Multitrace AdS/CFT and Master Field Dynamics

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ABSTRACT

We consider gauge theories with multitrace deformations in the context of certain AdS/CFT models with explicit breaking of conformal symmetry and supersymmetry. In particular, we study the standard four-dimensional confining model based on the D4-brane metric at finite temperature. We work in the self-consistent Hartree approximation, which becomes exact in the large-N limit and is equivalent to the AdS/CFT multitrace prescription that has been proposed in the literature. We show that generic multitrace perturbations have important effects on the phase structure of these models. Most notably they can induce new types of large-N first-order phase transitions.

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1. Introduction

In 't Hooft's large-N limit of gauge theories [1], the scaling of the bare gauge coupling $g^2 \sim 1/N$ is tuned so that the vacuum energy is proportional to N^2. This scaling generalizes to an arbitrary action according to the rule:

$$S = N^2 \, W(\mathcal{O}_1, \mathcal{O}_2, \ldots) , \tag{1.1}$$

where W is a general functional of operators of the symbolic form

$$\mathcal{O}_n = \frac{1}{N} \, \mathrm{Tr} \; F^n , \tag{1.2}$$

the set of single-trace gauge-invariant operators with expectation values of $O(1)$ in the large-N limit[2]. For more general theories, including scalar fields and fermions in the adjoint representation, we extend the basic family of gauge-invariant operators to include these fields as well. These operators become quasi-classical in the large-N limit, in the sense that

$$\lim_{N \to \infty} \langle \, \mathcal{O} \, \mathcal{O}' \, \rangle = \lim_{N \to \infty} \langle \, \mathcal{O} \, \rangle \, \langle \, \mathcal{O}' \, \rangle .$$

This means that there is a notion of saddle-point configuration $-$a "master field" defined up to gauge transformations, which makes the $1/N$ expansion into a semiclassical expansion [2].

Known or conjectured master fields are usually established for single-trace actions, i.e. for linear W in (1.1), such as the Yang–Mills action. However, the behaviour of master fields under perturbations by multitrace operators is of primary interest, especially in the context of the AdS/CFT correspondence [3]. In the holographic mapping, multitrace operators are associated to multiparticle states in the bulk theory. Hence they correspond to exotic deformations of the string background [4]. Moreover, truly non-perturbative effects in the bulk theory manifest themselves as finite-N multitrace effects on the CFT. This is simply the translation of the fact that only $O(N)$ elementary powers of the form $\mathrm{Tr} \, F^n$ are algebraically independent: for $n \gg N$ the single-trace operator decomposes as a sum of products of lower-order single-trace operators. Hence, the spectrum of the bulk theory must deviate significantly from a Fock space for states with $O(N)$ "particles".

It is then very interesting to study the effect of multitrace deformations on the AdS/CFT saddle point, particularly the effect of deformations that are non-polynomial

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[2] We shall not discuss here operators with anomalous large-N scaling. The most notable example is the theta-term with scaling $\langle \, \mathrm{Tr} \, F \wedge F \, \rangle = O(1)$.

in the traces. Recently, the AdS/CFT algorithm was modified to incorporate multitrace operators [5,6] (see also [7]). Here we elaborate on some points made in [6] to argue that this modification can be understood in rather general terms, as an application of the mean-field approximation.

In analysing the large-N master field, we could attempt a saddle-point approximation once we have managed to exactly integrate out $O(N^2)$ degrees of freedom. If we remain with $O(N)$ degrees of freedom, this sets the order of magnitude of the fluctuations. Since the action is of $O(N^2)$, we have a sharp saddle point. In practice, such a program only works in very restricted models in low dimensions, where we can integrate out explicitly the $O(N^2)$ angular variables (for a discussion of multitrace operators in these models, see [8]). Still, one can argue in great generality that in the leading large-N approximation W can be taken essentially linear.

Let us suppose that we have managed to change variables in the path integral from the gauge field A_μ to the set of gauge-invariant monomials $\mathcal{O}_n$ with $n < O(N)$. In the process we generate a complicated (non-local) effective action Γ. At the large-N saddle-point we have:

$$\frac{\partial \Gamma}{\partial \mathcal{O}_n}(\mathcal{O}_{cl}) + \frac{\partial W}{\partial \mathcal{O}_n}(\mathcal{O}_{cl}) = 0 \; , \tag{1.3}$$

where we have incorporated the fact that the solution of the saddle-point equations is nothing but the planar expectation values: $\mathcal{O}_{cl} \equiv \lim_{N\to\infty} \langle \mathcal{O}_n \rangle$.

In view of (1.3), it is clear that these equations are exactly the same as those that follow from a model with a single-trace action given by

$$\overline{W}(\mathcal{O}) = \sum_n \overline{\zeta}_n \, \mathcal{O}_n \; , \tag{1.4}$$

where the effective single-trace couplings $\overline{\zeta}_n$ are given by

$$\overline{\zeta}_n = \frac{\partial W}{\partial \mathcal{O}_n}(\mathcal{O}_{cl}) \; . \tag{1.5}$$

Therefore, provided we only consider the planar $N \to \infty$ limit, any quantity of the original theory (1.1) can be computed in the single-trace theory (1.4), with the expectation values $\langle \mathcal{O} \rangle$ being determined self-consistently.[3] Thus, in the AdS/CFT set-up, the combination $\partial W(\mathcal{O}_{cl})/\partial \mathcal{O}_n$ plays the role of the source for the single-trace operator $\mathcal{O}_n$, and this precisely determines the boundary conditions proposed in [5,6].

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[3] This argument assumes some explicit regularization, so that the path integral measure is defined independently of the details of the action.

Our discussion shows that the basic phenomenon is more general than the particular AdS/CFT set-up. Namely, it is a general consequence of the fact that the Hartree (or Thomas–Fermi) approximation becomes exact in the large-N limit (see for example [9]). In this limit, the interactions between the gauge-invariant variables $\mathcal{O}_n$ can be substituted by the interaction of each variable with a collective mean field that must be determined sefl-consistently.

We should emphasize that these rules are only valid in the strict $N = \infty$ limit. The $1/N$ corrections will alter the master equation (1.5) since the Hartree approximation obtains corrections. Equivalently, the AdS/CFT boundary conditions of [5,6] will receive $1/N$ corrections, in addition to the usual loop corrections in the bulk of AdS.

2. Master Field Dynamics

To be more specific, let us suppose that the deformations by a certain single-trace operator $\mathcal{O}$:

$$\delta S = N^2 \, \zeta \int d^d x \, \mathcal{O} \, , \tag{2.1}$$

are under control, in the sense that we are able to compute the planar one-point function $\langle \mathcal{O} \rangle_\zeta$ as a function of ζ and the other couplings of the Lagrangian. Then we can compute any planar expectation value of the more general theory with perturbation

$$N^2 \int d^d x \, \mu^d \, F(\mu^{-d_\mathcal{O}} \, \mathcal{O}) \, , \tag{2.2}$$

where F is general function, μ is a mass scale and $d_\mathcal{O}$ is the scaling dimension of the operator $\mathcal{O}$ in the single-trace model. We simply do our calculations in the single-trace theory (2.1) with perturbation

$$\delta \overline{S} = N^2 \, \overline{\zeta} \int d^d x \, \mathcal{O} \, , \tag{2.3}$$

where $\overline{\zeta}$ is given self-consistently by the solution of the "master equation":

$$G(\overline{\zeta}) \equiv \overline{\zeta} - \mu^{d-d_\mathcal{O}} \, F' \left[\mu^{-d_\mathcal{O}} \, \langle \mathcal{O} \rangle_{\overline{\zeta}} \right] = 0 \, , \tag{2.4}$$

where the prime denotes differentiation. In principle, we can give $\overline{\zeta}$ a space-time dependence so that (2.4) becomes a functional equation for an effective source. Such a generalization is appropriate to compute correlation functions in the multitrace-deformed

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theory. However, for the purposes of this paper we are only interested in vacuum properties of the master field, i.e. we consider only condensates and effective couplings that are translationally invariant in $\mathbf{R}^d$.

The "master equation" (2.4) implies that multitrace deformations whose single-trace "elementary" operator has a vanishing one-point function are equivalent (in the large-N limit) to single-trace deformations, i.e. a constant shift

$$\delta\zeta = \mu^{d-d_{\mathcal{O}}}\, F'(0)$$

of the coupling dual to the single-trace operator $\mathcal{O}$. Hence, in order to have specifically new phenomena associated to multitrace deformations we need non-vanishing condensates. This means that the auxiliary single-trace model with perturbation (2.3) must break conformal invariance either explicitly or spontaneously. Since the one-point function depends on the particular state that we are considering, it is plain that the physical properties of multiple-trace deformations have a strong dependence on the full physics of condensates of the associated single-trace model.

We may take F as a non-polynomial function of single-trace operators. However, we implicitly treat the non-linear terms as a perturbation since the scaling dimensions $d_{\mathcal{O}}$ are defined with respect to the single-trace theory. At any rate, it is interesting to evaluate (2.4) when the function F becomes non-polynomial.

Our main observation in this paper is the following. The function $G(\overline{\zeta})$ may have a complicated structure, being non-linear in both $\overline{\zeta}$ and the couplings of the bare Lagrangian W. In particular, if $G(\overline{\zeta})$ has various nodes, we have a set of solutions $\{\overline{\zeta}_\alpha\}$ for a given fixed value of the microscopic couplings in W. In this case we must select the master field that dominates the large-N dynamics among the various solutions $\overline{\zeta}_\alpha$.

By analogy with similar situations in large-N physics we characterize the dominating master field by requiring that the partition function be maximized:

$$\lim_{N\to\infty}\frac{1}{N^2}\log Z_W = \max_\alpha\left[\lim_{N\to\infty}\frac{1}{N^2}\log Z(\overline{\zeta}_\alpha)_{\overline{W}}\right]. \qquad (2.5)$$

Large-N phase transitions induced by the multitrace couplings will arise when the dominating zero of $G(\overline{\zeta})$ changes discontinuously as a function of the microscopic couplings in W. These phase transitions will be characterized by a "latent heat" release of $O(N^2)$. Typically, $Z(\overline{\zeta})_{\overline{W}}$ will be a monotonic function of $\overline{\zeta}$, so that a change of branch in (2.5) will require that the cardinality of the solution set $\{\overline{\zeta}_\alpha\}$ changes as a function of W.

Although the phenomena described so far are expected to be rather general, we will illustrate them in a specific example in the context of the AdS/CFT correspondence.

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3. Multitraces in Deformed QCD

As a concrete example along the previous lines, we consider a regularized version of four-dimensional non-supersymmetric Yang–Mills theory that has been introduced in [10]. In its most straightforward definition, the model is given by the low-energy theory on the world-volume of a stack of N parallel D4-branes at finite temperature T. Equivalently, we can view it as a Scherk–Schwarz compactification of the D4-branes on $\mathbf{S}^1 \times \mathbf{R}^4$, the compact circle having size $1/T$. At large distances on $\mathbf{R}^4$ the effective theory is a four-dimensional Yang–Mills theory modified at energies of $O(T)$ by remnants of the five-dimensional $\mathcal{N} = 4$ super Yang–Mills theory. The action is given by

$$\frac{N}{g_{\mathrm{YM}}^2} \int d^4x \, \mathcal{L} = \frac{1}{4 g_{\mathrm{YM}}^2} \int d^4x \, \left(\mathrm{Tr}\, F^2 + \dots \right) \;, \tag{3.1}$$

where the dots stand for other fields such as fermions and scalars of the parent $\mathcal{N} = 4$ theory, suppressed by powers of the cutoff scale $\mu = T$. Planar quantities are functions of the 't Hooft coupling λ, defined at the cutoff scale μ. In terms of the microscopic parameters of the parent D4-brane theory we have

$$\lambda \equiv g_{\mathrm{YM}}^2(\mu)\, N \sim g_s \, N \, \mu \, \sqrt{\alpha'} \,,$$

where g_s is the string coupling and α' is the string's Regge slope. For $\lambda \ll 1$ we have the standard planar perturbation theory of the four-dimensional Yang–Mills theory. On the other hand, for $\lambda \gg 1$ we have a good description in terms of the low-curvature expansion of the black D4-brane metric. In this case, the expansion parameter is controlled by the curvature of the near-horizon metric in string units:

$$\frac{\alpha'}{R_c^2} \sim \frac{1}{\lambda} \,,$$

with R_c the curvature radius. Defining

$$x \equiv \frac{1}{\lambda},$$

the supergravity description is good for $0 < x \ll 1$. At $x \sim 1$ we have the standard "correspondence point" in the sense of [11], which represents the matching to the perturbative regime. As long as we only look at energy scales of $O(1)$ in the large-N limit, we can neglect non-perturbative thresholds associated to large values of the dilaton, since these involve explicit powers of N.

The simplest multitrace perturbation in these models is a non-linear function of the Lagrangian density,

$$ S = \frac{N^2}{\lambda} \int \mathcal{L} + N^2 \int \mu^4 \, F(\mu^{-4} \, \mathcal{L}) \, . \tag{3.2} $$

According to (2.4), all physical quantities in this model, such as thermodynamic functions, condensates, Wilson loops, etc. , can be computed in the large-N limit in the auxiliary undeformed model

$$ \overline{S} = \frac{N^2}{\overline{\lambda}} \int \mathcal{L} \, , \tag{3.3} $$

with effective 't Hooft coupling $\overline{\lambda}$ given by the solution of the equation

$$ \overline{\lambda}^{-1} = \lambda^{-1} + F' \left[\mu^{-4} \langle \mathcal{L} \rangle_{\overline{\lambda}} \right] \, , \tag{3.4} $$

where the gluon condensate $\langle \mathcal{L} \rangle_{\overline{\lambda}}$ is determined by the expectation value of the action:

$$ \left\langle \int \mathcal{L} \right\rangle_{\overline{\lambda}} = \mathrm{Vol}\,(\mathbf{R}^4) \left\langle \frac{1}{4N} \, \mathrm{Tr} \, F^2 + \dots \right\rangle_{\overline{\lambda}} = \frac{1}{N^2} \, \overline{\lambda}^2 \, \frac{\partial}{\partial \overline{\lambda}} \, \log Z(\overline{\lambda}) \, . \tag{3.5} $$

The partition function in the planar supergravity approximation is defined in terms of the thermal free energy of the D4-brane (see, for example [12]):

$$ \frac{1}{\mathrm{Vol}(\mathbf{R}^4)} \, \log \, Z(\overline{\lambda}) = N^2 \, C \, \overline{\lambda} \mu^4 \, , \tag{3.6} $$

where C is a positive numerical constant. This expression for the partition function has been normalized to the Euclidean action of the wrapped D4-brane metric with supersymmetric boundary conditions, i.e. we define the five-dimensional thermal free energies with respect to the $T = 0$ vacuum.

Notice that, even if the general multitrace deformation of the $T = 0$ D4-brane theory may break supersymmetry, the $N = \infty$ effective theory (3.3) *does not*. Hence, the D4-brane theory reduced on a supersymmetric circle will be supersymmetric at $N = \infty$ and no condensates will be induced.[4] This implies that the condensates are entirely due to thermal effects of the D4-brane theory and our normalization of (3.6) is the physically correct one.

Combining (3.5) and (3.6) we find the value of the gluon condensate (c. f. [13]):

$$ \mu^{-4} \, \langle \mathcal{L} \rangle_{\overline{\lambda}} = C \, \overline{\lambda}^2 \, . \tag{3.7} $$

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[4] Casimir energies are not induced either, since the D4 world-volume is flat $\mathbf{S}^1 \times \mathbf{R}^4$.

This expectation value has the crucial property of diverging as $\overline{\lambda} \to \infty$. Since this is precisely the supergravity regime of the effective single-trace theory, we learn that multitrace deformations are potentially stronger in the region where AdS/CFT is under quantitative control and they may be reliably studied.

In terms of the dimensionless expansion parameters $x \equiv 1/\lambda$ and $\bar{x} \equiv 1/\overline{\lambda}$ the master equation reads

$$G(\bar{x}) \equiv \bar{x} - x - F' \left[C/\bar{x}^{\,2}\right] = 0. \tag{3.8}$$

Equation (3.8) was derived within the supergravity approximation to the near-horizon black D4-brane solution. In terms of the supergravity expansion parameter $\bar{x}$, this is the regime:

$$0 < \bar{x} \ll 1 \,. \tag{3.9}$$

As before, these limits ignore other thresholds that are related to large dilaton corrections and are of subleading order in the $1/N$ expansion.

One important property of (3.8) is the redundancy of the description in terms of the original variables in the microscopic Lagrangian, i.e. the coupling x and the multitrace couplings that define the function F. For a fixed value of $\bar{x}$ all models in the codimension-1 submanifold

$$\mathcal{M}_{\bar{x}}: \quad x - \bar{x} + F' = 0$$

have the same large-N properties. The region of the microscopic coupling space where supergravity is a good approximation is the union of these submanifolds for $0 < \bar{x} < 1$:

$$\mathcal{S} = \bigcup_{0 < \bar{x} < 1} \mathcal{M}_{\bar{x}} \,. \tag{3.10}$$

One component of the boundary is $\mathcal{M}_{\bar{x}=0}$ defined as

$$\mathcal{M}_0: \quad x + F'(\infty) = 0 \,. \tag{3.11}$$

It yields the strong-coupling (low-curvature) limit of the AdS/CFT background. On the other hand, the correspondence line (the matching to perturbative variables) occurs at $\bar{x} = 1$ or

$$\mathcal{M}_1: \quad x + F'(C) - 1 = 0 \,. \tag{3.12}$$

Although $\mathcal{M}_0 \cup \mathcal{M}_1$ are components of the boundary of $\mathcal{S}$, they do not exhaust it in general.

3.1. Multicritical Behaviour

For $0 < x \ll 1$ there is always a standard solution of (3.8) that is valid for very small multitrace couplings. This solution has $\bar{x} \approx x$ and can be obtained iteratively as the limit of the set $\{\bar{x}_{(k)}\}$ with

$$\bar{x}_{(k+1)} = x + F' \left[C/(\bar{x}_{(k)})^2 \right] , \qquad \bar{x}_{(0)} = x . \tag{3.13}$$

However, it is clear that there will be other solutions if $F'[C/\bar{x}^2]$ shows "bumps" in the supergravity interval $0 < \bar{x} < 1$.

Let us assume that F' admits a finite Laurent expansion around the origin, so that the master equation takes the form:

$$G(\bar{x}; \, x, \, f_j) = \bar{x} - x - \sum_{j \neq 0} \frac{f_j}{\bar{x}^{\,2j}} = 0 . \tag{3.14}$$

The $j = 0$ term is equivalent to a constant shift of x and has been removed from (3.14).

Our first result is a simple consequence of the divergence of (3.7). The pole part of F, corresponding to $j < 0$ in (3.14), has no dramatic effects in the supergravity interval $0 < \bar{x} \ll 1$. Thus, multitrace deformations that are completely singular in perturbation theory become rather tame in the supergravity approximation. This looks surprising at first sight, but it fits naturally with the character of AdS/CFT as a strong/weak coupling duality with respect to the 't Hooft coupling.

Conversely, perturbations that are polynomial in multitraces translate into non-analytic contributions to $G(\bar{x})$ and therefore dominate the supergravity regime at $\bar{x} \to 0$. In this limit $G(\bar{x})$ diverges with a sign that is correlated with that of f_J, J being the largest value of the index j. In particular, for $f_J < 0$ and small there is always a solution:

$$\bar{x}_- \approx \left(-\frac{f_J}{x} \right)^{\frac{1}{2J}} . \tag{3.15}$$

This solution disappears for $f_J > 0$, unless one also dials the microscopic 't Hooft coupling to negative values: $x < 0$.

We have found that for $0 < x < 1$ and small $|f_J|$, we have a discrete jump in the number of solutions of the master equation as f_J crosses zero. This is a source of possible phase transitions.

A more general multicritical behaviour in the vicinity of $\bar{x} \approx 0$ will depend on the higher multitrace powers. Let us consider a simple example of a deformation proportional to

$$\frac{g_1}{2} \int \mu^{-4} \left(\operatorname{Tr} F^2\right)^2 + \frac{g_2}{3N} \int \mu^{-8} \left(\operatorname{Tr} F^2\right)^3 \ . \tag{3.16}$$

The master equation (3.14) reads:

$$G(\bar{x};\, x,\, f_1,\, f_2) = \bar{x} - x - \frac{f_1}{\bar{x}^2} - \frac{f_2}{\bar{x}^4} = 0 \ ,$$

with $f_i \sim g_i$ up to numerical constants. Besides the standard solution $\bar{x}_+ \approx x$ for very small f_j there are other interesting solutions. Consider $x > 0$, $f_2 > 0$ and $f_1 < 0$, with $f_2 \ll |f_1| \ll x$ and furthermore $x f_2 \ll f_1^2$. Then, the master equation has two small solutions in the vicinity of

$$\bar{x}_- \sim \sqrt{-f_2/f_1}, \qquad \bar{x}'_- \sim \sqrt{-f_1/x} \ .$$

These solutions coincide for $f_1^2 \sim x f_2$ and disappear for larger values of f_2.

3.2. Phase Transitions

The previous discontinuities in the solution set of the master equation translate into large-N phase transitions. Since the partition function scales as

$$\log\, Z(\bar{x}) \propto \frac{N^2}{\bar{x}} \ , \tag{3.17}$$

we find that the dominant solutions in the supergravity approximation are those with the *smallest* value of $\bar{x}$ within the unit interval. The jump of the effective action across the transition from $\bar{x}_\alpha$ to $\bar{x}_\beta$ is given by

$$\frac{1}{\operatorname{Vol}(\mathbf{R}^4)}\, \log\, \left[\frac{Z(\bar{x}_\alpha)}{Z(\bar{x}_\beta)}\right] = N^2\, C\, \mu^4 \left(\frac{1}{\bar{x}_\alpha} - \frac{1}{\bar{x}_\beta}\right) \ . \tag{3.18}$$

Coming back to the examples in the previous subsection, we see that there is always a phase transition when f_J crosses zero from negative to positive values. In this case $\bar{x}_\alpha = 0$ and $\bar{x}_\beta \approx x > 0$. The density of "latent heat" in (3.18) is infinite. This phase transition is not hard to interpret. Since f_J is the coupling of the multitrace interaction of highest order, it dominates the limit of large field-strengths. Hence, the very strong singularity for $f_J \to 0^-$ reflects the fact that the microscopic action is not bounded below for $f_J < 0$.

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A more physical phase transition with finite "latent heat" takes place in the two-coupling model (3.16) with $x > 0$, $f_2 > 0$ and $f_1 < 0$, when the two solutions around $\bar{x}_- \sim \sqrt{-f_1/x}$ coalesce as we decrease the magnitude of $|f_1|/f_2$. For small values of this ratio the only solution is $\bar{x} \approx x$.

This example illustrates the general pattern of phase transitions in this class of models. When the minimal solution $\bar{x}_-$ of the master equation is separated from the first subleading one $\bar{x}'_-$ by a local maximum of $G(\bar{x})$, a variation of the parameters can bring the maximum to zero and make the two solutions coalesce $\bar{x}_- = \bar{x}'_-$. A further variation of the parameters can bring the maximum to negative values and make the double solution disappear. This generic situation is depicted in Fig. 1 below.
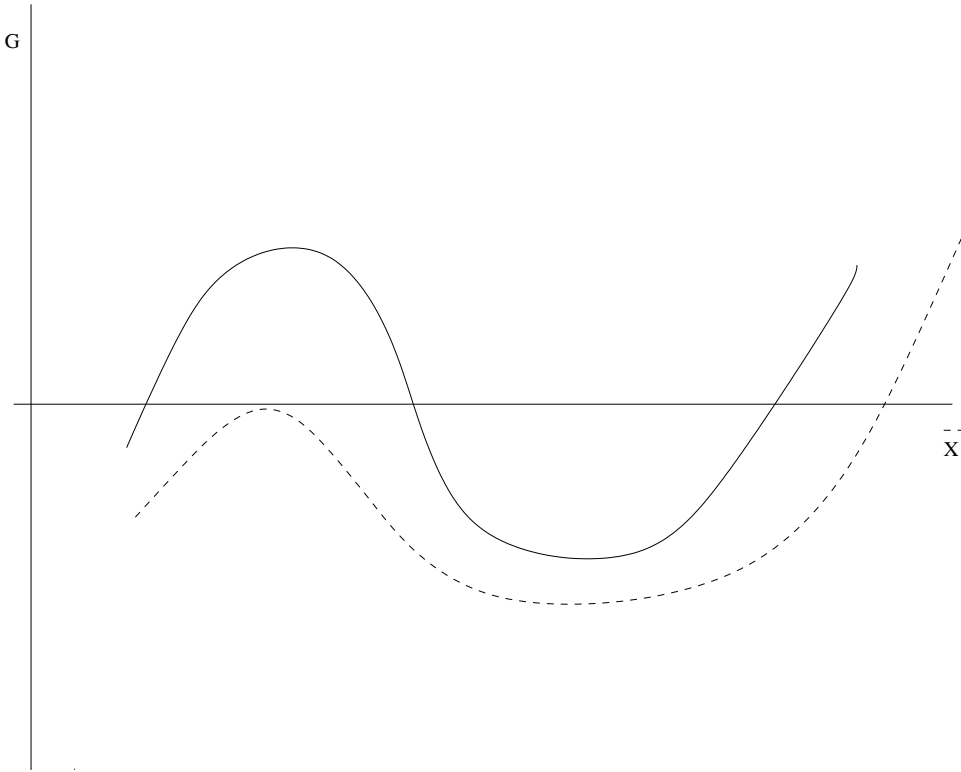

Fig. 1: *A depiction of a typical phase transition. The solid line shows the function $G(\bar{x})$ with three zeros, $\bar{x}_- < \bar{x}'_- < \bar{x}_+$. The dotted line shows the degeneration of the lower zeros $\bar{x}_- = \bar{x}'_-$ and their disappearance in favour of $\bar{x}_+$. When the partition function is dominated by the smallest solution this degeneration yields a large-N phase transition.*

4. Generalization to Other Dimensions

This set-up can be generalized to the regularized Yang–Mills model on $\mathbf{R}^p$, with $p < 5$, in terms of a hot Dp-brane model and the corresponding generalization of the AdS/CFT correspondence [14]. In this case, the effective dimensionless 't Hooft coupling normalized at the cutoff scale $\mu = T$ is given by

$$\lambda_p(\mu) \sim g_s N \, (\alpha')^{\frac{p-3}{2}} \, \mu^{p-3} \,.$$

This is the expansion parameter of the planar perturbative expansion. The expansion parameter of the supergravity approximation that arises at $\lambda_p(\mu) \gg 1$ is:

$$x \equiv \left(\frac{1}{\lambda_p(\mu)} \right)^{\frac{1}{5-p}} \sim \frac{\alpha'}{R_c^2} \,. \tag{4.1}$$

The large-N solution of these models perturbed by multitrace interactions of the form

$$N^2 \int d^p x \, \mu^p \, F\left(\mu^{-4} \, \mathcal{L} \right) \tag{4.2}$$

can be studied along lines similar to the $p = 4$ case above. Here, $\mathcal{L}$ denotes the Yang–Mills Lagrangian operator, corrected by regularization artefacts at the scale $\mu = T$. As before, one reduces the problem to the study of an effective single-trace model with supergravity expansion parameter

$$\bar{x} \equiv \left(1/\overline{\lambda}_p(\mu) \right)^{\frac{1}{5-p}} \tag{4.3}$$

that is determined self-consistently. The supergravity regime of the $N = \infty$ problem is then given by $0 < \bar{x} \ll 1$. The partition function in the single-trace model with effective coupling $\bar{x}$ is

$$\frac{1}{\mathrm{Vol}(\mathbf{R}^p)} \, \log \, Z(\bar{x}) = N^2 \, (5 - p) \, C_p \, \mu^p \, \bar{x}^{\,3-p} \,, \tag{4.4}$$

where C_p is a positive numerical constant. The gluon condensate is given by

$$\mu^{-4} \, \langle \mathcal{L} \rangle_{\bar{x}} = (p - 3) \, \frac{C_p}{\bar{x}^2} \,. \tag{4.5}$$

These expressions show that the $p = 3$ case, based on the hot D3-brane, yields trivial multitrace deformations in this approximation. This is a consequence of the free energy of D3-branes being very smooth for large 't Hooft coupling. Of course, this situation changes when considering subleading terms in the α' expansion of the supergravity background. It

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is interesting to study these corrections in more detail, although we will not attempt to do this here.

For $p < 3$ one finds a situation somewhat similar to that discussed before in the $p = 4$ case. The master equation for $\bar{x}$ reads:

$$\bar{x}^{5-p} = x^{5-p} + F'\left[(p-3)C_p/\bar{x}^2\right] = 0 . \tag{4.6}$$

Hence, the same qualitative properties follow, regarding the multiplicity of solutions at small $\bar{x}$. In particular, the crucial singularity at $\bar{x} = 0$ of the gluon condensate (4.5) still holds.

The main difference with $p = 4$ is that, according to (4.4), for $p < 3$ it is the *largest* solution $\bar{x}_+$ that dominates the partition function. As a result, we expect that the standard solution $\bar{x} \approx x$ will dominate and that sharp phase transitions will be more difficult to produce than in the $p = 4$ case.

5. Conclusions

In this paper we have studied some simple multitrace deformations of the basic non-supersymmetric QCD model in [10], as well as its generalizations to less than four dimensions. In particular we have considered deformations by a non-linear function of the Lagrangian operator.

Our main result is the emergence of new types of "multicritical" behaviour, similar in many ways to those studied in the context of matrix models [8]. There appear various competing master fields whose dynamics yields new examples of large-N phase transitions. It turns out that the dynamical effect of multitrace deformations is particularly strong in the supergravity approximation to the AdS/CFT master field.

These results suggest various avenues for further research. It would be interesting to study more examples of large-N phase transitions induced by multitrace deformations. Eventually, these phase transitions should be related to the breakdown of string perturbation theory in the geometrical description of the large-N master field. Another interesting question is the effect of multitrace deformations on other large-N phase transitions that have been identified in single-trace models, in particular, the phase transitions associated to theta-dependence in [15] or those related to finite-size effects, as in [16,10,12].

6. Acknowledgements

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