MEASUREMENTS OF COHERENT TUNE SHIFT AND HEAD-TAIL GROWTH RATES AT THE SPS.

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Abstract

Measurements of the coherent tune shifts with intensity and of head-tail growth rates have been performed with single proton bunches in the SPS at 26 GeV. From these measurements, the real and imaginary part of the transverse impedance can be estimated. A reproducibility at the 20% level was achieved for the value of the effective vertical impedance inferred from the coherent tune-shift measurements.

1 INTRODUCTION

Several measurements and calculations similar to those described in this paper have been performed in the past. Results are summarised in section 8. Most of them are quite old and moreover, there has been a significant spread in the vertical and horizontal broadband impedance parameters obtained (covering about a factor of 3, from 12 to 48 MΩ/m in Z_v/Q).

Our aim is to measure an observable with an uncertainty below 20%. This would allow us to monitor the improvements planned to reduce the impedance of the SPS as injector into the LHC. As far as possible, we try to perform the measurements with the same bunch dimensions. This minimises the model dependence and uncertainties due to variation in bunch parameters. Based on these measurements, and assuming a broad-band model, we estimate the impedance. For more detailed information on thes studies see [1].

2 BEAM CONDITIONS

The measurements were all performed using single and relatively short bunches ($\sigma_z \approx 16$ cm or 0.55 ns) injected at 26 GeV in the SPS-MD cycle. The fixed beam energy of 26 GeV, was rather imposed by beam availability. It would be useful in the future to confirm these measurements at a higher energy, to exclude any bias from space-charge effects [2].

The measurements were performed close to "standard tunes" ($Q_x = 26.62, Q_y = 26.58$). Chromaticity was carefully measured and corrected in order to be slightly positive. The octupole components in the machine were compensated. The damper was off. The tune measurements were done using 1 mm (nominal) kicks. With these settings and for small intensities ($\sim 10^{10}$ protons), one obtains rather clean sinusoidal oscillations with little damping, observable online over $2^{12} = 4096$ turns using the SPS tune application.

Ideally, the bunch dimensions and in particular the bunch length should not vary from one intensity to another. The best compromise was achieved by adjusting the beam in the PS for the highest intensity first (close to 10^{11} protons per bunch), and then reducing it by vertical scraping in the PS. In this way, the bunch length and horizontal beam size remained nearly constant (whereas the vertical size changes by a factor of 4 within this range of bunch population). In the future we plan to also try the new scraping facilities in the SPS.

On the SPS side, the 200 MHz rf was adjusted to obtain good capture and matching. Depending on intensity, this was achieved with voltages in the range of 0.5 - 0.8 MV.

In order to be independent of injection optimisation and to have shorter bunches with a larger effect on the coherent tune shift, the rf was ramped adiabatically to 3 MV nominal (corresponding to about 2.5 MV measured) just before the time of the measurements.

3 BUNCH DIMENSIONS

The vertical and horizontal bunch dimensions were recorded as a function of the proton intensity using wire-scanners. The results are shown in Fig. 1. The horizontal measurements are scattered with maximal variations of about 40%. The vertical emittance increases strongly with the bunch population, as expected from scraping.

An approximately constant voltage of $V_{\rm rf} = 0.8$ MV was used on the first MD on the 23/08/1999. A shorter, better controlled bunch length was obtained in the subsequent MD's using the voltage ramp described above. The bunch length was systematically recorded. A good knowledge of the bunch length σ is needed to extract the parameters of the broad band impedance model. Since the bunch length is not constant we will use the average $\langle\sigma\rangle$ of all individual length measurements in our calculations. The r.m.s. spread in the measured bunch length is used as the error in the determination of σ and will lead to an error in the impedance estimate. These values are summarised in Table 1. Note that the measurement on the 13/08/1999 was done without ramp of the rf-voltage, *i.e.* with longer bunches.

The bunch mode spectrum for these σ extends up to $f = 1/(2\pi \cdot \sigma) \approx 300$ MHz.

4 FREQUENCY ANALYSIS METHOD

The frequency analysis method is a refined Fourier analysis which can be applied on experimental or tracking data. More about the mathematical details of the method can be

* The work reported in this contribution has been done together with: G. Arduini, H. Burkhardt, K. Cornelis, Y. Papaphilippou and F. Zimmermann

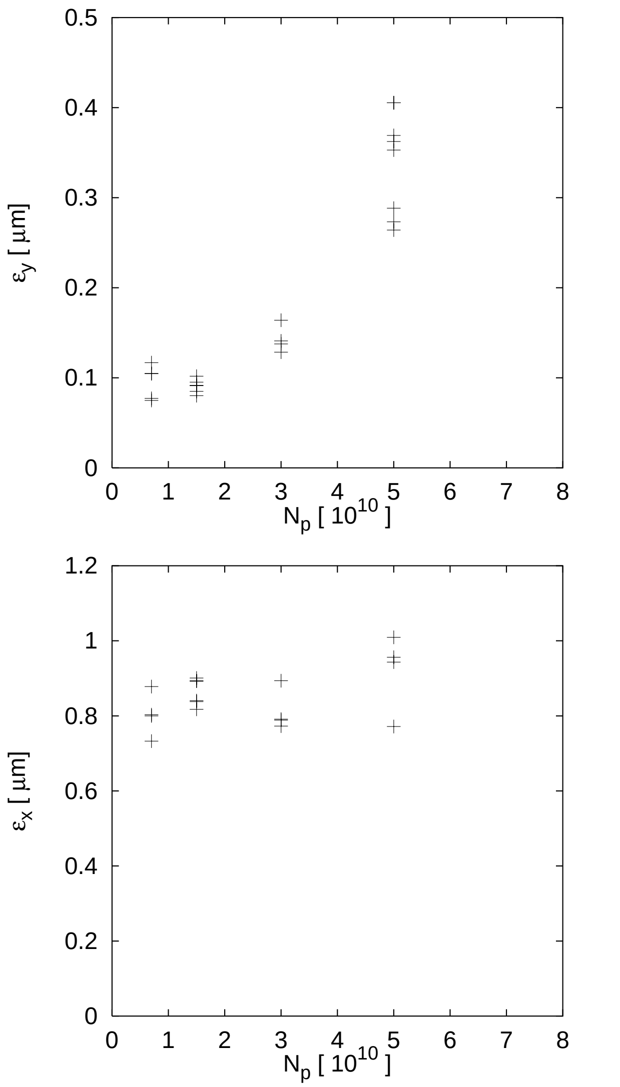

Figure 1: 17/9/1999 Proton emittance as a function of bunch population: vertical emittance $4\sigma_y^2/\beta_y$ (top) and horizontal emittance $4\sigma_x^2/\beta_x$ (bottom). Measured with the wire-scanner at a location with $\beta_y = 22$ m, $\beta_x = 97$ m and dispersion $D_x = 2.9$ m., when $V_{\rm rf} \approx 2.5$ MV.

Table 1: Bunch length measurements

date	σ [ns]
13/08/99	0.77 ± 0.14
23/08/99	0.47 ± 0.05
17/09/99	0.53 ± 0.02
10/11/99	0.58 ± 0.03

found in papers of Laskar who introduced it in celestial mechanics [3] and accelerator dynamics [4]. Recording the bunch oscillations over a period of N turns, and through an advanced filtering algorithm using the Hanning window, the method guarantees the asymptotic accuracy of the determination of the base tunes to be of the order of $1/N^4$ [5] for quasi-periodic signals, compared to $1/N$ of an FFT. Actually for the noisy signals associated with experimental data, we can expect an accuracy of the order of $1/N^2$ [6].

Another interesting application of the method is the determination of the damping or growth rates. A straightforward calculation of this rate can be achieved by estimating one of the amplitudes of the series (e.g. the one corresponding to the base frequency $a_1(t)$), for successive time spans (e.g. every 100 turns) and then fit an exponential to represent the function $a_1(t)$.

As an example, we present in Fig. 2 one of the measurements performed in the SPS when the vertical chromaticity was slightly negative, producing a growth from the head-tail instability in the vertical plane. The actual measurement from the SPS acquisition system and the exponential fit with the calculated growth rate are plotted. We may note the good accuracy with which the growth rate is obtained (the R^2 of the fit is very close to 1).

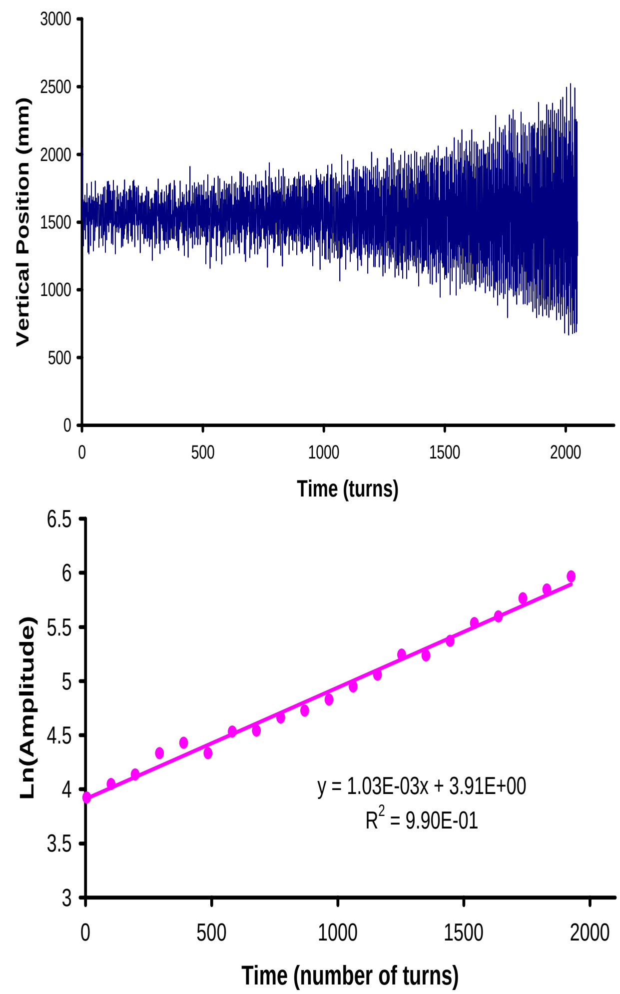

Figure 2: Vertical position of a bunch with slightly negative chromaticity, as measured by the SPS acquisition system (top) and growth rate obtained by fitting an exponential to the leading oscillating amplitude of the series issued by the frequency analysis method (bottom).

5 HORIZONTAL AND VERTICAL TUNE SHIFT VS. BUNCH POPULATION

The vertical and horizontal tunes were obtained by kicking the beam and post-processing the time sequence (1024 to 4096 turns) of the beam position. Using the Frequency analysis technique, the precision of the measurement was increased. We measure the tunes after the adiabatic ramp for bunch populations between 10^{10} and 5×10^{10}.

In Fig. 3 we show a typical measurement of the tune as a function of bunch population, for the horizontal and vertical plane. As expected from measurements performed in the past, with increasing current the vertical tune decreases and the horizontal tune increases. The slope of these plots is related to the imaginary part of the impedance. The difference in sign and magnitude between the two planes is due to the flat dimensions of the chamber. The horizontal mean radius is about 7 cm and the vertical mean radius of the SPS chamber is about 2.4 cm.

The data was fitted to a straight line $f(x) = a \cdot x + b$. To obtain realistic errors for the slope, the uncertainties in each tune point (e_x, e_y) were scaled to obtain $\chi^2 = 1$ for the fit.

5.1 Summary of tune shift measurements

In Table 2 we summarise the slopes found and the errors.

Table 2: Coherent tune shift measurements

date	$\Delta Q_x/\Delta N_p[10^{10}]$	$\Delta Q_y/\Delta N_p[10^{10}]$
13/08/99	$(+24 \pm 2) \times 10^{-5}$	$(-18 \pm 2) \times 10^{-4}$
23/08/99	$(+58 \pm 6) \times 10^{-5}$	$(-29 \pm 1) \times 10^{-4}$
17/09/99	$(+21 \pm 4) \times 10^{-5}$	$(-36 \pm 2) \times 10^{-4}$
10/11/99	$(+23 \pm 2) \times 10^{-5}$	$(-29 \pm 1) \times 10^{-4}$

6 GROWTH RATE VS. CHROMATICITY

For negative chromaticity, and operating above transition, the head-tail mode ($l = 0$) becomes unstable and drives the motion of the centroid of the beam. The amplitude of these oscillations increases exponentially in time. Analysing this exponential growth, we get the growth rate $1/\tau$ which increases with $|\xi|$. The slope of this plot is related to the real part of the impedance.

We studied the head-tail mode for low currents ($N_p = 1.6 \times 10^{10}$ protons per bunch). Changing the strength of the sextupoles the chromaticity was reduced by $\Delta\xi_y$ with respect to the settings used for the tune shift measurements. In Fig. 4 we show the vertical growth rate as a function of $\Delta\xi_y$. For the negative chromaticity measurements the bunch population was constant and equal to 1.6×10^{10}. The values at $\Delta\xi_y = 0$ were taken from the tune shift measurements which were performed with slightly positive chromaticity that lead to damping of the centroid motion. These points were measured with a bunch population of

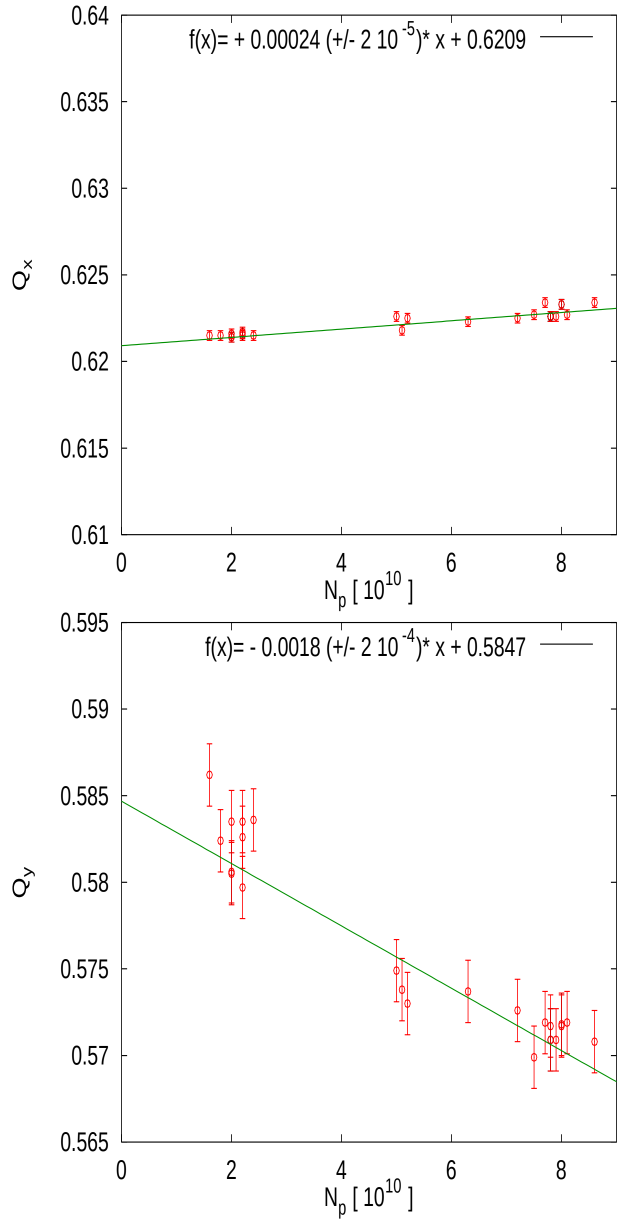

Figure 3: 13/08/1999 ($V_{\rm rf}$=0.8 MV). Horizontal (top) and vertical (bottom) tune as a function of the bunch population and fit with errors. Tune error bars $e_y = 1.8 \times 10^{-3}$ and $e_x = 3 \times 10^{-4}$.

$N_p = 10^{10}$ and $N_p = 2.2 \times 10^{10}$ and their values were rescaled by the intensity ratio to compare with the measurements at $N_p = 1.6 \times 10^{10}$.

The zero crossing of the linear fit suggests that our standard setting $\Delta\xi_y = 0$ corresponds to a slightly positive chromaticity of $\xi_y = 0.011$.

7 FITTING THE RESULTS WITH A BROAD-BAND IMPEDANCE MODEL

The transverse impedance is modelled by an equivalent LRC resonator with resonance frequency $w_R = 1/\sqrt{CL}$,

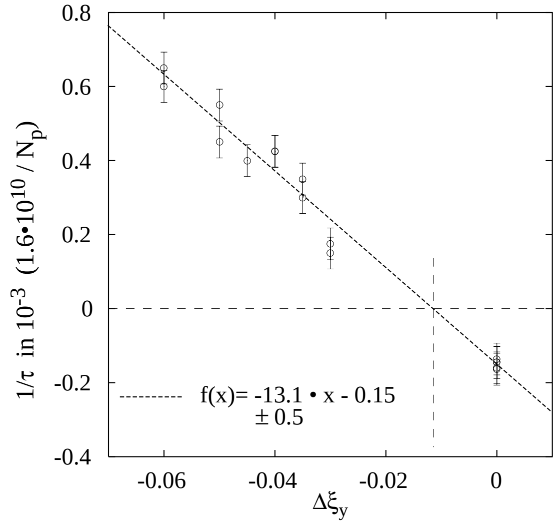

Figure 4: 10/11/1999. Growth rate of the vertical head-tail mode instability (in units of 10^{-3} turns^{-1}), as a function of the decrement of chromaticity. Error bars are $e = 0.043$ (in units of 10^{-3}).

resistance R_s and quality factor $Q = R_s\sqrt{C/L}$, according to:

$$Z_1^{\perp} = \frac{w_R}{w}\frac{Z_t}{1 + iQ\left(\frac{w_R}{w} - \frac{w}{w_R}\right)} \tag{1}$$

where $Z_t = c/w_R R_s$.

Let ξ be the chromaticity, η the slip factor ($\eta = 5.55 \times 10^{-4}$), w_0 the revolution frequency, $w_\beta = Q_\beta w_0$ the betatron frequency and $Q_\beta = 26.6$ the betatron tune. Defining $w_\xi = \frac{\xi w_\beta}{\eta}$ and $w_p = pw_0 + w_\beta$ with p an integer number we can evaluate the effective transverse impedance [7]

$$(Z_1^{\perp})_{eff} = \frac{\sum_{p=-\infty}^{\infty} Z_1^{\perp}(w_p) h_l(w_p - w_\xi)}{\sum_{p=-\infty}^{\infty} h_l(w_p - w_\xi)} \quad , \tag{2}$$

where the impedance is convoluted with the bunch spectrum h_l for the $l = 0$ head-tail mode as defined for a Gaussian beam model

$$h(w_p) = e^{-w_p^2\sigma_z^2/c^2} \tag{3}$$

with $\sigma_z = c\sigma$ the bunch length, c the speed of light and σ the r.m.s of the Gaussian distribution in units of time. Then the tune shift is given by

$$\Delta Q = \frac{\Omega - w_\beta}{w_0} \approx \frac{1}{w_0}\frac{N_p e c^2}{2E/eT_0 w_\beta 2\sqrt{\pi}\sigma_z}\Im(Z_1^{\perp})_{eff} \tag{4}$$

with N_p the number of particles per bunch, e charge of the particle, $E = 26.017$ GeV the particle energy, and $T_0 = 2\pi/w_0 = 23.05$ μs the revolution period. Similarly the growth rate (in turns^{-1}) is given by

$$\frac{1}{\tau} \approx -T_0\frac{N_p e c^2}{2E/eT_0 w_\beta 2\sqrt{\pi}\sigma_z}\Re(Z_1^{\perp})_{eff} \quad . \tag{5}$$

The real part of the effective impedance is different from zero if the chromaticity is not zero. Above transition, this leads to a negative growth rate (damping) for positive chromaticity, and to a positive growth rate otherwise.

7.1 Tune shift

We fit the broad band resonator with a quality factor $Q = 1$ and a resonance frequency $w_R = 2\pi \times 1.3$ GHz.

The ratio $\Delta Q/\Delta N_p[10^{10}]$ is directly proportional to the impedance $Z_1^{\perp}$. For each plane we determine the impedance such that $\Delta Q/\Delta N_p[10^{10}]$ equals the slope found in our measurements. In Table 3 we summarise the impedances inferred from the tune shifts. The uncertainty reflects both the error of the fitted slope and the spread in the measured bunch length σ.

Table 3: Impedance results obtained by fitting coherent tune shifts with a broad-band model.

date	Z_v in MΩ/m	Z_h in MΩ/m
13/08/1999	25 ± 6	-3.3 ± 0.7
23/08/1999	24 ± 2	-4.8 ± 0.7
17/09/1999	33 ± 3	-2.0 ± 0.4
10/11/1999	30 ± 2	-2.4 ± 0.3
weighted average	28 ± 2	-2.6 ± 0.2

The averages and uncertainties from combining the four measurements are also given. The four numbers of Z_v are all compatible with the weighted average within 20%. The effect in the horizontal plane is much smaller, and has clearly the opposite sign. The uncertainties given above, based only on the scatter in the data, are relevant for a comparison of data taken under similar conditions. To compare this impedance with other estimates based on different methods one should also take into account the dependence on the model and the variation of parameters such as E, σ, ξ etc.

7.2 Growth rate

On the experiment of the 10/11/1999 (see Fig. 4), we found that the growth rate increases linearly with the decrement of chromaticity. This can be understood as follows. If the bunch is longer than the range of the wake field ($\sigma c > c/w_R = 3.6$ cm for $w_R = 2\pi \times 1.3$ GHz) then $(Z_1^{\perp})_{eff} \approx Z_1^{\perp}(w_\xi)$. The growth rate $1/\tau$ which is proportional to $\Re(Z_1^{\perp})_{eff}$ is then

$$\frac{1}{\tau} \approx -T_0\frac{N_p e c^2 Z_1}{2E/eT_0 w_\beta 2\sqrt{\pi}\sigma_z w_R}\frac{\xi w_\beta}{\eta} \tag{6}$$

which increases linearly with $-\xi$.

Using the complete formula, Eq. (2), assuming $Q = 1$, the impedance that fits the measured dependence on the chromaticity is $Z_t = 8.3 \pm 0.6$ MΩ/m. This impedance is 3.7 times smaller than the impedance found by fitting the coherent tune shift.

We can fit both measurements simultaneously with $Z_t = 108$ MΩ/m by changing the quality factor to $Q = 3.6$. In this broad band model $Z_t/Q = 30$ MΩ/m.

8 SUMMARY OF PREVIOUS SPS IMPEDANCE MEASUREMENTS

Table 4: Broadband resonance parameters found in earlier transverse impedance measurements and calculations.

Z/Q in MΩ/m		reference	year
vertical	horizontal		
18		[8]	1980
47.7		[9]	1984
13	-8	[10]	1986
12.5	-5.2	[10]	1986
26.8	-16.88	[11]	1988
(23 ± 2)		[12]	1993

9 CONCLUSIONS

Using single proton bunches at 26 GeV we have succeeded to measure the coherent tune shift, with a 20% reproducibility. This will allow us to experimentally document the impedance update of the SPS ring. From the coherent tune shift and head-tail mode instability studies we estimate the broad-band component of the vertical impedance to be of the order of $Z_v/Q \approx 30$ MΩ/m (for a quality factor $Q \approx 3.6$). The horizontal impedance is negative and small.

In the future, further measurements at different energies, bunch lengths and bigger values of ξ will help understanding the role of space charge and probing the reliability of a broad-band impedance model. These measurements can be combined with the 1000 turns technique to localise in the ring the main sources of impedance.

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