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We examine the gluon propagator in the Laplacian gauge in quenched lattice QCD as a function of the number of colours. We observe a weak dependence on N_c over the whole momentum range. This implies an almost N_c -independent gluon pole mass in units of the string tension.

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I. INTRODUCTION

Lattice QCD provides a well suited theoretical framework for the study of nonperturbative quantities as a function of the number of colours. Although experimentally inaccessible, QCD for $N_c = 2$ and $N_c = 4$ is interesting from the theoretical point of view. Recently $SU(N_c)$ Yang-Mills theories were investigated for various N_c by two groups [1,2] with emphasis on the evaluation of string tensions and glueball masses and their large N_c limit. It is the purpose of the present work to examine the gluon propagator for different numbers of colours. Like in our previous work in $SU(3)$, we evaluate the gluon propagator in the Laplacian gauge which, unlike the more usual Landau gauge, is free of spurious Gribov copies on the lattice. Therefore no doubt can be cast on our results due to the lattice gauge fixing procedure. Furthermore theoretical arguments have been presented supporting gauge invariance of the pole mass of the transverse gluon propagator [3] and it is therefore an interesting quantity to investigate as a function of N_c even in a particular gauge. Studying the dependence of the gluon mass on N_c has theoretical implications for confinement and chiral symmetry breaking. It has been argued that a non-zero gluon mass is connected to vortex condensation [4] and to the glueball mass [5]. Since the N_c dependence of the glueballs has been investigated, a study of the gluon mass can shed light on their relation. In Dyson Schwinger studies the form of the gluon propagator affects that of the quark propagator, which in turn provides a determination of the quark mass and therefore is connected to chiral symmetry breaking.

In previous studies [6,7] we considered the gluon propagator in $SU(3)$, and presented evidence for a pole which survives the continuum limit. In this work we present re-

sults in $SU(2)$, $SU(3)$ and $SU(4)$ at similar couplings. We refer the reader to ref. [6] for our notation and the details of our approach.

II. LAPLACIAN GAUGE FIXING

The Laplacian gauge consists of rotating along a fixed orientation the local color frame built from the lowest-lying eigenvectors of the covariant Laplacian. It is well defined in the continuum theory but non-perturbative. On the lattice, it has the virtue of being unambiguous (except for *genuine* Gribov copies arising from accidental degeneracy of Laplacian eigenvalues), unlike the lattice Landau gauge.

In the case of $SU(2)$, the Laplacian eigenvalues are twofold degenerate [8] due to charge conjugation symmetry: if f is an eigenvector so is $\sigma_2 f^*$. At each lattice site, one can construct from these two vectors a 2×2 matrix which is then projected onto $SU(2)$ and defines the required gauge rotation.

In $SU(3)$ and $SU(4)$, we identify the $N_c - 1$ lowest-lying eigenvectors of the Laplacian. From these eigenvectors we construct at each lattice site an $SU(N_c)$ matrix by using a Gram-Schmidt orthogonalization, as described in detail in ref. [6].

III. RESULTS

In order to make the comparison of our results in $SU(2)$, $SU(3)$ and $SU(4)$ easier, we consider β values which yield similar lattice spacings in units of the string tension σ . We take $\beta = 2.4$, 6.0 and 10.9 for $SU(2)$, $SU(3)$ and $SU(4)$ respectively, which give $a\sqrt{\sigma} =$

0.2634(16) [13], 0.2265(55) [9] and 0.2429(14) [1] for the three gauge groups. We can then express our $SU(N_c)$ results in “physical” units using $\sqrt{\sigma} = 440$ MeV. The inverse lattice spacing is 1.67(1), 1.94(5) and 1.81(1) GeV for $SU(2)$, $SU(3)$ and $SU(4)$ respectively.

We first examine $SU(2)$. For this gauge group, several lattice studies of the gluon propagator in the Landau and Coloumb gauges exist [10,11]. We present here the first study in the Laplacian gauge where no issues of lattice Gribov copies arise. It is therefore important to have lattice artifacts under control. The volume dependence is investigated by considering the $SU(2)$ gluon propagator for lattices of size 4^4 , 8^4 , 16^4 , $16^3 \times 64$ and 32^4 . The quantity of interest is the transverse part, $D(q^2)$, of the gluon propagator. This is shown in Fig. 1 in lattice units for the various volumes after making a cylindrical cut [12] in the momenta which eliminates the leading lattice artifacts due to lack of rotational symmetry. For non-zero momenta, we subtract the longitudinal part of the propagator to project out its transverse part. For zero momentum this subtraction is not well defined, and we keep the complete propagator. We include results from the very asymmetric $16^3 \times 64$ lattice. Even in that case, only small finite-size effects are visible in the infrared (momenta less than ~ 0.5 GeV in “physical” units).

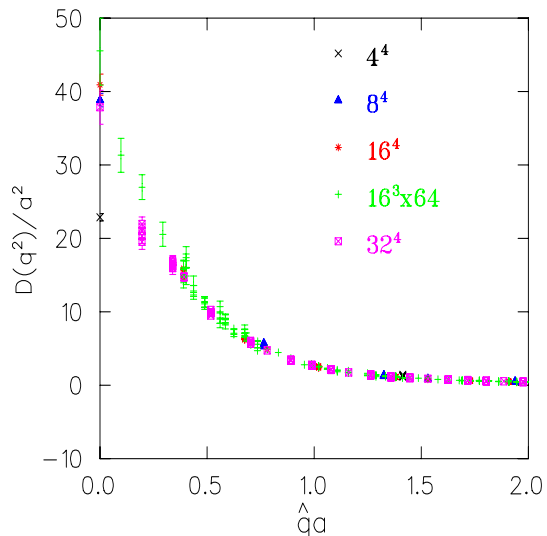


FIG. 1. The $SU(2)$ gluon propagator in lattice units for various volumes.

One conclusion to be drawn from Fig. 1 is that the zero momentum gluon propagator has a finite value as the volume is increased. In fact for lattices of spatial size greater than 0.8 fm it reaches a constant value. This behaviour also holds for $SU(3)$ [6,7]. For $SU(4)$ we have performed simulations for only one lattice size since the similarity of the results in $SU(2)$ and $SU(3)$ leads us to expect the same behaviour also in $SU(4)$.

In Fig. 2 we make a comparison of the renormalized zero momentum gluon propagator, $\mathcal{D}(0)$ in “physical” units, collecting all our results for the three gauge groups.

To relate the bare lattice propagator to the renormalized continuum propagator $D_R(q; \mu)$ one needs the renormalization constant $Z_3(\mu, a)$:

$$a^2 D(qa) = Z_3(\mu, a) D_R(q; \mu) \quad . \quad (1)$$

We take as our renormalization condition

$$D_R(q)|_{q^2=\mu^2} = \frac{1}{\mu^2} \quad (2)$$

at a renormalization scale μ which allows a determination of $Z_3(\mu, a)$. Taking the renormalization point at $\mu = 1.94$ GeV we find for $SU(3)$ $Z_3^{SU(3)}(\mu, a) = 2.51(1)$, and for $SU(2)$ and $SU(4)$ the ratios $Z_3^{SU(3)}(\mu, a)/Z_3^{SU(2)}(\mu, a) = 1.07(4)$ and $Z_3^{SU(3)}(\mu, a)/Z_3^{SU(4)}(\mu, a) = 0.96(2)$ respectively..

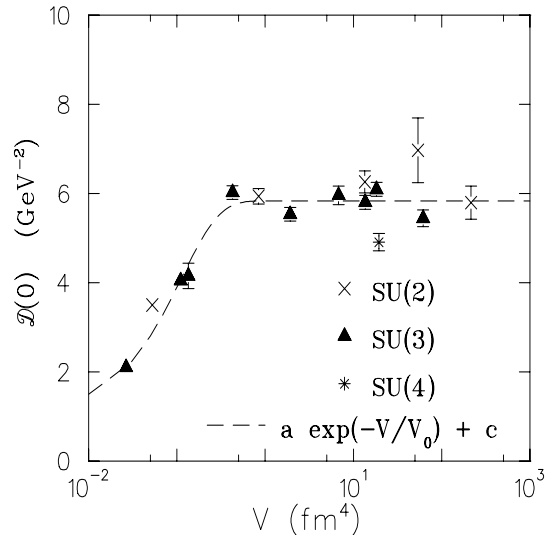


FIG. 2. Volume dependence of the renormalised zero momentum gluon propagator $\mathcal{D}(0)$ for $SU(2)$ (crosses), $SU(3)$ (triangles) and $SU(4)$ (stars). The dashed line is the best fit to the form $a \exp(-V/V_0) + c$ yielding $V_0 = 0.1 \text{ fm}^4$.

All the results for various volumes fall rather well on a universal curve demonstrating independence on N_c . In particular both in $SU(2)$ and $SU(3)$ they reach a plateau at approximately similar volumes. Since the zero momentum propagator gives a measure of the range over which the gauge fields are correlated, these results show that the gauge fields decorrelate in $SU(N_c)$ at a distance of ~ 0.8 fm or about 1.7 times the confining correlation length.

In Fig. 3 we plot the results for the renormalized transverse gluon propagator for lattice sizes for which we expect volume effects to be negligible, namely $16^3 \times 32$ for $SU(3)$ and $SU(4)$, and 32^4 for $SU(2)$. Remarkably, the overall shape of the gluon propagator shows no clear N_c dependence.

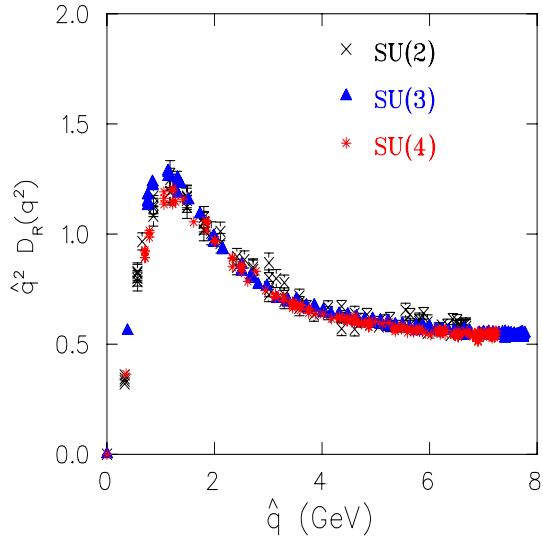


FIG. 3. The momentum dependence of the renormalised transverse gluon propagator for $SU(2)$ (crosses), $SU(3)$ (triangles) and $SU(4)$ (stars) .

However, infrared information is suppressed when showing $q^2 D(q^2)$. To make any low-momentum difference clearly visible, we plot the $SU(2)$ and $SU(4)$ propagators versus the $SU(3)$ propagator for various momenta. If there would be no N_c -dependence the results should fall on the diagonal $y = x$. Small, but systematic differences are visible between $SU(3)$ and $SU(2)$ – $SU(4)$ where the propagators are the largest, i.e. in the infrared, for “physical” momenta smaller than ~ 1 GeV. The $SU(3)$ propagator is slightly larger, which will translate into a slightly smaller gluon pole mass.

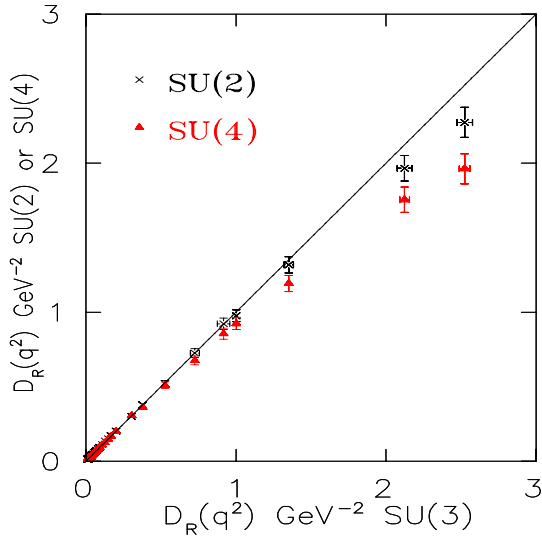


FIG. 4. The renormalised transverse gluon propagator for $SU(2)$ (crosses) and $SU(4)$ (squares) versus that for $SU(3)$.

To investigate more closely the q dependence of the gluon propagator we perform various fits to phenomeno-

logical Ansätze. We fit the transverse propagator to Cornwall’s Ansatz which allows for a dynamically generated gluon mass and is physically motivated. We also fit to two parametrizations which appeared recently in the literature, namely Model A [12] and the mass parametrisation of ref. [10]:

$$D_{\text{mass}}(q) = \frac{Z}{(q^2 + m_1^2)} \left[\frac{1}{1 + q^4/m_2^4} + \frac{s}{[\log(1 + q^2/m_L^2)]^{13/22}} \right] \quad (3)$$

In Fig. 5 we show the quality of the fits to the three Ansätze: Cornwall’s Ansatz has three fit parameters, Model A four and the mass Ansatz five.

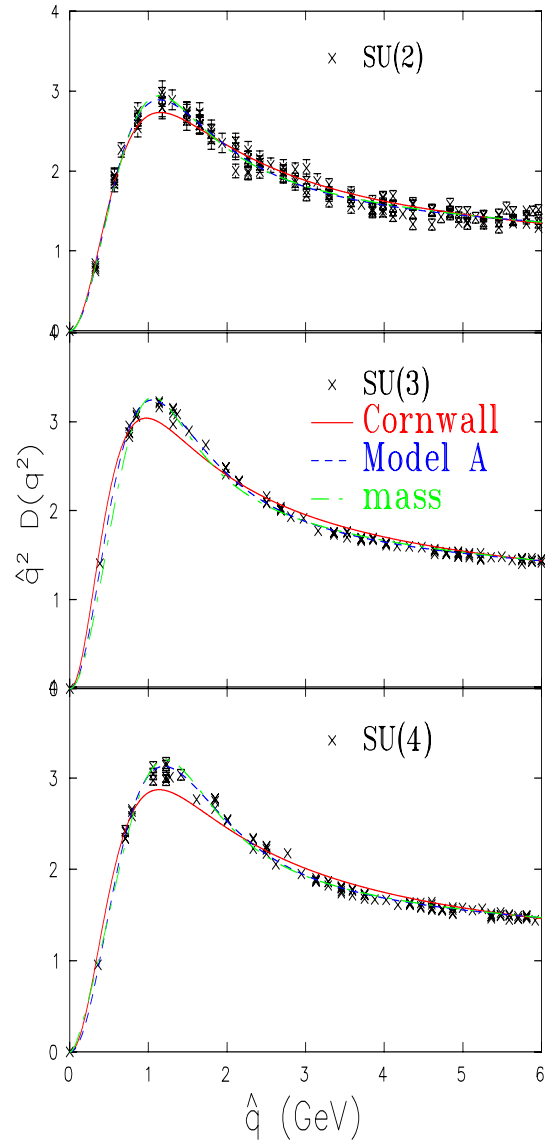


FIG. 5. Fits of the gluon propagator to Cornwall’s Ansatz (solid line) , Model A (dashed line) and the mass parametrisation (dash-dotted line), in $SU(2)$ (upper), $SU(3)$ (middle) and $SU(4)$ (lower).

In accord with ref. [10] which studied the $SU(2)$ Landau gauge, we find that the mass Ansatz works well for $SU(N_c)$. This is not so surprising since it has the largest number of parameters. We find no compelling reason to prefer this Ansatz to the other two. Cornwall's Ansatz is the only one to allow a smooth analytic continuation to $-q^2$ and we will use it for the determination of the pole mass.

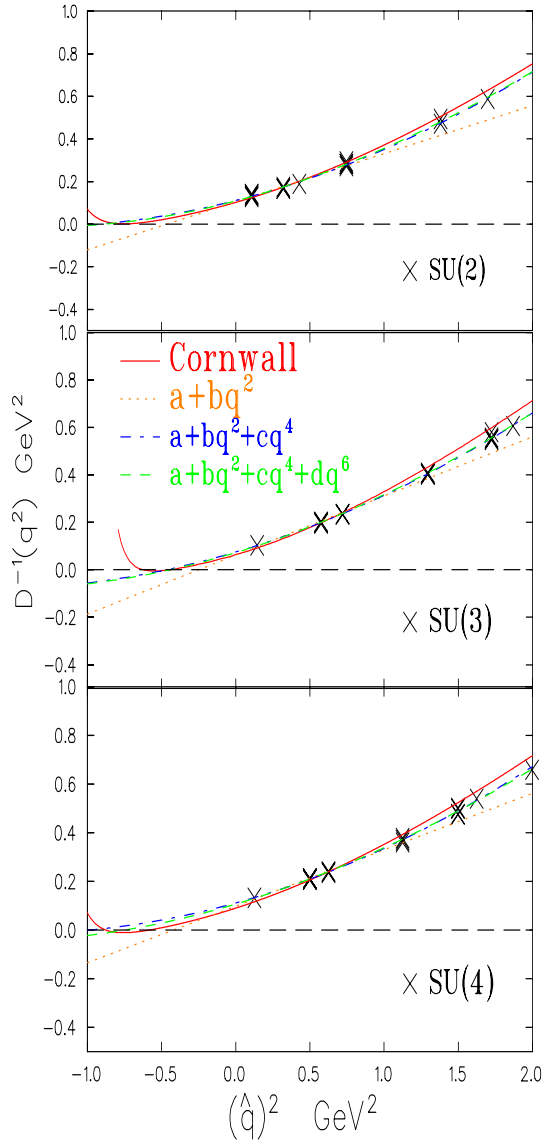


FIG. 6. Inverse propagator in the infrared, and analytic continuation to $-q^2$ for the determination of a pole mass: Upper graph for $SU(2)$, middle for $SU(3)$ and lower for $SU(4)$. We show Cornwall's Ansatz (solid line), a linear (dotted line), a quadratic (dashed-dotted line) and a cubic (dashed line) polynomial in q^2 .

The salient features of the infrared behaviour of the gluon propagator in the three gauge groups show up by plotting its inverse as a function of q^2 . From Fig. 6 it can be seen that the overall behaviour is very similar for

the three gauge groups although, as shown in Fig. 4, not identical. To make this observation more quantitative we investigate the gluon pole mass by analytic continuation to negative values of q^2 as shown in Fig. 6. In addition to Cornwall's Ansatz we use polynomials of increasing degree in q^2 to test the model dependence of the analytic continuation. Taking as a mean the value extracted from Cornwall's Ansatz and as a crude indication of the systematic error the results of the polynomial fit, we find for the pole mass $(1.85 \pm 0.30)\sqrt{\sigma}$, $(1.52 \pm 0.06)\sqrt{\sigma}$ and $(1.74 \pm 0.27)\sqrt{\sigma}$ in $SU(2)$, $SU(3)$ and $SU(4)$ respectively. A study of the gluon propagator for $SU(2)$ in Landau gauge [10] found that the low momentum behaviour supported a non-zero mass of $\sim (1.48 \pm 0.05)\sqrt{\sigma}$. Since only the mass parametrisation was used in that study and no analytic continuation was performed, this value can be regarded as a rough estimate consistent with our pole mass.

IV. CONCLUSIONS

We have extended a previous lattice study of the gluon propagator in the Laplacian gauge to $SU(N_c)$. In $SU(2)$, we have made an analysis of the finite-size effects, which indicates that the Laplacian gauge gluon propagator at zero momentum approaches a non-zero constant value in the infinite volume limit, as in $SU(3)$. In fact, this finite non-zero value is independent of N_c within our statistics. We find $\mathcal{D}(0) = 5.85(5) \text{ GeV}^{-2}$.

The overall dependence of the gluon propagator on N_c appears extremely weak, and is barely visible, even for infrared momenta, even for $N_c = 2$. For all $N_c = 2, 3, 4$, the infrared behaviour is consistent with a pole mass in the range 600 – 800 MeV.

Our results give further support to the view that $N_c = 2$ is a theory already close to the large N_c limit of QCD, a conclusion reached from the study of the string tension [1] and the glueball mass [13] in $SU(N_c)$.

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