SOFT AND COLLINEAR RADIATION AND FACTORIZATION IN PERTURBATION THEORY AND BEYOND

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Power corrections to differential cross sections near a kinematic threshold are analysed by Dressed Gluon Exponentiation. Exploiting the factorization property of soft and collinear radiation, the dominant radiative corrections in the threshold region are resummed, yielding a renormalization-scale-invariant expression for the Sudakov exponent. The interplay between Sudakov logs and renormalons is clarified, and the necessity to resum the latter whenever power corrections are non-negligible is emphasized. The presence of power-suppressed ambiguities in the exponentiation kernel suggests that power corrections exponentiate as well. This leads to a non-perturbative factorization formula with non-trivial predictions on the structure of power corrections, which can be contrasted with the OPE. Two examples are discussed. The first is event-shape distributions in the two-jet region, where a wealth of precise data provides a strong motivation for the improved perturbative technique and an ideal situation to study hadronization. The second example is deep inelastic structure functions. In contrast to event shapes, structure functions have an OPE. However, since the OPE breaks down at large x, it does not provide a practical framework for the parametrization of power corrections. Performing a detailed analysis of twist 4 it is shown precisely how the twist-2 renormalon ambiguity eventually cancels out. This analysis provides a physical picture which substantiates the non-perturbative factorization conjecture.

1 Dressed Gluon Exponentiation

A classical application of QCD is the evaluation of semi-inclusive differential cross sections of hard processes depending on several scales. We shall consider here cross sections that depend on a hard scale Q and an intermediate scale W, both in the perturbative regime $Q > W \gg \Lambda$. Here Λ represents the fundamental QCD scale. In case of a large hierarchy, $Q \gg W$, there are large perturbative corrections containing logarithms, $\ln Q/W$. Typically, non-perturbative corrections are suppressed by powers of the *lower* scale W. If the latter is not so large, such power corrections must be taken into account. From a theoretical point of view, power corrections are particularly interesting because of their relation to confinement[1].

157

The first example is provided by event-shape distributions near the two-jet limit. Here the hard scale Q is the centre-of-mass energy and the lower one is set by the shape variable, e.g. in the case of the thrust (T), it is $W^2 = Q^2(1-T)^2$, the sum of the squared invariant masses of the two hemispheres. For $T \longrightarrow 1$ hadrons are produced in two narrow jets and large perturbative and non-perturbative corrections appear due to soft gluon radiation and hadronization. The second example is structure functions in deep inelastic scattering (DIS) at $x \longrightarrow 1$, where the hard scale is the momentum transfer $Q = \sqrt{-q^2}$ and the lower scale is the invariant mass of the hadronic system $W^2 = (p+q)^2 = Q^2(1-x)/x$, where p and q are the momenta of the proton and γ^*, respectively. For $x \longrightarrow 1$ the recoiling quark [a] develops into a narrow jet, and large corrections appear from the jet fragmentation process.

From the outset it is clear that resummation must be applied. Two relevant types of radiative corrections can be computed to all orders: renormalons and Sudakov logs. Renormalons appear from integration over the running coupling. Using the large N_f limit, one calculates the diagrams where a *single* gluon is dressed by radiative corrections[2]. These contributions dominate at large orders and exhibit the strongest sensitivity to infrared physics. Consequently they are useful in detecting power corrections[2,3]. In the absence of an infrared cutoff, infrared renormalons make the perturbative expansion non-summable. The summation ambiguity is cancelled by non-perturbative corrections. Sudakov logs, on the other hand, emerge from *multiple* emission of soft and collinear gluons. These contributions dominate the perturbative coefficients at large x. At a fixed logarithmic accuracy, Sudakov logs can be summed to all orders[4,5], and, contrary to renormalons, they do not indicate non-perturbative corrections[6,7]. As shown schematically in fig. 1, these two classes of radiative corrections correspond to "orthogonal" sets of diagrams. Both
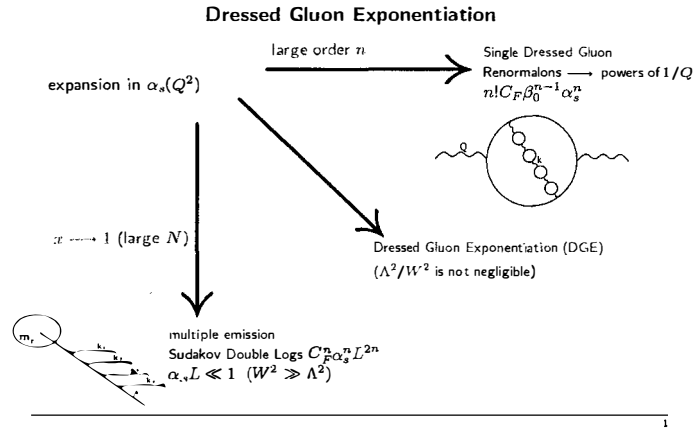
Dressed Gluon Exponentiation

Figure 1: Different directions in summing the perturbative expansion in a two-scale problem.

classes are relevant; however, none of the limits considered (large orders and large x, respectively) is appropriate in the threshold region, where both Sudakov logs and power corrections are important.

The gap is closed by[7,8,9] Dressed Gluon Exponentiation (DGE). This is a resummation method that incorporates both types of diagrams by resumming renormalons in the Sudakov exponentiation kernel. Thus, contrary to the standard approach to Sudakov resummation the result is renormalization-scale-invariant. In the Sudakov exponent, renormalons appear through the enhancement of subleading logs. At any given power of the coupling α_s^n, the leading

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[a] At large x the gluon distribution is small.

log [b], L^{n+1}, typically has a coefficient of order 1, whereas the L^k term (where $k \leq n$) appears with a larger numerical factor $\sim n!/k!$. Thus, the factorial growth in the exponent appears upon summing *all the logs*. In practice, subleading logs are not fully known and the state-of-the-art computation is restricted to a single dressed gluon (SDG) in the exponentiation kernel. In this case DGE reproduces the exact leading and next-to-leading logs (NLL) [c], but only generates a certain class of subleading logs (next-to-next-to-leading logs and beyond) that can be regarded as an approximation to these coefficients. Not much is known about the accuracy of this approximation in general. On the other hand, the factorial growth of subleading logs implies that a resummation with a fixed logarithmic accuracy [d] has a small range of validity. By construction it does not hold to power accuracy. If the latter is required, the additional resummation provided by DGE is necessary.

Analysing the large-order behaviour of the Sudakov exponent and the ambiguity associated with its resummation, one can access the dominant power corrections and obtain the information that is essential for their parametrization. The crucial difference in the way power corrections appear in this case, as compared with the standard OPE formulation, is that it is an overall factor multiplying [11,12,13,14] the resummed perturbative result in moment (or Laplace) space, rather than an additive term. In DGE, such non-perturbative factorization is unavoidable, since the perturbative exponent by itself contains power suppressed ambiguities. The non-perturbative corrections, which compensate these ambiguities, must therefore exponentiate together with the perturbative logs. This exponentiation reflects the effect of multiple soft emission at the non-perturbative level. From the OPE point of view, the resulting non-perturbative factorization is highly non-trivial. It amounts to assuming that the dominant contribution at each twist is proportional to the leading-twist matrix element. Moreover, the corresponding log-enhanced coefficient functions at higher twist must coincides with that of the leading twist. The matrix element information is essentially inaccessible by perturbative methods; however the higher-twist coefficient function can be computed, allowing one to check some of these far-reaching conclusions. A first step in this direction was recently taken [15] in the context of DIS structure functions.

2 Event-shape distributions in the two-jet region

A strong motivation for the improved resummation technique as well as for a systematic study of power corrections is provided by the very precise data on event-shape distributions in e^+e^- annihilation. The goal is to have a handle on the parametrization of hadronization effects and to understand how they change depending on the observable considered. This will hopefully lead to better understanding of the hadronization process itself.

The DGE result for the single-jet mass distribution [e] is given by a Laplace integral,

$$\frac{1}{\sigma}\frac{d\sigma}{d\rho}(\rho, Q^2) = \int_c \frac{d\nu}{2\pi i} \exp\left\{\rho\nu + \ln J_\nu(Q^2)\right\},\qquad(1)$$

where c is an integration contour parallel to the imaginary axis. Here, terms that are not enhanced by logarithms of ρ were discarded. Eventually, such terms are included by matching [5] the resummed expression to the fixed-order result, which is currently available [17] numerically to

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[b]$L \equiv \ln N$, where N is a moment index conjugate to $1 - x$, so N is large.
[c]NLL require using the "gluon bremsstrahlung" effective charge [10] with a 2-loop renormalization-group equation.
[d]Such resummation is derived in the $N \longrightarrow \infty$ limit with $\alpha_s(Q^2) \longrightarrow 0$, so that $\alpha_s(Q^2) \cdot \ln N$ is small.
[e]The single-jet mass distribution is used as an intermediate step in the evaluation of the thrust and the heavy-jet mass distributions. Note that this observable obtains additional non-global [16] corrections at the NLL level, which are absent in the thrust and the heavy-jet mass considered here.

next-to-leading order (NLO). The Sudakov exponent in (1) is

$$\ln J_\nu(Q^2) \;=\; \int d\rho \,\frac{1}{\sigma}\frac{d\sigma}{d\rho}(\rho,Q^2)\Big|_{\mathrm{SDG}} \left(e^{-\nu\rho}-1\right) \;=\; \frac{C_F}{2\beta_0}\int_0^\infty du\, B_\nu(u)\left(\frac{Q^2}{\bar\Lambda^2}\right)^{-u}\frac{\sin\pi u}{\pi u}\,\bar A_B(u), \quad (2)$$

where $\bar A_B(u)$ depends only on the renormalization group equation for the coupling and $\bar\Lambda$ corresponds to the "gluon bremsstrahlung" effective charge[10]. The Borel function is remarkably simple:

$$B_\nu(u) \;=\; \frac{2}{u}\left(\nu^{2u}-1\right)\Gamma(-2u) - \left(\frac{2}{u}+\frac{1}{1-u}+\frac{1}{2-u}\right)\left(\nu^u-1\right)\Gamma(-u). \quad (3)$$

Using this distribution, and the assumption that the hemisphere masses are independent[f], both the thrust (we define $t \equiv 1 - T$) and the heavy-jet mass (ρ_H) distributions are readily obtained,

$$\frac{1}{\sigma}\frac{d\sigma}{dt}(t,Q^2) \;=\; \frac{d}{dt}\int_c \frac{d\nu}{2\pi i\nu}\exp\left\{\nu t + 2\ln J_\nu(Q^2)\right\} \quad (4)$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\rho_H}(\rho_H,Q^2) \;=\; \frac{d}{d\rho_H}\left[\int_c \frac{d\nu}{2\pi i\nu}\exp\left\{\nu\rho_H + \ln J_\nu(Q^2)\right\}\right]^2. \quad (5)$$

These resummation formulae suggest a specific way in which power corrections should be included[7,9]. First of all, since renormalon ambiguities appear in the exponent, $\ln J_\nu^{\mathrm{PT}}(Q^2) \longrightarrow \ln J_\nu^{\mathrm{PT}}(Q^2) + \ln J_\nu^{\mathrm{NP}}(Q^2)$, power corrections appear as a factor in Laplace space, implying factorization and exponentiation of these terms, as previously suggested[11,12,13,14]. Moreover, the particular structure of Borel ambiguities from the renormalon singularities in (3) allows one to deduce the dependence of $\ln J_\nu^{\mathrm{NP}}(Q^2)$ on Q and ν. There are two classes of corrections: (a) *odd powers* of $\bar\Lambda\nu/Q$ from the first term in (3), which are related to large-angle soft emission; and (b) the *first two powers* of $\bar\Lambda^2\nu/Q^2$ from the second term in (3), which are associated with collinear emission. The leading corrections are (a) and they can be resummed into a shape function of a single variable[12]. The effect of the leading $\lambda_1\bar\Lambda\nu/Q$ power correction in the exponent is to shift the entire distribution[12,13], whereas higher (odd) power corrections modify the shape of the perturbative spectrum.

Fitting the thrust distribution in a large range of Q and t values with such a shape function provides a strong test of the approach. Good fits were obtained to the world data[7,9,20]. An example[9] is shown in fig. 2. An even more stringent test is the comparison of the extracted parameters from the thrust and the heavy-jet mass distributions, both defined in the decay scheme[21] in order to minimize the effect of hadron masses. Assuming that (4) and (5) hold non-perturbatively, the power corrections to the two distributions are associated with the same exponent and are therefore simply related. The agreement in the description of the two distributions is demonstrated in fig. 1, where the full line in the right frame shows not a fit but rather a calculated distribution for heavy-jet mass, where the parameters (α_s and the shape function) are fixed by the fit to the thrust. A comparison of the leading power correction extracted from the two distributions is shown in fig. 3. In addition to DGE, the figure shows a fit based on the NLL result. Contrary to DGE, in the NLL case there is no agreement between the two. This demonstrates the necessity to resum also the factorially enhanced subleading logs, as done by DGE, when a quantitative power correction analysis is done. Note also the significant impact on the extracted value of α_s. The best fit for the thrust distribution yields $\alpha_s^{\overline{\mathrm{MS}}}(M_Z) = 0.1086$. This value is consistent with that extracted from the average thrust upon performing renormalon resummation[22].

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[f]In the two-jet region, correlations between the hemispheres are suppressed perturbatively. It may play a more important role non-perturbatively[18,19]. We neglect this effect[9], still finding a good agreement with the data.
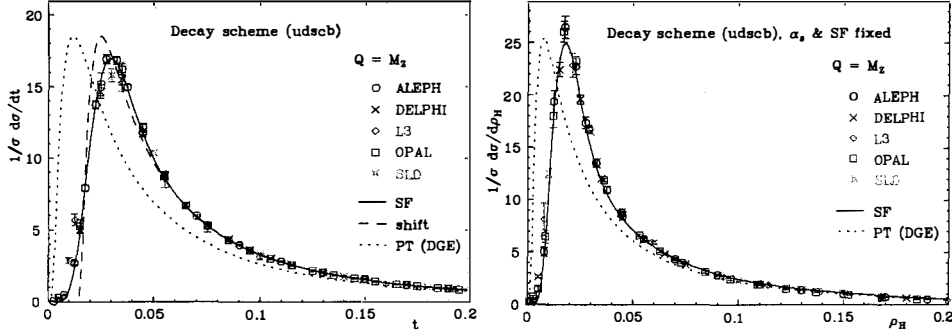
160

Figure 2: Left: fit to the thrust distribution data at $Q = M_Z$. The dotted line is the perturbative DGE result (principal-value regularization of the Borel sum), the dashed line shows a fit based on shifting the perturbative distribution (a single non-perturbative parameter) and the full line shows a shape-function-based fit. Right: the heavy-jet mass data compared with the predicted distribution based on the parameters fixed in the thrust fit. Here $\alpha_s^{\overline{MS}}(M_Z) = 0.1086$.
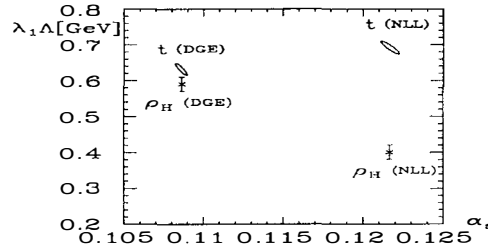

Figure 3: The leading non-perturbative correction on the scale Q/ν, extracted from the t and ρ_H distributions. Results based on DGE and on NLL resummation are shown. In each case α_s is fixed by the fit to the thrust.

3 DIS structure functions at large Bjorken x

In contrast to event-shape distributions, DIS structure functions have an OPE, by which power-suppressed contributions can be systematically identified and related to hadronic matrix elements[23]. The moments of the structure functions can be written as an expansion in inverse powers of the momentum transfer $q^2 \equiv -Q^2$,

$$\int_0^1 dx\, x^{N-1} F_2(x, Q^2) = C^{(2)}(N, \mu_F)\, \langle O^{(2)}(N)\rangle_{\mu_F} + \frac{1}{Q^2} \sum_j C_j^{(4)}(N, \mu_F)\, \langle O_j^{(4)}(N)\rangle_{\mu_F} + \ldots, \quad (6)$$

where $O_j^{(m)}$ are operators of twist m with the appropriate quantum numbers. The OPE can be expressed in terms on non-local operators[24,25,26,27] defined on the light cone $y^2 = 0$ (y is the Fourier conjugate of the γ^* momentum q), as demonstrated in fig. 4. Calculating the hadronic matrix elements of $O_j^{(m)}$ requires a full knowledge of the hadron structure. Thus, in practice these are simply parametrized and fixed by fit to experimental data. Moreover, most phenomenological analyses of structure functions are still restricted nowadays to the leading twist. On the other hand, the very existence of a non-perturbative definition puts the study of power corrections to structure functions on a firmer basis, as compared with event shapes. The OPE allows one to answer certain questions that are hard to address otherwise. In particular, within the OPE one can trace the cancellation of infrared renormalon ambiguities[2,28]. This is not only important in principle, but, in fact, *essential* for any reliable measurement of higher-twist
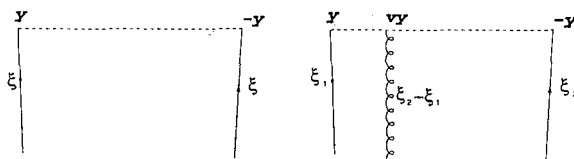
161

Figure 4: Twist-2 and twist-4 light-cone operators whose hadronic matrix elements are the quark distribution and a correlation between quark–gluon and quark states, respectively. At twist 4 there are several other operators.

matrix elements, since the renormalon ambiguity from the resummation of $C^{(2)}(N, \mu_F)$ in (6) is of the same order as the next term in the expansion. It is well known that by studying the renormalization properties of these operators one can determine their large Q^2 scaling violation. We shall also see that by calculating the higher-twist coefficient functions one can gain some insight into which partonic configurations may be relevant to certain kinematics. Both the anomalous dimension and the information encoded into the coefficient functions of the higher twist can be useful in identifying the dominant higher-twist contributions, without making too strong assumptions concerning the matrix elements themselves.

Let us now consider F_2 in the large-x limit. In particular, consider the limit where Q^2 gets large but $W^2 = Q^2(1 - x)/x$ is held fixed at some low (yet perturbative) scale. At the level of the leading twist ($m = 2$) the analysis of the structure function simplifies, since the gluon distribution can be neglected and only valance quarks contribute. The leading-twist factorization amounts to [9] incorporating the effect of gluons softer than some scale μ_F into the hadronic matrix element $\langle O^{(2)}(N) \rangle_{\mu_F}$, whereas the effect of gluons of higher virtualities go into the coefficient functions $C^{(2)}(N, \mu_F)$. Independence of the structure function of μ_F is guaranteed by an evolution equation stating that the logarithmic dependence of the renormalized operator on μ_F is cancelled by that of the coefficient function. Moreover, radiative corrections can be factorized into a hard subprocess, a jet subprocess, and a soft subprocess, which are mutually incoherent [4], as shown in fig. 5. Interaction between the remnants of the target and the recoiling
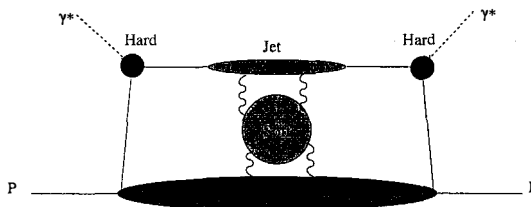

Figure 5: Factorization of DIS structure functions at large Bjorken x.

jet proceeds, at the level of the leading twist, only through the exchange of soft gluons (harder gluons contribute at higher twist), which cannot resolve the jet or the hadron structure.

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[9]The way factorization is implemented in practice (through dimensional regularization) removes infrared singularities from the coefficient functions, making them well defined order by order. However, it is not equivalent to a rigid cutoff. Infrared effects do penetrate into the coefficient functions at orders as renormalons.

Since non-perturbative corrections at large x go effectively in powers of $1/W^2$, the OPE in (6) tends to break down and needs to be summed up. At first sight this seems an incredibly complex task, in particular, since the number of parameters involved increases sharply at large N: at each twist $(m \geq 4)$ the number of local matrix elements [h] grows as a power of N.

Not only the need to compensate for the renormalon ambiguity at twist 2, but also the complexity of the twist expansion, and the difficulty to use it to parametrize power corrections, have led to renormalon-based phenomenology of structure functions. Renormalons resummation in the twist-2 coefficient function, at the level of a SDG (the large-N_f limit), leads to the following ambiguity [3,29,30,31] to order $1/Q^2$,

$$
\Delta \int_0^1 dx\, x^{N-2} F_2 = \frac{C_F}{2\beta_0} \left[4\psi(N+1) + 4\gamma + \frac{2}{1+N} + \frac{12}{2+N} - 8 - N \right] q_N \frac{\Lambda^2 \delta}{Q^2}, \tag{7}
$$

with $\delta \equiv \int_C du/(1 - u)$, where the contour C is the difference between different integration contours in the Borel plane, which avoid the singularity at $u = 1$. Since F_2 is a physical quantity the ambiguity must cancel, and thus there must exist at twist 4 a contribution of the form (7), which is proportional to the leading twist matrix element q_N. The renormalon-based model [3,29,30,31] amounts to assuming that this contribution dominates, and thus the entire twist 4 can be approximated by (7), replacing the ambiguous δ by a single fit parameter. No physical arguments have been given to support this dominance. From the OPE point of view it seems a very strong assumption: genuine multiparton correlations are absent here. One can also parametrize the twist-6 contribution based on the $1/Q^4$ renormalon ambiguity in the twist-2 coefficient function. At large x this model does not apply: since we know that multiple emission is very important perturbatively, it is unlikely that power corrections will be entirely associated with a *single* dressed gluon. Instead, at large x it is more appropriate to parametrize power corrections according to DGE, taking multiple gluon emission into account. But also in this case strong assumptions are made on the higher-twist contribution. To proceed we need to clarify the meaning of the renormalon dominance assumption, but we must also identify the dominant higher-twist contribution independently of any prejudice. Both these issues were recently addressed [15] at the level of twist 4.

To clarify the meaning of the renormalon dominance assumption we must trace the cancellation of renormalon ambiguities within the OPE. The infrared renormalon ambiguity cancels against another ambiguity in the definition of higher-twist matrix elements, due to the mixing of the corresponding operators with the leading-twist operator [32,33,2]. To make use of the OPE, one must regularize both sources of ambiguity in (6):

$$
C^{(2)}(N, \mu_{\mathrm{F}}) \quad \longrightarrow \quad C^{(2)}(N, \mu_{\mathrm{F}}) \Big|_{\mathrm{reg}} + \frac{\Lambda^2 \delta_{\mathrm{reg}}}{Q^2} \tag{8}
$$

$$
\langle O_j^{(4)}(N) \rangle_{\mu_{\mathrm{F}}} \quad \longrightarrow \quad \langle O_j^{(4)}(N) \rangle_{\mu_{\mathrm{F}}} \Big|_{\mathrm{reg}} + \langle O^{(2)}(N) \rangle_{\mu_{\mathrm{F}}} \Lambda^2 \delta_{\mathrm{reg}}^{(j)}. \tag{9}
$$

Here δ_{reg} and $\delta_{\mathrm{reg}}^{(j)}$ in eqs. (8) and (9) represent the effect of changing the regularization prescription in defining the sum of the series in the twist-2 coefficient functions and the ultraviolet-divergent integrals in the renormalized twist-4 operators, respectively. A consistent regularization guarantees the cancellation of all the δ terms leading to unambiguous predictions for the structure functions to power accuracy. It is clear from (9) that the mixing of twist 4 with twist 2 must be associated with *quadratic* divergence in the renormalization of $O_j^{(4)}(N)$. Indeed, contracting the gluon in fig. 4 to one of the quark lines, there appears a loop which is quadratically divergent in the ultraviolet, while the remaining operator is the twist-2 one. An ambiguity

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[h]As N increases, larger distances along the light cone become relevant. This makes the formulation of the OPE in terms of non-local matrix elements attractive. Indeed, using light-cone distributions, the analysis of twist 4 at large x becomes tractable.

emerges from the regularization of this loop. Summing the ambiguous ultraviolet contributions from all the relevant operators, each appearing with its own coefficient function, we recover an ambiguous expression, which is identical to (7) but has an opposite overall sign [i]. This way, when the two are summed in (6), a well-defined expression is obtained.

Note that while the renormalon ambiguity is associated with infrared scales, the presence of ambiguity in the higher twist is an ultraviolet property of the operator, which is not related to infrared physics. In conclusion, the renormalon ambiguity merely reflects the arbitrariness in separating contributions of different twists. The precise separation (or regularization of the renormalon sum) does not have any physical significance, and it should be regarded as complementary to the standard factorization used to define separately the coefficient functions and the operator matrix elements at each twist.

From this discussion it follows that the renormalon dominance assumption should be interpreted, within the OPE, as the assumption that the ultraviolet divergent contribution [32,33,2], the one which mixes under renormalization with the leading twist, dominates the higher twist. Independently of this assumption we conclude that any treatment of higher twist which fails to deal with renormalon resummation (and the corresponding regularization of higher twist) is bound to be ambiguous.

A priori, it is natural to expect that terms which mix with the leading twist will be of the same order of magnitude as other higher-twist effects. Ultraviolet dominance is the assumption that the former dominate. One can also imagine a scenario opposite to ultraviolet dominance, where the matrix elements are much larger than their ambiguous part. This point of view was adopted in the framework of QCD sum rules[1]. The success of renormalon-based phenomenology in various applications calls for reconsideration of this assumption also in the framework of the sum rules. In general, similarly to DIS higher twist, condensates should be extracted when performing renormalon resummation; they can be assigned numerical values only within a given regularization prescription for the renormalons.

Let us now return to the large-x limit and study the twist-4 contribution to F_2. Examining the coefficient functions of the corresponding light-cone operators at leading order we find [15] that a significant simplification occurs at large x. This simplification is due to the fact that certain partonic configurations dominate the entire twist-4 contribution. In particular, we find that the dominating final state is that of a single energetic quark, just as at twist 2. The final state in which a quark and a gluon share the momentum is subdominant – it is suppressed by a power of $1 - x$. Thus the difference between the leading twist and twist 4 at large x is restricted to the initial states. However, also here a great simplification occurs. Representing the coefficient functions of the twist-4 operator in fig. 4 in terms of the longitudinal momentum fractions of the quarks ξ_1 and $-\xi_2$, the dominant contribution arises from the region where the gluon momentum fraction is small $\xi_2 - \xi_1 \longrightarrow 0$, since the coefficient functions are singular at this point. Consequently the Heisenberg uncertainty principle implies that the quark–gluon–quark correlator $\langle p|O_j^{(4)}(v,y)|p\rangle_{\mu_F}$ is essentially independent of the position of the gluon field on the light cone (v), and it effectively becomes a function of the light-cone separation between the quarks (py), just as the leading-twist matrix element $\langle p|O_j^{(2)}(y)|p\rangle_{\mu_F}$. In conclusion, the configurations that dominate the twist-4 contribution to F_2 at large x, in both the final and initial states, make it twist-2-like. It is therefore natural to conjecture that ultraviolet dominance indeed holds, namely that the dominant ingredient in twist 4 is the part that mixes with the leading twist.

While justifying ultraviolet dominance at large x, this picture does not apply to moderate or small values of x, and therefore it does not support the application of the renormalon model

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[i]The first example where this cancellation was demonstrated is the longitudinal structure function [2,28]. Recently [15] it was demonstrated in the case of F_2.

where eq. (7) is assumed to represent the N dependence of twist 4.

The essence of the simplification we identified at twist 4 at large x is that the dominant multiparton correlation measured by F_2 in such kinematics is still associated with the leading twist. This is most naturally realized through ultraviolet dominance. Gluons of momentum scale of order W which are exchanged between the jet and the remnants of the target simply cannot resolve the full multiparton correlation function. Their interaction can be associated with "power-like evolution", similarly to the way the interaction of soft gluons at the leading twist is associated with logarithmic evolution.

Assuming that ultraviolet dominance holds to all orders in the twist expansion, a non-perturbative factorization formula (valid up to perturbative and non-perturbative corrections that are suppressed by $1/N$) emerges from the leading contributions to each twist:

$$\int_0^1 dx\, x^{N-1} F_2(x, Q^2) \;=\; H\left(Q^2\right) J\left(Q^2/N; \mu_F^2\right)\, q_N(\mu_F^2) \left[1 + \kappa_1 \frac{N\Lambda^2}{Q^2} + \cdots\right] \qquad (10)$$

$$\;=\; H\left(Q^2\right) J\left(Q^2/N; \mu_F^2\right)\, q_N(\mu_F^2)\, J_{\mathrm{NP}}\left(N\bar\Lambda^2/Q^2\right),$$

where q_N is the twist-2 quark matrix element, H and J are the hard and jet components in the twist-2 coefficient function, and κ_i are target-dependent non-perturbative parameters. In the second line all the non-perturbative corrections on the scale Q^2/N are resummed into a shape function of a single argument, similarly to the parametrization of non-perturbative effects in event-shape distributions.

DGE can now be applied to calculate J. Using the standard $\overline{\mathrm{MS}}$ factorization with $\mu_F = Q$, we obtain

$$J(Q^2/N, \mu_F = Q) \;=\; \exp\left\{ \frac{C_F}{2\beta_0} \int_0^\infty du\, B_N(u) \left(\frac{Q^2}{\bar\Lambda^2}\right)^{-u} \frac{\sin \pi u}{\pi u} \bar A_B(u) \right\}, \qquad (11)$$

with [8]

$$B_N(u) \;=\; -\left(\frac{2}{u} + \frac{1}{1-u} + \frac{1}{2-u}\right)(N^u - 1)\, \Gamma(-u) - \frac{2}{u}\ln(N), \qquad (12)$$

where the first term is similar to the collinear part in the event-shape case (3). Contrary to the latter, here the collinear singularity (appearing as a pole at $u = 0$) requires a subtraction. This is the rôle of the second term in (12), which can be identified as the large-N limit of the leading twist anomalous dimension. Having implemented the $\overline{\mathrm{MS}}$ factorization, J is regular at $u = 0$ and thus has a well defined perturbative expansion. Still, it has Borel singularities at $u = 1, 2$. These will be cancelled by the non-perturbative jet function $J_{\mathrm{NP}}(N\bar\Lambda^2/Q^2)$. Thus, the following ansatz suggests itself [8,15]

$$J_{\mathrm{NP}}\left(N\bar\Lambda^2/Q^2\right) = \exp\left\{ -\omega_1 \frac{C_F}{\beta_0} \frac{N\bar\Lambda^2}{Q^2} - \frac{1}{2}\omega_2 \frac{C_F}{\beta_0} \frac{N^2\bar\Lambda^4}{Q^4} \right\}, \qquad (13)$$

where ω_i are non-perturbative parameters.

4 Conclusions

Many interesting hard processes involve kinematic thresholds. Owing to the emission of soft and collinear radiation, the corresponding differential cross-sections tend to have large perturbative and non-perturbative corrections. As a result, a naive, fixed-order perturbative treatment is insufficient. Moreover, the OPE does not apply or tends to break down. Here we shortly reviewed

the case of event-shape distributions in e^+e^- annihilation, where we demonstrated the virtues of DGE both as a resummation method and as a way to study power corrections. This was followed by a deeper look into the case of DIS structure functions at large x, where the OPE was used to get additional insight into the problem. There are many other physical applications where DGE and the shape-function approach can be applied, including, for example, fragmentation functions of light and heavy quarks and Drell–Yan or heavy-boson production processes.

DGE is primarily a novel approach to resummation: the Sudakov exponent is calculated in a renormalization-scale-invariant manner by means of renormalon resummation. The criterion of a fixed-logarithmic accuracy becomes irrelevant when power corrections are being quantified: it is the subleading logs that carry the characteristic factorial growth of the coefficients, which is associated with the power corrections. Perturbatively, the additional resummation achieved by DGE with respect to the standard NLL resummation is significant. For event-shape distributions, it is $\sim 20\%$ at M_Z. In principle, power corrections cannot be quantified without renormalon resummation. Our analysis of the thrust and the heavy-jet mass shows that this has very practical implications: a consistent description of the two observables is possible only if renormalon resummation in the Sudakov exponent is performed (fig. 3). Another important consequence is the significant impact on the extracted value of α_s.

DGE and the shape-function approach can be applied in the case of DIS structure function at large x. However, here it can be contrasted with the OPE. As we have seen, a non-perturbative factorization can be consistent with the OPE, and it is supported by the OPE-based analysis. In spite of the fact that the OPE tends to break down in the large-x limit, it is very useful: a simple picture emerges from the analysis of twist 4 in terms of light-cone distributions. We have found that the dominant non-perturbative corrections at large x are associated with the formation of a narrow jet in the final state. These corrections are due to the exchange of gluons with momentum scale of the order W, which are insensitive to the details of multiparton correlations in the target. Instead of the full correlation, they measure a particular ingredient which is twist-2 like. It is therefore natural to conjecture that the dominant contributions at large x are associated with mixing with the leading twist. To $1/N$ accuracy the hadronization process of the jet involves a single target-dependent non-perturbative scale at each order in the twist expansion. These dominant corrections can be resummed into a shape function of a single argument: N/Q^2, defining a non-perturbative jet function. Thus, at large N the OPE collapses (to $1/N$ accuracy) into a factorized formula in which the leading twist is multiplied by a jet function[j]. The application of DGE is then quite natural.

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[j]It should be stressed that the factorization formula is not derived here from first principles. Its justification involves a strong assumption. Nevertheless, it is possible to check explicitly certain results. In particular, the formula predicts a common asymptotic behaviour of the logarithmic evolution in the large N limit for any twist. This can be verified at twist 4.

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