence is moduli independent and drops when one considers derivatives with respect to the moduli. Consider for instance the 3-point scattering amplitude involving one modulus  $T$  and two gauge bosons. As a consequence of supersymmetry, the  $CP$ -even and the  $CP$ -odd part of this amplitude are proportional,<sup>2</sup> implying:

$$\partial_T g^{-2}(T, \bar{T}) = i\Theta_T, \quad (4)$$

where  $\Theta_T$  is the axionic coupling of the pseudoscalar component of  $T$  to gauge bosons. If  $\Theta_T$  is the derivative with respect to  $T$  of a  $\Theta$ -angle which is a function of moduli, Eq. (4) implies that gauge couplings and the corresponding  $\Theta$ -angles are obtained as the real and the imaginary part, respectively, of analytic gauge functions  $f$ , consistently with the general form of  $N = 1$  supergravity.<sup>9</sup> However, due to the integration over massless particles in the physical amplitude,  $\Theta_T$  may not in general be integrable and a useful relation to examine is the integrability condition:

$$\partial_T \partial_{\bar{T}} g^{-2}(T, \bar{T}) = \frac{i}{2} (\partial_{\bar{T}} \Theta_T - \partial_T \Theta_{\bar{T}}) \stackrel{?}{=} 0. \quad (5)$$

Explicit one loop calculation in orbifolds shows that the integrability condition Eq. (5) is indeed violated and the full moduli dependence of threshold corrections to gauge couplings can be computed.<sup>2</sup> At higher loops, this dependence

satisfies a non-renormalization theorem implying that the one-loop result is exact.<sup>3</sup> For general compactifications, a general formula can be derived by integrating the one-loop expression of the axionic coupling  $\Theta_T$  in Eq. (4):<sup>4,5</sup>

$$\frac{1}{g^2} = -\frac{i}{32\pi^2} \int_{\Gamma} \frac{d^2\tau}{\tau_2} \bar{\eta}^{-2} \text{Tr}_R F(-1)^F \left(Q^2 - \frac{k}{4\pi\tau_2}\right), \quad (6)$$

where  $\eta$  is the Dedekind eta function,  $Q$  is the gauge group generator, and the trace is over the Ramond sector of the internal  $N = 2$  superconformal theory with  $U(1)$ -charge operator  $F$ . Note that the integral in Eq. (6) is infrared divergent. The coefficient of the logarithmic divergence  $\frac{d\tau_2}{\tau_2}$  is:

$$b = \frac{1}{32\pi^2} (-3\text{Tr}Q_V^2 + \text{Tr}Q_M^2), \quad (7)$$

where the two terms are the contribution of gauginos and matter fermions with  $U(1)$ -charges  $F = \pm 3/2$  and  $F = \pm 1/2$ , respectively. The expression of  $b$  in Eq. (7) coincides with the field-theoretical one-loop  $\beta$ -function of gauge couplings in  $N = 1$  supersymmetric Yang-Mills theory. It is worth noting that the same quantity  $\text{Tr}F(-1)^F$  was studied in the massive case, where  $F = F_L - F_R$ , as a new kind of “index” for  $N = 2$  theories which depends only on chiral deformations.<sup>12</sup>

#### 4. Gravitational couplings

The above results for gauge couplings are extended for the case of gravitational couplings.<sup>4</sup> In this case, there is no moduli dependent correction to the Planck mass, at least up to the one loop order, and the role of gauge couplings and  $\Theta$ -angles is played by the coefficients of the Gauss-Bonnet combination and the  $R\tilde{R}$  term, respectively. Eq. (6) is then valid for the gravitational coupling after replacing the gauge group generator  $Q^2$  with

$$Q_{\text{grav}}^2 \equiv -\frac{1}{i\pi} \partial_{\bar{\tau}} \ln(\bar{\eta}^2). \quad (8)$$

The infrared divergence can now be identify with the four-dimensional one-loop trace anomaly:

$$b_{\text{grav}} = \frac{1}{32\pi^2} \left[ \frac{1}{6} (-3N_V + N_S) - \frac{11}{3} (-3 + N_{3/2}) \right], \quad (9)$$

where  $N_S$ ,  $N_V$  and  $N_{3/2}$  denote the number of chiral, vector and spin-3/2 (massless) supermultiplets, while the first term in the second bracket accounts for the contribution of graviton and dilaton supermultiplets. This expression agrees with the field-theoretical calculations of trace anomaly coefficients, where the antisymmetric tensor contribution is *different* from that of a scalar.<sup>13</sup>

#### 5. Integrability condition and duality anomalies

Going back to the integrability condition (5), using the expression of Eq. (6)

one finds:<sup>4,5</sup>

$$\begin{aligned} \partial_T \partial_{\bar{T}} g^{-2}(T, \bar{T}) &= \frac{-i}{2(2\pi)^4} \int_{\Gamma} d^2\tau \partial_{\tau} \int d^2\zeta \bar{\eta}^{-2} \langle Q^2 \Psi_T(\zeta) \bar{\Psi}_{\bar{T}}(0) \rangle_{\text{odd}} \\ &\quad + \frac{k}{8(2\pi)^5} \int_{\Gamma} \frac{d^2\tau}{\tau_2^2} \int d^2\zeta \bar{\eta}^{-2} \langle \Psi_T(\zeta) \bar{\Psi}_{\bar{T}}(0) \rangle_{\text{odd}}, \end{aligned} \quad (10)$$

where  $\Psi_T$  ( $\bar{\Psi}_{\bar{T}}$ ) are the corresponding chiral (anti-chiral) primary fields of the underlying  $N = 2$  internal superconformal theory with dimension  $(\frac{1}{2}, 1)$ . Eq. (10) gives the non-harmonicity of gauge couplings and contains two parts:

1. The group-dependent part proportional to  $Q^2$  is a total derivative in  $\tau$  and its contribution to the integral comes only from the boundary of the moduli space, namely the degeneration limit  $\tau_2 \rightarrow \infty$ . It is therefore determined only from massless particles. In fact, all massless chiral fermions couple off-shell with the pseudoscalars<sup>9</sup> and generate one-loop anomalous couplings of pseudoscalars to gauge bosons or gravitons which are non-local. A detailed analysis of this term<sup>4</sup> was shown to reproduce the field theory computation of the anomalous graphs.<sup>14,15</sup>
2. The universal part (*i.e.* the term proportional to  $k$ ) can be identified with the one-loop correction  $G_{T\bar{T}}^{(1)}$  to the Kähler metric. The corresponding one-loop contribution to the Kähler potential is:

$$K^{(1)} = -\ln\left[1 - \frac{2}{(S + \bar{S})} G^{(1)}(Z, \bar{Z})\right]. \quad (11)$$

$G^{(1)}(T, \bar{T})$  gives rise to one-loop kinetic terms that mix the moduli  $T$  with the dilaton  $S$  which then couples universally to gauge bosons and gravitons at the tree-level. This is usually called the Green-Schwarz term because it can be interpreted as the compactification of the ten-dimensional term involved in the Green-Schwarz anomaly cancellation mechanism.<sup>14</sup>

The non-harmonicity of gauge couplings is also related to duality anomalies.<sup>14,15</sup> In fact the first term in Eq. (10), associated with one-loop anomalous graphs, generates anomalies in the duality transformations at the effective field theory level. These are partially cancelled by the contribution of the massive string modes which produce *local* one-loop corrections to the gauge kinetic functions  $f^{(1)}(T)$ , obtained after integrating Eq. (10) to Eq. (6). There remains a gauge group-independent anomaly cancelled by the second term of Eq. (10), which plays a similar role as the Green-Schwarz term in the cancellation of ten-dimensional gauge anomalies.

## 6. One loop Kähler metric and Yukawa couplings

As a consequence of the supersymmetric non-renormalization theorems in the background field method, the superpotential does not receive any loop corrections.

However, the physical Yukawa couplings defined by the fermion-scalar-fermion scattering amplitudes may receive loop corrections which arise from the wave function renormalization factors, *i.e.* from the corrections to the Kähler metric. The computation described in the previous Sections 3-5 remains valid if one replaces the modulus  $T$  with any matter field which is singlet with respect to the gauge group under consideration. Thus, the one-loop correction to the Kähler metric is given by the second term of Eq. (10) and corresponds to the variation of a Kähler potential given by the second term of Eq. (6):

$$G^{(1)} = \frac{i}{16(2\pi)^3} \int_{\Gamma} \frac{d^2\tau}{\tau_2^2} \bar{\eta}^{-2} \text{Tr}_R F(-1)^F. \quad (12)$$

This can be checked independently by an alternative computation of the one-loop three-point amplitude involving two complex scalars and the antisymmetric tensor field.<sup>5</sup> The result is:

$$G_{ij}^{(1)} = \frac{1}{8(2\pi)^5} \int_{\Gamma} \frac{d^2\tau}{\tau_2^2} \int d^2\zeta \bar{\eta}(\bar{\tau})^{-2} \langle \Psi_i(\zeta) \bar{\Psi}_j(0) \rangle_{\text{odd}}, \quad (13)$$

in agreement with the field-theoretical expression from Eq. (10).

In the case of matter metric, the integral over the Teichmüller parameter  $\tau$  in Eq. (13) is infrared divergent. As in the case of gauge couplings, these divergences are due to massless particles propagating in the loop. The coefficients of divergent terms correspond to the one-loop anomalous dimensions:<sup>5</sup>

$$\gamma_i = -\frac{1}{64\pi^2} \left\{ -\frac{4}{k} C_2(R_i) + \frac{1}{g^2} \sum_{j,k} |\lambda_{ijk}|^2 \right\}, \quad (14)$$

where  $C_2(R_i)$  is the quadratic Casimir of the representation  $R_i$  to which the  $i$ -th field belongs, and  $\lambda_{ijk}$  are the physical Yukawa couplings. The comparison with the field-theoretical anomalous dimensions shows that the string computation implicitly uses a gauge in which the superpotential remains unrenormalized. Again as in the case of gauge couplings, the momentum-dependence of the physical Yukawa couplings in string theory turns out to be determined by the corresponding field-theoretical  $\beta$ -functions.<sup>16</sup> The remaining finite part of  $G_{ij}^{(1)}$  gives the string threshold corrections to wave function factors. These corrections determine the boundary conditions for the physical Yukawa couplings  $\lambda_{ijk}$  at the unification scale:

$$\lambda_{ijk}(M_{SU}) = \lambda_{ijk}^{\text{tree}} [1 + g^2 (Y_i + Y_j + Y_k)]^{-1/2}, \quad (15)$$

where  $Y_i$  is defined as the finite part of  $G_{ii}^{(1)}/G_{ii}^{(0)}$ .

## 7. Explicit examples in orbifold models

As an example we will consider the symmetric orbifold models. These are a particular case of  $(2, 2)$  compactifications, which possess  $N = 2$  world-sheet supersymmetry in both left and right moving sectors. The gauge group is  $E_8 \otimes E_6 \otimes H_2$ ,

with all Kač-Moody levels  $k = 1$  and with  $H_2$  a model dependent factor of rank-2. The relevant matter fields transform as  $27$  or  $\overline{27}$  under  $E_6$  and they are in one-to-one correspondence with the moduli :  $27$ 's are related to  $(1, 1)$  moduli and  $\overline{27}$ 's to  $(1, 2)$  moduli. Furthermore, the moduli metric is block-diagonal with respect to these two types of moduli. An interesting consequence of the right-moving  $N = 2$  tree-level Ward-identities is the property of special geometry, which relates the tree-level moduli metric to the Yukawa couplings.<sup>1,10</sup>

Here, we mainly focus our attention on the case of the three untwisted moduli  $T_j$ ,  $j = 1, 2, 3$ , describing the size of the three internal compactified planes, which are in one-to-one correspondence with three untwisted families  $A_j$ . In this case, the superpotential and the tree-level Kähler potential are:

$$W = w_{123}A_1A_2A_3, \quad G^{(0)} = -\sum_{j=1}^3 \ln(T_j + \overline{T}_j - A_j\overline{A}_j), \quad (16)$$

where  $w_{123}$  are numerical constants. The tree-level physical Yukawa couplings are then  $\lambda_{123}^{\text{tree}} = \frac{g}{\sqrt{2}}w_{123}$ .

At the one-loop level, one has to sum over all sectors of boundary conditions of the orbifold group. The untwisted sector, which preserves  $N = 4$  space-time supersymmetry, gives vanishing contribution to all  $\beta$ -functions and threshold corrections. The twisted sector preserving only  $N = 1$  supersymmetry leads to moduli-independent corrections, since the masses of the corresponding string states do not depend on the geometry of the compactified space. Therefore, the gauge and untwisted Yukawa couplings receive moduli-dependent corrections only from sectors that preserve  $N = 2$  space-time supersymmetry, which appear when one of the three internal planes remains invariant under the boundary conditions. The variation  $\partial_T\partial_{\overline{T}}g_a^{-2}$  can be easily evaluated from Eq. (10) with the result:<sup>2,3</sup>

$$\partial_T\partial_{\overline{T}}g_a^{-2} = \tilde{b}_a G_{T\overline{T}}^{(0)}, \quad G_{T\overline{T}}^{(0)} = \frac{1}{(T + \overline{T})^2}, \quad (17)$$

where the coefficient  $\tilde{b}_a$  is proportional to the  $\beta$ -function of the gauge group factor  $G_a$  in the corresponding  $N = 2$  theory. It can be written as a sum of two terms,

$$\tilde{b}_a = \tilde{b}_a^{\text{FT}} + \tilde{b}^{\text{GS}}, \quad (18)$$

where  $\tilde{b}_a^{\text{FT}}$  is the field-theoretical group-dependent contribution and  $\tilde{b}^{\text{GS}}$  is the universal Green-Schwarz term, in correspondence with the two terms of Eq. (10). Explicit evaluation<sup>5</sup> of the second integral shows that  $\tilde{b}^{\text{GS}}$  does indeed subtract the contribution of the  $N = 1$  sector from the field-theoretical coefficient, so that  $\tilde{b}_a$  is given entirely by  $N = 2$  sectors. The same equation (17) is also valid for the gravitational couplings with a coefficient  $\tilde{b}_{\text{grav}}$  proportional to the trace anomaly of the corresponding  $N = 2$  theory.<sup>4</sup>



After integrating the differential equation (17) using the  $SL(2, Z)$  duality symmetry one finds:

$$\begin{aligned} \frac{1}{g_a^2(M_{SU})} &= \frac{1}{g^2} - \tilde{b}_a \ln[|\eta(iT)|^4(T + \bar{T})] + c_a \\ &+ \frac{B\bar{B}}{T + \bar{T}} \left[ \frac{1}{2} C_2(G_a) + \mathcal{T}(27) \left( 1 + \frac{\chi}{6} - \frac{2}{g^2} \sum_{k,l} |\lambda_{Bkl}|^2 \right) \right] + \dots, \end{aligned} \quad (19)$$

where  $c_a$  are moduli-independent constants and we also included the contribution of the blowing-up moduli  $B$  at the lowest order.<sup>4</sup> The latter deform orbifolds to smooth Calabi-Yau manifolds and are in one-to-one correspondence with twisted families.  $\chi$  is the number of generations given by the Euler number, and  $\mathcal{T}(27)$  is the Dynkin index of the matter representation which equals 3 and 0 for  $E_6$  and  $E_8$ , respectively.

The one-loop Kähler metric and threshold corrections to Yukawa couplings can be explicitly computed from Eqs. (13) and (15). For the untwisted moduli one obtains  $G_{T\bar{T}}^{(1)} = \tilde{b}^{\text{GS}} G_{T\bar{T}}^{(0)}$ , where  $\tilde{b}^{\text{GS}}$  was defined in Eq. (18). A consequence of this multiplicative finite renormalization of the moduli metric is that special geometry is in general violated beyond the tree approximation. This is expected since from field-theoretical point of view special geometry is a consequence of  $N = 2$  space-time supersymmetry and  $\tilde{b}^{\text{GS}}$  is non vanishing only in  $N = 1$  sectors.<sup>5</sup>

The wave function threshold corrections of an untwisted field  $A_j$  (27 or  $\bar{27}$ ) associated with the  $j$ -th plane depends only on its moduli  $T_j$ , and *not* on the moduli of other planes:<sup>5</sup>

$$Y_j = 2\tilde{\gamma}_j \ln[|\eta(iT_j)|^4(T_j + \bar{T}_j)] + y_j, \quad (20)$$

where  $y_j$  is a moduli-independent constant and the coefficient  $\tilde{\gamma}_j$  is the anomalous dimension of the  $A_j$ -field in the corresponding  $N = 2$  supersymmetric theory. Since this field belongs to an  $N = 2$  vector supermultiplet, one has  $\tilde{\gamma}_j = -\tilde{b}_j/2$ , where  $\tilde{b}_j/2$  is the corresponding beta function coefficient of any gauge subgroup that transforms  $A_j$  non-trivially in the embedding  $N = 2$  theory. The threshold corrections to Yukawa couplings can be computed by substituting Eq. (20) into Eq. (15). The boundary conditions for the one-loop untwisted Yukawa couplings are then given by:

$$\lambda_{123}(M_{SU}) = \frac{g_{E_6}(M_{SU})}{\sqrt{2}} w_{123} [1 + g_{E_6}^2(M_{SU}) y_{123}]^{-1/2}, \quad (21)$$

where  $y_{123}$  are moduli-independent constants, and  $g_{E_6}(M_{SU})$  is the one-loop  $E_6$  gauge coupling at the unification scale, *cf.* Eq. (19):

$$\frac{1}{g_{E_6}^2(M_{SU})} = \frac{1}{g^2} - \sum_{j=1}^3 \tilde{b}_{E_6}^j \ln[|\eta(iT_j)|^4(T_j + \bar{T}_j)] + c_{E_6}. \quad (22)$$

As a result, the boundary relation between the untwisted Yukawa couplings and the  $E_6$  gauge coupling at the unification scale does not receive any moduli-dependent

corrections at the one-loop level.

## 8. Conclusions

Superstring theory provides a unique example of a perfectly consistent unification of gauge and gravitational interactions, within the framework of local supersymmetry. The efforts described here have been motivated by the desire to understand the low-energy limit of such a consistent theory. As a result, we have now a very good understanding of the low-energy physics of supergravity theory that describes not only the classical limit of string theory, but also some interesting string loop effects, in particular the threshold corrections. From the three functions which determine the effective  $N = 1$  supergravity, the two analytic ones are subject to non-renormalization theorems. The superpotential is given entirely at the tree-level, while the gauge kinetic function at the one loop. On the other hand, the one-loop correction to the real Kähler potential is closely related to the new “index” of  $N = 2$  superconformal theories. The infrared divergences were identified with the field-theoretical  $\beta$ -functions of gauge and Yukawa couplings, while the finite part define the moduli-dependent threshold corrections which are calculable in any given model. Here, we presented explicit examples in the case of orbifolds. In addition to possible applications for low-energy physics, this “bottom-up” approach to superstring theory may well provide some new insights into the Planck-scale physics.

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