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A NEW TECHNIQUE TO COMPUTE LONG-RANGE WAKEFIELDS IN ACCELERATING STRUCTURES

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A new technique is proposed to compute the coupling impedances and the long-range wakefields based on a scattering-matrix formalism which relies heavily on post-processed data from the commercial finite-element code HFSS. To illustrate the speed of this technique, the procedures to compute the long-range wakefields of conventional constant-impedance structures and of structures damped with waveguides are presented. The efficiency and accuracy of the technique is achieved because the characteristics of periodic structures can be computed using single-cell data. Damping and synchronism effects are determined from such a computation.

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A new technique is proposed to compute the coupling impedances and the long-range wakefields based on a scattering-matrix formalism which relies heavily on post-processed data from the commercial finite-element code HFSS. To illustrate the speed of this technique, the procedures to compute the long-range wakefields of conventional constant-impedance structures and of structures damped with waveguides are presented. The efficiency and accuracy of the technique is achieved because the characteristics of periodic structures can be computed using single-cell data. Damping and synchronism effects are determined from such a computation.

1 INTRODUCTION

For future linear colliders, very strict constraints are placed on the long- and short-range wakefields in the high-gradient normal-conducting accelerating structures. These requirements have resulted for CLIC in the development of a heavily damped and detuned accelerating structure (TDS) [1]. The method used to compute the transverse wakefields was based on a circuit model [2] and on time-domain computations performed with the code MAFIA. This paper discusses a technique fundamentally different from the previous approaches which is based on the computation of the propagation constants and of the associated travelling-wave field patterns of an infinitely long periodic structure [3].

This paper is organized as follows. The second section describes how to compute the coupling impedances of a lossless periodic structure from the travelling-wave normal modes. An analytical expression in which the group velocity at the synchronism points appears explicitely is obtained for the longitudinal coupling impedance. In the third section, the dispersive properties associated with the reference cell of the TDS are given and an intuitive approach to compute its wake potential is presented. A final section makes some concluding remarks.

2 COUPLING IMPEDANCES OF A LOSSLESS PERIODIC STRUCTURE

Consider a point charge q travelling in a lossless periodic accelerating structure along the z-axis at (x_0, y_0) with velocity c. The current density is then: $\tilde{J}_z(\mathbf{r}_\perp, z; t) = qc\delta(z - ct)\delta(x - x_0)\delta(y - y_0)$. In the frequency domain,

$J_z(\mathbf{r}_\perp, z; \omega) = q\delta(x - x_0)\delta(y - y_0)\exp(-jk_0 z)$ where $k_0 = \omega/c$. The longitudinal coupling impedance is then [4]:

$$Z_\parallel(\omega) = -\frac{1}{q} \int_{-\infty}^{+\infty} E_z(z; \omega)e^{jk_0 z} \, dz,$$

where the integration path is along the coordinates (x_0, y_0). The fields $(\mathbf{E}(\mathbf{r}_\perp, z; \omega), \mathbf{H}(\mathbf{r}_\perp, z; \omega))$ radiated by the point charge can be written in the form of a series of forward- and backward-travelling waves [5]:

$$
\begin{aligned}
\mathbf{E} &= \sum_m C_m^+ \mathbf{E}_m^+ + C_m^- \mathbf{E}_m^-, \\
\mathbf{H} &= \sum_m C_m^+ \mathbf{H}_m^+ + C_m^- \mathbf{H}_m^-,
\end{aligned}
$$

where $\mathbf{E}_m^- \equiv \mathbf{E}_m^-(\mathbf{r}_\perp, z; \omega)$ and $\mathbf{E}_m^+ \equiv \mathbf{E}_m^+(\mathbf{r}_\perp, z; \omega)$ are respectively the backward- and forward-travelling electric eigenfields of the periodic structure, $\mathbf{H}_m^- \equiv \mathbf{H}_m^-(\mathbf{r}_\perp, z; \omega)$ and $\mathbf{H}_m^+ \equiv \mathbf{H}_m^+(\mathbf{r}_\perp, z; \omega)$ are the backward- and forward-travelling magnetic eigenfields, $C_m^+ \equiv C_m^+(z; \omega)$ and $C_m^- \equiv C_m^-(z; \omega)$ are the associated complex excitation coefficients and m is the mode index.

To obtain the general expression for the excitation coefficients, the Lorentz reciprocity theorem is applied to an infinitesimal volume of length dz. Taking into account the above expression of the current density, for the frequencies which are not associated with band edges, they can be shown to be given by:

$$C_m^+(z; \omega) = -\frac{q}{P_m} \int_{-\infty}^{z} E_{m,z}^-(z'; \omega)e^{-jk_0 z'} \, dz'$$

and:

$$C_m^-(z; \omega) = -\frac{q}{P_m} \int_{z}^{+\infty} E_{m,z}^+(z'; \omega)e^{-jk_0 z'} \, dz'.$$

The quantities P_m correspond to normalization factors and are given by:

$$P_m = -\int_{S_t} \left[\mathbf{E}_m^+ \times \mathbf{H}_m^- - \mathbf{E}_m^- \times \mathbf{H}_m^+ \right] \cdot \mathbf{z} \, dS.$$

These quantities are constant along the structure. Moreover, it can be shown that:

$$P_m = j\frac{4w_m}{d\gamma_m/d\omega}$$

where $\gamma_m = \alpha_m + j\beta_m$ is the propagation constant associated with the forward-travelling eigenmode m and:

$$w_m = \frac{1}{4d} \int_{V_{period}} (\epsilon_0 \mathbf{E}_m^+ \cdot \mathbf{E}_m^- - \mu_0 \mathbf{H}_m^+ \cdot \mathbf{H}_m^-) \, dV.$$

In the passbands, $\mathbf{E}_m^- = [\mathbf{E}_m^+]^*$ and $\mathbf{H}_m^- = -[\mathbf{H}_m^+]^*$. w_m is then the energy density in a unit cell and $P_m = 4v_{g,m}w_m$, $v_{g,m}$ being the group velocity.

The impedance and the excitation coefficients can be expressed as infinite sums of intervals of integration. Taking into account the following properties of the eigenfields:

$$E_{m,z}^\pm(z + nd; \omega) = e^{\mp\gamma_m nd}E_{m,z}^\pm(z; \omega),$$

the impedance can be written as infinite sums of products of double integrals, the integrations being performed over intervals of the type $[-d/2, +d/2]$, $[z, +d/2]$ and $[-d/2, z]$ $(-d/2 \leq z \leq +d/2)$, and of coefficients of the type $\exp[\pm(jk_0 - \gamma_m)nd]$ and $\exp[\pm(jk_0 + \gamma_m)nd]$. The double integrals with integration over $[z, +d/2]$ and $[-d/2, z]$ are shown to vanish for all frequencies. Ultimately, for the frequencies which fall into the passbands, the final expression for the longitudinal impedance is:

$$Z_\parallel(\omega) = \pi \sum_m \frac{|V_m^+|^2\delta(\omega - \beta_{mk}c) + |V_m^-|^2\delta(\omega + \beta_{mk}c)}{2(w_md)(v_{g,m}/c)}$$

with:

$$V_m^+(\omega) = \int_{-d/2}^{+d/2} E_{m,z}^+(z; \omega)e^{jk_0 z}\, dz$$

and:

$$V_m^-(\omega) = \int_{-d/2}^{+d/2} E_{m,z}^-(z; \omega)e^{jk_0 z}\, dz.$$

The term $\beta_{mk} = \beta_m + 2\pi k/d$ where $0 < \beta_m d < \pi$ takes into account potential synchronism at other space harmonics than the fundamental. Note that the dispersive properties of the structure appear explicitly through the group velocity at the synchronism point.

All the terms in the expression of the longitudinal coupling impedance can be computed with the method presented in [3]. With the Panofsky-Wenzel theorem in the frequency domain reading:

$$Z_\perp(\mathbf{r}_\perp; \omega) = \frac{c}{\omega}\nabla_\perp Z_\parallel(\mathbf{r}_\perp; \omega),$$

the transverse coupling impedance is easily derived. The transverse wake potential is obtained by performing an inverse Fourier transform.

3 DISPERSIVE PROPERTIES AND COUPLING IMPEDANCES OF A DAMPED PERIODIC STRUCTURE

When each cell of a periodic structure is coupled to waveguides, the dispersion diagram is substantially more complicated. It is distorted in such a manner that complex modes appear. Figure 1 shows such a diagram (first Brillouin zone) computed with post-processed data from the code HFSS for the reference cell of the TDS [1] in the frequency range where the two first dipole bands in the

undamped configuration are located. The presented four branches appear in pairs. Figure 2 shows the 2-D projection in the $(\beta d, f)$ plane of this dispersion diagram and the dispersive properties of the structure in the undamped case restricted to one half of the first Brillouin zone. In the frequency range which corresponds to the first dipole band in the undamped configuration, the dipole modes are so heavily damped that their associated dispersion curves are relatively far from the speed-of-light line.
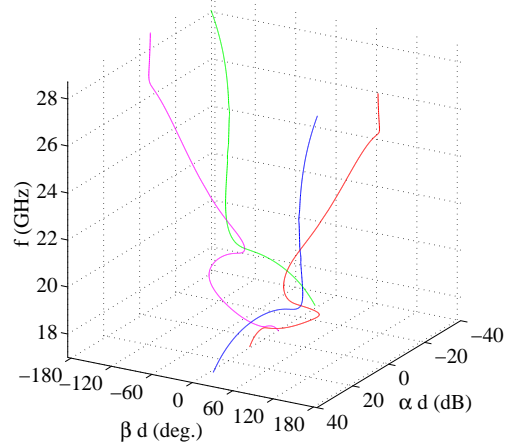

Figure 1: Dispersion diagram for the first two pairs of dipole modes of the reference cell of the TDS
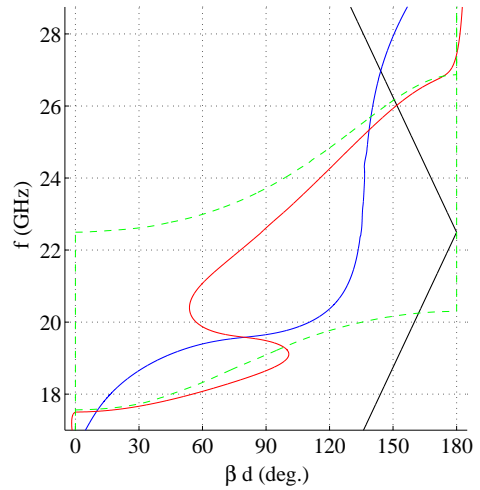

Figure 2: Speed-of-light line and projection in the $(\beta d, f)$ plane of the dispersion diagram for the first two dipole modes - undamped and damped cases

A criterion to quickly estimate the quality of the damping consists in evaluating at each frequency the distance of the different branches from the speed-of-light line (see Figure 3). Equivalently, the local maxima in the variation with the frequency of the inverse of the distance between branches and the speed-of-light line can be observed. This is illustrated in Figure 4 where such a distance is plotted for the least damped dipole mode. Such a criterion was actually hinted in [6].

To compute the coupling impedances, a direct application of the method described in the previous section proved

to be far more complex. Due the high losses, the eigenmodes are not orthogonal anymore, and the excitation coefficients cannot be expressed in a simple way. A physically intuitive method to calculate the transverse wake using the travelling-wave fields and propagation constants computed by the scattering matrix technique has therefore been developed. The method is based on the observation that the final expression for the longitudinal impedance presented in the previous section is very similar to the definition of shunt impedance. In this method, the power lost to the cavity walls in the definition of shunt impedance, is replaced by the power lost to the damping waveguides. This impedance is calculated as a continuous function of frequency, so it must be modulated by the phase between the excited travelling wave and the Fourier component of the beam at each frequency. This angle represents a measure of synchronism (or spatial resonance) and is given by the phase of the quantity S defined by:

$$ S = \frac{1}{1 - e^{(jk_0 - \gamma_m)d}} - \frac{1}{1 - e^{(jk_0 + \gamma_m)d}}. $$
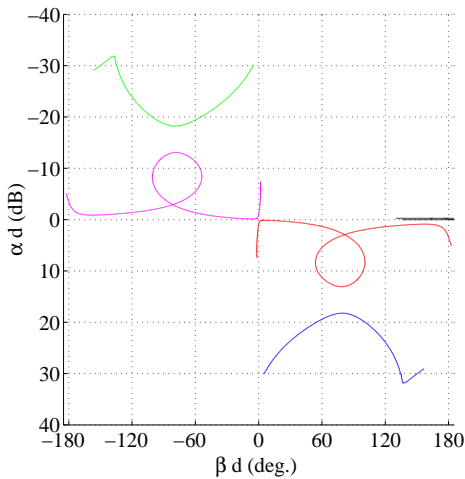

Figure 3: Speed-of-light line and projection in the $(\beta d, \alpha d)$ plane of the dispersion diagram for the first two pairs of dipole modes
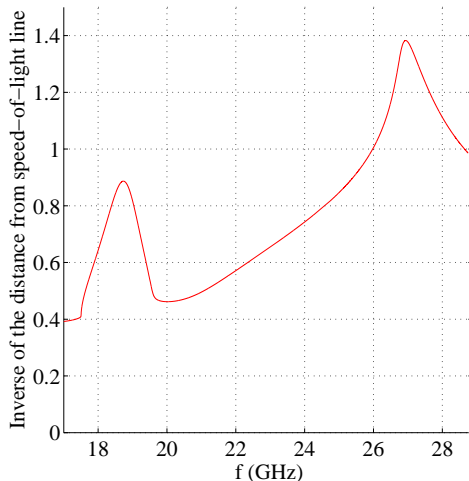

Figure 4: Inverse of the distance from the speed-of-light line for the main damped dipole mode

The transverse impedance spectrum derived with the shunt impedance for the TDS reference cell is shown in Figure 5. The accuracy of the impedance has been demonstrated by comparing this result with a direct time-domain computation using MAFIA. The lower peak in the impedance spectrum shows an effective Q of 19 at 19.7 GHz while MAFIA gives a Q of 21 at 18.9 GHz [7].
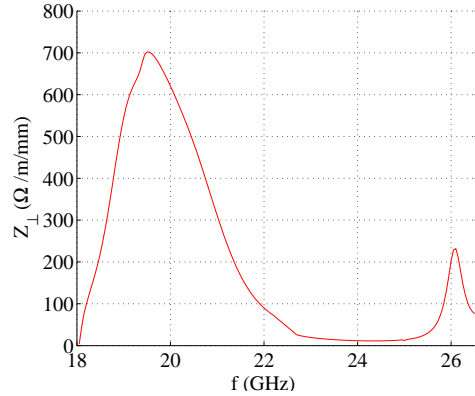

Figure 5: Transverse impedance versus frequency - reference cell of the TDS

4 CONCLUSIONS

A new technique to compute the coupling impedances and the long-range wakefields has been developed for constant-impedance structures and for waveguide damped structures. Contrary to previous approaches, it is directly based on the computation of the propagation constants and the travelling-wave field patterns of the normal modes. The analytical expression of the longitudinal coupling impedance for the lossless structures shows an explicit dependence on the group velocity at the synchronism points. For heavily damped structures, an intuitive method to compute the impedance leads to a fairly good agreement with time-domain computation using MAFIA.

5 REFERENCES

[1] I. Wilson and W. Wuensch, "The CLIC Main Linac Accelerating Structure", Proc. LINAC 2000, pp. 419-421, Monterey, 2000.

[2] M. Dehler, "Modeling a 30 GHz Loaded Detuned Structure for the Compact Linear Collider (CLIC)", CLIC Note 358, CERN, 1998.

[3] J.-Y. Raguin, "Network Representation of Multi-Cell Accelerating Cavities", Proc. PAC 2001, pp. 3027-3029, Chicago, 2001.

[4] R. L. Glückstern, "Analytic Methods for Calculating Coupling Impedances", CERN 2000-011, CERN, 2000.

[5] V. A. Solntsev, "Excitation of Uniform and Periodic Waveguides by External Currents", Sov. Phys. Tech. Phys., vol. 13, pp. 70-75, 1968.

[6] W. Wuensch, "Calculation of the Transverse Wake Function of a Highly Damped Periodic Structure", Proc. PAC 99, pp. 827-829, New York, 1999.

[7] M. Luong, private communication.