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Duality and Spontaneously Broken Supergravity in Flat Backgrounds

L. Andrianopoli[♯ ♮], R. D'Auria[♯ ♮], S. Ferrara[♯ ♭] and M. A. Lledó[♯ ♮].

[♯] *Dipartimento di Fisica, Politecnico di Torino, Corso Duca degli Abruzzi 24, I-10129 Torino, Italy.*

[♮] *INFN, Sezione di Torino, Italy.*

[♭] *CERN, Theory Division, CH 1211 Geneva 23, Switzerland, and INFN, Laboratori Nazionali di Frascati, Italy.*

Abstract

It is shown that the super Higgs mechanism that occurs in a wide class of models with vanishing cosmological constant (at the classical level) is obtained by the gauging of a flat group which must be an electric subgroup of the duality group. If the residual massive gravitinos which occur in the partial supersymmetry breaking are BPS saturated, then the flat group is non abelian. This is so for all the models obtained by a Scherk-Schwarz supersymmetry breaking mechanism. If gravitinos occur in long multiplets, then the flat groups may be abelian. This is the case of supersymmetry breaking by string compactifications on an orientifold $T^6/\mathbb{Z}_2$ with non trivial brane fluxes.

1 Introduction

Spontaneously broken supergravity is a plausible scenario for a unified theory of particle interactions below the Planck scale. Among all the possible models, no-scale supergravities [1, 2] have received much attention in the literature because of the possibility of creating a hierarchy of scales much lower than the Planck or string scale.

Very recently, no-scale supergravities have emerged from certain superstring compactifications in presence of brane fluxes [3] - [10]. In these models, the scalar potential and a non trivial gauging appear as a consequence of the non trivial flux along 3-cycles of the internal space. If $\hat{H}_{\hat\mu\hat\rho\hat\sigma}$ is a 3-form field strength (the "$\hat{}$" will always denote variables and indices in the higher dimensional space), then the integral over the 3-cycle is

$$\Phi(x^\mu) = \int_{3-\mathrm{cycle}} dx^{\hat\mu} dx^{\hat\rho} dx^{\hat\sigma} \hat{H}_{\hat\mu\hat\rho\hat\sigma} \neq 0.$$

In the Scherk-Schwarz [11] mechanism, the dimensional reduction ($D = 5 \to D = 4$) ansatz on the 1-form potentials is

$$\hat{A}_{\hat\mu}(x^\nu, x^5) = e^{Mx^5} A_{\hat\mu}(x_\nu), \qquad \hat\mu, \hat\nu = 1, \dots 5, \;\; \mu, \nu = 1, \dots 4. \tag{1}$$

x^5 is compactified to a circle S^1, and M is a generic element of the Cartan subalgebra of $\mathfrak{usp}(8)$. The 2-form field strengths $F^\Lambda_{\hat\mu\hat\nu}$ have also a non trivial flux through the 1-cycle S^1,

$$\Phi^\Lambda_\nu = \int_{S^1} (e^{-Mx_5}\hat{F})^\Lambda_{\hat\mu\nu} dx^{\hat\mu} = M^\Lambda_\Sigma A^\Sigma_\nu - \partial_\nu a^\Lambda, \qquad a^\Lambda = A^\Lambda_5. \tag{2}$$

The combination in the right hand side of (2) appears in the four dimensional Lagrangian when making the generalized dimensional reduction (1). Since it is a flux, it must be covariant under gauge transformations. One can explicitly check it. Under the infinitesimal transformations [12]

$$\begin{aligned} \delta A^\Lambda_\mu &= \partial_\mu \Xi^\Lambda + M^\Lambda_\Sigma A^\Sigma_\mu \Xi^0 + a^\Lambda \partial_\mu \Xi^0 \\ \delta a^\Lambda &= M^\Lambda_\Sigma \Xi^\Sigma + M^\Lambda_\Sigma a^\Sigma \Xi^0, \end{aligned}$$

the flux transforms as $\delta\Phi^\Lambda_\mu = \Xi^0 M^\Lambda_\Sigma \Phi^\Sigma_\mu$.

For $M = 0$ the term proportional to A^Λ_μ in (2) is zero. This implies that in the four dimensional Lagrangian there are no gauge couplings for A^Λ_μ, and the gauge group of the theory is trivial in the sense that none of the fields is charged. So an $M \neq 0$ in the dimensional reduction ansatz is the origin of the gauged gravity in four dimensions.

We note that the genuine gauge fields in $D = 4$ are $Z_\mu^\Lambda = A_\mu^\Lambda - B_\mu a^\Lambda$ and B_μ, with B_μ the vector field coming from the metric (graviphoton). As shown in Ref. [12], they are gauge connections of a non-abelian Lie algebra, whose generators $\{X_\Lambda, X_0\}$ obey the commutation relations

$$[X_\Lambda, X_0] = M_\Lambda^\Sigma X_\Sigma \qquad [X_\Lambda, X_\Sigma] = 0.$$

M_Λ^Σ are the matrix elements of M in (1). This is a subalgebra of the U-duality algebra [1], which for $N = 8$ is $\mathfrak{e}_{7(7)}$. The subalgebra has dimension 28, and it has a linear realization on the vector potentials[2]. This is not a particular case of any of the gaugings studied in the literature on four dimensional supergravity [13, 14].

A crucial point for the definition of an electric subgroup of the U-duality group G is the fact that G can be embedded in the symplectic group $\mathrm{Sp}(2n, \mathbb{R})$, where n is the number of electric potentials in the theory. As we will see, G admits different symplectic embeddings, which generically are related by a conjugation with a symplectic transformation. The electric subgroup is not invariant under this conjugation, then giving rise to the possibility of non isomorphic maximal electric subgroups. The standard formulation of supergravity [15] (where the R-symmetry of the theory is manifest) implies the choice of one particular symplectic embedding. The non standard gaugings considered here correspond to other choices of the symplectic embedding which give partial supersymmetry breaking with vanishing cosmological constant. The symmetry that was electric in the standard formulation is not fully electric in the other choices. For example, in the $N = 8$ Scherk–Schwarz supergravity, SO(8) mixes electric with magnetic field strengths, and so it does SO(6,n) in the case of $N = 4$ Scherk–Schwarz supergravity. Another consequence of the change of a symplectic basis for the duality group is that the R-symmetry is no longer manifest. In the above examples, only $\mathrm{USp}(8) \subset \mathrm{SU}(8)$ and $\mathrm{USp}(4) \subset \mathrm{SU}(4)$ are manifest symmetries.

The paper is organized as follows. In Section 2 we give some mathematical preliminaries on electric subgroups of the U-duality group G. In Section

[1] We call U-duality the continuous symmetry of the low energy effective action that gives rise to the discrete U-duality symmetry in the full quantum theory.

[2] We will call such subalgebras *electric* subalgebras.

3 we discuss two different choices of maximal electric subgroup in $N = 8$ supergravity, one giving rise to the standard gaugings and the other realizing the Scherk–Schwarz mechanism from a purely four dimensional point of view. In Section 4 we consider 3 different choices of electric subalgebras in $N = 4$ supergravity, corresponding to different physical theories and in Section 5 we do a similar analysis for $N = 2$ supergravity.

2 Symplectic embeddings and electric subalgebras

We consider $D = 4$ supergravity in absence of fluxes. Let G be the U-duality group of the theory and n the number of vectors in the theory, A^Λ_μ, $\Lambda = 1 \dots n$. The field strengths $F^\Lambda_{\mu\nu}$ and their duals $G^{\mu\nu}_\Lambda = \partial\mathcal{L}/\partial F^\Lambda_{\mu\nu}$ together carry a linear representation of G. When G is considered as embedded in $\mathrm{Sp}(2n, \mathbb{R})$, the representation carried by $\{F^\Lambda, G_\Lambda\}$ is promoted to a representation of the full symplectic group [16].

An arbitrary matrix of the Lie algebra $\mathfrak{sp}(2n, \mathbb{R})$ can be written in terms of blocks of size $n \times n$

$$X = \begin{pmatrix} a & b \\ c & -a^T \end{pmatrix}, \qquad b = b^T,\ c = c^T, \tag{3}$$

and a an arbitrary matrix of $\mathfrak{gl}(n, \mathbb{R})$. The Lie algebra $\mathfrak{sp}(2n, \mathbb{R})$ admits then the decomposition

$$\mathfrak{sp}(2n, \mathbb{R}) = \tilde{\mathfrak{g}}^0 + \tilde{\mathfrak{g}}^{+1} + \tilde{\mathfrak{g}}^{-1},$$

with

$$\tilde{\mathfrak{g}}^0 = \{ \begin{pmatrix} a & 0 \\ 0 & -a^T \end{pmatrix} \}, \quad \tilde{\mathfrak{g}}^{+1} = \{ \begin{pmatrix} 0 & 0 \\ c & 0 \end{pmatrix} \}, \quad \tilde{\mathfrak{g}}^{-1} = \{ \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} \}.$$

So $\tilde{\mathfrak{g}}^0 \approx \mathfrak{gl}(n, \mathbb{R})$, $\tilde{\mathfrak{g}}^{+1}$ carries the representation of $\mathfrak{gl}(n, \mathbb{R})$ $\mathrm{sym}(\mathbf{n}' \otimes \mathbf{n}')$ and $\tilde{\mathfrak{g}}^{-1}$ the representation $\mathrm{sym}(\mathbf{n} \otimes \mathbf{n})$. $\mathbf{n}'$ denotes the contragradient representation of $\mathbf{n}$. The subalgebra $\mathfrak{so}(1,1)_z \subset \mathfrak{gl}(n, \mathbb{R})$ (the subindex z is to distinguish it from other subalgebras $\mathfrak{so}(1,1)$ that we will consider in the following), whose generator is the element $-\frac{1}{2}\mathbb{1}$, acts with charge $+1$ on $\tilde{\mathfrak{g}}^{+1}$ and with charge -1 on $\tilde{\mathfrak{g}}^{-1}$ (so the upper indices indicate this charge). In fact, $\mathfrak{so}(1,1)_z$ defines a grading of $\mathfrak{sp}(2n, \mathbb{R})$ and consequently $\tilde{\mathfrak{g}}^{\pm 1}$ are abelian subalgebras. The matrices of the subalgebra $\tilde{\mathfrak{g}}^0 + \tilde{\mathfrak{g}}^{+1}$ have lower block-triangular form ($b = 0$). The vector space carrying the fundamental representation of the symplectic algebra inherits also a grading and it decomposes as $V = V^+ \oplus V^-$. Notice that V^+ carries a representation of $\tilde{\mathfrak{g}}^0 + \tilde{\mathfrak{g}}^{+1}$.

We consider now the U-duality group G with Lie algebra $\mathfrak{g} \subset \mathfrak{sp}(2n)$. Any subalgebra of $\mathfrak{g}$ which is a subalgebra of the lower triangular matrices $\tilde{\mathfrak{g}}^0 + \tilde{\mathfrak{g}}^{+1}$ will transform the field strengths (in V^+) without involving their magnetic duals. Then, the vector potentials themselves carry a linear representation of this subalgebra which will be called an *electric subalgebra* of $\mathfrak{g}$ (and generically denoted by $\mathfrak{g}_{\mathrm{el}}$). The corresponding group, G_{el}, is an *electric subgroup* of G. The gauge group $\mathrm{G}_{mathrmgauge}$ is a subgroup of the G_{el} such that its action on the vector potentials is the adjoint action. As we will see, there is not a unique maximal electric subgroup of G, and this gives rise to many different gaugings of the supergravity theory.

In the next sections, we will discuss the examples of $N = 8, 4, 2$ supergravity from this new point of view.

3 $N = 8$ supergravity

The U-duality group of $N = 8$ supergravity is $\mathrm{G} = \mathrm{E}_{7,7}$ [17], which can be embedded in several ways in $\mathrm{Sp}(56, \mathbb{R})$ (there are 28 vector fields). The electric subalgebras will always be subalgebras of $\mathfrak{g}_{\mathrm{el}} \subset \tilde{\mathfrak{g}}^0 + \tilde{\mathfrak{g}}^{+1}$, being in this case $\tilde{\mathfrak{g}}^0 \approx \mathfrak{sl}(28, \mathbb{R}) + \mathfrak{so}(1,1)_z$.

Consider the following decomposition of $\mathfrak{e}_{7,7}$

$$\mathfrak{e}_{7,7} = \mathfrak{sl}(8, \mathbb{R}) + \mathbf{70}. \qquad (4)$$

$\mathfrak{sl}(8, \mathbb{R})$ is a maximal subalgebra of $\mathfrak{e}_{7,7}$ and $\mathbf{70}$ is an irreducible representation of $\mathfrak{sl}(8, \mathbb{R})$, the four-fold antisymmetric.

The representation $\mathbf{56}$ of $\mathfrak{e}_{7,7}$ decomposes under the subgroup $\mathfrak{sl}(8, \mathbb{R})$ as

$$\mathbf{56} \longrightarrow \mathbf{28} + \mathbf{28}',$$

where $\mathbf{28}$ and $\mathbf{28}'$ are two-fold antisymmetric representations of $\mathfrak{sl}(8, \mathbb{R})$.

The embedding $\mathfrak{e}_{7,7} \subset \mathfrak{sp}(56, \mathbb{R})$ is constructed as follows. We have that $\mathfrak{sl}(8, \mathbb{R}) \subset \mathfrak{sl}(28, \mathbb{R})$ by means of the two-fold antisymmetric representation (the $\mathbf{28}$) of $\mathfrak{sl}(8, \mathbb{R})$. With a two-fold antisymmetric tensor we can construct a four-fold antisymmetric tensor by taking the symmetrized tensor product. In this way the generators in the **70** of (4) are realized in the two-fold symmetric representation of $\mathfrak{sl}(28,\mathbb{R})$, sym($\mathbf{28}\otimes\mathbf{28}$), forming the b matrix of (3), $b_{\{[AB][CD]\}}$. Since in $\mathfrak{sl}(8,\mathbb{R})$ there is an invariant, totally antisymmetric tensor $\epsilon_{A_1\ldots A_8}$, we have another symmetric matrix

$$c^{\{[A_1A_2][A_3A_4]\}} = \frac{1}{4!}\epsilon^{A_1\ldots A_8} b_{\{[A_5A_6][A_7A_8]\}}.$$

This is the standard embedding of $\mathfrak{e}_{7,7}$ in $\mathfrak{sp}(56,\mathbb{R})$. $\mathfrak{sl}(8,\mathbb{R})$ is a maximal electric subalgebra. The gauging of different electric subalgebras of $\mathfrak{sl}(8,\mathbb{R})$ and of its contractions gives rise to all the theories described in [13, 14]. In this choice, SO(8) is the maximal compact electric subgroup.

We will now consider a different embedding, which is the one relevant for the Scherk–Schwarz mechanism. Consider the decomposition

$$\mathfrak{e}_{7,7} = \mathfrak{e}_{6,6} + \mathfrak{so}(1,1)_k + \mathbf{27}_{-2} + \mathbf{27}'_{+2},$$

where $\mathfrak{e}_{6,6}+\mathfrak{so}(1,1)_k$ is a maximal reductive subalgebra[3]. Notice that it is not a maximal subalgebra. In five dimensions the U-duality group is $\mathrm{E}_{6,6}$ and it is totally electric. If we want to see the four dimensional theory as the dimensional reduction of a five dimensional one, this is the natural decomposition to consider, and the $\mathfrak{so}(1,1)_k$ rescales the modulus of the compactification radius of the 5th dimension. The normalization of the generator of $\mathfrak{so}(1,1)_k$ has been chosen in such way that the fundamental representation decomposes as

$$\mathbf{56} \to \mathbf{27}_{+1} + \mathbf{1}_{+3} + \mathbf{27}'_{-1} + \mathbf{1}_{-3}.$$

(Note that the ratio one to three of the $\mathfrak{so}(1,1)_k$ charges is what one obtains for the relative charges of the 27 five-dimensional vectors versus the graviphoton in the standard Kaluza–Klein reduction).

We have that $\mathfrak{e}_{6,6}+\mathfrak{so}(1,1)_k \subset \mathfrak{gl}(28,\mathbb{R})$, by decomposing the fundamental representation

$$\mathbf{28} \to \mathbf{27}_{+1} + \mathbf{1}_{+3}$$

but $\mathfrak{so}(1,1)_\mathrm{k}$ does not correspond to the trace generator $\mathfrak{so}(1,1)_z$ in $\mathfrak{gl}(28,\mathbb{R})$, since all the vectors in the representation **28** have the same charge under

[3]Reductive means that it is the direct sum of a semisimple algebra plus abelian factors.

$\mathfrak{so}(1,1)_z$, $\mathbf{27_z} + \mathbf{1_z}$. Indeed there is another subalgebra $\mathfrak{so}(1,1)_r$ in $\mathfrak{gl}(28,\mathbb{R})$ which commutes with $\mathfrak{e}_{6,6}$. This comes from the sequence of embeddings

$$\mathfrak{e}_{6,6} \subset \mathfrak{sl}(27,\mathbb{R}) \subset \mathfrak{gl}(27,\mathbb{R}) = \mathfrak{sl}(27,\mathbb{R}) + \mathfrak{so}(1,1)_r \subset \mathfrak{sl}(28,\mathbb{R})$$

corresponding to the fact that only 27 of the 28 vectors are transformed by $\mathfrak{e}_{6,6}$. The charges of the 28 vectors are $\mathbf{27_r} + \mathbf{1_{-27r}}$. Then $\mathfrak{so}(1,1)_k$ turns out to be a combination of $\mathfrak{so}(1,1)_z$ and $\mathfrak{so}(1,1)_r$.

In this setting the symplectic embedding of $\mathfrak{e}_{7,7}$ in $\mathfrak{sp}(56,\mathbb{R})$ is different from the standard one previously considered. It was explicitly worked out in [12]. Here $\mathfrak{g}^0 = \mathfrak{e}_{6,6} + \mathfrak{so}(1,1)_k$ is the block diagonal part and

$$\mathfrak{g}_{\text{el}} = \mathfrak{e}_{6,6} + \mathfrak{so}(1,1)_k + \mathbf{27'_{+2}}$$

is lower block-triangular. Then, it is an electric subalgebra.

Note that in order to have a physical theory the unbroken gauge group in the $\mathfrak{g}^0$ part must belong to the maximal compact subgroup of $\mathfrak{g}^0$.

The Scherk–Schwarz [11, 18] mechanism corresponds to the gauging of an electric subgroup (a "flat group") with algebra $\mathfrak{g}_{\text{el}} = \mathfrak{u}(1)\circledS\mathbf{27'_{+2}}$ (semidirect sum), where $\mathfrak{u}(1)$ is a generic element of the Cartan subalgebra of the maximal compact subgroup $\mathfrak{usp}(8)$ of $\mathfrak{e}_{6,6}$ [12]. The gauging of this electric group breaks spontaneously the supersymmetry. Partial breaking is allowed, and the unbroken supersymmetry algebra has a central charge. Central charges are $\mathfrak{u}(1)$ symmetries which belong to the CSA of G, so if they belong to $\mathfrak{g}_{\text{el}}$ they must belong to the maximal compact subalgebra of $\mathfrak{g}^0 \subset \mathfrak{g}_{\text{el}}$. In our case we have one central charge which is identified with the $\mathfrak{u}(1)$ factor in the semidirect sum $\mathfrak{g}_{\text{el}}$.

In fact, the Scherk–Schwarz mechanism allows partial breakings $N = 8 \rightarrow N' = 6, 4, 2, 0$, and the spin 3/2 multiplets are 1/2 BPS, which means that only one central charge is present. The number of unbroken translational symmetries in the phases $N' = 6, 4, 2, 0$ is, respectively, 15,7,3,3.

4 $N = 4$ supergravity.

As we are going to see, a richer structure emerges in the $N = 4$ theory because in this case we can have both non abelian and abelian flat groups, depending on the particular model we consider.

Let us consider the $N = 4$ theory with $n_v + 1$ vector multiplets. The U-duality group is $\mathrm{G} = \mathrm{SO}(6, n_v + 1) \times \mathrm{SL}(2,\mathbb{R})$, embedded (in different ways) in $\mathrm{Sp}(2(6+n_v+1),\mathbb{R})$, so any electric subalgebra must have block diagonal part a subalgebra $\mathfrak{g}^0 \subset \mathfrak{sl}(6+n_v+1,\mathbb{R}) \times \mathfrak{so}(1,1)_z$.

The standard embedding corresponds to take

$$\mathfrak{so}(6,n_v+1) + \mathfrak{so}(1,1)_q \subset \mathfrak{gl}(6+n_v+1,\mathbb{R}).$$

Here $\mathfrak{so}(1,1)_q$ is the Cartan subalgebra of $\mathfrak{sl}(2,\mathbb{R})$ and it is identified with $\mathfrak{so}(1,1)_z$. The only off-diagonal elements are the other two generators of $\mathfrak{sl}(2,\mathbb{R})$, say $X^{\pm}$. The electric subalgebra is then the lower triangular subalgebra $(\mathfrak{so}(6,n_v+1)+\mathfrak{so}(1,1)_q)\circledS\{X^+\}$. This embedding appears when doing the compactification of the heterotic string on T^6. (See for example the review of Ref. [19]).

We analyze now another symplectic embedding. We take $n_v = 5$ (the embedding is possible only for $n_v \geq 5$). Then we have the following decomposition

$$\mathfrak{so}(6,6) = \mathfrak{sl}(6,\mathbb{R}) + \mathfrak{so}(1,1)_s + \mathbf{15}'^{+} + \mathbf{15}^{-},$$

where $\mathbf{15}$ is the two-fold antisymmetric representation. Since $\mathfrak{sl}(n)+\mathfrak{sl}(m) \subset \mathfrak{sl}(nm)$, we have that

$$\mathfrak{sl}(6,\mathbb{R}) + \mathfrak{sl}(2,\mathbb{R}) + \mathfrak{so}(1,1)_s \subset \mathfrak{gl}(12,\mathbb{R}).$$

The representation $(\mathbf{15}',\mathbf{1})$ is symmetric (the singlet of $\mathfrak{sl}(2,\mathbb{R})$ is the two-fold antisymmetric), so we have that $\mathbf{15}'^{+} \subset \tilde{\mathfrak{g}}^{+} \subset \mathfrak{sp}(24,\mathbb{R})$. This defines the symplectic embedding. The $\mathfrak{so}(1,1)_s$ is identified with $\mathfrak{so}(1,1)_z$ of the symplectic algebra.

The representation $\mathbf{12}$ of $\mathfrak{so}(6,6)$ decomposes, with respect to $\mathfrak{sl}(6,\mathbb{R}) + \mathfrak{so}(1,1)_z$, as

$$\mathbf{12} \to \mathbf{6}_{+1} + \mathbf{6}_{-1},$$

thus containing six electric and six magnetic fields, and the bifundamental of $\mathfrak{so}(6,6)+\mathfrak{sl}(2,\mathbb{R})$ decomposes as

$$(\mathbf{12},\mathbf{2}) = (\mathbf{6}_{+1},\mathbf{2})_{\mathrm{electric}} + (\mathbf{6}_{-1},\mathbf{2})_{\mathrm{magnetic}}.$$

In particular, we see that $\mathfrak{sl}(2,\mathbb{R})$ is totally electric.

The twelve vectors gauge an abelian 12-dimensional subgroup of the $\mathbf{15}'^{+}$ translations.

This model was investigated in Ref. [20], but from our point of view it comes from the general analysis of gauging flat groups. In this case the flat group is completely abelian, since no central charge is gauged. The theory has four independent mass parameters, the four masses of the gravitinos. This allows a partial supersymmetry breaking without cosmological constant from $N = 4 \to N' = 3, 2, 1, 0$, where the massive gravitinos belong to long (non BPS) massive representations in all the cases, as it is implied by the fact that the central charge is not gauged and the fields are not charged under it.

From the analysis of the consistent truncation of $N = 4 \to N' = 3$ supergravity, it is known that $N = 4$ supergravity coupled to 6 matter vector multiplets can indeed be consistently reduced to an $N = 3$ theory coupled to 3 matter multiplets [21]. Correspondingly we have, for the scalar manifolds of such models,

$$\mathrm{SU}(3,3)/(\mathrm{SU}(3)\times\mathrm{SU}(3)\times\mathrm{U}(1)) \subset \mathrm{SO}(6,6)/(\mathrm{SO}(6)\times\mathrm{SO}(6)).$$

In fact, the Higgs effect in this theory needs the gauging of a group of dimension 12, spontaneously broken to a group of dimension 6. The scalar manifolds of the broken phases are

$$\begin{array}{rcl} \mathrm{SU}(3,3)/(\mathrm{SU}(3)\times\mathrm{SU}(3)\times\mathrm{U}(1)), & \text{for} & N'=3 \\ (\mathrm{SU}(1,1)/\mathrm{U}(1))\times\mathrm{SU}(2,2)/(\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)), & \text{for} & N'=2 \\ (\mathrm{SU}(1,1)/\mathrm{U}(1))^3, & \text{for} & N'=1. \end{array}$$

A detailed analysis of this model has been given in [22].

These models are also analyzed in Ref. [9, 10] from another point of view. There, Type IIB supergravity is compactified on the $T_6/\mathbb{Z}_2$ orientifold with brane fluxes turned on and the same pattern of spontaneous symmetry breaking is found.

There is still a third symplectic embedding. An $N = 4$ theory coupled to $n_v + 1$ vector multiplets can be obtained by dimensional reduction of $N = 4$ supergravity in $D = 5$ with n_v vector multiplets. This theory is described by a σ-model $\mathrm{SO}(1,1)\times\mathrm{SO}(5,n_v)/\,(\mathrm{SO}(5)\times\mathrm{SO}(n_v))$ [23].

In $D = 5$, the U-duality group (totally electric) is $\mathrm{SO}(5,n_v)\otimes\mathrm{SO}(1,1)_{d_5}$. By dimensional reduction from $D = 5$ to $D = 4$ we know that a symplectic embedding must exist where the four dimensional electric group contains $\mathrm{SO}(5,n_v)\otimes\mathrm{SO}(1,1)_{d_5}\otimes\mathrm{SO}(1,1)_k$, where, as in the $N = 8$ case, $\mathrm{SO}(1,1)_k$

rescales the radius of the fifth dimension. We will write explicitly this embedding later.

The $5+n_v$ vectors A^Λ_μ belonging to the fundamental representation of $\mathrm{SO}(5,n_v)$ in $D=5$ and the singlet C_μ have charges -1 and 2 respectively under $\mathrm{SO}(1,1)_{d_5}$. When the dimensional reduction from $D=5$ to $D=4$ is performed we obtain the extra vector B_μ coming from the fifth vielbein. It is neutral under $\mathrm{SO}(1,1)_{d_5}$ but has charge 3 with respect to $\mathrm{SO}(1,1)_k$, while all the other $6+n_v$ five-dimensional vectors have charge 1. Summarizing, the electric and magnetic field strengths have the following charges (k,d_5) (in a given normalization)

$$\begin{aligned}&(\mathbf{5}+\mathbf{n_v})^{+1,-1}_{\text{electric}}+\mathbf{1}^{+1,+2}_{\text{electric}}+\mathbf{1}^{+3,0}_{\text{electric}}\,,\\&(\mathbf{5}+\mathbf{n_v})^{-1,+1}_{\text{magnetic}}+\mathbf{1}^{-1,-2}_{\text{magnetic}}+\mathbf{1}^{-3,0}_{\text{magnetic}}.\end{aligned}\tag{5}$$

From a purely four dimensional perspective, the full duality group is $\mathrm{SO}(6,n_v+1)\otimes\mathrm{SL}(2,\mathbb{R})$. We have the following decomposition

$$\begin{aligned}\mathfrak{so}(6,n_v+1)&=\mathfrak{so}(5,n_v)+\mathfrak{so}(1,1)_s+(\mathbf{5}+\mathbf{n_v})^{+}+(\mathbf{5}+\mathbf{n_v})^{-}\\\mathfrak{sl}(2,\mathbb{R})&=\mathfrak{so}(1,1)_q+\mathrm{span}\{X^+,X^-\}\end{aligned}\tag{6}$$

where the $(5+n_v)^+$ Peccei-Quinn symmetries appear explicitly. The representation of the full duality group is the bifundamental, a doublet of $\mathfrak{sl}(2,\mathbb{R})$ tensor product with the fundamental of $\mathfrak{so}(6,n_v+1)$. We denote by $\mathfrak{so}(1,1)_q\subset\mathfrak{sl}(2,\mathbb{R})$ the CSA of $\mathfrak{sl}(2,\mathbb{R})$. Then the field strengths (electric and magnetic) have the following charges (s,q)

$$\begin{aligned}\big(\mathbf{2},(\mathbf{6}+\mathbf{n_v}+\mathbf{1})\big)\ \ \rightarrow\ \ &(\mathbf{5}+\mathbf{n_v})^{0,+1/2}+\mathbf{1}^{-1,+1/2}+\mathbf{1}^{+1,+1/2}+\\&(\mathbf{5}+\mathbf{n_v})^{0,-1/2}+\mathbf{1}^{-1,-1/2}+\mathbf{1}^{+1,-1/2}.\end{aligned}\tag{7}$$

In the standard formulation, $\mathfrak{so}(6,n_v+1)$ is fully electric, but in the symplectic embedding that we are considering this is not the case.

We consider now the subspace

$$(\mathbf{5}+\mathbf{n_v})^{0,+1/2}+\mathbf{1}^{+1,-1/2}+\mathbf{1}^{+1,+1/2}$$

in (7). We want to identify it with the electric subspace in (5). Indeed, let us denote (X_k,X_{d_5}) and (X_s,X_q) the generators of the respective algebras $\mathfrak{so}(1,1)$. From the charges written above we have that this identification is possible if we assume the relations

$$\frac{1}{2}X_k = X_s + X_q, \qquad X_{d5} = X_s - 2X_q$$
$$X_s = \frac{1}{3}(X_k + X_{d5}), \quad X_q = \tfrac{1}{3}(\tfrac{1}{2}X_k - X_{d5}).$$

So we see that under this identification the generators (X_k, X_{d5}) are linear combinations of (X_s, X_q).

We now give the precise embedding of the U-duality algebra $\mathfrak{so}(6, n_v + 1) \times \mathfrak{sl}(2,\mathbb{R})$ in $\mathfrak{sp}(2(7+n_v),\mathbb{R})$. We consider first the diagonal subalgebra, $\mathfrak{gl}(7+n_v),\mathbb{R})$. We have

$$\begin{aligned}\mathfrak{gl}(7+n_v) \quad & \simeq \mathfrak{so}(1,1)_z \otimes \mathfrak{sl}(7+n_v) \\ & \supset \mathfrak{so}(1,1)_z \otimes \mathfrak{so}(1,1)_r \otimes \mathfrak{so}(1,1)_t \otimes \mathfrak{sl}(5,n_v) \\ & \supset \mathfrak{so}(1,1)_z \otimes \mathfrak{so}(1,1)_r \otimes \mathfrak{so}(1,1)_t \otimes \mathfrak{so}(5,n_v). \end{aligned} \tag{8}$$

Since $A^\Lambda_\mu, C_\mu, B_\mu$ have to be electric potentials, we assume they have all the same charge under $\mathfrak{so}(1,1)_z$. From (8) we can compute their charges (r,t,z) under $\mathfrak{so}(1,1)_r + \mathfrak{so}(1,1)_t + \mathfrak{so}(1,1)_z$:

$$(\mathbf{5}+\mathbf{n_v})_{(\frac{r}{6+n_v},\frac{t}{5+n_v},z)}, \qquad \mathbf{1}_{(\frac{r}{6+n_v},-t,z)}; \quad \mathbf{1}_{(-r,0,z)}. \tag{9}$$

It is then straightforward to show that there exist linear combinations of these three charges which reproduce the charges of $\mathfrak{so}(1,1)_{d_5} \otimes \mathfrak{so}(1,1)_k \subset \mathfrak{so}(6,6) + \mathfrak{sl}(2,\mathbb{R})$. We have proved that

$$\mathfrak{so}(5,n_v) + \mathfrak{so}(1,1)_s + \mathfrak{so}(1,1)_q \subset \mathfrak{gl}(7+n_v).$$

The generators in $(\mathbf{5}+\mathbf{n_v})^+$ are embedded in the symplectic group as (see (3))

$$a(t) = \begin{pmatrix} 0^\Lambda_\Sigma & 0^\Lambda & t^\Lambda \\ 0_\Sigma & 0 & 0 \\ 0_\Sigma & 0 & 0 \end{pmatrix}, \qquad c(t) = \begin{pmatrix} 0_{\Lambda\Sigma} & t_\Lambda & 0_\Lambda \\ t_\Sigma & 0 & 0 \\ 0_\Sigma & 0 & 0 \end{pmatrix}, \qquad \Lambda, \Sigma = 1, \ldots 5 + n_v.$$

Note that the difference with the $N=8$ case is that the matrix c here is symmetric off-diagonal while for the $\mathfrak{e}_{7(7)}$ case it was symmetric diagonal. The generators $X^\pm$ have also an appropriate embedding in $\mathfrak{sp}(2(7+n_v),\mathbb{R})$, which depends on the linear combination of X_r, X_t, X_z (generators of the corresponding $\mathrm{SO}(1,1)$ groups) which gives X_q.

From the symplectic embedding it follows that the transformations of the electric vectors under the Peccei-Quinn translational symmetries are:

$$\delta A^\Lambda_\mu = t^\Lambda B_\mu; \qquad \delta C_\mu = \delta B_\mu = 0 \tag{10}$$

while their magnetic counterparts have field strengths transforming as follows:

$$\begin{aligned} \delta F^{(m)}_{\Lambda\mu\nu} &= t_\Lambda C_{\mu\nu} \\ \delta C^{(m)}_{\mu\nu} &= t_\Lambda F^\Lambda_{\mu\nu} \\ \delta B^{(m)}_{\mu\nu} &= -t^\Lambda F^{(m)}_{\Lambda\mu\nu} \end{aligned} \tag{11}$$

where $t_\Lambda = \eta_{\Lambda\Sigma} t^\Sigma$.

Note that the charges of t^Λ with respect to $\mathfrak{so}(1,1)_k$ and $\mathfrak{so}(1,1)_{d_5}$ are -2 and -1 respectively so that the equations (10) and (11) respect the charge assignements.

In this symplectic embedding we can gauge flat groups with algebra $\mathfrak{g}_{\mathrm{el}} = \mathfrak{u}(1) \circledS \mathfrak{g}^+$, where $\mathfrak{g}^+$ is an abelian subalgebra contained in $(\mathbf{5}+\mathbf{n_v})_+$. The commutation rules are of the form

$$[X_0, X_\Lambda] = f_{0\Lambda}{}^\Sigma X_\Sigma, \qquad \Lambda, \Sigma = 1, \dots 5+n_v.$$

X_0 is a generic element of the Cartan subalgebra of $\mathfrak{usp}(4)+\mathfrak{so}(n_v)$, the maximal compact subalgebra of $\mathfrak{so}(5,n_v)$. If n_v is even, this element depends on $2+n_v/2$ parameters.

Let us assume that X_0 is in the CSA of $\mathfrak{usp}(4)$, so it depends only on two parameters,

$$X_0 = \begin{pmatrix} m_1\epsilon & 0 \\ 0 & m_2\epsilon \end{pmatrix}, \qquad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

The model obtained by gauging this algebra reproduces the Scherk–Schwarz mechanism applied to $N=4$ supergravity. The analysis of the explicit symmetry breaking pattern follows the lines of the $N=8$ supergravity treated in Ref. [18]. The results are as follows: m_1 and m_2 are the masses acquired by the gravitinos and the gauginos. The non zero eigenvalues of the Cartan element in the two-fold antisymmetric representation of $\mathfrak{usp}(4)$ are $\pm(m_1 \pm m_2)$, and this gives the masses of the vectors in the gravity multiplet. At most four vectors can become massive and four translational symmetries can be broken.

If $m_1, m_2 \neq 0$, then all supersymmetries are broken. If $m_2 = 0$, we have a spontaneous breaking $N = 4 \to N' = 2$. The massive gravitinos are short, 1/2 BPS multiplets with central charge given by X_0. All the vector multiplets remain massless and all hypermultiplets acquire a mass m_1. The moduli space of this theory is

$$(\mathrm{SU}(1,1)/\mathrm{U}(1)) \times \big(\mathrm{SO}(2, n_v + 1)/(\mathrm{SO}(2) \times \mathrm{SO}(n_v + 1))\big).$$

5 $N = 2$ supergravity.

We would like to discuss now the Scherk–Schwarz mechanism in $N = 2$ supergravity coupled to matter.

Let us first discuss $N = 2$ supergravity in $D = 5$, with n_v vector multiplets and no hypermultiplets. The scalars of the vector multiplets are coordinates of a real manifold of dimension n_v.

The global symmetry of this theory is $\mathrm{USp}(2) \times \mathrm{K}$, where USp(2) is the R-symmetry of the $D = 5$ theory and K is the isometry group of the scalar manifold.

In absence of such isometries ($\mathrm{K} = \mathbb{1}$) then USp(2) is the only global symmetry which can be used to perform the Scherk–Schwarz mechanism. The Cartan generator X_0 of USp(2) is gauged by the graviphoton.

In the theory dimensionally reduced to $D = 4$ the manifold parametrized by the scalars of the vector multiplets is a complex special manifold $\mathcal{M}_v$ of complex dimension $n_v + 1$. We denote by t^A, $A = 1, \ldots n_v + 1$ its complex coordinates. The Kähler potential of $\mathcal{M}_v$ is defined as [24]

$$\mathcal{K} = -\log\left(d_{ABC}\Im(t^A)\Im(t^B)\Im(t^C)\right).$$

The scalar manifold has $n_v + 1$ translational isometries, however, since the scalars are inert under USp(2), these isometries are not gauged.

Let $m\epsilon$ be a generic element of the CSA of USp(2) with

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Under the Scherk–Schwarz generalized dimensional reduction all gravitinos and gauginos, which are USp(2) doublets, acquire a mass $e^{\mathcal{K}/2}m$. All vectors and scalars are massless, so that no gauge symmetry is broken and the scalar potential vanishes identically.

The Scherk–Schwarz model, in absence of hypermultiplets and with a special manifold without isometries, corresponds then to $N = 2$ supergravity with a Fayet–Iliopoulos term and gives the model discussed in [25].

Let us now consider the case in which the special manifold $\mathcal{M}_v$ has a non trivial isometry group K. Then one could consider a model where the X_0 generator has a non zero component in the Cartan subalgebra of the maximally compact subgroup of K. In this case the gauge algebra would become non-abelian and some of the vector fields would acquire a mass.

As an example consider the model based on the special manifold

$$\frac{\mathrm{SU}(3,3)}{\mathrm{SU}(3)\times\mathrm{SU}(3)\times\mathrm{U}(1)}.$$

In the Scherk–Schwarz mechanism the five dimensional ancestor of this model is based on the coset $\mathrm{SL}(3,\mathbb{C})/\mathrm{SU}(3)$. Then, in analogy with the $N = 8$ case, we have the decomposition

$$\mathfrak{su}(3,3) = \mathfrak{sl}(3,\mathbb{C}) + \mathfrak{so}(1,1)_k + \mathbf{9'}^{+2} + \mathbf{9}^{-2}.$$

The electric subalgebra in $\mathfrak{su}(3,3)$ is [21] $\mathfrak{g}_{\mathrm{el}} = \mathfrak{g}^0 + \mathfrak{g}^+$ where

$$\begin{aligned} g^0 &= \mathfrak{sl}(3,\mathbb{C}) + \mathfrak{so}(1,1)_k \\ g^+ &= \mathbf{9'} = (\mathbf{3'},\bar{\mathbf{3}}') \text{ of } \mathfrak{sl}(3,\mathbb{C}). \end{aligned}$$

The vector field strengths and their magnetic duals are in the **20** (three-fold antisymmetric) of $\mathfrak{su}(3,3)$ [26]. This representation decomposes as follows under $\mathfrak{sl}(3,\mathbb{C}) + \mathfrak{so}(1,1)_k$

$$\mathbf{20} \to \mathbf{9}^{+1} + \mathbf{1}^{+3} + \mathbf{9'}^{-1} + \mathbf{1}^{-3}$$

The gauge algebra is therefore $X_0 \circledS \mathfrak{g}^+$ where X_0 is a Cartan element of $\mathrm{SU}(3) \subset \mathrm{SL}(3,\mathbb{C})$, under which g^+ is in the $(\mathbf{8}+\mathbf{1})$ representation.

The 10 dimensional Lie algebra

$$[X_\Lambda, X_0] = M_\Lambda^\Sigma X_\Sigma\,; \qquad [X_\Lambda, X_\Sigma] = 0$$

has structure constants M_Λ^Σ given by a diagonal matrix with entries

$$\{\pm(a_1 - a_2), \pm(2a_1 + a_2), \pm(a_1 + 2a_2), 0, 0, 0\}\,. \tag{12}$$

In this case the central charge does not break the supersymmetry, but the nine vector multiplets will organize in three massive 1/2-BPS multiplets with masses proportional to $|a_1 - a_2|$, $|2a_1 + a_2|$, $|a_1 + 2a_2|$, and three massless ones, the fourth massless vector being the graviphoton.

In the broken phase, the moduli space of this theory is [27, 21]

$$\left(\frac{SU(1,1)}{U(1)}\right)^3 . \tag{13}$$

The mass matrix is

$$\tilde{M}_\Lambda^\Sigma = e^{\mathcal{K}/2} M_\Lambda^\Sigma .$$

where $\mathcal{K}$ is the Kähler potential of the manifold (13). There are six broken symmetries, corresponding to the difference $(9 - 3 = 6)$ in the number of translational symmetries between the unbroken and broken phase. Since the masses of the gravitinos vanish, the scalar potential is positive definite and it vanishes at the minimum.

This model can be easily generalized by taking X_0 to be a linear combination of a Cartan element of SU(3) with the one of USp(2) (with eigenvalue m). In this case the gravitinos get a mass m (so that the supersymmetry is completely broken), vectors and scalars still have masses given by the eigenvalues (12) of M_Λ^Σ, while the gauginos have masses shifted by m with respect to the bosons.

Let us briefly discuss the Scherk–Schwarz mechanism when also hypermultiplets are present. In this case, since the hyperscalars transform under USp(2), to perform the gauging of X_0 in the CSA of USp(2) it is required that the quaternionic manifold $\mathcal{M}_Q$ parametrized by them has at least an USp(2) isometry. This would be for instance the case for all the quaternionic symmetric spaces [28, 29, 30] but would not be true for more general manifolds as for instance the general quaternionic spaces obtained through c-map [31, 32].

If the quaternionic manifold $\mathcal{M}_Q$ has some larger isometry K′, we may use K′ for the Scherk–Schwarz mechanism.

In the case where only USp(2) is used as a global symmetry, then the hyperscalars would acquire a mass set by m, while the hyperinos would remain massless. Then, a scalar potential would appear.

The Scherk–Schwarz mechanism allows models either with the $N = 2$ supersymmetry left unbroken, or completely broken to $N = 0$.

To obtain partial breaking $N = 2 \to N = 1$ we need hypermultiplets to be present and to gauge at least two translational isometries of the quaternionic manifold. The minimal model [33, 34] has one vector multiplet whose complex scalar parametrizes the special manifold

$$\mathcal{M}_v = \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \tag{14}$$

with Kähler potential $\mathcal{K} = -\log(z+\bar{z})$, and one hypermultiplet whose scalars parametrize the quaternionic manifold

$$\mathcal{M}_Q = \frac{\mathrm{USp}(2,2)}{\mathrm{USp}(2)\times\mathrm{USp}(2)}. \tag{15}$$

This contains three axions with charges (g_1, g_2) with respect to the two gauge fields (B_μ, Z_μ).

Since $\mathcal{M}_Q$ has three translational isometries, the unbroken $N = 1$ theory has still one translational isometry coming from the quaternionic manifold. The moduli space of the $N = 1$ theory is

$$\mathcal{M}_v \times \left(\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}\right)_Q .$$

$(\mathrm{SU}(1,1)/\mathrm{U}(1))_Q$ is a sub-manifold of $\mathcal{M}_Q$ and the spectrum contains a massive gravitino multiplet and two massless chiral multiplets. Since no compact generators are gauged, the gauge algebra is purely abelian and this model is similar to the $N = 4$ one with abelian gauging of the translational isometries considered in Section 4 [20]. As in that model, the two gravitinos acquire masses

$$m_{1,2} = |g_1 \pm g_2| e^{\tilde{\mathcal{K}}/2} \tag{16}$$

where $\tilde{\mathcal{K}}$ is the Kähler potential of

$$\left(\frac{SU(1,1)}{U(1)}\right)_v \times \left(\frac{SU(1,1)}{U(1)}\right)_Q .$$

An $N = 1$ theory is found for $|g_1| = |g_2|$.

This model has vanishing potential since the only scalars which become massive are the two Goldstone bosons of the $\mathbb{R}^2$ gauged isometry, which are eaten by the vectors that become massive.

Note that the number of massive vectors is given by $4 - 2 = 2$, that is the difference in number between the translational isometries of the $N = 2$ scalar manifolds and the corresponding ones in the $N = 1$ moduli space. This agrees with the general analysis of [35]. A possible generalization of this model to many hyper and vector multiplets has been given in [36].

6 Summary

In this paper we have explored new gaugings of extended supergravity in which different maximal subgroups of the duality group have a diagonal embedding in the electric-magnetic symplectic duality rotations.

The gauging of “flat groups” allows to obtain a class of models which are the extension of no-scale supergravity to higher N-theories. These models encompass the Scherk–Schwarz generalized dimensional reduction and supergravity or M-theory compactifications in presence of brane-fluxes.

Among these models, of particular interest for phenomenological studies are the $N = 2 \rightarrow N = 1$ models, which can be obtained, for example, in IIB orientifolds with three-form fluxes turned on.

Finally, another possibility of obtaining an $N = 2 \rightarrow N = 1$ breaking is to compactify a five dimensional theory on a $S_1/\mathbb{Z}_2$ orientifold à la Scherk–Schwarz, by introducing a $\mathbb{Z}_2$ parity which truncates the second gravitino [37] - [42]. This is the mechanism used in the supergravity version of the Randall-Sundrum scenario and will be discussed elsewhere.

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