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EVOLUTION AND TUNNELING TIME OF WAVE PACKETS IN A SUPERLATTICE

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Abstract

A formulation of the evolution of a wave packet inside and outside a scattering superlattice is presented. Time series of specific Gaussian packets, centered at an arbitrary energy E_o, exhibit interesting back-scattering, trapping and transmission effects. These effects depend on whether the energy E_o is in a gap, resonance, or an arbitrary point in a band of the superlattice. The time evolution depends strongly on the transmission coefficient and the superlattice tunneling time. It is shown that the back-scattered wave could provide much more information than the transmitted wave on the observation of tunneling time.

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I. INTRODUCTION

The evolution of massive and non-massive particles, inside and outside a potential or scattering region, dealing with both quantum and electromagnetic systems has been a problem of great and relevant interest in the scattering theory[1,2]. The distortion of a wave packet is the most evident and at the same time the less understood effect. In quantum systems the reflected and transmitted wave packet components depend in a very precise way on the tunneling probability amplitudes and on the phase coherence and interference phenomena. The calculation of this fundamental propertY requires, in the theoretical description, either appropriate or powerful methods to solve directly or indirectly the wave equations. In general, most if not all of the existing approaches study the outgoing wave functions in the asymptotic regions. Here we are interested in describing the time evolution of wave packets both outside and inside the scattering potential region. The interest in studying the scattering amplitudes in the asymptotic limit is suggested by the experimental set up. The fact that the calculated scattering amplitudes are precisely in the asymptotic region is actually not due to intrinsic limitations of the scattering approach, as is generally believed, but is a consequence of the need to account for physical quantities and results, determined precisely in that region. Besides the topological and geometrical properties of the scattering process, the lack of detailed information on the scattering potential forces one to study this field in rather general terms without much reference or details to the specific potential function. The quantum analog of classical particle motion is a complex process that can be understood only in terms of very precise analytical calculations that will allow a rigorous and exhaustive analysis of the overwhelming variety of possible effects. As will be seen in the present work, the evolution and the way in which a wave packet gets distorted by a given scatterer system is an extremely sensitive effect and absolutely dependS on all the physical parameters characterizing the scattering process. When the scattering region has a known potential shape, or the refraction properties are known, it is possible, in principle, to follow the dynamics of the transmitted and reflected components of the wave packets when the theory is able to determine the transmission and reflection amplitudes as well as the wave functions along the scatterer system, for any value of the incoming particle energy, i.e. for each k-component of the wave packet. This has been shown to be possible for an important class of systems: *the locally periodic systems.* The general results of the theory of finite periodic systems (TFPS) provide analytical expressions for the calculation of scattering amplitudes, through an arbitrary periodic region. We can also determine precisely resonance energies in the bands[3]. Knowing these properties of a superlattice (SL) we are now able to study their effects on the dispersion of a wave packet. Among these effects, we mention the complete or partial back-scattering and the trapping of a wave packet for the resonance energies. Some particular examples will be chosen in such a way that the energy is in a gap, a resonance energy $E_{\mu,\nu}$ or at an arbitrary energy in an allowed band. Here μ is the label for a band and ν is the label for the intraband state. In each case it will be interesting to establish the relation with the transmission and reflection amplitudes, and also to understand the formation of the stationary pattern and its persistance in connection with the corresponding tunneling time, which issue has been revived lately both theoretically[4–10] and experimentally[11–13]. The aim of this work is to determine the time evolution on a Gaussian wave packet, centered at an arbitrary k_o, when it approaches and gets scattered by a locally periodic potential. The case of scattering by a rectangular barrier was studied in reference [14]. We shall present here explicit derivations of the principal theoretical expressions that will then be applied to a number of particular cases. In Section II we start with the description of a stationary wave function. In section III we present the evolution of a Gaussian wave packet in all regions of the SL (outside and inside). In section IV we apply our formulation for specific cases which will be continued with a discussion on the results.

II. STATIONARY WAVE FUNCTION

Before studying the time-dependent wave function, we shall first consider the stationary state of the wave function in a superlattice by using the transfer matrix method. Let us consider a finite periodic superlattice with n cells where each cell has length l_c. We represent the superlattice by having a square potential barrier of height V_o and width b located in the middle of each cell, while the remaining part of the cell has zero potential of length $a/2$ on the left and right of the barrier so that $l_c = a + b$; see figure 1. We take the origin $z = 0$ at the left corner of the superlattice. In the formalism of transfer matrix, the wave function is represented by a state vector as

$$\hat{\psi}(z) \;=\; \begin{pmatrix} \psi_a(z) \\ \psi_b(z) \end{pmatrix} \tag{1}$$

where $\psi_a(z)$ and $\psi_b(z)$ are the right-moving (or increasing) and left-moving (or decreasing) wave functions. The state vector $\hat{\psi}(z)$ at an arbitrary point z can be related to the one at an arbitrary point z' by

$$\hat{\psi}(z) \;=\; T(z,z')\hat{\psi}(z') \tag{2}$$

where $T(z,z')$ is the transfer matrix of the form

$$T(z,z') \;=\; \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}. \tag{3}$$

Here, in the one-propagating mode approximation, α ,β ,γ ,δ are complex functions of the potential parameters and of z, z'. With the multiplicative property of the transfer matrix, the state vector $\psi_{j+1}(z)$ at any point z in cell $j+1$ of the superlattice can then be written as

$$\hat{\psi}_{j+1}(z) \; = \; T(z, jl_c)\, [T(l_c, 0)]^j \, \hat{\psi}_1(0) \; , \qquad j = 0, 1, \ldots, (n-1) \tag{4}$$

where $T(z, jl_c)$ is the transfer matrix from jl_c to z in cell $j+1$. $T(l_c, 0)$ is the transfer matrix for one cell and $[T(l_c, 0)]^j$ is the transfer matrix for j cells. For square barrier potentials, the transfer matrix $T(l_c, 0)$ of a cell is given by

$$T(l_c, 0) \; \equiv \; \begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix} \; = \; \begin{pmatrix} \alpha_b e^{ika} & \beta_b \\ \beta_b^* & \alpha_b^* e^{-ika} \end{pmatrix} \tag{5}$$

where the elements $\alpha = \alpha_b e^{ika}$ and $\beta = \beta_b$ are given as

$$\alpha_b = \cosh(\kappa b) - \frac{i}{2}\left(\frac{\kappa}{k} - \frac{k}{\kappa}\right) \sinh(\kappa b) \tag{6}$$

$$\beta_b = -\frac{i}{2}\left(\frac{\kappa}{k} + \frac{k}{\kappa}\right) \sinh(\kappa b) \tag{7}$$

with

$$k \; = \; \sqrt{\frac{2mE}{\hbar^2}} \; , \qquad \qquad \kappa \; = \; \sqrt{\frac{2m(V_o - E)}{\hbar^2}} \; . \tag{8}$$

The transfer matrix $[T(l_c, 0)]^j$ for j full cells is given by

$$T^j(l_c, 0) \equiv \; [T(l_c, 0)]^j = \begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix}^j \; \equiv \; \begin{pmatrix} \alpha_j & \beta_j \\ \beta_j^* & \alpha_j^* \end{pmatrix} \tag{9}$$

which elements α_j and β_j were shown in reference [3] to be

$$\alpha_j \; = \; U_j(\alpha_R) - \alpha^* U_{j-1}(\alpha_R) \; , \qquad \beta_j \; = \; \beta U_{j-1}(\alpha_R). \tag{10}$$

$U_j(\alpha_R)$ is the Chebyshev polynomial of the second kind[15] evaluated at α_R, which is the real part of α, i.e., $\alpha = \alpha_R + i\alpha_I$.

Consider now the situation with an incoming wave from the left of the superlattice so that $\psi(z)$ at $z < a/2$ is given by

$$\psi(z) = Ae^{ikz} + Be^{-ikz}. \tag{11}$$

With the condition of outgoing (transmitted) wave moving only to the right, and by using $\hat{\psi}_n(z) = T^n(z, 0)\hat{\psi}_1(0)$, we find that the amplitude $r_n(k)$ of reflection for the whole superlattice is

$$r_n(k) \equiv \frac{B}{A} = -\frac{\beta_n^*}{\alpha_n^*} \, . \tag{12}$$

Substituting Eq. (12) into (((to))) Eq. (4), we have the state vector $\hat{\psi}_{j+1}(z)$ in cell $(j+1)$ as

$$\hat{\psi}_{j+1}(z) = A \begin{pmatrix} \alpha_p \left(\alpha_j - \beta_j \frac{\beta_n^*}{\alpha_n^*}\right) + \beta_p \left(\beta_j^* - \alpha_j^* \frac{\beta_n^*}{\alpha_n^*}\right) \\ \\ \gamma_p \left(\alpha_j - \beta_j \frac{\beta_n^*}{\alpha_n^*}\right) + \delta_p \left(\beta_j^* - \alpha_j^* \frac{\beta_n^*}{\alpha_n^*}\right) \end{pmatrix} \equiv A \begin{pmatrix} \psi_{j+1,a}(z) \\ \\ \psi_{j+1,b}(z) \end{pmatrix}. \tag{13}$$

where α_p ,β_p ,γ_p ,δ_p are the elements of $T(z, jl_c)$, i.e.,

$$T(z, jl_c) = \begin{pmatrix} \alpha_p & \beta_p \\ \\ \gamma_p & \delta_p \end{pmatrix}. \tag{14}$$

The actual wave stationary function in cell $(j+1)$ is then[16]

$$\psi_{j+1}(z) = A \left[(\alpha_p + \gamma_p) \left(\alpha_j - \beta_j \frac{\beta_n^*}{\alpha_n^*}\right) + (\beta_p + \delta_p) \left(\beta_j^* - \alpha_j^* \frac{\beta_n^*}{\alpha_n^*}\right) \right]. \tag{15}$$

In this function, the components $\psi_{j+1,a}(z)$ and $\psi_{j+1,b}(z)$ (clearly expressed in Eq. (13)) are mixed just for compactness.

To illustrate, we have plotted in figure 2 the electron wave function in the superlattice GaAs($Al_{0.3}Ga_{0.7}As$/GaAs)[6]. In this case, the number of cells is $n = 6$, $a = 50$ Å and $b = 30$ Å so that $l_c = 130$ Å. For the figure, we have used the barrier height as $V_o = 0.23$ eV and the energy of the electron as $E = E_{2,4} = 0.1318$ eV, which is the fourth resonance in the second band.

III. EVOLUTION OF A WAVE PACKET

Having found the stationary state, we now study the evolution of a wave packet as a function of time t. For clarity of the natural physical differences, we will divide the presentation in three characteristic scattering regionS, ouside (the left- and right-hand side of the superlattice) and inside the superlattice.

A. The reflected wave packet

We assume the packet to be located at some position $z_o < a/2$ at an arbitrary time t_o that will be taken as $t_o = 0$. With the possibility of having a reflected wave the total wave function for $z < a/2$ will be

$$\psi(z, t = 0) = \int dk\; g(k) \left[e^{ik(z+z_o)} + |r_n(k)| e^{-i[k(z-z_o)+\theta_r(k)]} \right] \tag{16}$$

where

$$r_n(k) \equiv |r_n(k)| e^{-i\theta_r(k)}\,, \qquad \text{and} \qquad \theta_r(k) = \frac{\pi}{2} - \tan^{-1}\left(\frac{\alpha_I U_{n-1}(\alpha_R)}{U_n(\alpha_R) - \alpha_R U_{n-1}(\alpha_R)} \right). \tag{17}$$

For a concrete study, we take the wave packet to be Gaussian distributed and centered at $k = k_o$ (with energy E_o), i.e.,

$$g(k) = e^{-\gamma(k-k_o)^2}. \tag{18}$$

At any time $t > 0$, the wave packet on the left-hand side of the superlattice ($z < a/2$) is given by

$$\psi(z,t) = \int dk\, e^{-\gamma(k-k_o)^2} \left[e^{ik(z+z_o)} + |r_n(k)| e^{-i[k(z-z_o)+\theta_r(k)]}\right] e^{-i\omega(k)t} \tag{19}$$

where $\omega(k) = \hbar k^2/(2m)$. Performing the Gaussian integration, and using

$$v_g \equiv \left(\frac{d\omega(k)}{dk}\right)_{k_o} = \frac{\hbar k_o}{m}\,, \qquad \zeta \equiv \frac{1}{2}\left(\frac{d^2\omega(k)}{dk^2}\right)_{k_o} = \frac{\hbar}{2m} \tag{20}$$

we obtain, for $z < a/2$, the wave function

$$\begin{aligned}\psi(z,t) \approx & \left(\frac{\pi}{\gamma + i\zeta t}\right)^{1/2} \exp\left[-\frac{(z+z_o-v_g t)^2}{4(\gamma+i\zeta t)}\right] e^{i[k_o(z+z_o)-\omega(k_o)t]} + \\ & + \left(\frac{\pi}{\gamma + i\zeta t + \frac{i}{2}\frac{d^2\theta_r(k_o)}{dk^2}}\right)^{1/2} |r_n(k_o)| \exp\left[-\frac{\left(z - z_o + v_g t + \frac{d\theta_r(k_o)}{dk}\right)^2}{4\left(\gamma + i\zeta t + \frac{i}{2}\frac{d^2\theta_r(k_o)}{dk^2}\right)}\right] \times \\ & \times e^{-i[k_o(z-z_o)+\omega(k_o)t+\theta_r(k_o)]}\,.\end{aligned} \tag{21}$$

Here, the first term represents the incoming wave centered at $z = -z_o + v_g t$, while the second term represents the reflected wave centered at $z = z_o - v_g t - (d\theta_r(k)/dk)_{k_o}$. Therefore, there is a time delay $\tau(k_o)$ for the back-scattered component that is given by

$$\tau(k_o) = -\frac{1}{v_g}\left[a + \left(\frac{d\theta_r(k)}{dk}\right)_{k_o}\right]. \tag{22}$$

Notice that τ is a phase time[17,11]. It will be seen below that this is the same as the superlattice tunneling time[10]. In figure 3, we have plotted the time delay $\tau(k)$ as a function of the energy E of an electron in a superlattice of GaAs($Al_{0.3}Ga_{0.7}As$/GaAs)[6] where the number of cells is $n = 6$, $a = 50$ Å and $b = 30$ Å (i.e., $l_c = 130$ Å) and the barrier height is $V_o = 0.23$ eV.

B. Wave packet inside the superlattice

Let us now consider the time-dependent wave packet inside the superlattice. With the stationary wave function $\psi_{j+1}(z)$ in Eq. (15), the time dependent wave packet $\psi_{j+1}(z,t)$ becomes

$$\psi_{j+1}(z,t) = \int dk\; g(k)\,[\psi_{j+1,a}(z,k) + \psi_{j+1,b}(z,k)]\, e^{i[kz_o - \omega(k)t]} \tag{23}$$

where we have defined $\psi_{j+1,a}(z,k) \equiv \psi_{j+1,a}(z)$ and $\psi_{j+1,b}(z,k) \equiv \psi_{j+1,b}(z)$. Performing the Gaussian integration as before, we get

$$\psi_{j+1}(z,t) \approx \left(\frac{\pi}{\gamma + i\zeta t - \frac{i}{2}\frac{d^2\theta_{j+1,a}(k_o)}{dk^2}}\right)^{1/2} |\psi_{j+1,a}(z,k_o)| \exp\left[-\frac{\left(v_g t - z_o - \frac{d\theta_{j+1,a}(k_o)}{dk}\right)^2}{4\left(\gamma + i\zeta t - \frac{i}{2}\frac{d^2\theta_{j+1,a}(k_o)}{dk^2}\right)}\right]$$
$$\times e^{ik_o z_o} e^{-i[\omega(k_o)t - \theta_{j+1,a}(k_o)]}$$
$$+ \left(\frac{\pi}{\gamma + i\zeta t - \frac{i}{2}\frac{d^2\theta_{j+1,b}(k_o)}{dk^2}}\right)^{1/2} |\psi_{j+1,b}(z,k_o)| \exp\left[-\frac{\left(v_g t - z_o - \frac{d\theta_{j+1,b}(k_o)}{dk}\right)^2}{4\left(\gamma + i\zeta t - \frac{i}{2}\frac{d^2\theta_{j+1,b}(k_o)}{dk^2}\right)}\right]$$
$$\times e^{ik_o z_o} e^{-i[\omega(k_o)t - \theta_{j+1,b}(k_o)]}. \tag{24}$$

Here, we have defined

$$\theta_{j+1,a}(k_o) \equiv \tan^{-1}\left(\frac{Re\ \psi_{j+1,a}(z,k_o)}{Im\ \psi_{j+1,a}(z,k_o)}\right), \qquad \theta_{j+1,b}(k_o) \equiv \tan^{-1}\left(\frac{Re\ \psi_{j+1,b}(z,k_o)}{Im\ \psi_{j+1,a}(z,k_o)}\right) \tag{25}$$

where Re and Im stand for the real and imaginary parts, respectively. The result in Eq. (24) certainly goes back to Eq. (21) for $j = 0$ and $z < a/2$.

C. The time-dependent transmitted wave packet

The transmitted wave packet is given by Eq. (24) with $j = n - 1$ and $z > [(n-1)l_c + \frac{a}{2} + b]$, which is

$$\psi_n(z,t) \approx \left(\frac{\pi}{\gamma + i\zeta t - \frac{i}{2}\frac{d^2\theta_t(k_o)}{dk^2}}\right)^{1/2} \frac{1}{|\alpha_n|} \exp\left[-\frac{\left(v_g t - z_o - [z - n\ell_c] - \frac{d\theta_t(k_o)}{dk}\right)^2}{4\left(\gamma + i\zeta t - \frac{i}{2}\frac{d^2\theta_t(k_o)}{dk^2}\right)}\right] \times$$
$$\times e^{ik_o z_o} e^{-i[\omega(k_o)t - k(z - n\ell_c) - \theta_t(k_o)]}, \qquad \text{for} \quad z > [(n-1)\ell_c + \frac{a}{2} + b]. \tag{26}$$

Here, $\alpha_n \equiv |\alpha_n| e^{i\theta_t}$, i.e.,

$$\theta_t = \tan^{-1}\left(\frac{\alpha_I U_{n-1}(\alpha_R)}{U_n(\alpha_R) - \alpha_R U_{n-1}(\alpha_R)}\right). \tag{27}$$

Equation (26) gives the center of $|\psi_n(z,t)|^2$ at the position $z = v_g t - z_o + nl_c - (d\theta_t/dk)_{k_o}$ so that it also gives tunneling time t_T for the wave packet through the superlattice as

$$t_T \;=\; -\frac{1}{v_g}\left[a - \frac{d\theta_t}{dk}\right] \;=\; \tau. \tag{28}$$

We see here that, as stated earlier, the tunneling time t_T is the same as the time delay τ for reflection in Eq. (22). Furthermore, we have now recovered the tunneling time derived in reference [10], which agreed extremely well with the experiments in references [11,12].

IV. WAVE PACKET DISPERSION AND DISCUSSION

We will apply our formulation to study the time and space evolution of a wave packet which gets scattered and moves through a multilayer GaAs($Al_{0.3}Ga_{0.7}As/GaAs)^6$ superlattice. We work in the effective mass approximation and take the electron effective mass $m_A^* = 0.067 m_e$ in GaAs layers and $m_B^* = 0.1 m_e$ in the $Al_{0.3}Ga_{0.7}As$ layers, with m_e the bare electron mass. We shall start considering the wave packet with its energy centroid lying in a gap. A particular case will be taken between the first and the second allowed energy bands (see figure 3). For $E = 0.075$ eV, the stationary wave function is shown in figure 4(a). It is clear that the particle prefers to be localized in the first cell. For this reason it is possible to distribute the wave packet near the SL without much distortion (see figure 4(b)). This is not the case for energies in the bands, as will be seen below. As time increases, the packet approaches the SL and gets back-scattered almost completely. Eventually, the packet will try to recover its previous Gaussian shape with intermediate steps with very strong phase interference. We notice from figures 4(c)-(f) that the probability distribution inside the SL resembles very much the shape of that of the stationary states.

We now move on to a resonance energy, which will be taken arbitrarily as the second resonance in the second band, i.e., $E_{2,2} = 0.1225$ eV. From figures 5(a) we observe that the stationary probability is symmetrically distributed around the center of the SL. Contrary to the earlier case, figure 5(b) shows that the Gaussian tail is distorted in the SL. This happens because the wave components with energies around $E_{2,2}$ are allowed to be transmitted and trapped in the SL with a probability distribution that again resembles the stationary distribution. When the main part of the packet enters the system (see figure 5(d)) the trapping gets more pronounced. As most of the non-resonant components have left the SL, the resonant component keeps trying to remain with the stationary distribution for a very long time which is of the order of the tunneling time and of the decay time $\Delta t \sim \hbar/\Delta E_{\mu,\nu}$ with $\Delta E_{\mu,\nu}$ the resonance width.

Another characteristic example will be an energy around a local minimum in a band, taken arbitrarily as $E = 0.1303$ eV which lies between $E_{2,3}$ and $E_{2,4}$. a stationary wave function is an extented asymmetric function. In this case, as is in the previous ones, the tail inside the SL is distorted and the local probability distribution tends to follow the pattern of the stationary one. This type of response of the SL is also present in the previous cases showing a kind of universal behavior. For a wave packet with narrow width in energy, the centroid energy component dominates and contributes the most to the pattern of the trapped distribution. It is remarkable that the centroid stationary wave function stays for a very long time, as seen in figures 6(c)-(e). While in the case of resonance energy the persistance is due to the long tunneling time, in this case it is due to the small transmission coefficient.

Although scattering processes are very complicated, the procedure presented here allows us to follow all details of the process clearly.

The ratio of the amount of flux leaving the SL to the right and to the left depends on the ratio of the transmission and reflection coefficients for a particle with energy in the centroid of the wave packet.

V. CONCLUSIONS

We have deduced a general expression for Gaussian wave packets moving through a periodic potential of arbitrary shape, that has been applied to rectangular potentials with arbitrary number n of cells. We have derived the tunneling time, defined as the phase time. It is shown that for a wave packet with its centroid energy E_o in a gap, the back-scattered wave will provide much more information on the tunneling time. To summarize, we have studied the evolution of a Gaussian wave packet in a superlattice of an arbitrary number of cells.

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FIGURES

FIG. 1. A superlattice with n cells and square-barrier periodic potential with periodicity $l_c = a + b$.

FIG. 2. The absolute value of the stationary wave function, $|\psi_{j+1}(z)|$ (in arbitrary units), of an electron with energy $E = E_{2,4} = 0.1381$ eV, in a superlattice of $GaAs(Al_{0.3}Ga_{0.7}As/GaAs)^6$ with $n = 6$ and $l_c = 130$ Å.

FIG. 3. The time delay $\tau(k)$ as a function of energy E of an electron in a $GaAs(Al_{0.3}Ga_{0.7}As/GaAs)^6$ superlatice with $n = 6$ and $l_c = 130$ Å.

FIG. 4. The time-series of wave packets with centroid energy E_o in a gap of $GaAs(Al_{0.3}Ga_{0.7}As/GaAs)^6$. $\gamma = 12,133$ Å^2. In (a) the absolute value of the stationary state (in arbitrary units) for an electron with energy $E_f = 0.075$ eV. In (b) the approaching wave packet centered at $z_o = -3l_c = -390$ Å. In (c) the wave packet which center has not yet reached the superlattice. The tail of the packet starts building the stationary partern of the E_f-component. The wave packet when the center just reaches the first barrier at $a/2$ is shown in (d). After a time delay $\tau = 6.33 \times 10^{-15}$s the E_f-component of the wave packet gets reflected as shown in (e). In (f) the reflected wave packet has its center back to z_o. Compare with (b).

FIG. 5. The time-series for wave packets with E_o in a resonance. $\gamma = 185,686.8$ Å^2. In (a) the absolute value of the stationary state (in arbitrary units) for an electron with energy $E_{2,2} = 0.1225$ eV. In (b) the approaching wave packet is centered at $z_o = -l_c = -130$ Å. Notice that the right-hand side tail is already distorted. In (d) most of of the wave packet is already inside the superlattice. The trapped $E_{2,2}$-component, with negligible reflection, can be noticed in (e), (f). In the last figure this component remains after a very long time compared to $\tau = 9.54 \times 10^{-13}$ s).

FIG. 6. The time-series for wave packets with centroid energy E_o around a local minimum in a band. $\gamma = 6,983.75$ Å^2. In (a) the absolute value of the stationary state (in arbitrary units) for an electron with energy $E_{lm} = 0.1303$ eV. In (b) the approaching wave packet centered at $z_o = -l_c = -130$ Å. In (c) most of the wave packet is already inside the superlattice. The trapped $E_{2,2}$-component, with negligible reflection, can be noticed in (d), (e). In the last figure this component remains after a very long time compared to $\tau = 5.04 \times 10^{-13}$ s).