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$N = 4$ gauged supergravity and a IIB orientifold with fluxes

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Abstract

We analyze the properties of a spontaneously broken $D = 4$, $N = 4$ supergravity without cosmological constant, obtained by gauging translational isometries of its classical scalar manifold.
This theory offers a suitable low energy description of the super–Higgs phases of certain Type-IIB orientifold compactifications with 3–form fluxes turned on.
We study its $N = 3, 2, 1, 0$ phases and their classical moduli spaces and we show that this theory is an example of no–scale extended supergravity.

1 Introduction

Spontaneously broken supergravities have been widely investigated over the last 25 years, as the supersymmetric analogue of the Higgs phase of spontaneously broken gauge theories [1]–[7].

We recall that when N supersymmetries are spontaneously broken to $N' < N$ supersymmetries, then $N - N'$ gravitini acquire masses by absorption of $N - N'$ Goldstone fermions. The theory in the broken phase, will then have N' manifest supersymmetries with $N - N'$ gravitini belonging to massive multiplets of the residual N' supersymmetries. However, unlike gauge theories, the super–Higgs phases of local supersymmetries, require more care because these theories necessarily include gravity.

Therefore, by broken and unbroken supersymmetry, we mean the residual global supersymmetry algebra in a given gravitational background solution of the full coupled Einstein equations.

A particularly appealing class of spontaneously broken theories are those which allow a Minkowski background, because in this case the particle spectrum is classified in terms of Poincaré supersymmetry, and the vacuum energy (cosmological constant) vanishes in this background.

It is usually not easy to obtain [5], [6], [7], in a generic supergravity theory, a broken phase with vanishing cosmological constant, even at the classical level. Few examples of isolated Minkowski vacua with broken supersymmetry were found in the context of gauged extended supergravities [8] with non compact gaugings [9], [10], [11], [12]. However, it was realized [13], first in the context of $N = 1$ supergravity, that there are particular classes of supergravity theories, called no–scale supergravities [14], in which the vacuum energy, parametrized by the scalar potential, is always non negative, as is the case of rigid supersymmetry, then offering the possibility of having either positive or vanishing cosmological constant.

The euristic structure of these models, at the classical level, is that the supersymmetry breaking is mediated by some degrees of freedom, while some other degrees of freedom do not feel the supersymmetry breaking. The latter are responsible to the positive potential, which, however, vanishes when extremized, reflecting the fact the those degrees of freedom do not participate to the supersymmetry breaking.

Examples of no–scale supergravities in the case of extended supersymmetries, were first given in the case of $N = 2$ supergravity [15], [16], by introduction of an $N = 2$ Fayet–Iliopoulos term and, for special geometries [17], [18], [19], with a purely cubic holomorphic prepotential.

These models did admit a $N = 2$ or $N = 0$ phase, but not a $N = 1$ phase. Later models with $N = 2$ spontaneously broken supergravity to either $N = 1$ or $N = 0$ in presence of hypermultiplets were obtained [20], [21], [22].

The no–scale structure of these models, resulted in the fact that the broken phases had a non trivial moduli space, with sliding gravitino (and other massive fields) masses dependent on the moduli.

Moreover, a severe restriction on the allowed broken phases comes from the constrained geometry of the moduli space of the supergravity with a given number of supersymmetries. In fact, these moduli spaces, are described by manifolds of restricted holonomy and therefore, the interpretation of massive degrees of freedom, which allows to describe a

given broken phase, must be compatible with this requirement [23], [24] [25].
Consistent truncation of extended supergravities to theories with lower supersymmetries, was studied in [26] and the super–Higgs phases was shown to be consistent with this analysis.
The first model with $N = 2 \rightarrow 1 \rightarrow 0$ breaking showed an unusual feature [20], [21], namely that in order to have a theory with different gravitino masses and vanishing cosmological constant, two out of the three translational isometries of the $USp(2, 2)/USp(2) \times USp(2)$ quaternionic manifold [27] must be gauged, the relative gauge fields being the graviphoton and the vector of the only vector multiplet present in this model.
Recently, J. Louis observed [25] that in the context of a generic $N = 2$ theory, spontaneously broken to $N = 1$ in Minkowski space, the two massive vectors, superpartners of the massive gravitino, must correspond to a spontaneously broken $\mathbb{R}^2$ symmetry, irrespectively of the other matter fields, thus confirming the relevance of the gauging of translational isometries in spontaneously broken supergravity.
A wide class of spontaneously broken supergravities with a no–scale structure, is provided by the Scherk–Schwarz generalized dimensional reduction [28], [29]. The four dimensional description of these models [30], [31] is obtained by gauging " flat groups " [28], which are a semidirect product of an abelian group of translational isometries with a compact $U(1)$ generators of the Cartan subalgebra of the maximal compact subgroup of the global symmetry of the five dimensional theory.
In all these models, the massive vector bosons, partners of the massive gravitini, are again associated to spontaneously broken translational isometries ($\mathbb{R}^{27} \subset E_{7(7)}$ in the case of $N = 8$ supergravity) of the scalar manifold of the unbroken theory.
Many variants of the Scherk–Schwarz breaking and their stringy realization have been studied in the literature [32], [33], [34].
Spontaneously broken supergravities, by using dual versions of standard extended supergravities, where again translational isometries of the scalar manifold of the ungauged theory are gauged, were studied in reference [35] as a $N > 2$ generalization [36] of the original model which allowed the $N = 2 \longrightarrow N = 1$ hierarchical breaking of supersymmetry.
In the string and M–theory context, no–scale supergravity models, were recently obtained as low energy description of orientifold compactification with brane fluxes turned on [37]–[52]. The natural question arises which low energy supergravity corresponds to their description and how the Higgs and super–Higgs phases are incorporated in the low energy supergravity theory.
It was shown in a recent investigation [53], extending previous analysis [15],[35], [36], that the main guide to study new forms of N–extended gauged supergravities, is to look for inequivalent maximal lower triangular subgroups of the full duality algebra (the classical symmetries of a four dimensional N–extended supergravity) inside the symplectic algebra of electric–magnetic duality transformations [54].
Indeed, different maximal subgroups of the full global (duality) symmetry of a given supergravity theory, allow in principle to find all possible inequivalent gaugings [10], [55], [56].
In the case of $Type-IIB$ superstring compactified on a $T_6/\mathbb{Z}_2$ orientifold [42], [43]the relevant embedding of the supergravity fields corresponds to the subgroup $SO(6, 6) \times SL(2, \mathbb{R})$ which acts linearly on the gauge potentials (six each coming from the $N-S$ and the $R-R$

2–forms $B_{\mu i}$, $C_{\mu i}$ $i = 1\dots 6$). It is obvious that this group is $GL(6,\mathbb{R}) \times SL(2,\mathbb{R})$ where $GL(6,\mathbb{R})$ comes from the moduli space of T^6 while $SL(2,\mathbb{R})$ comes from the $Type-IIB$ $SL(2,\mathbb{R})$ symmetry in ten dimensions. This means that the twelve vectors are not in the fundamental $\mathbf{12}$ of $SO(6,6)$ but rather a $(\mathbf{6}_+, \mathbf{2})$ of $GL(6,\mathbb{R}) \times SL(2,\mathbb{R})$ where the " + " refers to the $O(1,1,)$ weight of $GL(6,\mathbb{R}) = O(1,1) \times SL(6,\mathbb{R})$. Their magnetic dual are instead in the $(\mathbf{6}_-, \mathbf{2})$ representation. Note that instead in the heterotic string, the twelve vectors $g_{\mu i}$, $B_{\mu i}$ $i = 1\dots 6$ are in the $(\mathbf{6}_+^+, \mathbf{6}_-^+)$ and their magnetic dual in the $(\mathbf{6}_+^-, \mathbf{6}_-^-)$ representation, where the lower plus or minus refer to the $\mathbb{R}$ of $GL(6,\mathbb{R})$ and the upper plus or minus refer to the $\mathbb{R}$ of $SL(2,\mathbb{R})$.
The symplectic embedding of the Lie algebra of $SO(6,6) \times SL(2,\mathbb{R})$ inside $Sp(24,\mathbb{R})$ is therefore realized as follows [35], [53]

$$SO(6,6) = GL(6,\mathbb{R}) + T_{15}^+ + T_{15}^- \tag{1.1}$$

$$(12,2) \longrightarrow (6_+,2) + (6_-,2) \tag{1.2}$$

where $GL(6,\mathbb{R}) \times SL(2,\mathbb{R})$ is block diagonal and $T_{15}^\pm$ are lower and upper off–diagonal generators respectively.
The gauged supergravity, corresponding to this symplectic embedding was constructed in reference [35], but the super–Higgs phases were not studied.
In the present paper we study these phases, derive the mass spectrum in terms of the four complex gravitino masses and analyze the moduli space of these phases and their relative unbroken symmetries.
Connection with supergravity compactification on the $T^6/\mathbb{Z}_2$ orbifold with brane fluxes is discussed.
The major input is that the fifteen axion fields $B^{\Lambda\Sigma} = -B^{\Sigma\Lambda}$, $\Lambda, \Sigma = 1\dots 6$ related to the fifteen translational isometries of the moduli space $SO(6,6)/SO(6) \times SO(6)$ are dual to a compactified $R-R$ 4–form scalars ($B^{\Lambda\Sigma} = \frac{1}{4!}\epsilon^{\Lambda\Sigma\Delta\Gamma\Pi\Omega} C_{\Delta\Gamma\Pi\Omega}$). Moreover the charge coupling of $N=4$ dual supergravity of reference [35]

$$\nabla_\mu B^{\Lambda\Sigma} = \partial_\mu B^{\Lambda\Sigma} + f^{\Lambda\Sigma\Delta\alpha} A_{\Delta\alpha} \quad \alpha = 1,2; \quad \Delta = 1\dots 6 \tag{1.3}$$

identifies the supergravity coupling $f^{\Lambda\Sigma\Delta\alpha}$ with the 3–form fluxes coming from the term [42] [43]

$$dC + H^\alpha \wedge B^\beta \epsilon_{\alpha\beta} \tag{1.4}$$

of the covariant 5–form field strength, where H^α is taken along the internal directions and integrated over a non trivial 3–cycle[1].

The paper is organized as follows:

in Section 2 we describe the geometry underlying the $N=4$ supergravity in the dual basis chosen by the Type-IIB orientifold compactification.

In Section 3 we describe the ungauged and gauge theory in this basis: the main ingredient is to rewrite the supergravity transformation laws in an unconventional way in terms of the reduced manifold $GL(6)/SO(6)$ and the fifteen axion fields $B^{\Lambda\Sigma}$. This allows us to compute the fermion shifts in terms of which the potential can be computed.

[1] The gauge coupling in (1.3) are actually "dual" to the fluxes of references [42],[43]

In Section 4 we analyze the potential. We show that it is semidefinite positive and find the extremum which stabilizes the dilaton and the $GL(6)/SO(6)$ scalar fields, except three fields related to the radii of $T^6 = T^2 \times T^2 \times T^2$.

In Section 5 we compute the mass spectrum of the gravitini and the vector fields. It is shown that the four complex gravitino masses precisely correspond to the $(3,0) + 3(2,1)$ decomposition of the real 3–form flux matrix $F^{\Lambda\Sigma\Delta} = L_\alpha f^{\Lambda\Sigma\Delta\alpha}$ (L_α coset representative of $SL(2,\mathbb{R})/SO(2)$.

In Section 6 the reduction of the massive and massless sectors of the different super–Higgs phases are described. In particular it is shown that by a given choice of the complex structure, the $N = 3$ supergravity corresponds to taking as nonzero only the $(3,0)$ part of the holomorphic components of $F^{\Lambda\Sigma\Delta}$.

In Section 7 we give the conclusions, while in the Appendix A we give the explicit representation of the $SU(4)$ Gamma matrices used in the text.

2 Geometry of the $N = 4$ scalar manifold

We start from the coset representative of $SO(6,6)/SO(6)\times SO(6)$ written in the following form [57], [58]

$$L = \begin{pmatrix} E^{-1} & -BE \\ 0 & E \end{pmatrix} \tag{2.5}$$

Here $E \equiv E^I_\Lambda$ and $E^{-1} \equiv E^\Lambda_I$ are the coset representative and its inverse of $GL(6)/O(6)_d$, $SO(6)_d$ being the diagonal subgroup of $SO(6)\times SO(6)$ .The indices $\Lambda, I = 1 \dots 6$ are in the fundamental of $GL(6)$ and $SO(6)_d$ respectively (indices I, J can be raised or lowered with the Kronecker delta). Finally $B^{\Lambda\Sigma}$ parametrizes the 15 translations. This corresponds to the following decomposition

$$\mathfrak{so}(6,6) = \mathfrak{sl}(6,\mathbb{R}) + \mathfrak{so}(1,1) + \mathbf{15}'^{+} + \mathbf{15}^{-} \tag{2.6}$$

Note that the representation $\mathbf{12}$ of $\mathfrak{so}(6,6)$ decomposes as $\mathbf{12} \to \mathbf{6_{+1}} + \mathbf{6_{-1}}$, thus containing six electric and six magnetic fields, and the bifundamental of $\mathfrak{so}(6,6)+\mathfrak{sl}(2,\mathbb{R})$ decomposes as $(\mathbf{12},\mathbf{2}) = (\mathbf{6_{+1}},\mathbf{2})_{\text{electric}} + (\mathbf{6_{-1}},\mathbf{2})_{\text{magnetic}}$. In particular, we see that $\mathfrak{sl}(2,\mathbb{R})$ is totally electric. The 12 vectors gauge an abelian subgroup of the $\mathbf{15}'^{+}$ translations. The left invariant 1–form $L^{-1}dL \equiv \Gamma$ turns out to be

$$\Gamma = \begin{pmatrix} EdE^{-1} & -EdBE \\ 0 & E^{-1}dE \end{pmatrix} \tag{2.7}$$

Now we extract the connections ω_d and $\widehat{\omega}$, where ω_d is the connection of the diagonal $SO(6)_d$ subgroup and $\widehat{\omega}$ is its orthogonal part. We get

$$\omega_d = \frac{1}{2}\begin{pmatrix} EdE^{-1} - dE^{-1}\,E & 0 \\ 0 & EdE^{-1} - dE^{-1}\,E \end{pmatrix} \tag{2.8}$$

$$\widehat{\omega} = \frac{1}{2}\begin{pmatrix} 0 & -EdBE \\ -EdBE & 0 \end{pmatrix} \tag{2.9}$$

so that the total connection $\Omega = \omega_d + \widehat{\omega}$ is

$$\Omega = \frac{1}{2}\begin{pmatrix} EdE^{-1} - dE^{-1}\,E & -EdBE \\ -EdBE & EdE^{-1} - dE^{-1}\,E \end{pmatrix} \tag{2.10}$$

By definition the vielbein P is defined as

$$\mathcal{P} = \Gamma - \Omega \tag{2.11}$$

so that we get

$$\mathcal{P} = \frac{1}{2}\begin{pmatrix} EdE^{-1} + dE^{-1}\,E & -EdBE \\ EdBE & -(EdE^{-1} + dE^{-1}\,E) \end{pmatrix} \tag{2.12}$$

In the following we will write Ω and $\mathcal{P}$ as follows

$$\Omega = \frac{1}{2}\begin{pmatrix} \omega^{IJ} & -P^{[IJ]} \\ -P^{[IJ]} & \omega^{IJ} \end{pmatrix}; \qquad \mathcal{P} = \frac{1}{2}\begin{pmatrix} P^{(IJ)} & -P^{[IJ]} \\ P^{[IJ]} & P^{(IJ)} \end{pmatrix} \tag{2.13}$$

where

$$\omega^{IJ} = (EdE^{-1} - dE^{-1}\,E)^{IJ} \tag{2.14}$$
$$P^{(IJ)} = (EdE^{-1} + dE^{-1}\,E)^{IJ} \tag{2.15}$$
$$P^{[IJ]} = (EdBE)^{IJ} \tag{2.16}$$

Note that

$$\nabla^{(SO(6)_d)} E^I_{\ \Lambda} = \frac{1}{2} E^J_{\ \Lambda} P^{(JI)} \tag{2.17}$$

For the $SU(1,1)/U(1)$ factor of the $N=4$ σ–model we use the following parameterizations

$$S = \begin{pmatrix} \phi_1 & \overline{\phi}_2 \\ \phi_2 & \overline{\phi}_1 \end{pmatrix} \qquad (\phi_1\overline{\phi}_1 - \phi_2\overline{\phi}_2 = 1) \tag{2.18}$$

Introducing the 2-vectors

$$L^\alpha \equiv \begin{pmatrix} L^1 \\ L^2 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + \phi_2 \\ -i(\phi_1 - \phi_2) \end{pmatrix} \tag{2.19}$$
$$L_\alpha \equiv \epsilon_{\alpha\beta} L^\beta \tag{2.20}$$

the identity $\phi_1\overline{\phi}_1 - \phi_2\overline{\phi}_2 = 1$ becomes

$$L^\alpha \overline{L}^\beta - \overline{L}^\alpha L^\beta = i\epsilon^{\alpha\beta} \tag{2.21}$$

Introducing the left-invariant $\mathfrak{sl}(2, \mathbb{R}$ Lie algebra valued 1-form:

$$\theta \equiv S^{-1} dS = \begin{pmatrix} q & \overline{p} \\ p & -q \end{pmatrix} \tag{2.22}$$

one easily determine the coset connection 1-form q and the vielbein 1-form p:

$$q = i\epsilon_{\alpha\beta} L^\alpha d\overline{L}^\beta \tag{2.23}$$
$$p = -i\epsilon_{\alpha\beta} L^\alpha dL^\beta \tag{2.24}$$

Note that we have the following relations

$$\nabla L^\alpha \equiv dL^\alpha + qL^\alpha = -\overline{L}^\alpha p \tag{2.25}$$
$$\nabla \overline{L}^\alpha \equiv d\overline{L}^\alpha - q\overline{L}^\alpha = -L^\alpha \overline{p} \tag{2.26}$$

3 The gauging (turning on fluxes)

In the ungauged case the supersymmetry transformation laws of the bosonic and fermionic fields can be computed from the closure of Bianchi identities in superspace and turn out to be:

$$\begin{aligned}
\delta V^a_\mu &= -i\bar{\psi}_{A\mu}\gamma^a\varepsilon^A - i\bar{\psi}^A_\mu\gamma^a\varepsilon_A && (3.27)\\
\delta A_{\Lambda\alpha\mu} &= -L_\alpha E^I_\Lambda(\Gamma_I)^{AB}\bar{\psi}_{A\mu}\varepsilon_B - \overline{L}_\alpha E_{\Lambda\, I}(\Gamma^I)_{AB}\bar{\psi}^A_\mu\varepsilon^B + \\
&\quad + i\overline{L}_\alpha E^I_\Lambda(\Gamma_I)^{AB}\bar{\chi}_A\gamma_\mu\varepsilon_B + iL_\alpha E^I_\Lambda(\Gamma^I)_{AB}\bar{\chi}^A\gamma_\mu\varepsilon^B + && (3.28)\\
&\quad + i\overline{L}_\alpha E^I_\Lambda\bar{\lambda}_{IA}\gamma_\mu\varepsilon^A + iL_\alpha E_{\Lambda\, I}\bar{\lambda}^{IA}\gamma_\mu\varepsilon_A && (3.29)\\
p_\beta\delta L^\beta &\equiv -i\epsilon_{\alpha\beta}L^\alpha\delta L^\beta = 2\bar{\chi}_A\varepsilon^A \Longrightarrow \delta L^\alpha = 2\overline{L}^\alpha\bar{\chi}_A\varepsilon^A && (3.30)\\
P^{IJ}_m\delta a^m &= (\Gamma^I)^{AB}\bar{\lambda}^J_A\varepsilon_B + (\Gamma^I)_{AB}\bar{\lambda}^{JA}\varepsilon^B && (3.31)\\
\delta\psi_{\mu A} &= \mathcal{D}_\mu\varepsilon_A - \overline{L}^\alpha E^\Lambda_I\Gamma^I_{AB}\mathcal{F}^-_{\mu\nu\Lambda\alpha}\gamma^\nu\varepsilon^B && (3.32)\\
\delta\chi^A &= \frac{i}{2}p_\mu\gamma^\mu\varepsilon^A + \frac{i}{4}\overline{L}^\alpha E^\Lambda_I(\Gamma^I)^{AB}\mathcal{F}^-_{\mu\nu\Lambda\alpha}\gamma^{\mu\nu}\varepsilon_B && (3.33)\\
\delta\lambda_{IA} &= \frac{i}{2}(\Gamma^J)_{AB}P_{JI\,\mu}\gamma^\mu\varepsilon^B - \frac{i}{2}L^\alpha E^\Lambda_I\mathcal{F}^-_{\mu\nu\Lambda\alpha}\gamma^{\mu\nu}\varepsilon_A && (3.34)
\end{aligned}$$

where $p_\mu \equiv p_\alpha\partial_\mu L^\alpha$ and $P^{IJ}_\mu \equiv P^{IJ}_m\partial_\mu a^m$, $m = 1\ldots 21$, a^m being the scalar fields parametrizing the coset $GL(6)/SO(6)$.
The position of the $SU(4)$ index A on the spinors is related to its chirality as follows: $\psi_{\mu A}, \varepsilon_A, \chi^A, \lambda_{IA}$ are left-handed spinors, while $\psi^A_\mu, \varepsilon^A, \chi_A, \lambda^{IA}$ are right-handed. Furthermore Γ_I, $I = 1\ldots 6$ are the four dimensional gamma matrices of $SO(6)$ (see Appendix). Note that $(\Gamma_I)_{AB} = -(\Gamma_I)_{BA}$ and $(\Gamma_I)^{AB} = \overline{(\Gamma_I)}_{AB}$

The previous transformations leave invariant the ungauged Lagrangian that will be given elsewhere together with the solution of the superspace Bianchi identities. Our interest is however in the gauged theory where the gauging is performed on the Abelian subgroup T_{15} of translations.

It is well known that when the theory is gauged, the transformation laws of the fermion fields acquire extra terms called fermionic shifts which are related to the gauging terms in the Lagrangian and enter in the computation of the scalar potential [5], [7], [59].
Let us compute these extra shifts for the gravitino and spin $\frac{1}{2}$ fermions in the supersymmetry transformation laws. Since we want to gauge the translations, according to the general rules, we have to perform the substitution

$$dB^{ij} \longrightarrow \nabla B^{ij} = dB^{ij} + k^{ij\Lambda\alpha}A_{\Lambda\alpha} \qquad (3.35)$$

where $k^{ij\Lambda\alpha}$ are the Killing vectors corresponding to the 15 translations, with ij a couple of antisymmetric world indices and $\Lambda\alpha$ denoting the adjoint indices of $GL(6)\times SL(2,R)$. Since the "coordinates" B^{ij} are related to the axion $B^{\Lambda\Sigma}$ by

$$dB^{\Lambda\Sigma} = E^\Lambda_I E^\Sigma_J P^{[IJ]}_{ij} dB^{ij} \qquad (3.36)$$

we get

$$\nabla B^{\Lambda\Sigma} = E^\Lambda_I E^\Sigma_J P^{[IJ]}_{ij}\nabla B^{ij} = dB^{\Lambda\Sigma} + E^\Lambda_I E^\Sigma_J P^{[IJ]}_{ij}k^{ij\Gamma\alpha}A_{\Gamma\alpha} \equiv dB^{\Lambda\Sigma} + f^{\Lambda\Sigma\Gamma\alpha}A_{\Gamma\alpha} \qquad (3.37)$$

where $f^{\Lambda\Sigma\Gamma\alpha}$ are numerical constants.
Therefore, the gauged connection affects only $\widehat{\omega}$ and not ω and we have

$$\widehat{\omega}^{IJ}_{gauged} = -\frac{1}{2} E^I_\Lambda \nabla B^{\Lambda\Sigma} E^J_\Sigma = \widehat{\omega}^{IJ} - \frac{1}{2} E^I_\Lambda f^{\Lambda\Sigma\Gamma\alpha} A_{\Gamma\alpha} E^J_\Sigma \tag{3.38}$$

Therefore, if we take the Bianchi identities of the gravitino

$$\nabla \rho_A + \frac{1}{4} R^{ab} \gamma_{ab} \psi_A + \frac{1}{4} R_A{}^B(\omega_1) \psi_B = 0 \tag{3.39}$$

where ω_1 is the composite connection of the $SU(4) \sim SO(6)$ R-symmetry acting on the gravitino $SU(4)$ index, and

$$R_A{}^B = R^{IJ} (\Gamma_{IJ})_A{}^B \tag{3.40}$$

then, since

$$\omega_1 = \omega_d + \widehat{\omega}_{gauged} \tag{3.41}$$

the gauged $SU(4)$ curvatures becomes

$$R_A{}^B(\omega_{1gauged}) = R_A{}^B(\omega_d + \widehat{\omega}_{gauged}) = R_A{}^B(\omega_1) - \frac{1}{2} (\Gamma_{IJ})_A{}^B E^I_\Lambda f^{\Lambda\Sigma\Gamma\alpha} dA_{\Gamma\alpha} E^J_\Sigma \tag{3.42}$$

As for the supersymmetry transformations (3.27)we do not report here the procedure used to determine the fermion shifts in the gauged Bianchi identities which, as mentioned before, will be given elsewhere. It is sufficient to say that, according to a well known procedure, the cancellation of the extra term appearing in (3.42) requires an extra term in the superspace parametrization of the gravitino curvature. This in turn implies a modification of the space-time supersymmetry transformation law of the gravitino, obtained by adding the following extra term to $\delta\psi_{A\mu}$:

$$\delta\psi^{(shift)}_{A\mu} = S_{AB} \gamma_\mu \varepsilon^B = -\frac{i}{48} \overline{L}^\alpha f^{IJK}_\alpha (\Gamma_{IJK})_{AB} \gamma_\mu \epsilon^B \tag{3.43}$$

where we have defined $f^{IJK\alpha} = f^{\Lambda\Sigma\Gamma\alpha} E^I_\Lambda E^J_\Sigma E^K_\Gamma$ and the symmetric matrix S_{AB} is (one-half) the gravitino mass matrix entering the Lagrangian.
Recalling the selfduality relation $\Gamma_{IJK} = \frac{i}{3!} \epsilon_{IJKLMN} \Gamma^{LMN}$ and introducing the quantities

$$F^{\pm\, IJK} = \frac{1}{2} \left(F^{IJK} \pm i\, {}^* F^{IJK} \right) \tag{3.44}$$

where

$$F^{IJK} = L^\alpha f^{IJK}_\alpha, \qquad \overline{F}^{IJK} = \overline{L}^\alpha f^{IJK}_\alpha \qquad (F^{\pm\, IJK})^* = \overline{F}^{\mp IJK} \tag{3.45}$$

the gravitino gauge shift can be rewritten as

$$\delta\psi^{(shift)}_{A\mu} = S_{AB} \gamma_\mu \varepsilon^B = -\frac{i}{48} \overline{F}^{-IJK} (\Gamma_{IJK})_{AB} \gamma_\mu \epsilon^B \tag{3.46}$$

Analogous computations in the Bianchi identities of the left handed gaugino and dilatino fields give the following extra shifts

$$\delta\chi^{A\,(shift)} = N^{AB} \epsilon_B = -\frac{1}{48} \overline{L}^\alpha f_{IJK\alpha} (\Gamma^{IJK})^{AB} \epsilon_B = -\frac{1}{48} \overline{F}^+_{IJK} (\Gamma^{IJK})^{AB} \epsilon_B \tag{3.47}$$

$$\delta\lambda^{(shift)I}_A = Z^{I\,B}_A \epsilon_B = \frac{1}{8} L^\alpha f_{IJK\alpha} (\Gamma^{JK})_A{}^B \epsilon_B = \frac{1}{8} F_{IJK} (\Gamma^{JK})_A{}^B \epsilon_B \tag{3.48}$$

These results agree, apart from normalizations, with reference [35].

4 The scalar Potential

The Ward identity of supersymmetry [5], [7].

$$V\,\delta^B_A = -12 S_{AC}\overline{S}^{CB} + 4 N_{AC}\overline{N}^{CB} + 2 Z^{IC}{}_A Z^I_C{}^B \tag{4.49}$$

allows us to compute the scalar potential from the knowledge of the fermionic shifts S_{AB}, N^{AB}, Z^{IB}_A computed before, equations (3.43), (3.47), (3.48).[2]. We obtain:

$$\begin{aligned} V &= \frac{1}{24}(L^\alpha \overline{L}^\beta f^{IJK}_\alpha f_{IJK\beta} - \frac{1}{2}\epsilon^{\alpha\beta} f^{IJK}_\alpha {}^* f_{IJK\beta}) = \\ &= \frac{1}{24}\left(L^\alpha \overline{L}^\beta \mathcal{N}_{\Lambda\Pi}\mathcal{N}_{\Sigma\Delta}\mathcal{N}_{\Gamma\Omega} f^{\Lambda\Sigma\Gamma}_\alpha f^{\Pi\Delta\Omega}_\beta - \frac{1}{2}\epsilon^{\alpha\beta} f^{\Lambda\Sigma\Gamma}_\alpha {}^* f_{\Lambda\Sigma\Gamma\beta}\, det(E)\right) \end{aligned} \tag{4.50}$$

where we have made explicit the dependence on the $GL(6)/SO(6)$ scalar fields and $\mathcal{N}_{\Lambda\Sigma}$ is defined by

$$\mathcal{N}_{\Lambda\Sigma} = E^I_\Lambda E^I_\Sigma \tag{4.51}$$

Another useful form of the potential, which allows the discussion of the extrema in a simple way is to rewrite equation (4.50) as follows

$$V = \frac{1}{48} L^\alpha \overline{L}^\beta (f_{\alpha IJK} - i\,{}^* f_{\alpha IJK})(f^{IJK}_\beta + i\,{}^* f^{IJK}_\beta) = \frac{1}{12}|F^{-IJK}|^2 \tag{4.52}$$

where we have used equations (3.44), (3.45).
From (4.52) we see that the potential has an absolute minimum with vanishing cosmological constant when $F^{-IJK} = 0$.
In order to have a theory with vanishing cosmological constant the two $SL(2,\mathbb{R})$ components of $f^{\alpha\Lambda\Sigma\Gamma}$ cannot be independent. A general solution of $F^{-IJK} = 0$ is given by setting:

$$f^{-\Lambda\Sigma\Delta}_1 = i\alpha f^{-\Lambda\Sigma\Delta}_2 \tag{4.53}$$

where α is a complex constant. In real form we have :

$$f^{\Lambda\Sigma\Delta}_1 = +\Re\alpha\, {}^* f^{\Lambda\Sigma\Delta}_2 - \Im\alpha f^{\Lambda\Sigma\Delta}_2 \tag{4.54}$$

. The solution of (4.53) is :

$$L^\alpha = +i\alpha L^\beta \epsilon^{\alpha\beta} \longrightarrow \frac{L_2}{L_1} = -i\alpha \Longrightarrow \frac{\phi_2}{\phi_1} = \frac{1-\alpha}{1+\alpha} \tag{4.55}$$

In the particular case $\alpha = 1$, (4.53) reduces to

$$f^{\alpha\Lambda\Sigma\Delta} = \frac{1}{3!}\epsilon^{\alpha\beta}\epsilon^{\Lambda\Sigma\Delta\Gamma\Pi\Omega} f^{\beta\Gamma\Pi\Omega} \Longrightarrow f^{\Lambda\Sigma\Delta}_1 = -{}^* f^{\Lambda\Sigma\Delta}_2 \Longrightarrow f^{-\Lambda\Sigma\Delta}_1 = i f^{-\Lambda\Sigma\Delta}_2 \tag{4.56}$$

which is the constraint imposed in reference [35]. In this case the minimum of the scalar potential is given by

$$\phi^2 = 0 \Longrightarrow |\phi^1| = 1 \tag{4.57}$$

[2] The complete gauged Lagrangian will be given elsewhere

or, in terms of the L^α fields, $L^1 = \frac{1}{\sqrt{2}}$, $L^2 = -\frac{i}{\sqrt{2}}$.
If we take a configuration of the $GL(6)/SO(6)$ fields where all the fields $a^m = 0$ except the three fields φ_1, φ_2, φ_3, parametrizing $O(1,1)^3$, then the matrix E^I_Λ has the form given in the section 6 (equation (6.117)) and in this case

$$f^{IJK}_\alpha = e^{-(\varphi_1+\varphi_2+\varphi_3)} f^{\Lambda\Sigma\Gamma}_\alpha \tag{4.58}$$

so that

$$V(\phi_1, \phi_2, \varphi_1, \varphi_2, \varphi_3; a^m = 0) = \frac{1}{12} e^{-2(\varphi_1+\varphi_2+\varphi_3)} |F^{-\Lambda\Sigma\Gamma}|^2 \tag{4.59}$$

The minimum condition can be also retrieved in the present case by observing that in the case $\alpha = 1$ the potential takes the simple form

$$V = \frac{1}{48}(L^\alpha \overline{L}^\beta - \frac{1}{2}\delta_{\alpha\beta}) f^{IJK}_\alpha f_{IJK\beta} \tag{4.60}$$

Using the explicit form of f^{IJK} as given in the next section (equations (5.73), (5.74)), the potential becomes

$$\begin{aligned} V(\phi_1, \phi_2, \varphi_1, \varphi_2, \varphi_3; a^m = 0) &= \frac{1}{192} \sum m_i^2 e^{-(2\varphi_1+2\varphi_2+2\varphi_3)} (L^\alpha \overline{L}^\alpha - 1) \\ &= \frac{1}{96} \sum m_i^2 e^{-(2\varphi_1+2\varphi_2+2\varphi_3)} |\phi_2|^2. \end{aligned} \tag{4.61}$$

Therefore $V = 0$ implies

$$|L^1|^2 + |L^2|^2 \equiv |\phi_1|^2 + |\phi_2|^2 = 1 \tag{4.62}$$

which is satisfied by equation (4.57) where we have taken into account equation (2.18).
Note that equation (4.62), (4.57) giving the extremum of the potential, fixes the dilaton field. On the contrary, the extremum of the potential with respect to φ_1, φ_2, φ_3, doesn't fix these fields, since the corresponding extremum gives the condition $V \equiv 0$.
This shows the no–scale structure of the model. The a^m fields are instead stabilized at $a^m = 0$. All the corresponding modes get masses (for $N = 1$, 0)
When $\alpha \neq 1$, a simple solution of equation (4.53) is to take $f_1^{-\Lambda\Sigma\Delta}$ non vanishing only for a given value of $\Lambda\Sigma\Delta$, e.g $\Lambda = 1$, $\Sigma = 2$, $\Delta = 3$. Then we have a four real parameter solution in term of f^{123}_α and f^{456}_α as in reference [42]. The general solution, contains, besides α, four complex parameters, since $f_1^{\Lambda\Sigma\Delta}$ has at most eight non vanishing components.
In string theory, f_1 and f_2 satisfy some quantization conditions which restrict the value of α [42], [43].
It is interesting to see what is the mechanism of cancellation of the negative contribution of the gravitino shift to the potential which makes it positive semidefinite. For this purpose it is useful to decompose the gaugino shift (3.48) in the 24 dimensional representation of $SU(4)$ into its irreducible parts $\mathbf{20} + \overline{\mathbf{4}}$. Setting:

$$\lambda^I_A = \lambda^{I\,(20)}_A + \frac{1}{6}(\Gamma_I)_{AB} \lambda^{B(4)} \tag{4.63}$$

where

$$\lambda^{A(4)} = (\Gamma_I)^{AB} \lambda^I_B; \qquad (\Gamma_I)^{AB} \lambda^{I\,(20)}_B = 0 \tag{4.64}$$

we get

$$\delta\lambda_A^{I\,(20)} = \frac{1}{8}\left(F^{IJK}(\Gamma_{JK})_A^{\ B} + \frac{1}{6}F^{+JLM}(\Gamma^I)_{AC}(\Gamma_{JLM})^{CB}\right)\epsilon_B \tag{4.65}$$

$$\delta\lambda^{A(4)} = \frac{1}{8}F^{+IJK}(\Gamma_{IJK})^{AB}\epsilon_B \tag{4.66}$$

Performing some Γ–matrix algebra, equation (4.65) reduces to

$$\begin{aligned}\delta\lambda_A^{I\,(20)} &= Z_A^{I(20)\ B}\epsilon_B \\ Z_A^{I\,(20)\ B} &= +\frac{1}{16}(F^{IJK} - i\,{}^*F^{IJK})(\Gamma_{JK})_A^{\ B} = \frac{1}{8}F^{-IJK}(\Gamma_{JK})_A^{\ B}\end{aligned} \tag{4.67}$$

In this way the irreducible parts of the fermion shifts are all proportional to $F^{\pm IJK}\Gamma_{IJK}$,namely:

$$\delta\psi_{A\mu}^{(shift)} = S_{AB}\gamma_\mu\varepsilon^B = -\frac{i}{48}\overline{F}^{IJK-}(\Gamma_{IJK})_{AB}\gamma_\mu\epsilon^B \tag{4.68}$$

$$\delta\chi^{A\,(shift)} = N^{AB}\epsilon_B = -\frac{1}{48}\overline{F}^{IJK+}(\Gamma_{IJK})^{AB}\epsilon_B \tag{4.69}$$

$$\delta\lambda^{A\,(shift)(4)} = Z^{(4)AB}\epsilon_B = \frac{1}{8}F^{IJK+}(\Gamma_{IJK})^{AB}\epsilon_B \tag{4.70}$$

$$\delta\lambda_A^{I\,(shift)(20)} = Z_A^{I(20)\,B}\epsilon_B = \frac{1}{8}F^{IJK-}(\Gamma_{JK})_A^{\ B}\epsilon_B. \tag{4.71}$$

When one traces the indices AB in (4.49) one sees that the contributions from the gravitino shifts and from the $\overline{\mathbf{4}}$ of the gaugino shifts are both proportional to $|F^{-IJK}|^2$ and since on general grounds they have opposite sign, they must cancel against each other. Viceversa the square of the gaugino shift in the **20** representation and the square of the dilatino shift are both proportional to $|F^{-IJK}|^2$ that is to the scalar potential. Indeed

$$Z^{(20)IB}_{\ \ \ \ \ A}\, Z_B^{(20)I\ A} = 6N^{AB}N_{BA} = \frac{1}{64}\left(L^\alpha\overline{L}^\beta f_\alpha^{IJK} f_{IJK\beta} - \frac{1}{2}\epsilon^{\alpha\beta} f_\alpha^{IJK\ *} f_{IJK\beta}\right) = \frac{3}{8}V. \tag{4.72}$$

It then follows that the $\chi^{A(4)}$ are the four Goldstone fermions of spontaneously broken supergravity. These degrees of freedom are eaten by the four massive gravitini in the superHiggs mechanism. This cancellation, reflects the no–scale structure of the orientifold model as discussed in references [42], [43]. It is the same kind of cancellation of F–and D–terms against the negative (gravitino square mass) gravitational contribution to the vacuum energy that occurs in Calabi–Yau compactification with brane fluxes turned on [38], [41], [48].

5 Mass Spectrum of the Gravitini and Vector Fields

It is clear that not all the $f^{\Lambda\Sigma\Gamma\alpha}$ are different from zero; indeed we have only twelve vectors which can be gauged, while the axion field $B^{\Lambda\Sigma}$ has fifteen components. Therefore, some of the components of the axion field must be invariant under the gauging. From equation (3.37) one easily realizes that the components B^{14}, B^{25}, B^{36} are inert under gauge transformations. This can be ascertained using the explicit form of $f^{IJK\alpha}$

$$
\begin{aligned}
f_1^{IJK} = \quad & f^{123}\delta^{[IJK]}_{1\,2\,3} + f^{156}\delta^{[IJK]}_{1\,5\,6} + f^{246}\delta^{[IJK]}_{2\,4\,6} + f^{345}\delta^{[IJK]}_{3\,4\,5} + \\
& + f^{456}\delta^{[IJK]}_{4\,5\,6} + f^{234}\delta^{[IJK]}_{2\,3\,4} + f^{135}\delta^{[IJK]}_{1\,3\,5} + f^{126}\delta^{[IJK]}_{1\,2\,6} \qquad (5.73) \\
f_2^{IJK} = \quad & \frac{1}{|\alpha|^2}\left(-\Re\alpha\ {}^*f_1^{IJK} - \Im\alpha f_1^{IJK}\right) \xrightarrow{\alpha=1} f_2^{IJK} = -{}^*f_1^{IJK} \qquad (5.74)
\end{aligned}
$$

which implies

$$
f^{14k} = f^{25k} = f^{36k} = 0, \quad \forall k \tag{5.75}
$$

Let us now compute the masses of the gravitini. As we have seen in the previous section the extremum of the scalar potential is given by

$$
F^{-IJK} = L^1 f_1^{-IJK} + L^2 f_2^{-IJK} = 0 \tag{5.76}
$$

It follows that the gravitino mass matrix S_{AB} at the extremum takes the values

$$
\begin{aligned}
S_{AB}^{(extr)} &= -\frac{i}{48}\left(\overline{L}^1 f_1^{-IJK} + \overline{L}^2 f_2^{-IJK}\right)(\Gamma_{IJK})_{AB} \qquad (5.77) \\
&= -\frac{i}{48\ L^2}\left(L^2\overline{L}^1 - L^1\overline{L}^2\right) f_1^{-IJK}\Gamma_{IJK} = \frac{1}{48\ L^2} f_1^{-IJK}\Gamma_{IJK} \qquad (5.78)
\end{aligned}
$$

From (5.77) we may derive an expression for the gravitino masses at the minimum of the scalar potential very easily, going to the reference frame where S_{AB} is diagonal. Indeed it is apparent that this corresponds to choose the particular frame corresponding to the diagonal Γ_{IJK}. As it is shown in the Appendix, the diagonal Γ_{IJK} correspond to Γ_{123}, Γ_{156}, Γ_{246} Γ_{345} and their dual.

It follows that the four eigenvalues $\mu_i + i\mu_i^{'}$, $i = 1, \ldots, 4$ of S_{AB} are:

$$
\begin{aligned}
\mu_1 + i\mu_1^{'} &= \frac{1}{24L^2}(f^{-123} - f^{-156} - f^{-246} + f^{-345}) \\
\mu_2 + i\mu_2^{'} &= \frac{1}{24L^2}(f^{-123} + f^{-156} + f^{-246} + f^{-345}) \\
\mu_3 + i\mu_3^{'} &= \frac{1}{24L^2}(f^{-123} + f^{-156} - f^{-246} - f^{-345}) \\
\mu_4 + i\mu_4^{'} &= \frac{1}{24L^2}(f^{-123} - f^{-156} + f^{-246} - f^{-345}) \qquad (5.79)
\end{aligned}
$$

Here we have set $f_1^{-IJK} \equiv f^{-IJK}$.
Furthermore L^2, computed at the extremum (see equation (4.55)) is a function of α.
The gravitino mass squared m_i^2 are given by $m_i^2 = \mu_i^2 + \mu_i^{'2}$.
The above results, (5.79), (5.75) take a more elegant form by observing that if use a complex basis:

$$
e_1 + ie_4 = E_x; \quad e_2 + ie_5 = E_y; \quad e_3 + ie_6 = E_z \tag{5.80}
$$

$$
e_1 - ie_4 = \overline{E}_x; \quad e_2 - ie_5 = \overline{E}_y; \quad e_3 - ie_6 = \overline{E}_z \tag{5.81}
$$

the tensor $f_{IJK1} \equiv f_{IJK}$ takes the following components:

$$f_{xyz} = \frac{1}{8}\{f_{123} - f_{156} + f_{246} - f_{345} + i\,({}^*f_{123} -^* f_{156} +^* f_{246} -^* f_{345})\} \quad (5.82)$$
$$f_{x\overline{y}\overline{z}} = \frac{1}{8}\{f_{123} - f_{156} - f_{246} + f_{345} + i\,({}^*f_{123} -^* f_{156} -^* f_{246} +^* f_{345})\} \quad (5.83)$$
$$f_{xy\overline{z}} = \frac{1}{8}\{f_{123} + f_{156} - f_{246} - f_{345} + i\,({}^*f_{123} +^* f_{156} -^* f_{246} -^* f_{345})\} \quad (5.84)$$
$$f_{x\overline{y}k} = \frac{1}{8}\{f_{123} + f_{156} + f_{246} + f_{345} + i\,({}^*f_{123} +^* f_{156} +^* f_{246} +^* f_{345})\} \quad (5.85)$$

while

$$f^{x\overline{x}y} = f^{x\overline{x}z} = f^{y\overline{y}x} = f^{y\overline{y}z} = f^{z\overline{z}x} = f^{z\overline{z}y} = 0 \quad (5.86)$$

Therefore, the twenty entries of $f_1^{\Lambda\Sigma\Delta}$ are reduced to eight.
In this holomorphic basis the gravitino mass eigenvalues assume the rather simple form:

$$\mu_1 + i\mu_1^{'} = \frac{1}{6L^2} f_{x\overline{y}\overline{z}} \quad (5.87)$$
$$\mu_2 + i\mu_2^{'} = \frac{1}{6L^2} f_{x\overline{y}z} \quad (5.88)$$
$$\mu_3 + i\mu_3^{'} = \frac{1}{6L^2} f_{xy\overline{z}} \quad (5.89)$$
$$\mu_4 + i\mu_4^{'} = \frac{1}{6L^2} f_{xyz} \quad (5.90)$$

(Note that the role of $\mu_1+i\mu_1^{'}$, $\mu_2+i\mu_2^{'}$, $\mu_3+i\mu_3^{'}$, $\mu_4+i\mu_4^{'}$ can be interchanged by changing the definition of the complex structure, (5.80), that is permuting the roles of $E_{x,y,z}$ and $\overline{E}_{x,y,z}$).

Let us now compute the masses of the 12 vectors. We set here for simplicity $\alpha = 1$. Taking into account that the mass term in the vector equations can be read from the kinetic term of the vectors and of the axions in the Lagrangian, namely

$$2L^{\alpha}\overline{L}^{\beta}\mathcal{N}^{\Lambda\Sigma}\mathcal{F}^{\mu\nu}_{\Lambda\alpha}\mathcal{F}_{\Sigma\beta\mu\nu} + \mathcal{N}_{\Lambda\Gamma}\mathcal{N}_{\Sigma\Delta}(\partial_\mu B^{\Lambda\Sigma} + f^{\Lambda\Sigma\Omega\alpha}A_{\Omega\alpha\mu})(\partial^\mu B^{\Gamma\Delta} + f^{\Gamma\Delta\Pi\beta}A^{\mu}_{\Pi\beta}) \quad (5.91)$$

where $\mathcal{N}^{\Lambda\Sigma} \equiv E^{\Lambda}_I\, E^{\Sigma}_I$ is the kinetic matrix of the vectors and $\mathcal{N}_{\Lambda\Sigma} = (\mathcal{N}^{-1})^{\Lambda\Sigma}$. At zero scalar fields ($E^{\Lambda}_I = \delta^{\Lambda}_I, \quad \alpha = 1 \longrightarrow (L^1,\, L^2) = \frac{1}{\sqrt{2}}(1, -i)$) the vector equation of motion gives a square mass matrix proportional to $Q_{\Lambda\alpha,L\Sigma\beta}$:

$$Q_{\Lambda\alpha,\Sigma\beta} = f_{\Gamma\Delta(\alpha\Lambda}f_{\beta\Sigma)\Gamma\Delta} \quad (5.92)$$

which is symmetric in the exchange $\Lambda\alpha \longleftrightarrow \Sigma\beta$. The eigenvalues of $Q_{\Lambda\alpha,\Sigma\beta}$ can be easily computed and we obtain that they are twice degenerate. In terms of the four quantities

$$\ell_0 = (m_1^2 + m_2^2 + m_3^2 + m_4^2)$$
$$\ell_1 = (m_1^2 - m_2^2 - m_3^2 + m_4^2)$$
$$\ell_2 = (m_1^2 - m_2^2 + m_3^2 - m_4^2)$$
$$\ell_3 = (m_1^2 + m_2^2 - m_3^2 - m_4^2)$$

the six different values turn out to be proportional to:

$$\ell_0 + \ell_1 = 2(m_1^2 + m_4^2); \qquad \ell_0 - \ell_1 = 2(m_2^2 + m_3^2) \tag{5.93}$$
$$\ell_0 + \ell_2 = 2(m_1^2 + m_3^2); \qquad \ell_0 - \ell_2 = 2(m_2^2 + m_4^2) \tag{5.94}$$
$$\ell_0 + \ell_3 = 2(m_1^2 + m_2^2); \qquad \ell_0 - \ell_3 = 2(m_3^2 + m_4^2) \tag{5.95}$$

Note that for $N = 3, 2$ six and two vectors are respectively massless, according to the massless sectors of these theories described in Section 6.

6 Reduction to lower Supersymmetry $N = 4 \longrightarrow N = 3,\ 2,\ 1,\ 0$

Since the supergravities with $1 \leq N < 4$ are described by σ–models possessing a complex structure, it is convenient to rewrite the scalar field content of the $N = 4$ theory in complex coordinates as already done for the computation of the gravitino masses.
We recall that we have 36 scalar fields parametrizing $SO(6,6)/SO(6) \times SO(6)$ that have been split into 21 fields $g_{IJ} = g_{JI}$ parametrizing the coset $GL(6)/SO(6)$ plus 15 axions $B_{IJ} = -B_{JI}$ parametrizing the translations. As we have already observed, since we have only 12 vectors, the three axions B_{14}, B_{15}, B_{36} remain inert under gauge transformations. When we consider the truncation to the $N = 3$ theory we expect that only 9 complex scalar fields become massless moduli parametrizing $SU(3,3)/SU(3) \times SU(3) \times U(1)$. Moreover, it is easy to see that if we set e.g. $\mu_1 = \mu_2 = \mu_3 = 0$ ($\mu_1^{'} = \mu_2^{'} = \mu_3^{'} = 0$)which implies $f_{345} = f_{156} = -f_{123} = -f_{246}$ (${}^*f_{345} ={}^* f_{156} = -{}^*f_{123} = -{}^*f_{246}$) in the $N = 3$ theory, we get that also the 6 fields $B_{12} - B_{45}$, $B_{13} - B_{46}$, $B_{24} - B_{15}$, $B_{34} + B_{16}$, $B_{23} + B_{56}$, $B_{35} + B_{26}$ are inert under gauge transformations.
We may take advantage of the complex structure of this manifold, by rotating the real frame $\{e_I\}$, $I = 1 \ldots 6$ to the complex frame defined in (5.80). In this frame we have the following decomposition for the scalar fields in terms of complex components:

$$B_{IJ} \longrightarrow B_{ij},\ B_{i\bar{\jmath}},\ B_{\bar{\imath}j},\ B_{\bar{\imath}\bar{\jmath}} \tag{6.96}$$
$$g_{IJ} \longrightarrow g_{ij},\ g_{i\bar{\jmath}},\ g_{\bar{\imath}j},\ g_{\bar{\imath}\bar{\jmath}} \tag{6.97}$$
$$\tag{6.98}$$

In presence of the translational gauging, the differential of the axionic fields become covariant and they are obtained by the substitution:

$$dB_{ij} \to dB_{ij} + (\Re f_{ijk})A^{k1} + (\Re f_{ij\bar{k}})A^{\bar{k}1} + (\Im f_{ijk})A^{k2} + (\Im f_{ij\bar{k}})A^{\bar{k}2} \tag{6.99}$$
$$dB_{i\bar{\jmath}} \to dB_{i\bar{\jmath}} + (\Re f_{i\bar{\jmath}k)1}A^{k1} + (\Re f_{i\bar{\jmath}\bar{k})}A^{\bar{k}1} + (\Im f_{i\bar{\jmath}k)1}A^{k2} + (\Im f_{i\bar{\jmath}\bar{k})}A^{\bar{k}2} \tag{6.100}$$

Since in the $N = 4 \longrightarrow N = 3$ truncation the only surviving massless moduli fields are $B_{i\bar{\jmath}} + ig_{i\bar{\jmath}}$, then the 3+3 axions $\{B_{ij},\ B_{\bar{\imath}\bar{\jmath}}\}$ must become massive, while $\delta B_{i\bar{\jmath}}$ must be zero. We see from equation (6.100) we see that we must put to zero the components

$$f_{i\bar{\jmath}k} = f_{i\bar{\jmath}\bar{k}} = f_{ij\bar{k}} = 0 \tag{6.101}$$

while

$$f_{ijk} \equiv f\epsilon_{ijk} \neq 0 \tag{6.102}$$

Looking at the equations (5.87) we see that these relations are exactly the same which set $\mu_1 + i\mu_1^{'} = \mu_2 + i\mu_2^{'} = \mu_3 + i\mu_3^{'} = 0$ and $\mu_4 + i\mu_4^{'} \neq 0$, confirming that the chosen complex structure corresponds to the $N = 3$ theory. Note that the corresponding $g_{i\overline{\jmath}}$ fields partners of $B_{i\overline{\jmath}}$ in the chosen complex structure parametrize the coset $O(1,1) \times SL(3,\mathbb{C})/SU(3)$. Actually the freezing of the holomorphic g_{ij} gives the following relations among the components in the real basis of g_{IJ}:

$$g_{14} = g_{25} = g_{36} = 0 \tag{6.103}$$
$$g_{11} - g_{44} = 0, \quad g_{22} - g_{55} = 0, \quad g_{33} - g_{66} = 0 \tag{6.104}$$
$$g_{12} - g_{45} = 0, \quad g_{13} - g_{46} = 0, \quad g_{23} - g_{56} = 0 \tag{6.105}$$
$$g_{15} + g_{24} = 0, \quad g_{16} + g_{34} = 0, \quad g_{26} + g_{35} = 0 \tag{6.106}$$

The freezing of the axions B_{ij} in the holomorphic basis give the analogous equations:

$$B_{12} - B_{45} = 0, \quad B_{13} - B_{46} = 0, \quad B_{23} - B_{56} = 0 \tag{6.107}$$
$$B_{15} + B_{42} = 0, \quad B_{16} + B_{43} = 0, \quad B_{26} + B_{53} = 0 \tag{6.108}$$
$$B_{14} = B_{25} = B_{36} = 0 \tag{6.109}$$

The massless $g_{i\overline{\jmath}}$ and $B_{i\overline{\jmath}}$ are instead given by the following combinations:

$$g_{x\overline{x}} = \frac{1}{2}(g_{11} + g_{44}), \quad g_{y\overline{y}} = \frac{1}{2}(g_{22} + g_{55}), \quad g_{z\overline{z}} = \frac{1}{2}(g_{33} + g_{66}) \tag{6.110}$$

$$B_{x\overline{x}} = \frac{i}{2}B_{14}, \quad B_{y\overline{y}} = \frac{i}{2}B_{25}, \quad B_{z\overline{z}} = \frac{i}{2}B_{36} \tag{6.111}$$

$$g_{x\overline{y}} = \frac{1}{2}(g_{12} + ig_{15}), \quad g_{x\overline{z}} = \frac{1}{2}(g_{13} + ig_{16}), \quad g_{y\overline{z}} = \frac{1}{2}(g_{23} + ig_{26}) \tag{6.112}$$
$$B_{x\overline{y}} = \frac{1}{2}(B_{12} + iB_{15}), \quad B_{x\overline{z}} = \frac{1}{2}(B_{13} + iB_{16}), \quad B_{y\overline{z}} = \frac{1}{2}(B_{23} + iB_{26}) \tag{6.113}$$
$$B_{xx} = B_{yy} = B_{zz} = 0 \tag{6.114}$$

Let us now consider the reduction $N = 4 \longrightarrow N = 2$ for which the relevant moduli space is $SU(2,2)/\left(SU(2) \times SU(2) \times U(1)\right) \otimes SU(1,1)/U(1)$. Setting $\mu_2 + i\mu_2^{'} = \mu_3 + i\mu_3^{'} = 0$ we find:

$$f_{x\overline{y}z} = f_{xy\overline{z}} = 0 \tag{6.115}$$

which , in real components implies:

$$f_{123} + f_{156} = 0; \qquad f_{246} + f_{345} = 0 \tag{6.116}$$

and analogous equations for their Hodge dual. This implies that in the $N = 2$ phase two more axions are gauge inert namely $B_{23} + B_{56} = 2B_{23}$ and $B_{26} + B_{35} = 2B_{26}$ or, in holomorphic components, $B_{y\overline{z}}$. The five fields $B_{14}, B_{25}, B_{36}, B_{23}, B_{26}$ parametrize the coset $SO(1,1) \times SO(2,2)/SO(2) \times SO(2)$.
If we now consider the truncation $N = 4 \longrightarrow N = 1$ the relevant coset manifold is $(SU(1,1)/U(1))^3$ which contains 3 complex moduli. To obtain the corresponding complex

structure, it is sufficient to freeze $g_{i\bar{\jmath}}$, $B_{i\bar{\jmath}}$ with $i \neq j$. In particular the $SU(1,1)^3$ can be decomposed into $O(1,1)^3 \otimes_s T_3$ where the three $O(1,1)$ and the three translations T_3 are parametrized by $g_{x\bar{x}}$, $g_{y\bar{y}}$, $g_{z\bar{z}}$ and $B_{x\bar{x}}$, $B_{y\bar{y}}$, $B_{z\bar{z}}$ respectively.
These axions are massless because of equation (5.86) (Note that the further truncation $N = 1 \longrightarrow N = 0$ does not alter the coset manifold $SU(1,1)^3$ since we have no loss of massless fields in this process). In this case we may easily compute the moduli dependence of the gravitino masses. Indeed, $O(1,1)^3$, using equations (6.104), (6.110), will have as coset representative the matrix

$$E^I_\Lambda = \begin{pmatrix} e^{-\varphi_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{-\varphi_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-\varphi_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-\varphi_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-\varphi_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-\varphi_3} \end{pmatrix} \tag{6.117}$$

where we have set $g_{11} = e^{2\varphi_1}$, $g_{22} = e^{2\varphi_2}$, $g_{33} = e^{2\varphi_3}$, the exponentials representing the radii of the manifold $T^2_{(14)} \times T^2_{(25)} \times T^2_{(36)}$.
We see that in the gravitino mass formula (3.43) the vielbein E^I_Λ reduces to the diagonal components of the matrix (6.117) A straightforward computation then gives:

$$S_{AB}\overline{S}^{AB} = \frac{1}{(48)^2} e^{-(2\varphi_1+2\varphi_2+2\varphi_3)} \begin{pmatrix} m_1^2 & 0 & 0 & 0 \\ 0 & m_2^2 & 0 & 0 \\ 0 & 0 & m_3^2 & 0 \\ 0 & 0 & 0 & m_4^2 \end{pmatrix} \tag{6.118}$$

We see that the square of the gravitino masses goes as $\frac{1}{R_1^2 R_2^2 R_3^2}$. Note that this is different from what happens in the Kaluza–Klein compactification, where the gravitino mass square goes as $\frac{1}{R^2 \Im S \Im \tau}$, where $\frac{1}{\Im S} = g^2_{string}$ and the complex structure, $\tau = i$ is a constant, so that $< \mu^2 > \simeq \frac{g^2_{string}}{R^2}$
We note that in the present formulation where we have used a contravariant $B^{\Lambda\Sigma}$ as basic charged fields, the gravitino mass depends on the T^6 volume. However if we made use of the dual 4-form $C_{\Lambda\Sigma\Gamma\Delta}$, as it comes from Type IIB string theory, then the charge coupling would be given in terms of ${}^*f^\alpha_{\Lambda\Sigma\Gamma}$ and the gravitino mass matrix would be trilinear in E^Λ_I instead of E^I_Λ. Therefore all our results can be translated in the new one by replacing $R_i \rightarrow R_i^{-1}$.

7 Conclusions

In this paper we have shown that a non standard form of $N = 4$ supergravity, where the full $SO(6,n)$ symmetry is not manifest, nor even realized linearly on the vector field strengths [53] is the suitable description for a certain class of IIB compactifications in presence of 3–form fluxes. Since the super–Higgs phases of N–extended supergravities solely depend on their gauging, it is crucial here the use of a dual formulation [35] where the linear symmetry acting on the vector fields ($n = 6$) is $GL(6,\mathbb{R}) \times SL(2,\mathbb{R})$ rather than $SO(6,6)$, thus allowing the gauging of a subalgebra T_{12} inside the T_{15} (see equation (1.1)), the latter being a nilpotent abelian subalgebra of $SO(6,6)$.

For a choice of complex structures on $T^6 = T^2 \times T^2 \times T^2$ the four complex gravitino masses are proportional to the $(3,0)$ and three $(2,1)$ fluxes of 3–forms. $N = 3$ supergravity corresponds to setting to zero the three $(2,1)$–form fluxes, $N = 2$ and $N = 1$ supergravities correspond to the vanishing of two or one $(2,1)$–form.
The scalar potential is non negative and given by the square of the supersymmetry variation of the component **20** of the **24** $SU(4)$ (reducible) representation of the six gaugini ($\mathbf{6} \times \mathbf{4} = \mathbf{20} + \overline{\mathbf{4}}$) of the six matter vector multiplets of the $SO(6,6)$ symmetric supergravity. Indeed the positive contribution of the component **4** of the gaugino just cancel in the calculation of the potential the negative contribution of the spin $\frac{3}{2}$ gravitini.
The classical moduli space of the $N = 3$, 2, 1 (or 0) are respectively the following three complex manifold

$$N = 3: \qquad \frac{SU(3,3)}{SU(3) \times SU(3) \times U(1)} \tag{7.119}$$

$$N = 2: \qquad \frac{SU(1,1)}{U(1)} \times \frac{SU(2,2)}{SU(2) \times SU(2) \times U(1)} \tag{7.120}$$

$$N = 1,\, 0: \qquad \left(\frac{SU(1,1)}{U(1)}\right)^3 \tag{7.121}$$

with six, two, or zero massless vector respectively.
Note in particular that the $N = 2 \longrightarrow N = 1$ phases correspond to a spontaneously broken theory with one vector and two hypermultiplets, which is the simplest generalization [22] of the model in [20], [21].
It is curious to observe that the moduli space of the $N = 0$ phase is identical to the moduli space of the $N = 0$ phase of $N = 8$ spontaneously broken supergravity via Scherk–Schwarz dimensional reduction [28]–[30], [31], [53]. The moduli spaces (7.119), (7.120) of the Scherk–Schwarz $N = 8$ dimensional reduced case, occur as $N = 2$ broken phases (depending on the relations among the masses of the gravitini).
The main difference is that in Scherk–Schwarz breaking, the gravitini are $\frac{1}{2} - BPS$ saturated, while here they belong to long massive multiplets [42], [53]. This is related to the fact that the "flat group" which is gauged is abelian in the $N = 4$ (orientifold) theory and non abelian in the Scherk–Schwarz dimensional reduced $N = 8$ theory.
We have considered here the effective of supergravity for the IIB orientifold only for the part responsible for the super–Higgs phases. If one adds n $D3$ branes, that will correspond to add n matter vector multiplets [35] which, however, will not modify the supersymmetry breaking condition. Then, the σ–model of the $N = 3$ effective theory will be $SU(3, 3 + n)/SU(3) \times SU(3 + n) \times U(1)$ [60] and will also contain, as moduli, the "positions" of the n $D3$ branes [42].

Acknowledgements

S. F. would like to thank the Dipartimento di Fisica, Politecnico di Torino for its kind hospitality during the completion of this work.
R. D'A. would like to thank the Theoretical division of CERN for its kind hospitality during the completion of this work.
Work supported in part by the European Community's Human Potential Program under

contract HPRN-CT-2000-00131 Quantum Space-Time, in which R. D'A. and S. V. are associated to Torino University. The work of S. F. has also been supported by the D.O.E. grant DE-FG03-91ER40662, Task C.

Appendix A: The $SU(4)$ Gamma–matrices

We have used the following $(\Gamma^I)_{AB} = -(\Gamma^I)_{BA}$-matrix representation

$$\begin{aligned}
\Gamma^1 &= \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \qquad \Gamma^4 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \\
\Gamma^2 &= \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \qquad \Gamma^5 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix} \\
\Gamma^3 &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \qquad \Gamma^6 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix}
\end{aligned} \tag{8.1}$$

while

$$(\Gamma^I)^{AB} = (\overline{\Gamma}^I)_{AB} = \frac{1}{2}\epsilon^{ABCD}(\Gamma^I)_{CD} \tag{8.2}$$

Note that

$$(\Gamma^{IJ})_A{}^B = \frac{1}{2}\left[(\Gamma^{[I})_{AC}(\Gamma^{J]})^{CB}\right] \tag{8.3}$$

$$(\Gamma^{IJK})_{AB} = \frac{1}{3!}\left[(\Gamma^I)_{AC}(\Gamma^J)^{CD}(\Gamma^K)_{DB} + perm.\right] \tag{8.4}$$

Here the matrices Γ^{IJK} are symmetric and satisfy the relation

$$\Gamma^{IJK} = \frac{i}{6}\varepsilon^{IJKLMN}\Gamma^{LMN} \tag{8.5}$$

In this representation, the following matrices are diagonal:

$$\Gamma^{123} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \Gamma^{156} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$
$$\Gamma^{246} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \Gamma^{345} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{8.6}$$

as well as the matrices Γ^{456}, Γ^{234}, Γ^{135} and Γ^{126} related with them through the relation (8.5)[3].

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