Available at: http://www.ictp.trieste.it/~pub_off IC/2001/93

United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency

THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

FINITE SIZE SCALING IN DISORDERED SYSTEMS: MEAN-FIELD ANALYSIS AND SELF-AVERAGING

H. Chamati
G. Nadjakov Institute of Solid State Physics, Bulgarian Academy of Sciences, 1784 Sofia, Bulgaria,

E. Korutcheva[1]
G. Nadjakov Institute of Solid State Physics, Bulgarian Academy of Sciences, 1784 Sofia, Bulgaria,[2]
Dep. Física Fundamental, Universidad Nacional de Educación a Distancia, c/ Senda del Rey No 9 , 28080 Madrid, Spain[3]
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy

and

N.S. Tonchev[4]
G. Nadjakov Institute of Solid State Physics, Bulgarian Academy of Sciences, 1784 Sofia, Bulgaria
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.

MIRAMARE – TRIESTE

August 2001

[1] Regular Associate of the Abdus Salam ICTP.
[2] Permanent address.
[3] Current address.
[4] Senior Associate of the Abdus Salam ICTP. E-mail: tonchev@issp.bas.bg

Abstract

The critical behavior of a quenched random hypercubic sample of linear size L is considered, within the "random-T_c" field-theoretical model and the mean-field approximation. A finite-size scaling behavior is established and analyzed and the problem of self-averaging is clarified for different critical regimes.

I. INTRODUCTION

During the last few decades, the description of effects of disorder on the critical behavior of finite-size systems has attracted a lot of interest [1–8]. Up to now the discussion takes place of whether the introduced disorder influences the finite-size scaling (FSS) results [3,8], compared to the standard ones, known for pure systems [9–11]. A formulation of general FSS concepts for the case of disorder is strongly complicated due to the additional averaging over the different random samples. For a random sample with volume L^d, where L is a linear dimension, any observable property X, singular in the thermodynamic limit, has different values for different realizations of the randomness and can be considered as stochastic variable with mean $\overline{X}$ and variance $(\Delta X)^2 := \overline{X^2} - \overline{X}^2$, where the over line indicates an average over all realizations of the randomness. Here, an important theoretical problem of interest is related with the property of self-averaging (SA) [12]. If the system does not exhibit SA a measurement performed on a single sample does not give a meaningful result and must be repeated on many samples. A numerical study of such a system will also be quite difficult. This point has been studied recently by means of FSS arguments [1,4], renormalization group (RG) analysis [2,5] and Monte Carlo simulations [4,6]. Quantities of physical interest are the Binders cumulant B and the relative variance $R_X(L) := (\Delta X)^2/\overline{X}^2$. A system is said to exhibit "strong SA" if $R_X(L) \sim L^{-d}$ as $L \to \infty$. This is the case if the system is away from criticality, i.e. if $L \gg \xi$. At the criticality, i.e. when $L \ll \xi$, the situation depends upon whether the randomness is irrelevant ($\nu_{pure} > 2/d$, e.g. "pure", P-case) or relevant ($\nu_{random} > 2/d$, e.g. "random" R-case) [2]. One calls the former case "weak SA", since $R_X(L) \sim L^{(\alpha/\nu)_{pure}}$, and the latter case "no SA", since $R_X(L)$ is fixed nonzero universal quantity even in the thermodynamic limit [2].

In the present paper we are analyzing the mean-field regime $d \geq 4$ of a N-component ($N \geq 1$) model of randomly diluted magnet with hypercubic geometry of linear size L and we are dealing with conditions for the presence or absence of a SA.

The paper is organized as follows. In Section II we define the model and the effective Hamiltonian. In Section III we perform the analysis in the zero-mode approximation. The analysis of the problem of SA is given in Section IV. Finally in Section V we present our main conclusions.

II. MODEL

We consider the "random - T_c" Ginzburg-Landau-Wilson model of disordered ferromagnets (see, e.g. [13–18])

$$\mathcal{H}_r = -\frac{1}{2}\int_{L^d} d^d x[t|\psi(\boldsymbol{x})|^2 + \varphi(\boldsymbol{x})|\boldsymbol{\psi}(\boldsymbol{x})|^2 + c|\nabla\psi(\boldsymbol{x})|^2 + \frac{u}{12}|\psi(\boldsymbol{x})|^4], \tag{2.1}$$

where $\psi(\boldsymbol{x})$ is a N-component field with $\psi^2(\boldsymbol{x}) = \sum_{i=1}^{N}\psi_i^2(\boldsymbol{x})$ and the random variable $\varphi(\boldsymbol{x})$ has a Gaussian distribution

$$P(\varphi(\boldsymbol{x})) = \frac{\exp[-\frac{\varphi(\boldsymbol{x})^2}{2\Delta}]}{\sqrt{2\pi\Delta}} \tag{2.2}$$

with mean

$$\overline{\varphi(\boldsymbol{x})} = 0 \tag{2.3}$$

and variance

$$\overline{\varphi(\boldsymbol{x})\varphi(\boldsymbol{x'})} = \Delta\delta^d(\boldsymbol{x} - \boldsymbol{x'}). \tag{2.4}$$

The over line in (2.4) indicates a random average performed with the distribution $P(\varphi(\boldsymbol{x}))$. Here we will consider a system in a finite cube of volume L^d with periodic boundary conditions. This means that the following expansion takes place

$$\psi(\boldsymbol{x}) = \frac{1}{L^d}\sum_{\boldsymbol{k}} \tilde{\psi}(\boldsymbol{k}) \exp(i\boldsymbol{k}.\boldsymbol{x}) \tag{2.5a}$$

and

$$\varphi(\boldsymbol{x}) = \frac{1}{L^d}\sum_{\boldsymbol{k}} \tilde{\varphi}(\boldsymbol{k}) \exp(i\boldsymbol{k}.\boldsymbol{x}), \tag{2.5b}$$

where $\boldsymbol{k}$ is a discrete vector with components $k_i = 2\pi n_i/L$, $n_i = 0, \pm1, \pm2, ...,$ $i = 1, ..., d$ and a cutoff $\Lambda \sim a^{-1}$ (a is the lattice spacing). In this paper, we are interested in the continuum limit, i.e. $a \to 0$.

In our case of quenched randomness, one must average the logarithm of the partition function over the Gaussian distribution (2.2) to produce the free energy

$$\mathcal{F}[\mathcal{H}_r] = -\int_{-\infty}^{\infty} D\varphi(\boldsymbol{x}) P(\varphi(\boldsymbol{x})) \ln \mathcal{Z}_r, \tag{2.6}$$

where

$$\mathcal{Z}_r = Tr_\psi \exp[\mathcal{H}_r]. \tag{2.7}$$

It is well known that the direct average of $\mathcal{H}_r$ over the Gaussian leads to equivalent results for the critical behavior as the $n = 0$ limit of the following "pure" translationally invariant model [19]:

$$\mathcal{H}_p(n) = -\frac{1}{2}\sum_{\alpha=1}^{n}\int_{L^d} d^dx[t|\psi_\alpha(\boldsymbol{x})|^2 + c|\nabla\psi_\alpha(\boldsymbol{x})|^2 + \frac{u}{12}|\psi_\alpha(\boldsymbol{x})|^4]$$
$$+\frac{\Delta}{8}\sum_{\alpha,\beta=1}^{n}\int_{L^d} d^dx|\psi_\alpha(\boldsymbol{x})|^2|\psi_\beta(\boldsymbol{x})|^2]. \tag{2.8}$$

Here $\psi_\alpha(\boldsymbol{x})$, $\alpha = 1, \ldots, n$ (n being the number of replicas) are components of an $(n\times N)$-components field $\vec{\psi}(\boldsymbol{x})$. Because of this equivalence, the model $\mathcal{H}_p$ has been the object of intensive field-theoretical studies (see [20] and refs. therein) in the bulk case. Much less is known for the equivalence of $\mathcal{H}_r$ and the $n = 0$-limit of $\mathcal{H}_p$ in the finite - size case. Problems may arise when finite-size techniques are used, since both procedures $L \to \infty$ and removing the disorder by the "trick" $n \to 0$ may not commute.

III. THE FSS EXPRESSIONS FOR THE FREE ENERGY AND CUMULANTS

In our case we have two possibilities: to consider the random model Eq. (2.1) or to consider the replicated pure model Eq. (2.8). The last one is closer to the case treated in [21] and [22] by getting around the difficulties due to random average performed with $P(\varphi(\boldsymbol{x}))$ and is used in the present study. For this case, the replicated partition function is given by

$$\mathcal{Z}_p(n) = \int \mathcal{D}\vec{\psi} \exp\left[\mathcal{H}_p(n)\right]. \qquad (3.1)$$

We decompose the field $\vec{\psi}(\boldsymbol{x})$ into a zero momentum component $\vec{\phi} = L^{-d}\int d^dx\vec{\psi}(\boldsymbol{x})$, which plays the role of the uniform magnetization and a second part depending upon the non-zero modes $\vec{\sigma} = L^{-d}\sum_{\boldsymbol{k}\neq 0}\vec{\psi}(\boldsymbol{k})\exp(-i\boldsymbol{k}.\boldsymbol{x})$. Neglecting fluctuations completely one has the mean-filed result

$$\mathcal{Z}_p(n) = \int \mathcal{D}\vec{\phi} \exp\left\{-\frac{L^d}{2}\sum_{\alpha=1}^{n}\left(t\phi_\alpha^2 + \frac{u}{12}\phi_\alpha^4\right) + \frac{L^d\Delta}{8}\left(\sum_{\alpha=1}^{n}\phi_\alpha^2\right)^2\right\} \qquad (3.2)$$

With the help of the identity

$$\exp\left(\frac{aA^2}{2}\right) = \frac{1}{(2\pi a)^{1/2}}\int_{-\infty}^{\infty} dy \exp[-(1/2a)y^2 + yA], \qquad (3.3)$$

we get

$$\mathcal{Z}_p(n) = \int_{-\infty}^{\infty} dy P\left(y\right)\left[S_N\int_0^{\infty} d\phi\phi^{N-1}\exp(\mathcal{H}_r^{\mathrm{eff}})\right]^n, \qquad (3.4)$$

where

$$\mathcal{H}_r^{\mathrm{eff}} = -\frac{1}{2}L^d\left[\left(t + \frac{y}{L^{d/2}}\right)\phi^2 + \frac{1}{12}u\phi^4\right] \qquad (3.5)$$

is an effective Hamiltonian with a random variable y with Gaussian distribution (depending on Δ)

$$P(y) = \frac{1}{\sqrt{2\pi\Delta}}\exp\left(-\frac{y^2}{2\Delta}\right) \qquad (3.6)$$

and $S_N = \frac{2\pi^{N/2}}{\Gamma(N/2)}$ is the surface of a N-dimensional unit sphere.

Let us note that the above mentioned equivalence between the models (2.1) and (2.8) may be mathematically expressed, within the used approximation, by the following relation:

$$\mathcal{F}\left[\mathcal{H}_r\right] = -\left.\frac{\partial}{\partial n}\mathcal{Z}_p(n)\right|_{n=0}. \qquad (3.7)$$

From Eqs. (3.4) and (3.7), and by using the identity

$$\left.\frac{\partial}{\partial n}A^n\right|_{n=0} = \ln A \qquad (3.8)$$

for the free energy, we get

$$\mathcal{F}\left[\mathcal{H}_r\right] = -\int_{-\infty}^{\infty} dy P(y) \ln \mathcal{Z}_r(0), \tag{3.9}$$

where

$$\mathcal{Z}_r(0) = S_N \int_0^{\infty} d\phi \phi^{N-1} \exp[\mathcal{H}_r^{\mathrm{eff}}] \tag{3.10}$$

is the partition function for the random system (3.5). The obtained effective "random-T_c model" (3.5), distributed with Gaussian weight (3.6), is the analytic basis of this paper. For describing the finite-size properties of the initial model (2.1), as follows from Eqs. (3.9) and (3.10), it is necessary to set n to zero.

Now using an appropriate rescaling of the field $\phi = (uL^d)^{-1/4}\Phi$ and introducing the scaling variable

$$\mu = tL^{d/2}u^{-1/2}, \tag{3.11}$$

for the partition function Eq. (3.10) in the mean-field approximation we obtain

$$\mathcal{Z}_r^{MF}(0) = \left(\frac{12\pi^2}{uL^d}\right)^{N/4} \exp\left[\frac{3}{4}(\mu + y/u^{1/2})^2\right] D_{-N/2}\left[\sqrt{3}(\mu + y/u^{1/2})\right]. \tag{3.12}$$

Here we have expressed the partition function in terms of the parabolic cylinder function $D_p(z)$ trough the identity [23]

$$\int_0^{\infty} x^{\nu-1} e^{-\beta x^2 - \gamma x} dx = (2\beta)^{-\nu/2}\Gamma(\nu) \exp\left(\frac{\gamma^2}{8\beta}\right) D_{-\nu}\left(\frac{\gamma}{\sqrt{2\beta}}\right). \tag{3.13}$$

From Eqs. (3.9) and (3.12) we get for the free energy

$$\mathcal{F}\left[\mathcal{H}_r^{MF}\right] = -\frac{1}{\sqrt{2\pi\Delta}} \int_{-\infty}^{\infty} dy \exp\left(-\frac{1}{2}\frac{y^2}{\Delta}\right) \ln\left\{(uL^d)^{-N/4}\mathcal{I}_N(\mu + y/u^{1/2})\right\}. \tag{3.14}$$

where

$$\mathcal{I}_N(z) = (12\pi)^{N/4} \exp\left(\frac{3}{4}z^2\right) D_{-N/2}(\sqrt{3}z), \tag{3.15}$$

If we introduce a second scaling variable

$$\lambda = \frac{\Delta}{u}, \tag{3.16}$$

Eq. (3.14) takes its final form

$$\begin{aligned}\mathcal{F}\left[\mathcal{H}_r^{MF}\right] &= -\frac{1}{\sqrt{2\pi\lambda}} \int_{-\infty}^{\infty} dx e^{-(x-\mu)^2/(2\lambda)} \ln\left[D_{-N/2}(\sqrt{3}x)\right] - \frac{3}{4}(\lambda + \mu^2) \\ &+ \frac{N}{4} \ln\left(\frac{uL^d}{12\pi^2}\right).\end{aligned} \tag{3.17}$$

In the large N limit ($N \to \infty$), the behavior of the model is expected to be similar to that of the spherical model [11]. Similar result for the density of the free energy in this case can be obtained after a suitable rescaling by a factor N of the interaction parameters u and Δ in the Hamiltonian (2.8). The corresponding expression in the limit $\lambda \to 0$ is given by

$$f = \frac{1}{2} \ln \left[\frac{tL^d}{2\pi} \right] + \frac{1}{24} \frac{u}{t^2 L^d}. \tag{3.18}$$

Note that the large N limit of the present model has meaning only for the pure model, i.e. $\lambda = 0$, since the renormalization group arguments reveal non physical behavior for $N > 4$.

In addition to the free energy, one also needs to know the correlation functions. Within the replica method the averages of the fields $\{\phi_\beta\}$ are defined by (see e.g. [12])

$$\overline{\langle |\phi_\beta|^{2m} \rangle}_{\mathcal{H}_r^{MF}} = \lim_{n \to 0} \left[\mathcal{Z}_p^{MF}(n)^{-1} S_N^n \int \left(\prod_{\alpha=1}^{n} d|\phi_\alpha| \right) (|\phi_\alpha|)^{N-1} (|\phi_\beta|)^{2m} \exp(\mathcal{H}_p^{MF}) \right], \tag{3.19}$$

where

$$\mathcal{Z}_p^{MF}(n) = S_N^n \int \left(\prod_{\alpha=1}^{n} d|\phi_\alpha| \right) (|\phi_\alpha|)^{N-1} \exp(\mathcal{H}_p^{MF}). \tag{3.20}$$

After taking the limit $n \to 0$, we end up with the following expression

$$\overline{\mathcal{M}_{2m}} := \overline{\langle |\phi_\beta|^{2m} \rangle}_{\mathcal{H}_r^{MF}} = \frac{(uL^d)^{-m/2}}{\sqrt{2\pi\lambda}} \int_{-\infty}^{\infty} dx \frac{\mathcal{I}_{N+2m}(x)}{\mathcal{I}_N(x)} e^{-(x-\mu)^2/2\lambda}. \tag{3.21}$$

In a similar way

$$\overline{(\mathcal{M}_2)^2} := \overline{\langle |\phi_\alpha|^2 |\phi_\beta|^2 \rangle}_{\mathcal{H}_r^{MF}} = \frac{(uL^d)^{-1}}{\sqrt{2\pi\lambda}} \int_{-\infty}^{\infty} dx \left[\frac{\mathcal{I}_{N+2}(x)}{\mathcal{I}_N(x)} \right]^2 e^{-(x-\mu)^2/2\lambda}. \tag{3.22}$$

From Eqs. (3.21) and (3.22), when $\mu = 0$ and $N = 1$ we obtain the results of Ref. [6] .

In terms of the second moment, the susceptibility is given by

$$\chi = L^d \overline{\mathcal{M}_2}. \tag{3.23}$$

Another quantity of importance for numerical analysis is the Binder cumulant defined by

$$B = 1 - \frac{1}{3} \frac{\overline{\mathcal{M}_4}}{\overline{\mathcal{M}_2}^2} \tag{3.24}$$

and the cumulant, specific for the random system, defined as

$$R = \frac{\overline{(\mathcal{M}_2)^2} - \overline{\mathcal{M}_2}^2}{\overline{\mathcal{M}_2}^2}. \tag{3.25}$$

Since the parameter R is the relative variance of the observable (the susceptibility), as we said in the Introduction, it is a measure of the self-averaging in the random system. If self-averaging takes place, this quantity should be zero in the thermodynamic limit.

Now for the evaluation of the above correlation functions, the integrand in Eq. (3.21) can be rewritten in a very simple form

$$\mathcal{M}_{2m}(x) := \frac{\mathcal{I}_{N+2m}(\mu+\sqrt{\lambda}x)}{\mathcal{I}_N(\mu+\sqrt{\lambda}x)} = \left(12\pi^2\right)^{\frac{m}{2}} \frac{D_{-m-N/2}((\mu+\sqrt{\lambda}x)\sqrt{3})}{D_{-N/2}((\mu+\sqrt{\lambda}x)\sqrt{3})}. \tag{3.26}$$

For small $\mu \ll 1$ (i.e. in the vicinity of the critical point) the asymptotic form of the ratio (3.26) is given by

$$\begin{aligned}\mathcal{M}_{2m}(x) = \left(12\pi^2\right)^{\frac{m}{2}} \Bigg\{ & \frac{D_{-m-N/2}(x\sqrt{3\lambda})}{D_{-N/2}(x\sqrt{3\lambda})} \\ & + \frac{\mu\sqrt{3}}{2}\left[N\frac{D_{-N/2-1}(x\sqrt{3\lambda})D_{-m-N/2}(x\sqrt{3\lambda})}{\left(D_{-N/2}(x\sqrt{3\lambda})\right)^2} - (2m+N)\frac{D_{-m-N/2-1}(x\sqrt{3\lambda})}{D_{-N/2}(x\sqrt{3\lambda})}\right] \\ & + \mathcal{O}(\mu)^2\Bigg\}.\end{aligned} \tag{3.27}$$

At the critical point, we have $\mu = 0$ and $\mathcal{M}_{2m}$ is equal to the first term in the r.h.s. of Eq. (3.27).

For large $\mu \gg 1$, the asymptotic behavior of the ratio $\mathcal{M}_{2m}$ is obtained with the help of the well-known Watson's Lemma (see for example [24]). According to it, we have

$$\mathcal{M}_2(x) = \left(12\pi^2\right)^{1/2} \frac{N}{\mu}\left[1 - \frac{x\sqrt{\lambda}}{\mu} + \frac{6x^2\lambda - N - 2}{6\mu^2} + \mathcal{O}\left(\frac{1}{\mu^3}\right)\right] \tag{3.28}$$

and

$$\mathcal{M}_4(x) = 12\pi^2 \frac{N(N+2)}{\mu^2}\left[1 - \frac{2x\sqrt{\lambda}}{\mu} + \frac{9x^2\lambda - N - 3}{3\mu^2} + \mathcal{O}\left(\frac{1}{\mu^3}\right)\right]. \tag{3.29}$$

Using the asymptotics of $\mathcal{M}_2$ and $\mathcal{M}_4$ for large μ, we can get the behavior of the cumulants R and B in the case $d \geq 4$. They are given by

$$B = 1 - \frac{1}{3}\left(1 + \frac{2}{N}\right)\left[1 + \frac{3\lambda - 1}{3\mu^2}\right] + \mathcal{O}\left(\frac{1}{\mu^3}\right) \tag{3.30}$$

and

$$R = \frac{\lambda}{\mu^2} + \mathcal{O}\left(\frac{1}{\mu^3}\right). \tag{3.31}$$

IV. CUMULANTS AND SELF-AVERAGING

Let us concentrate on the calculation of the Binder cumulant B (3.24) and the variance R (3.25) in the case $d \geq 4$. In Table I we present the corresponding universal numbers for B and R at $d \geq 4$ in the region $Lt^{\nu_R} = \frac{L}{\xi} \ll 1$, i.e. in the vicinity of the critical point. The calculations are performed with variables $\mu = 0$ and $\lambda = \frac{4-N}{12}$. The last expression comes from renormalization-group arguments [25]. The numerical values of B and R in the random case and for $N = 1$, presented in Table I, are in full agreement with those obtained in Ref. [6], while those of B for the pure case and $N = 1$ (Table I) are in full agreement with Ref. [21]. The finite value of R confirms the statement of the lack of SA [6].

The finite size correction to the bulk critical behavior of the cumulants B and R in the region $Lt^{\nu_R} = \frac{L}{\xi} \gg 1$, i.e. away from the critical point, are obtained with the help of the asymptotics for $\mu \gg 1$ leading to Eqs. (3.30) and (3.31). As one can see, the asymptotics of R for finite disorder ($\Delta \neq 0$) and large $\mu \gg 1$ confirms the statement that away from the critical point, a strong SA emerges in the system [2].

V. CONCLUSIONS

In the present paper we propose a scheme for the FSS scaling analysis of a finite disordered $\mathcal{O}(N)$ system within the mean field approximation. The method, we use here, is an extension of the field theoretical methods used to analyze FSS properties in pure systems. The nature of the symmetry of the model complicates the perturbative structure of the theory in comparison with the corresponding $\mathcal{O}(N)$ pure one. The meaning to consider the case $d \geq 4$ is in its simple analytical non perturbative treatment.

Our main results are related to the formulation of the problem for some numbers of components N of the fluctuating field for dimensions $d \geq 4$. Our results are a generalization of those obtained in [6] for $N = 1$. Due to the presence of randomness, it is shown that we are dealing with two variables problem with scaling variables $\mu = tL^{d/2}u^{-1/2}$ and $\lambda = \Delta/u$. Evaluating numerically the corresponding analytic expressions for the Binder's cumulant B and the relative variance R, we demonstrate a monotonic increase of B as a function of N in both pure and random cases and a monotonic decrease of R (to zero for $N = 4$) in the random case (see Table I), showing finally the validity of the SA in the different regimes. In FIG. 1, we present the plot of Binder's cumulant as a function of the number of component of the order parameter N. The tendency of $B \to 2/3$ for N large is clearly seen.

The formulation of the problem for the case of a large number of the components N of the order parameter relates our findings to the exact results for the pure spherical model [11] when $\Delta = 0$.

In our opinion, the present FSS study can also be applied in the "canonical" case [5], where the disorder is characterized by a constant total number of the occupied sites (or bonds), instead of the constant average density. We hope that the above results will also hold in the case of finite geometry, relating in this way our theoretical findings with the Monte-Carlo simulations.

ACKNOWLEDGMENTS

E.K. and N.S.T. acknowledge the hospitality and the financial support of the Abdus Salam International Centre for Theoretical Physics, Trieste. This work was done within the framework of the Associateship Scheme of the Abdus Salam ICTP. E.K. is also supported by the Spanish DGES Contract No. PB97-0076.

REFERENCES

[1] S. Wiseman and E. Domany, Phys. Rev. **E52** (1995) 3469.

[2] A. Aharony and A.B. Harris, Phys. Rev. Lett. **77** (1996) 3700.

[3] F. Pázmándi, R.T. Scalettor and G.T. Zimányi, Phys. Rev. Lett. **79** (1997) 5130.

[4] S. Wiseman and E. Domany, Phys. Rev. **E81** (1998) 2938.

[5] A. Aharony, A.B. Harris and S. Wiseman, Phys. Rev. Lett. **81** (1998) 252.

[6] H.G. Ballesteros, L.A. Fernández, V. Martin-Mayor, A.M. Sudupe, G. Parisi and J.J. Ruiz-Lorenzo, Nucl. Phys. **B512** [FS] (1998) 681.

[7] M.L. Marques and J.A. Gonzalo, Phys. Rev. **E60** (1999) 2394.

[8] K. Bernardet, F. Pázmándi and G.G. Batruini, Phys. Rev. Lett. **84** (2000) 4477.

[9] M. Barber in:*Phase Transitions and Critical Phenomena*, Eds. C. Domb and J.L. Lebowitz, Academic, London, 1983, vol.8.

[10] V. Privman, Ed., *Finite-size Scaling and Numerical Simulations of Statistical Systems*, World Scientific, Singapore, 1990.

[11] J.G. Brankov, D.M. Danchev and N.S. Tonchev *Theory of Critical Phenomena in Finite-Size Systems; Scaling and Quantum Effects*, World Scientific, Singapore, 2000.

[12] K. Binder and A.P. Young, Rev. Mod. Phys. **58** (1986) 801.

[13] D.E. Khemel'nitskii, Zh. Eksp. Theor. Fiz. **68** (1975) 1960 [Sov. Phys. JETP **41** (1975) 981].

[14] T. Lubensky, Phys. Rev. **B11** (1975) 3575.

[15] G. Grinstein and A. Luther, Phys. Rev. **B13** (1976) 1329.

[16] C. Jayaprakash and H.J. Katz, Phys. Rev. **B16** (1977) 3987.

[17] B.N. Shalaev, Zh. Eksp. Theor. Fiz. **73** (1977) 2301 [Sov. Phys. JETP **46** (1977) 1244].

[18] G. Jug, Phys. Rev. **B27** (1983) 609.

[19] The indices "r" and "p" will refer to the "random" and the effective "pure" system.

[20] A. Pelissetto and E. Vicari, Phys. Rev. **B62** (2000) 6393.

[21] E. Brézin and J. Zinn-Justin, Nucl. Phys. **B257** [FS14] (1985) 867.

[22] J. Rudnick, H. Guo and D. Jasnow, J. Stat. Phys. **41** (1985) 751.

[23] I.S. Gradsteyn and I. M. Ryzhik, Academic, New York and London, 1965.

[24] J.P. Keener, *Principles of Applied Mathematics*, Pereus Books, 2000.

[25] H. Chamati, E. Korutcheva and N. Tonchev, submitted to Phys. Rev. E, 2001.

TABLES

TABLE I. Numerical values for the Binder cumulant B from Eq. (3.24) and the relative variance R from Eq. (3.25) in the mean-field regime i.e. $d \geq 4$.

	Random		Pure	
N	B	R	B	R
1	0.216368	0.310240	0.270520	0
2	0.451486	0.111381	0.476401	0
3	0.533513	0.038365	0.543053	0
4	0.575587	0	0.575587	0

FIGURES

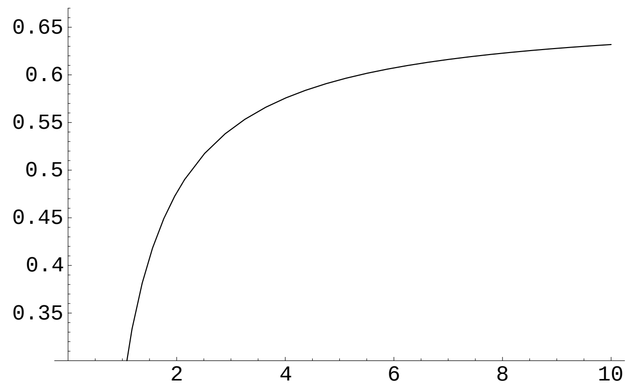

FIG. 1. Behavior of the Binder's cumulant as a function of N in the pure limit.