A Tool for Simulating Rotating Coil Magnetometers

L. Bottura, M. Buzio, P. Schnizer, and N. Smirnov

***Abstract*—When investigating the quality of a magnetic measurement system, one observes difficulties to identify the "trouble maker" of such a system as different effects can yield similar influences on the measurement results. We describe a tool in this paper that allows to investigate numerically the effects produced by different imperfections of components of such a system, including, but not limited to: vibration and movements of the rotating coil, influence of electrical noise on the system, angular encoder imperfections. This system can simulate the deterministic and stochastic parts of those imperfections.**

We outline the physical models used that are generally based on experience or first principles. Comparisons to analytical results are shown. The modular structure of the general design of this tool permits to include new modules for new devices and effects.

***Index Terms*—Harmonic coil measurement, LHC magnets, measurement system simulation.**

I. INTRODUCTION

MAIN dipoles and quadrupoles being built for the LHC will be measured magnetically at CERN using equipment mostly based on the rotating coils technique [1]. While searching for the source of errors (e.g., electrical noise, imperfection of the rotation of the coils, spurious marks in the angular encoder) in rotating coil magnetometers one can see that different malfunctioning components can produce similar effects. The aim of this tool is to identify the weakest points of the measurement system and to improve the quality of series LHC magnet tests. An analytical treatment of nearly any effect is available and only perfect magnets are considered. However, considering more than one effect renders the analysis already more difficult. Previously codes were developed using a purely numerical approach [2], [3]. This tool, however, uses a semi-numerical approach which calculates the effects of coil imperfections (e.g., coil vibration) analytically.

The key design features of this tool are modularity, flexibility and simplicity:

- Modularity: Follow the "real system" setup as closely as possible. The preferred design is that an independent element of the experimental system is described as one entity in the software. Each element of the real system is a module encapsulating its description and key parameters.
- Flexibility: Not all effects can be foreseen during the design stage. Some effects can become important in future systems which are negligible nowadays. Therefore the system must allow the exchange and enhancement of all elements describing the experimental systems.
- Simplicity: The code must offer simple interfaces to the user to allow handling the system without having to know every detail of the total implementation.

This paper describes schematically the implementation possibilities and the basic physical principles used. The validation of the system will be shown and finally examples of applications will be given.

II. MEASUREMENT OF FIELD COMPONENTS

The final aim of a rotating coil system is to measure the field of a magnet and to describe its multipoles C_n. The magnetic field B of accelerator magnets can be expanded in a series as follows:

$$B(z) = B_y + iB_x = \sum_{n=1}^{\infty} [B_n + iA_n] \left(\frac{z}{R_{Ref}}\right)^n \qquad (1)$$

where $B_n + iA_n = C_n$ are the multipole components [4], and z the transverse position. The flux Φ seen by a rotating coil is given by [5]:

$$\Phi(t) = \sum_{n=1}^{M} K_n C_n \cos[n\beta(t)]. \qquad (2)$$

M is the highest harmonic taken into account, β is the angular position and K_n the coil's sensitivity to the nth harmonic given by:

$$K_n = \left(\frac{N_w L R_{Ref}}{n}\right) \left[\left(\frac{z_2}{R_{Ref}}\right)^n - \left(\frac{z_1}{R_{Ref}}\right)^n\right] \qquad (3)$$

where N_w is the number of windings, L the length and z_2 and z_1 the inner and outer radii respectively of the coil. R_{Ref} is the reference radius (for LHC 17 mm). The voltage $V(t)$ induced in the coil is given by:

$$V(t) = -n\dot{\beta}(t) \sum_{n=1}^{N} K_n C_n \sin[n\beta(t)]. \qquad (4)$$

Equation (4) is obtained applying the induction law on (2) assuming a magnetic field which is constant in time. The systems used for the magnetic measurements at CERN is based on digital integrators. The C_ns are obtained as measurements:

$$C_n = \frac{1}{K_n} FT \left(\int_0^P V(t')\, dt'\right) \qquad (5)$$

where FT denotes the Fourier transform, P the time needed for one revolution of the coil.

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III. Approach

Equation (5) can be solved using different approaches, a pure numerical approach in the time domain, a semi-numerical approach in the time domain, or in the frequency domain. The main advantages and disadvantages of these approaches are given below:

1) *A pure numerical approach in the time domain*. The movement of each winding of the coils is traced and the radii [z_2, z_1 in (3)] are corrected at each time step to describe non ideal behavior. The induced voltage is then calculated numerically by differentiating (2). This allows a simple modeling of the coil's motion irregularities, but the calculation is computationally expensive. Moreover, the geometry of the coils must be known in full detail. In a compensated system each induced voltage has to be calculated separately for each coil and compensated afterwards.
2) *A semi-numerical approach in the time domain*. Rather than the coil geometric parameters, in this approach the sensitivities K_n are directly used. Coil imperfections are then described calculating a correction to the voltage given by (4). Therefore the sensitivities need not be differentiated. In addition a compensated system can be simulated using the sensitivity factors obtained by calibration.
3) *In the frequency domain*. The magnetic field B is described by its multipoles C_n. Considering each harmonic in (2) individually it can be interpreted as a spectrum of $n\beta$. Electrical noise can also be described by a spectrum. So it seems natural to calculate in the frequency domain. Each component affects the spectrum of the previous one. However the imperfections of certain components such as an angular encoder with a missing mark, a non linear integrator or a drifting power supply can be difficult to model.

The fact that sensitivities K_n are well known and can be used directly was considered a sufficient advantage of the semi-numerical approach to accept the tradeoff of the more complicated formalisms. The approach in frequency space was not considered further as some components are difficult to model.

IV. Physical Models

Using physical models for the simulation, first principles are used to describe each element and its imperfections. The implementation of each element was kept as simple as possible but still allowing input of experimental experience. As an example the torsional vibrations of mechanical connections are modeled using a series of decaying frequencies. In the following the components of a rotating coil system are described which typically consists of rotating coils, a motor, an angular encoder, mechanical connections, cables and electronics:

1) *The motor* that drives the coil shaft following a time dependent angle $\theta(t) = \omega t + \sum_{i=1}^{N} A_i \sin(\alpha_i t + \phi_i)$ where the first term describes the ideal case and the second term adds a motor vibration with A_i the amplitude of the ith eigenfrequency of the system with frequency α_i and phase ϕ_i.
2) *Mechanical connections* that couple the coil to the motor. These connections can vibrate with their eigenfrequencies which can influence the measurement. This can be modeled as $\beta(t) = \theta(t) + \sum_i e^{-d_i t} a_i \sin(c_i t + \psi_i)$ with β the angle of the connection on the coils side, θ the angle on the motors side, a_i the amplitude of its ith eigenfrequency c_i with phase ψ_i. This torsion allows a simulation of a system consisting of similar coils with connections in between. (Such a system is described in [6].) There the torsional vibration of each coil can be simulated using one coil with different connections.
3) *Rotating coils*; their voltage is given by (4) for the ideal case setting $\beta(t) = \omega t$ with ω the nominal speed of the motor. A torsional vibration is modeled setting $\beta(t)$ to $\beta(t) + T(\beta(t))$ with $T(\beta)$ the function describing the vibration.
4) *The angular encoder* that triggers the measurement. In the simulation it is used to determine the appropriate times for the trigger. These events will then be used to calculate the flux Φ.
5) *Cables* that transfer signals from source to receiver and can pick up electromagnetic noise from the environment. Its output voltage U_O is given as: $U_O = U_I + U_N$ with U_I the input voltage and U_N the electromagnetic noise. An example of such noise is given later.
6) *Electronics* consisting of compensation circuitry, preamplifier and integrator. The ideal behavior is given as $U_{OE} = \int_{t_0'}^{t_0'+P} G_P \sum_i G_{C_i} U_{IC_i}\, dt$ where U_{OE} is the output voltage, G_P the gain of the preamplifier, U_{IC_i} the input voltage and G_{C_i} the gain of the ith channel of the compensation circuitry. These devices can have input and output voltage offsets and non linear amplification.

V. Implementation

In the real system the behavior of each component is influenced by the attached devices and the surrounding environment. In order to reflect reality as closely as possible an object oriented hierarchy was chosen. In Section IV the parameters allowing to simulate imperfections were described. These parameters are encapsulated in each module and allow a simple switch between "ideal" and "real" behavior.

We are mainly interested in the effects on the field at the level of the higher order harmonics. These are generally of the order of 100 ppm of the main field. Therefore the accuracy of the calculation must be in the range of 1 ppm or 6 digits. Equation (4) shows that the contribution of each harmonic is summed up in the time domain. This means that the contribution to the voltage of any non main harmonic starts at the forth digit of the main harmonic. The accuracy requirements can be fulfilled calculating with a 15 digits' accuracy.

From the numerical point of view the most challenging part is the implementation of the integrator because an accuracy of better than 10^{-6} has to be achieved. It was found that an Ordinary Differential Equation solver matched best our needs. Using the function *lsoda* from the ODEPACK package [7] it is possible to achieve a precision better than 10^{-10} with an eight byte floating point representation. This routine offers internal quality control of the calculation which allows a fast integration at a known accuracy. A typical run of one turn of the coil takes only a few seconds on a Pentium III 600 MHz. The computation time scales roughly linearly with the highest relevant frequency. The aforementioned number is valid for a frequency of 15 $\omega/2\pi$ as the harmonics are normally only considered up to this value.

VI. Comparison to Analytical Results

To validate our system we have compared the numerical results to known analytical solutions in the case of a coil vibrating either in the transverse direction or torsionally during a measurement. These calculations were done considering only one coil. In this case the system is more sensitive to mechanical imperfections compared to a system where the main harmonic is compensated by an array of coils. The goal of this comparison is to show that the simulation is accurate enough.

A. Transversally Vibrating Coil

The coils movement was assumed to be $l\sin(p\theta(t))$ with l the amplitude, and p the frequency index of the movement. In a pure quadrupole field the harmonics generated due to the vibration are given by [8]:

$$C_{p+1} = \frac{K_1}{K_{p+1}}\,\frac{l}{2R_{Ref}}\,C_2 \tag{6}$$

$$C_{p-1} = \frac{\overline{K}_1}{K_{p-1}}\,\frac{\bar{l}}{2R_{Ref}}\,C_2. \tag{7}$$

Variables with a bar denote the complex conjugate. In accelerator magnet measurement systems the coil movement $|l|$ must be at least 1μm to produce a signal comparable to the typical resolution (≈ 1 μT @ R_{Ref}). For $l = 1$ μm and $p = 1$ and $C_2 = 1.5$ T @ R_{Ref} we obtain that the analytical and numerical values match for the first six digits. $C_1/C_2 = 1 \cdot 10^{-6}$ and $C_3/C_2 = 1.3 \cdot 10^{-6}$.

B. Torsionally Vibrating Coil

Assuming a torsional vibration $T(\theta)$ of the type $T(\theta) = \zeta\cos(\tau\theta)$ and substituting $\theta(t)$ in (4) with $\omega t + T(\omega t)$ and approximating $\sin(n\theta + T(\theta)) = \mathrm{Im}[\exp(in\{\theta + T(\theta)\})]$ with $\mathrm{Im}[\exp(in\theta)\{1 + nT(\theta)\}]$ [8] one gets as spurious harmonics:

$$C_{\tau+n} \approx \frac{nK_n}{K_{\tau+n}}\,\zeta\,\overline{C}_n \tag{8}$$

$$C_{\tau-n} \approx -\frac{n\overline{K}_n}{K_{\tau-n}}\,\bar{\zeta}\,C_n. \tag{9}$$

τ is the frequency index. For the numerical case $\theta(t)$ is set to $\omega t + T(\omega t)$. In Fig. 1 the comparison between the analytical formula and the calculation of the fake octupole is shown. The two curves match well for realistic cases (typically in the order of mrad torsion). The deviation of the analytical solution from the numerical one is due to the aforementioned approximation.

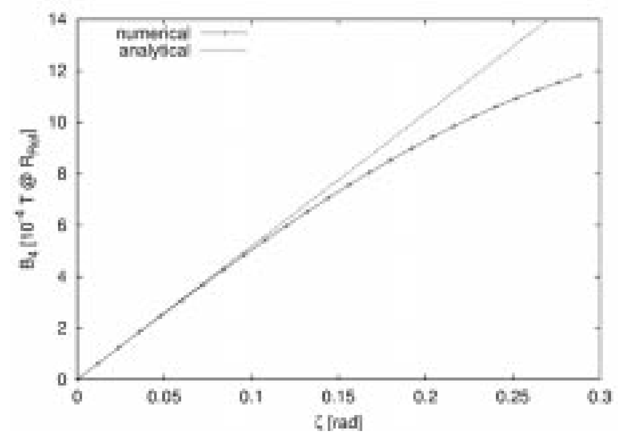

Fig. 1. Example of the influence of a torsional vibrating coil on the measurement of the octupole in a pure quadrupole field for a vibration $p = 2$. ζ represents the maximum amplitude of the vibration in radians and B4 the normal octupole due to the coil vibration in Tesla at reference radius.

TABLE I
Calculated Requirements to the Axis Vibration.
$unit = C_n/C_{main} * 10^4$

Measurement type	target [unit]	[μm]	
main field systematic	5	l	10
main field random	1	Δl	2
harmonics systematic	0.05	l	50
harmonics random	0.01	Δl	10

VII. Example of Applications

We have used the calculational tool to explore the sensitivity of the quadrupole measurement system described in [1] to mechanical and electrical perturbations.

A. Transversally Vibrating Coil

Due to mechanical tolerances in the support, the coil can vibrate during a measurement. We describe such a vibration by a horizontal displacement $l + \Delta l$ and we compute the maximum allowable amplitude l and the spread Δl that give systematic and random errors comparable with the accuracy targets set in [1]. The targets are reported in Table I together with the results of the calculation assuming a coil vibration $(l + \Delta l)\sin(\omega t)$. The table values were calculated assuming a quadrupole field of $C_2 = 1.5$ T @ R_{Ref}. For the random parts the limit seems to be very tight. However we remark that they were calculated assuming only one frequency. Typically the vibrations affecting the random error consist of a whole spectrum of frequencies which affect different harmonics resulting in a much softer requirement to the total maximum deviation. Therefore the results discussed here must be taken as pessimistic estimates. The figures of Table I broadly comply with the mechanical tolerances set for the moving components.

B. Torsionally Vibrating Coil

Due to the size of the LHC cryostated magnets the shaft of the system [1] is 15 meters long. Therefore torsional vibrations have to be considered. Only a frequency index $\tau = 4$ can influence the absolute measurement (9). Using the targets shown in

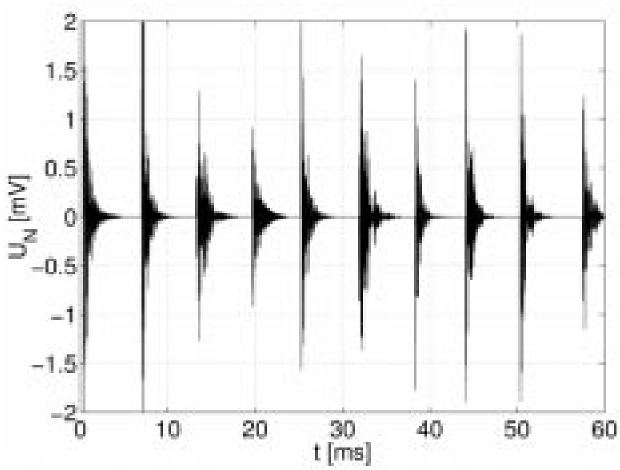

Fig. 2. Noise in the cable.

TABLE II
INFLUENCE OF THE ELECTRICAL NOISE ON THE HARMONICS

voltage q_{max} [mV]	0.1	1	10
$[Cn/C2 \cdot 10^4]$ $C_2 = 1.5T @ R_{Ref}$	10^{-4}	10^{-3}	10^{-2}
$[Cn/C2 \cdot 10^4]$ $C_2 = 0.2T @ R_{Ref}$	$7 \cdot 10^{-4}$	$7 \cdot 10^{-3}$	$7 \cdot 10^{-2}$

Table I one obtains $\zeta \pm \Delta\zeta = 0.1 \pm 0.02$ mrad for the absolute measurement. For the compensated measurement one obtains $\zeta \pm \Delta\zeta = 0.8 \pm 0.2$ mrad. These are worst case estimates as the vibration contains a whole spectrum of frequencies. Therefore the requirement to the maximum amplitude can be much softer.

C. Noise Coupled Into the Cables

A source of spurious measurement results can be electrical noise coupled into the cables feeding the coils signal to the electronics. Here the influence of a switching power supply generating medium range frequencies is considered.

A thyristor driven power supply can induce a voltage noise as shown in Fig. 2 inside the cables. This voltage U_N is given by:

$$U_N(t) = \sum_{i=1}^{N} q_i e^{-r_i(t-t_{0i})} \sin[s_i(t-t_{0i})]. \qquad (10)$$

We have chosen the parameters for (10) to reproduce Fig. 2. We have injected this noise after the analog compensation, where the signal is smallest and the effect largest. The influence of this noise on the system described in [1] is given in Table II. The influence on the harmonics is correlated with the maximum amplitude of the noise and shows roughly the same amplitude for all harmonics. As Fourier series need the whole spectrum of frequencies to describe discontinuities as shown in Fig. 2, the series contains low frequencies as well. Due to aliasing [9] (the maximal resolvable frequency in the system [1] is 125 $\omega/2\pi$) these frequencies are amplified and therefore an influence on the harmonic measurement is visible. However in a realistic range (1 to 5 mV) the effect on the harmonics is still within the targets given in Table I.

VIII. CONCLUSION

A system, simulating a rotating coil magnetometer has been described. It allows to study the main components and imperfections of such a system. The basic structure is given and comparisons to analytical results were shown. Due to its modularity it is straightforward to add new components to study additional imperfections. It was shown that for a system as described in [1] the mechanical vibrations of the coils need a tight control as their transversal vibration l has to be controlled to 10 μm. The maximal acceptable torsional vibration between the angular encoder and the coils is 0.1 mrad. These requirements are barely fulfilled by the aforementioned system. However these numbers were obtained using assumption based on worst case studies and conservative estimates. An electrical noise as discussed above is acceptable as long as the peak to peak voltage is smaller than 2.5 mV.

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