ETHER, LUMINOSITY AND GALACTIC SOURCE COUNTS

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Abstract. An interpretation of the cosmological redshift in terms of a cosmic ether is given. We study a Robertson-Walker cosmology in which the ether is phenomenologically defined by a homogeneous and isotropic permeability tensor. The speed of light becomes so a function of cosmic time like in a dielectric medium. However, the cosmic ether is dispersion free, it does not lead to a broadening of spectral lines. Locally, in Euclidean frames, the scale factors of the permeability tensor get absorbed in the fundamental constants. Mass and charge scale with cosmic time, and so do atomic energy levels. This substantially changes the interpretation of the cosmological redshift as a Doppler shift. Photon frequencies are independent of the expansion factor; their time scaling is determined by the permeability tensor. The impact of the ether on the luminosity-distance, on the distance-redshift relation, and on galactic number counts is discussed. The Hubble constant is related to the scale factors of the metric and the permeability tensor. We study the effects of the ether at first in comoving Robertson-Walker coordinates, and then, in the context of a flat but expanding space-time, in the globally geodesic rest frames of galactic observers.

1. Introduction

We consider the possibility to generate the cosmological redshift by means of a cosmic ether (Whittaker, 1951; Tomaschitz, 1998,b,c), contrary to the commonly accepted explanation in terms of a space expansion. The spacetime geometry is described by a Robertson-Walker (RW) line element, but the dynamics of light rays is determined by a permeability tensor. This tensor is homogeneous and isotropic, and defined by two scale factors depending on cosmic time. The speed of light becomes so a function of cosmic time, but phase and group velocity still coincide. There is no dispersion in the direction of propagation, as in vacuum electrodynamics.

We study the effects of the ether on distance measurement and source counts. This can be done in the framework of ray optics. The eikonal equation reads like in a curved space, but the spacetime metric is replaced by the permeability tensor. So the dynamics of rays gets completely detached from the spacetime metric.

In the cosmology presented here the semiclassical approximation for light rays is exact. The eikonal is the phase of the plane wave solutions of Maxwell's equations. Thus we can derive from the eikonal the exact cosmic time dependence of the frequency, and, via the Einstein relation, the energy of photons. The photon frequency as well as the Hubble constant depend on the scale factors of the perme-

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ability tensor. The Hubble constant is defined via the asymptotic distance-redshift relation $d_0 \sim c_0 H_0^{-1} z$, where d_0 is the metric distance between source and observer at arrival time, and c_0 is the present value of the speed of light.

In Section 2 electrodynamics in the ether is defined. We derive a condition on the scale factors of the permeability tensor and the metric which renders the ether dispersion free. The time evolution of the electromagnetic energy is shown to be strictly proportional to frequency, and we demonstrate in this way that the Planck constant is independent of cosmic time. The Coulomb potential of a static point source in the ether is calculated, and it is demonstrated that in the Kepler problem the scale factors of the permeability tensor can be absorbed in the fundamental constants, which become functions of cosmic time. The scaling laws for the speed of light, mass, and charge are derived. The fine structure constant, a moderate dimensionless ratio (Dirac, 1938; 1973) does not scale in cosmic time, contrary to atomic energy levels.

In Section 3 we discuss the redshift. It is an effect caused by the cosmic scaling of atomic energy levels as well as the scaling of the photon frequencies. The photon frequency is independent of the expansion factor in the RW metric, the ether is perfectly capable of producing redshifts in a static spacetime geometry. We identify the Hubble constant, which is inversely proportional to the age of the universe. The luminosity-distance is derived, and the deceleration parameter is related to the exponents of the scale factors of metric and permeability tensor.

In Sections 4 and 6 we study various source counting functions, in particular the number of galaxies of redshift smaller than z. This function gives information on the time evolution of the galactic density. Due to evolutionary effects, this density need not scale with the inverse cube of the expansion factor, and so $N(z)$ need not be monotonous. Redshift surveys of quasars (Hartwick and Schade, 1990) indicate a peak of $N(z)$. We discuss under which conditions a maximum can emerge, and relate it to the scaling exponents of the galactic density and the scale factors.

In Sections 3 and 4 we consider the luminosity-distance and number counts in comoving RW coordinates. In this frame all galaxies have constant space coordinates, and by virtue of these coordinates a universal rest frame and a unique cosmic time shared by all galactic observers is defined. In Sections 5 and 6 individual geodesic rest frames of galactic observers are investigated. Galaxies or galactic observers are not affected by the ether, unlike other massive particles, as they are at rest in the universal rest frame. We study a RW cosmology with linear expansion factor and negatively curved 3-space. The spacetime geometry is isometric to the forward light cone, and so globally geodesic rest frames can be introduced for galactic observers. In each of these frames the galactic background is radially receding, every galaxy with constant speed. The geodesic rest frames are related by Lorentz boosts.

In Section 5 we study the world lines of photons in globally geodesic frames, and how the ether effects their speed and energy. The ether generates double images of photons in individual galactic rest frames. Redshifts depend on the speed of the receding galaxies, but unlike Doppler shifts, also on the proper time of the observer. We derive the luminosity-distance relation in globally geodesic coordinates, and show its equivalence with that obtained in the universal frame of rest. In Section 6 we study the galactic density in globally geodesic coordinates, the spatial density as well as velocity and redshift distribution functions. In Section 7 we present our conclusions.

2. Electromagnetism and Massive Particles in the Ether

We discuss electrodynamics in the ether, the hypothetical material substance of space (Whittaker, 1951; Tomaschitz, 1998b,c), which macroscopically manifests by a symmetric permeability tensor $g^P_{\mu\nu}$. Quite analogously to a dielectric medium, the following formalism is based on two symmetric tensor fields, the space-time metric $g_{\mu\nu}$ (inverse $g^{\mu\nu}$, determinant g), and the permeability tensor $g^P_{\mu\nu}$ (inverse $g^{P-1\mu\nu}$, determinant g^P). Action and Lagrangian for the electromagnetic potentials we define as

$$S = \int (L + A_\mu j^\mu)\sqrt{-g}\,dx\,d\tau, \quad L = -\frac{1}{4}F_{\mu\nu}F_{\alpha\beta}g^{P-1\mu\alpha}g^{P-1\nu\beta}, \tag{2.1}$$

and $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$. Introducing the tensor

$$H^{\mu\nu} := g^{P-1\mu\alpha}g^{P-1\nu\beta}F_{\alpha\beta}, \tag{2.2}$$

we may write the field equations as

$$\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}H^{\mu\nu})}{\partial x^\nu} = j^\mu, \quad \frac{1}{\sqrt{-g}}\varepsilon^{\lambda\alpha\beta\gamma}F_{\alpha\beta;\gamma} = 0. \tag{2.3}$$

The continuity equation, $j^\mu{}_{;\mu} = 0$, follows from the inhomogeneous equations in (2.3). The energy-momentum tensor reads

$$T_\nu{}^\mu := -F_{\nu\alpha}H^{\mu\alpha} + \frac{1}{4}\delta^\mu_\nu F_{\alpha\beta}H^{\alpha\beta}, \tag{2.4}$$

which is the usual definition of electromagnetic energy in a dielectric medium.

We consider electromagnetic waves freely propagating in a perfectly isotropic and homogeneous cosmic ether, in the context of an open RW universe with line elements

$$ds^2 = -c^2d\tau^2 + a^2(\tau)d\sigma^2, \tag{2.5}$$

$$ds_P^2 = -c^2h^2(\tau)d\tau^2 + b^2(\tau)d\sigma^2, \tag{2.6}$$

defining $g_{\mu\nu}$ and $g^P_{\mu\nu}$, respectively. We assume that the 3-space is open and negatively curved, but this is not really essential for most of the following. We use the

Poincaré half-space representation of hyperbolic geometry; $d\sigma^2 = R^2 u^{-2}(|d\xi|^2 + du^2)$ in the half-space: H^3: (ξ, u), $\xi \in \mathbb{C}$, $u > 0$, cf., e.g., Magnus (1974). $x^\mu = (\tau, \xi, u)$. R is a dummy constant of the dimension of length, needed for dimensional reasons, like c in the line elements (2.5) and (2.6); H^3 endowed with $d\sigma^2$ has constant sectional curvature $-1/R^2$. The dimensionless scale factors a, b and h actually depend on $\Lambda\tau$, $\Lambda := cR^{-1}$, but in the following calculations we put $c = R = 1$.

We solve Equations (2.3) ($j^\mu = 0$) with the transversality condition $(-g)^{-1/2}\partial(\sqrt{-g}g^{\mu\nu}A_\nu)/\partial x^\mu = 0$ and the Coulomb gauge $A_0 = 0$. The homogeneous equations in (2.3) are already satisfied by the potential representation of $F_{\mu\nu}$. We are interested in plane waves propagating along the u-semi-axis, and use the separation ansatz $A_1 = \varphi(\tau)u^{is}$, $A_0 = A_2 = A_3 = 0$. (There is a second transversal set of plane waves obtained by interchanging A_1 and A_2; these two sets are orthogonal.) With this ansatz the transversality condition is satisfied. The $\mu = 0, 2, 3$ components of the inhomogeneous equations (2.3) are identically satisfied, and the $\mu = 1$ component gives the equation for $\varphi(\tau)$,

$$\frac{b^4}{a^3}\frac{d}{d\tau}\left(\frac{a^3}{h^2 b^2}\frac{d}{d\tau}\varphi\right) + s^2\varphi = 0, \tag{2.7}$$

$\varphi(\tau)u^{is}(1, 0, 0)$ and $\varphi(\tau)u^{is}(0, 1, 0)$ constitute a complete set of tansversal plane waves propagating along the u-axis. A complete set propagating in any other direction is obtained by applying to them symmetry transformations of H^3. The energy density of a plane wave has the time dependence

$$T_0{}^0\sqrt{\gamma} \sim \frac{a^3}{b^2}\left(\frac{|\varphi'|^2}{h^2} + \frac{|\varphi|^2}{b^2}\right). \tag{2.8}$$

A wave packet composed of a Gaussian average (with respect to the spectral variable s) over the plane waves $\varphi(\tau, s)u^{is}(1, 0, 0)$ will in general show dispersion (Tomaschitz, 1992a, 1997b) since group and phase velocity differ in a dielectric medium. This is very contrary to the vacuum formalism, which is recovered in the case $h = 1$ and $b = a$ (Tomaschitz, 1993b). In vacuum the phase of the electromagnetic spectral waves linearly depends on the spectral parameter, therefore phase and group velocities coincide and are identical with the speed of rays obtained from the eikonal equation; there is no dispersion along the direction of propagation. Dispersion leads to a broadening of spectral lines, which is not observed (Sandage, 1988). If

$$h(\tau) \sim a^3(\tau)/b^3(\tau), \tag{2.9}$$

then

$$\varphi = \exp\left(\mp is\int hb^{-1}d\tau\right), \tag{2.10}$$

is an exact solution of (2.7), and the phase is linear in the spectral variable like in vacuum electrodynamics. Thus there is no dispersion in the ether if the proportionality (2.9) holds. The phase of the spectral elementary waves evidently reads

$$\psi = s\left(\log u \mp \int hb^{-1} d\tau\right). \tag{2.11}$$

This coincides with the exact solution of the eikonal equation

$$g^{P-1\mu\nu}\psi_{,\mu}\psi_{,\nu} = 0, \tag{2.12}$$

for rays along the u-semi-axis. From (2.11) we obtain the frequency of the spectral waves as $\omega = \Lambda|s|hb^{-1}(\Lambda = cR^{-1})$. From (2.8) and (2.10) we obtain $T_0{}^0\sqrt{\gamma} \sim \omega$, which suggests for the photon energy the relation $E(\tau) = \hbar\omega(\tau)$, with $\hbar$ independent of cosmic time. For the wave vector we then obtain from (2.11) $k_u = s/u$; photon momentum and wave length read $p_u = \hbar k_u$ and $\lambda\!\!\!^{-} = |k_u|^{-1} = aR|s|^{-1}$, respectively, so that the speed of light in the ether is $\hat{c}(\tau) = \omega\lambda\!\!\!^{-} = cab^{-1}h$.

Next we calculate the potential of a static point source e_s. Because of the spherical symmetry of the potential we use as coordinate representation of the cosmic 3-space the ball model B^3 of hyperbolic geometry. We then have in (2.5) and (2.6) $d\sigma^2 = 4(1 - |\mathbf{x}|^2/R^2)^{-2}d\mathbf{x}^2$ (Cartesian coordinates in the ball $|\mathbf{x}| < R$). B^3 is isometric to H^3 (Magnus, 1974). We put $R = c = 1$, and $x^\mu = (\tau, \mathbf{x})$. The potential must solve (2.3) with the current $j^0 = e_s\gamma^{-1/2}\delta(\mathbf{x})$, $j^k = 0$, of a static point source e_s located at $\mathbf{x} = 0$. (γ denotes the determinant of the 3-metric $a^2 d\sigma^2$). We try the ansatz $A_0 = h^2b^2a^{-3}\varphi(r)$, $A_k = 0$. The homogeneous equations and the spatial components of the inhomogeneous set of equations in (2.3) are identically solved in this way. The $\mu = 0$ component gives

$$\frac{\partial}{\partial x^k}\left(\frac{2}{1-r^2}\frac{\partial}{\partial x^k}\varphi(r)\right) = e_s\delta(\mathbf{x}), \tag{2.13}$$

and so we obtain asymptotically

$$A_0(\tau, \mathbf{x}) = -\frac{e_s}{4\pi}\frac{h^2b^2}{a^2}\frac{1}{d(\tau, \mathbf{x})} + O(d/(Ra)). \tag{2.14}$$

Here $d(\tau, \mathbf{x})$ denotes the distance of $\mathbf{x}$ from the point source e_s at $\mathbf{x} = 0$,

$$d(\tau, \mathbf{x}) = Ra(\tau)\log\frac{1 + |\mathbf{x}|/R}{1 - |\mathbf{x}|/R}. \tag{2.15}$$

$Ra(\tau)$ is the curvature radius of the cosmic 3-space, whereas $d(\tau, \mathbf{x})$ is, for example, an atomic length if we study the scaling in the Rutherford model, therefore the asymptotic formula (2.14) is by all standards sufficient.

From the eikonal equation (2.12) it is clear how to define the mechanics of classical particles in the ether, namely by the Hamilton-Jacobi equation

$$g^{P-1\mu\nu} S_{,\mu} S_{,\nu} = -m^2, \tag{2.16}$$

(Tomaschitz, 1998b,c). This corresponds to the Lagrange function

$$L(s) = -m\sqrt{-g^P_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}, \tag{2.17}$$

with the action $S = \int L ds$. We study geodesic motion along the u-axis of H^3. This does not mean any restriction, as H^3 is homogeneous. From the Lagrange equations we immediately have two integrals of motion along the u-axis,

$$h^2 \dot{\tau}^2(s) - b^2(\dot{u}(s)/u)^2 = 1, \quad b^2\dot{u}/u = \nu, \tag{2.18}$$

with a real integration constant ν, so that

$$d \log u = \nu h b^{-1}(b^2+\nu^2)^{-1/2} d\tau, \quad ds = hb(b^2+\nu^2)^{-1/2} d\tau. \tag{2.19}$$

The generalized 4-momentum reads

$$p_\nu = \partial L/\partial \dot{x}^\nu = \partial S/\partial x^\nu = m\dot{x}^\mu(s) g^P_{\mu\nu}, \tag{2.20}$$

$x^\mu := (\tau, \xi = 0, u)$, and the energy of a particle moving in the ether we define as $E = -p_0 = p^0$. From (2.18) and (2.19) we obtain (after restoring the natural units c and R)

$$E = \frac{\hat{m}\hat{c}^2}{\sqrt{1-|\mathbf{v}|^2/\hat{c}^2}} = mc^2 h\sqrt{1+\nu^2 b^{-2}}, \tag{2.21}$$

$$\hat{c} = ca(\tau)b^{-1}(\tau)h(\tau), \quad \hat{m} = ma^{-2}(\tau)b^2(\tau)h^{-1}(\tau), \tag{2.22}$$

$$|\mathbf{v}| = aRu^{-1}du/d\tau = ch\frac{a}{b}\frac{\nu}{\sqrt{b^2+\nu^2}}. \tag{2.23}$$

Equations (2.22) constitute the scaling laws for speed of light and mass. R, c and m are bare constants, whereas $\hat{R} = Ra(\tau)$ (curvature radius of the 3-space), $\hat{c}$ and $\hat{m}$ are the measured quantities. $\hat{c}$ is of course the same as already derived after (2.12).

The coupling of a particle of charge e to the electromagnetic potential is effected by minimal substitution, $S_{,\mu} \to S_{,\mu} - eA_\mu$, in (2.16). This amounts to add the term $eA_\mu \dot{x}^\mu$ to the Lagrangian (2.17). The zero-component of the generalized momentum now reads, with A_0 as in (2.14), $p_0 = -E$,

$$E = \frac{\hat{m}\hat{c}^2}{\sqrt{1-|\mathbf{v}|^2/\hat{c}^2}} + \frac{\hat{e}\hat{e}_s}{4\pi}\frac{1}{d(\tau, \mathbf{x})}, \tag{2.24}$$

$$\hat{e} = eh(\tau)b(\tau)a^{-1}(\tau). \tag{2.25}$$

This is the scaling law for the electric charge. The fine structure constant $\alpha = \hat{e}^2/4\pi\hbar\hat{c}$ evidently does not scale, because $\hbar$ is independent of cosmic time and because of the proportionality (2.9), which renders the ether dispersion free.

E in (2.24) is the Hamiltonian of the relativistic Kepler problem if $d(\tau, \mathbf{x})$ and $|\mathbf{v}|$ are identified with the Euclidean radial coordinate and velocity. Clearly, E is not any more an integral of motion because of the time dependence of the constants, but on the time scale of an orbital period the time variation of the fundamental constants is adiabatic. The angular momentum is still a constant of motion. By means of the Bohr quantization rules for the hydrogen atom, we obtain for the energy levels, the Bohr radii, the orbital velocity and period the scaling laws

$$E_n = -\frac{\hat{m}}{2\hbar^2}\left(\frac{\hat{e}\hat{e}_s}{4\pi}\right)^2 \frac{1}{n^2} \sim h^3 \frac{b^6}{a^6}, \quad r_n = 4\pi \frac{\hbar^2 n^2}{\hat{m}\hat{e}\hat{e}_s} \sim \frac{1}{h}\frac{a^4}{b^4}, \tag{2.26}$$

$$v_n = \frac{\hbar n}{\hat{m} r_n} \sim h^2 \frac{b^2}{a^2}, \quad T_n = 2\pi \frac{r_n^2 \hat{m}}{\hbar n} \sim \frac{1}{h^3}\frac{a^6}{b^6}. \tag{2.27}$$

Because of relation (2.9), they all scale with powers of the scale factor $h(\tau)$. These scaling laws can as well be derived from the Schrödinger equation, which reads as usual, but with time dependent mass and charge. This scaling of atomic energy levels holds for all atomic spectra because of the adiabatic variation of the scale factors, which means that time derivatives of the constants are negligible in leading asymptotic order.

In the following we choose the scale factors of metric and permeability tensor as power laws,

$$a(\tau) = A(\Lambda\tau)^\alpha, \quad b(\tau) = B(\Lambda\tau)^\beta, \quad h(\tau) = H(\Lambda\tau)^\gamma. \tag{2.28}$$

Remark: Apart from the fine structure constant there is a second dimensionless ratio, $\hbar^2 H_0/(kcm^3)$, which is of moderate magnitude, if we take for m the mass of an elementary particle (Dirac, 1938). If we require this ratio to be constant, then we must choose $\gamma = -1$. This follows from the scaling of the gravitational constant, $\hat{k} = kh^4(\tau)a^2(\tau)b^{-2}(\tau)$. A detailed derivation of this scaling law, based on the gravitational theory developed in Tomaschitz (1998c), will be given elsewhere. We will not make explicit use of the gravitational constant in this paper, but we will mainly discuss the choice $\gamma = -1$ in (2.28), which is strongly suggested by the indicated scaling of k. We also assume that $\beta > 0$, which is the condition for redshifts to occur; a negative β would result in blueshifts, as we will see in the next Section.

Integrating (2.19) with the scale factors (2.28) and $\gamma = -1$, we obtain

$$u = \kappa \exp(-\tilde{A}\sqrt{\nu^2\tau^{-2\beta} + B^2}), \quad \tilde{A} := H/(B\beta\nu), \tag{2.29}$$

which can also be derived from the action

$$S(\tau, u) = \nu m \log u - \frac{mH}{B}\int \frac{1}{\tau}\sqrt{\nu^2\tau^{-2\beta} + B^2}\, d\tau. \tag{2.30}$$

The eikonal equation (2.12) is solved by

$$\psi = s(\log u + \hat{A}\tau^{-\beta}), \quad \hat{A} := \pm H/(B\beta). \tag{2.31}$$

Here s is an integration parameter identical with the spectral variable in (2.11), and the rays read

$$u = \kappa \exp(-\hat{A}\tau^{-\beta}). \tag{2.32}$$

As the semiclassical approximation is exact, we attach to the rays a photon energy ($\hbar = c = 1$)

$$E = g^{00}\psi_{,\tau} = s\hat{A}\beta\tau^{-\beta-1}, \tag{2.33}$$

see the discussion following (2.12). For E to be positive, we choose the integration parameter s in (2.31) so that $s\hat{A} > 0$.

3. Effects of the Ether on the Redshift-Distance Relation in a General RW Cosmology

In traditional RW cosmology the redshift can be defined by $1 + z = E(\tau_{em})/E(\tau_{rec})$, with $E(\tau) = \hbar\omega(\tau)$. However, this is not the case in the ether, even though $\hbar$ remains independent of cosmic time. Not only the photon frequency scales in cosmic time, but also the measuring rods, the atomic energy levels. This is taken into account by expressing the photon energy in units of these varying rods, i.e., if we replace $E(\tau)$ in the redshift-energy relation by $E(\tau)/E_n(\tau)$, where $E_n(\tau) \sim h^3b^6a^{-6}$ is some atomic energy level, cf. (2.26). Since $\omega(\tau) = \Lambda|s|h(\tau)b^{-1}(\tau)$ as pointed out after (2.12), we have

$$1 + z = \tilde{R}(\tau_{rec})/\tilde{R}(\tau_{em}), \quad \tilde{R}(\tau) := h^2b^7a^{-6}. \tag{3.1}$$

This is a crucial relation, and very different from the standard theory, since $b(\tau)$ is a scale factor of the permeability tensor. The energy of the photon emitted by the source at time τ_{em} is $\Delta E_n(\tau_{em})$, namely the difference of two atomic energy levels. Its energy when received at time τ_{rec} is $\Delta E_n(\tau_{em})\omega(\tau_{rec})/\omega(\tau_{em})$; this is a consequence of light propagation through the ether. The redshift (3.1) is then obtained by comparing this energy to the energy of a photon emitted by a reference atom in the same transition. The energy of this reference photon is $\Delta E_n(\tau_{rec})$.

In the following we use the Poincaré half-space H^3 as coordinate representation of the hyperbolic 3-space, see after (2.6). We consider a galactic source of radiation at $(\xi = 0, u = 1)$, and a galactic observer sitting at $(\xi = 0, u = u_0)$. A photon is emitted at time τ_{em} and reaches at a later instant τ_{rec} the observer; it has the world-line

$$u(\tau) = \exp\left[\mathrm{sign}(u_0 - 1)\int_{\tau_{em}}^{\tau} R_P^{-1}(\tau)d\tau\right], \quad R_P(\tau) := b(\tau)h^{-1}(\tau). \tag{3.2}$$

Clearly, u_0 and τ_{rec} are related by $u(\tau_{rec}) = u_0$. Also note that we have to choose the sign of $\hat{A}$ in (2.32) so that $\mathrm{sign}(\hat{A}) = \mathrm{sign}(u_0 - 1)$, otherwise the photon cannot reach the observer. If a second photon is emitted by the source a little later, at $\tau_{em} + \Delta\tau_{em}$, it will arrive at $\tau_{rec} + \Delta\tau_{rec}$, and we have

$$\Delta\tau_{rec} R_P^{-1}(\tau_{rec}) = \Delta\tau_{em} R_P^{-1}(\tau_{em}) \tag{3.3}$$

(McVittie, 1965). In the following we specify the scale factors of metric and permeability tensor as in (2.28). We obtain from (3.1)

$$\tau_{rec}/\tau_{em} = (1+z)^{1/\delta}, \quad \delta := 2\gamma + 7\beta - 6\alpha, \tag{3.4}$$

$$a(\tau_{rec})/a(\tau_{em}) = (1+z)^{\alpha/\delta}, \quad R_P(\tau_{rec})/R_P(\tau_{em}) = (1+z)^{(\beta-\gamma)/\delta},$$

$$E_n(\tau_{rec})/E_n(\tau_{em}) = (1+z)^{(3\gamma+6\beta-6\alpha)/\delta}, \tag{3.5}$$

and from (3.2)

$$|\log u_0| = \int_{\tau_{em}}^{\tau_{rec}} R_P^{-1}(\tau)d\tau = D(\tau_{rec})((1+z)^{(\beta-\gamma-1)/\delta} - 1), \tag{3.6}$$

$$D(\tau_{rec}) := \frac{c_0}{H_0} \frac{1}{Ra(\tau_{rec})} \frac{\delta}{\beta - \gamma - 1}, \tag{3.7}$$

$$c_0 := c\frac{a(\tau_{rec})}{R_P(\tau_{rec})}, \quad H_0 := \dot{\tilde{R}}(\tau_{rec})/\tilde{R}(\tau_{rec}) = \delta/\tau_{rec}. \tag{3.8}$$

$\delta > 0$ is evidently the condition that a redshift occurs. In (3.7) and (3.8) we have restored the units c and R. We assume that $D(\tau_{rec}) > 0$, so that $\beta - \gamma - 1 > 0$. This means $|\log u_0| \to \infty$ for $z \to \infty$, and thus $\beta - \gamma - 1 > 0$ is the condition that no horizon appears. The metric distance [with respect to the line element $a(\tau)d\sigma$ on the cosmic 3-space] between source (at $u = 1$) and observer (at $u = u_0$) reads

$$d(\tau) = a(\tau)|\log u_0|, \quad d(\tau_{em}) = (1+z)^{-\alpha/\delta} d(\tau_{rec}). \tag{3.9}$$

With these prerequisites the redshift-distance relation (Tolman, 1930; 1934; Robertson, 1938; Sandage, 1961; Robertson and Noonan, 1968; Weinberg, 1972) can easily be derived. The source located at $u = 1$ is isotropically emitting at time τ_{em} (within an interval $\Delta\tau_{em}$) a swarm of photons. Its absolute luminosity, i.e., the total emitted energy per unit time in the rest frame of the source is thus ($\hbar = 1$)

$$L = \frac{E^{rad}(\tau_{em})/E_n(\tau_{em})}{\Delta\tau_{em}/T_n(\tau_{em})}, \quad E^{rad}(\tau_{em}) = \sum_k n_k \omega_k(\tau_{em}). \tag{3.10}$$

Here n_k denotes the number of emitted photons of frequency $\omega_k(\tau) = |s_k| R_P^{-1}(\tau)$. Energy and time are measured in units of some atomic energy level $E_n(\tau)$ and period $T_n(\tau)$, cf. (2.26) and (2.27). The apparent luminosity is defined as the energy absorbed by the observer's antenna (placed perpendicular to the u-axis) per unit time and unit area,

$$L_{app} = \frac{1}{\mathrm{area}(u_0, \tau_{rec})} \frac{E^{rad}(\tau_{rec})/E_n(\tau_{rec})}{\Delta\tau_{rec}/T_n(\tau_{rec})}. \tag{3.11}$$

Here $\mathrm{area}(u_0, \tau_{rec})$ denotes the area of the hyperbolic sphere centered at the source at $u = 1$ and passing through the observer at $u = u_0$. The antenna is assumed to be a tiny cap of this sphere, and the radiation is isotropic. The light rays emitted at $u = 1$ pass of course orthogonally through the sphere. Because of the space expansion the area of the sphere scales with the square of the expansion factor. For the area we obtain

$$\mathrm{area}(u_0, \tau) = 4\pi a^2(\tau)|\log u_0|^2 \tilde{d}_L^2, \quad \tilde{d}_L := |\log u_0|^{-1} \sinh(|\log u_0|). \tag{3.12}$$

[This is the area of a sphere with hyperbolic center at $(\xi = 0, u = 1)$, intersecting the u-axis at $u_0^{\pm 1}$. It is easiest calculated in the B^3-model, see after (2.12).] Assembling equations (3.10)–(3.12) and (3.3), we arrive at

$$L_{app} = \frac{L}{4\pi d_L^2}, \tag{3.13}$$

$$d_L^2 = \frac{1}{4\pi} \mathrm{area}(u_0, \tau_{rec}) \frac{R_P(\tau_{rec}) T_n(\tau_{em}) E^{rad}(\tau_{em}) E_n(\tau_{rec})}{R_P(\tau_{em}) T_n(\tau_{rec}) E^{rad}(\tau_{rec}) E_n(\tau_{em})}. \tag{3.14}$$

The luminosity distance d_L may be written as

$$d_L = Ra(\tau_{rec})(1 + z)|\log u_0|\tilde{d}_L, \tag{3.15}$$

with $|\log u_0|$ as in (3.6) and (3.7). Note that we can safely put $\tilde{d}_L \approx 1$, since

$$d(\tau_{rec}) = Ra(\tau_{rec}) D(\tau_{rec})((1 + z)^{(\beta-\gamma-1)/\delta} - 1) \tag{3.16}$$

[cf. (3.9)], and since $d(\tau_{rec}) \ll Ra(\tau_{rec})$ (curvature radius of the 3-space), and because the redshift factor is $O(1)$ for presently accessible redshifts. Therefore $D(\tau_{rec}) \ll 1$ holds, and $\tilde{d}_L \approx 1$ follows from (3.6) and (3.12). By substituting (3.6) for $|\log u_0|$ in (3.15), we can identify the deceleration parameter q,

$$d_L = \frac{c_0}{H_0} z \left(1 + \frac{1}{2}(1 - q)z + O(z^2)\right), \quad q := \frac{(1 + \gamma - \beta)}{\delta}. \tag{3.17}$$

Because of condition (2.9) on the scale factors and because of $\gamma = -1$, cf. the Remark following (2.28), we have among the exponents the relations $\beta - \alpha = 1/3$ and $\delta = \beta$. Thus the prediction is $q = -1$ for the deceleration parameter, like in steady state cosmology. However, the universe has a finite age, cf. (3.8). If the expansion is linear then $\delta = 4/3$, which means that its age is just twice that predicted by the standard theory with $q = 1/2$, cf. Sandage (1988). Linear expansion is extensively discussed in Sections 5 and 6.

4. Source Counts in the Ether

For the following considerations we use as coordinate representation of the cosmic 3-space the ball model B^3 of hyperbolic space as introduced after (2.12). We denote by $\rho(\tau)$ the galactic density, i. e., the number of galaxies per unit volume in the 3-space, and consider a thin spherical shell in B^3, centered at $\mathbf{x} = 0$, with Euclidean radius r_{em} and thickness dr_{em}. At time τ_{em} the number of galaxies in this shell is

$$dN(\tau_{em}, r_{em}) = \rho(\tau_{em}) d\mathrm{vol}(r_{em}, \tau_{em}) = 32\pi a^3(\tau_{em})\rho(\tau_{em}) \frac{r_{em}^2 dr_{em}}{(1 - r_{em}^2)^3}. \tag{4.1}$$

The light emitted by these galaxies at τ_{em} reaches an observer sitting at $\mathbf{x} = 0$ at time τ_{rec}. The rays in B^3 are calculated from $ds_P^2 = 0$, cf. (2.6), and we obtain in this way

$$\int_{\tau_{em}}^{\tau_{rec}} R_P^{-1}(\tau) d\tau = -2 \int_{r_{em}}^{0} (1 - r^2)^{-1} dr = \log \frac{1 + r_{em}}{1 - r_{em}} =: r_H. \tag{4.2}$$

We keep τ_{rec} fixed in (4.2) and regard r_{em} and τ_{em} as functions of the parameter r_H. Equation (4.1) may thus be written

$$dN(\tau_{em}(r_H), r_{em}(r_H)) = \hat{\rho}(\tau_{em}(r_H))\mathrm{area}(r_H) dr_H, \tag{4.3}$$

$$\hat{\rho}(\tau) := a^3(\tau)\rho(\tau), \quad \mathrm{area}(r_H) := 4\pi \sinh^2(r_H). \tag{4.4}$$

Since $r_H = |\log u_0|$, we obtain from (3.4) and (3.6)

$$\frac{\tau_{rec}}{\tau_{em}} = \left(1 + \frac{r_H}{D(\tau_{rec})}\right)^{1/(\beta-\gamma-1)}. \tag{4.5}$$

We assume for $\hat{\rho}$ a power law,

$$\hat{\rho} \sim \tau^{2\lambda}, \quad \hat{\rho}(\tau_{em}) = \hat{\rho}(\tau_{rec}) \left(1 + \frac{r_H}{D}\right)^{2\lambda/(1+\gamma-\beta)}. \tag{4.6}$$

The number of galaxies of redshift smaller than z reads

$$N(z) = \int_0^{r_H(z)} dN(r_H), \quad r_H(z) = D(\tau_{rec})((1+z)^{(\beta-\gamma-1)/\delta} - 1), \tag{4.7}$$

$$dN(r_H) = 4\pi \hat{\rho}(\tau_{rec}) \tilde{d}_L^2 r_H^2 \left(1 + \frac{r_H}{D}\right)^{2\lambda/(1+\gamma-\beta)} dr_H. \tag{4.8}$$

[The source counting function $N(z)$ is extensively discussed for RW cosmology by, e.g., Sangdage (1961), McVittie (1965), and Weinberg (1972).] The extrema of the integrand $dN(r_H)/dr_H$ are determined by the zeros of

$$1 + \frac{r_H}{D} = \frac{1}{D} \frac{\lambda}{\beta - \gamma - 1} \tanh(r_H). \tag{4.9}$$

It is geometrically evident that this equation has either none or two solutions. As $D(\tau_{rec})$ is very small, the first solution will be a maximum at $r_{H(\max)} = O(D)$ and the second a minimum at $r_{H(\min)} = O(1)$, before dN/dr_H exponentially diverges because of the $\tilde{d}_L^2$-factor. However, only the maximum is observationally accessible. [We can always assume $\tilde{d}_L \approx 1$, as pointed out after (3.16).] To calculate $r_{H(\max)}$, we may replace in (4.9) $\tanh r_H$ by r_H, i.e., we put $\tilde{d}_L = 1$ in dN/dr_H. So we obtain as solution of (4.9)

$$\frac{r_{H(\max)}}{D} = \frac{\beta - \gamma - 1}{\lambda - (\beta - \gamma - 1)} + O(D). \tag{4.10}$$

This maximum corresponds via (4.7) to a redshift

$$(1 + z_{\max})^{(\gamma+1-\beta)/\delta} = 1 + (\gamma + 1 - \beta)/\lambda. \tag{4.11}$$

As $\delta > 0$ and $\beta - \gamma - 1 > 0$ [pointed out after (3.8)], we have as condition on the density exponent λ for a maximum to occur

$$\lambda/(\beta - \gamma - 1) > 1. \tag{4.12}$$

Redshift surveys of quasars show a peak of dN/dz in the range $2.2 < z_{\max} < 2.4$ (Hartwick and Schade, 1990). If we choose $\alpha = 1$, $\beta = 4/3$, and $\gamma = -1$, cf. the end of Section 3 and the beginning of Section 5, we obtain $\lambda \approx 1.9$ for $z_{\max} \approx 2.3$. The galactic density then scales as $\rho(\tau) \sim \tau^{2\lambda-3}$, cf. (4.4) and (4.6).

The integral in (4.7) is elementary if we put $\tilde{d}_L = 1$,

$$N(z) = 4\pi\hat{\rho}(\tau_{rec})(RD)^3 \left[\frac{1}{\mu+3}((1+z)^{(\mu+3)\nu} - 1) \right.$$

$$\left. - \frac{2}{\mu+2}((1+z)^{(\mu+2)\nu} - 1) + \frac{1}{\mu+1}((1+z)^{(\mu+1)\nu} - 1) \right] (1 + O(D^2)),$$

$$\mu := 2\lambda(1 + \gamma - \beta)^{-1}, \quad \nu := (\beta - \gamma - 1)/\delta, \quad \nu > 0. \tag{4.13}$$

If $\mu = -1, -2, -3$, $N(z)$ is defined by an obvious limit procedure. We obtain for $z \to 0$ the Euclidean result

$$N(z) \sim \frac{4\pi}{3}\left(RD(\tau_{rec}) z \frac{\beta - \gamma - 1}{\delta}\right)^3 \hat{\rho}(\tau_{rec}) \sim \frac{4\pi}{3} d^3(\tau_{rec})\rho(\tau_{rec}). \tag{4.14}$$

[The second asymptotic equivalence is evident from (3.16) and (4.4).] If $\beta = \delta = 4/3$, $\gamma = -1$, and $\lambda = 2$, then we obtain from (4.13)

$$N(z) = 4\pi\hat{\rho}(\tau_{rec})(RD(\tau_{rec}))^3 \left(\log(1+z) - z\left(1 + \frac{3}{2}z\right)(1+z)^{-2}\right). \tag{4.15}$$

Remark: An observationally more tractable quantity than $N(z)$ is $N(z, L_{app})$, the number of galaxies with redshift smaller than z and luminosity greater than L_{app}. In this case both density and luminosity evolution are taken into account (Sandage, 1988). The galactic density becomes in this way an unknown function of two variables, $\rho(\tau, L_{abs})$, which requires a further integration over the absolute luminosity in the integral representation of $N(z, L_{app})$ (Weinberg, 1972). In this paper we content ourselves with $N(z) = N(z, L_{app} = 0)$, which evidently is an upper bound on $N(z, L_{app})$, and disregard the possibility of luminosity evolution.

5. Ether and Luminosity-Distance in an Expanding Minkowskian Universe

The universe defined by the RW line element (2.5) is isometric to the forward light cone $t^2 - |\mathbf{x}|^2 > 0$, $t > 0$, $\mathbf{x} = (x, y, z)$, provided the expansion factor is linear [$A = \alpha = 1$ in (2.28)] and the cosmic 3-space is negatively curved (Infeld and Schild, 1945). In this way globally geodesic coordinates can be introduced for every galactic observer. The coordinate change mapping the line element (2.5) into the Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ is given by

$$\begin{pmatrix} t \\ x \end{pmatrix} = \frac{\tau}{2u}(|\xi|^2 + u^2 \pm 1), \quad (y, z) = \frac{\tau}{u}(\xi_1, \xi_2);$$

$$\tau = \sqrt{t^2 - |\mathbf{x}|^2}, \quad u = \frac{\sqrt{t^2 - |\mathbf{x}|^2}}{t - x}, \quad (\xi_1, \xi_2) = \frac{(y, z)}{t - x}, \tag{5.1}$$

so that the u-semi-axis of H^3 [defined after (2.6)] is mapped onto the x-axis. Because of the homogeneity of the light cone it is sufficient to focus on world-lines along the x-axis. (All other trajectories can be obtained by applying Lorentz transformations onto them). We therefore put $\xi = 0$ in (5.1). The world line $u = u_0$ of a galactic observer [see Section 4] is mapped by (4.1) onto $x = \mathbf{v}t$, $\mathbf{v} = (u_0^2 - 1)(u_0^2 + 1)^{-1}$. In particular the light source at $u = 1$ is mapped onto $x = 0$.

The action $S(t, x)$ for massive particles moving in the ether reads as in (2.30), with (5.1) ($\xi = 0$) substituted. After all, S is a scalar, and it is understood that the permeability tensor as defined in (2.6) is mapped by (5.1) into the forward light cone (Tomaschitz, 1998b). Energy and momentum is then defined in the (t, x)-frame as $E = \eta^{00} S_{,t}$, $p = S_{,x}$. In particular, if a particle is at rest [$\nu = 0$ in (2.19) and (2.23)], we obtain for its rest energy

$$E_0 = m\frac{Ht}{t^2 - x^2}. \tag{5.2}$$

At the galaxy $x = 0$ we thus have the scaling $E_0 = mc^2 h(t)$. [$\gamma = -1$ in (2.28).] Since at $x = 0$ we have $\tau = t$, $u = 1$, all scaling laws derived in Section 2 remain valid at the galaxy, just by replacing τ by t. (The forward light cone coordinates are also the locally geodesic ones, of course.) Let us consider that for the speed of light.

The world-lines of photons as calculated in (2.32) are mapped by (5.1) into

$$\sqrt{(t+x)(t-x)^{-1}} = \kappa \exp(-\hat{A}(t^2 - x^2)^{-\beta/2}), \tag{5.3}$$

with $\hat{A} = \pm H/(B\beta)$. The general solution $\psi(t, x)$ of the eikonal equation in the forward light cone is obtained by the same substitution in (2.31). For photon energy and momentum we obtain

$$E(t, x) = -\psi_{,t} = \frac{st}{t^2 - x^2}\left(\frac{x}{t} + \beta\hat{A}(t^2 - x^2)^{-\beta/2}\right), \tag{5.4}$$

$$p = \psi_{,x} = \frac{st}{t^2 - x^2}\left(1 + \beta\hat{A}\frac{x}{t}(t^2 - x^2)^{-\beta/2}\right). \tag{5.5}$$

The speed of light reads $c(t, x) = -\psi_{,t}/\psi_{,x}$. Clearly,

$$|c(t, x)| = \bar{\lambda}\omega, \quad \omega = |\psi_{,t}|, \quad \bar{\lambda} = |\psi_{,x}|^{-1}, \tag{5.6}$$

$$c(t, x = 0) = \beta\hat{A}t^{-\beta}. \tag{5.7}$$

This is the scaling already derived in (2.22), if τ is identified with t. [Note that the exponents in (2.28) now read $\alpha = -\gamma = 1$]. For the energy of a photon at $x = 0$ we obtain

$$E = s\hat{A}\beta t^{-\beta-1}. \tag{5.8}$$

As pointed out after (2.33), we choose the sign of s in a way that E is positive.

A photon is emitted by the source $x = 0$ at some time t_{em}. Then the integration constant in (5.3) is determined as $\kappa = \exp(\hat{A}t_{em}^{-\beta})$. Inserting $x = \mathbf{v}t$ into (5.3), we obtain for the absorption of the photon by the observer the coordinates

$$t_{rec}(t_{em}, \mathbf{v}) = \gamma\left(t_{em}^{-\beta} - \frac{1}{2\hat{A}}\log\frac{1+\mathbf{v}}{1-\mathbf{v}}\right)^{-1/\beta}, \quad x_{rec} = \mathbf{v}t_{rec}(t_{em}, \mathbf{v}), \tag{5.9}$$

with $\gamma := (1 - \mathbf{v}^2)^{-1/2}$. Note that $\mathrm{sign}(\hat{A}) = \mathrm{sign}(\mathbf{v}) = \mathrm{sign}(u_0 - 1)$, otherwise the ray cannot reach the observer in the comoving frame, see the discussion after (3.2) and the relation between $\mathbf{v}$ and u_0 pointed out after (5.1).

The rest frame of the galactic observer is denoted by coordinates (t', x'), so that

$$t' = \gamma(t - \mathbf{v}x), \quad x' = \gamma(x - \mathbf{v}t). \tag{5.10}$$

[The (t, x)-frame is the rest frame of the source placed at $x = 0$.] This observer is located at $x' = 0$. For emission and absorption event we have in this frame the coordinates

$$t'_{em} = \gamma t_{em}, \quad x'_{em} = -\gamma\mathbf{v}t_{em}; \quad t'_{rec} = \gamma^{-1}t_{rec}, \quad x'_{rec} = 0. \tag{5.11}$$

Let us suppose that a second photon is emitted by the source a little later, at $(t_{em} + \Delta t_{em}, x = 0)$. In the rest frame of the source the two photons arrive at the galactic observer separated by the time interval (proper time of the source)

$$\Delta t_{rec} = \frac{\partial t_{rec}(t_{em}, \mathbf{v})}{\partial t_{em}} \Delta t_{em}. \tag{5.12}$$

In the proper time of the observer the time differences at which the two photons are emitted, and absorbed read

$$\Delta t'_{em} = \gamma \Delta t_{em}, \quad \Delta t'_{rec} = \gamma^{-1} \frac{\partial t_{rec}(t_{em}, \mathbf{v})}{\partial t_{em}} \Delta t_{em}. \tag{5.13}$$

At the time of emission the distance between observer and source is, in the rest frame (t, x) of the source, $R_{em} = |\mathbf{v}| t_{em}$. At the time of absorption t_{rec}, the distance is $R_{rec} = |\mathbf{v}| t_{rec}(t_{em}, \mathbf{v})$ in this frame. In the rest frame (t', x') of the galactic observer we have at emission time t'_{em} the distance $R'_{em} = |\mathbf{v}| \gamma t_{em}$. The trajectory of the source reads in these coordinates $x' = -\mathbf{v} t'$. Thus, at the time of absorption t'_{rec} the source is located at $-\mathbf{v} t'_{rec}$, and the distance between source and observer is $R'_{rec} = \gamma^{-1} |\mathbf{v}| t_{rec}(t_{em}, \mathbf{v})$. Inverting (5.9), we have

$$R'_{em} = |\mathbf{v}| \gamma t_{em}(t_{rec}, \mathbf{v}), \quad t_{em}(t_{rec}, \mathbf{v}) = \gamma^{-1} \left(t_{rec}^{-\beta} + \frac{1}{2\hat{A}} \gamma^{-\beta} \log \frac{1 + \mathbf{v}}{1 - \mathbf{v}} \right)^{-1/\beta}. \tag{5.14}$$

From (5.8) we know that $E_{em} \sim t_{em}^{-1-\beta}$, $E'_{rec} \sim t_{rec}'^{-1-\beta}$. If we calculate the redshift, we have to normalize these energies with the energy of atomic energy levels, cf. (3.1), which scale as $E_n \sim h(t) \sim t^{-1}$. (We assume $\alpha = 1$, $\beta = 4/3$, $\gamma = -1$, cf. the end of Section 3.) For the normalized energies we have $\hat{E}_{em} \sim t_{em}^{-\beta}$, $\hat{E}'_{rec} \sim t_{rec}'^{-\beta}$, so that the redshift reads as

$$1 + z = \frac{\hat{E}_{em}}{\hat{E}'_{rec}} = \left(\frac{t'_{rec}}{t_{em}} \right)^{\beta}. \tag{5.15}$$

From (5.11) and (5.14) we obtain

$$z = \frac{1}{2\hat{A}} t_{rec}'^{\beta} \log \frac{1 + \mathbf{v}}{1 - \mathbf{v}}. \tag{5.16}$$

Clearly, $z > 0$ since $\beta > 0$ and $\mathrm{sign}(\hat{A}\mathbf{v}) = 1$, see after (5.9). We may now write

$$t'_{rec} = (1 + z)^{1/\beta} t_{em}, \quad \Delta t'_{rec} = (1 + z)^{1+1/\beta} \Delta t_{em}. \tag{5.17}$$

Remark: The redshift definition (5.15) coincides with that in comoving coordinates, namely with $1 + z = (\tau_{rec}/\tau_{em})^{\beta}$, cf. (3.4). [In (3.4) we have $\delta = \beta$ as pointed out at the end of Section 3]. The photon is emitted at $(\tau_{em}, u = 1)$ and received by the observer at (τ_{rec}, u_0); therefore

$$u_0 = \exp(-\hat{A}(\tau_{rec}^{-\beta} - \tau_{em}^{-\beta})) = \exp(\hat{A}\tau_{rec}^{-\beta} z). \tag{5.18}$$

Moreover, $u_0 = \sqrt{(1+\mathbf{v})(1-\mathbf{v})^{-1}}$ and $x = \mathbf{v}t$, as pointed out after (5.1). The signs of $\hat{A}$, $\mathbf{v}$ and $u_0 - 1$ coincide, see after (5.9). Thus (5.18) is equivalent to (5.16) provided $\tau_{rec} = t'_{rec}$. This identification follows from $t'_{rec} = \gamma(t_{rec} - \mathbf{v}x_{rec})$, if we express (t_{rec}, x_{rec}) in terms of (τ_{rec}, u_0) via the transformation (5.1). [τ_{em} coincides with t_{em} at $x = 0$, cf. (5.1).]

With these prerequisites it is now very easy to compile the luminosity-distance relation, as we already did in Section 3 for a general RW cosmology without specifying the exponents of the scale factors in (2.28). We measure time in units of $T_n \sim h^{-1}(t) \sim t$, cf. (2.27), and we write $\hat{\Delta}t = \Delta t/t$. So we have

$$\hat{\Delta}t'_{rec} = (1+z)\hat{\Delta}t_{em}. \tag{5.19}$$

Energy we normalize as in the redshift definition (5.15), $\hat{E} = E/E_n$.

The source at $x = 0$ isotropically emits at time t_{em} during the interval Δt_{em} a swarm of photons of total energy $E_{em}^{rad} = \sum_k n_k\omega_k$ and frequencies $\omega_k = s_k\hat{A}\beta t_{em}^{-\beta-1}$, cf. (5.8) and after (5.14). The observer is assumed to carry an antenna, a disk of radius r_0 placed orthogonal to the x-axis, which therefore collects a very tiny fraction $r_0^2\pi/(4\pi R_{rec}^2)$ of the isotropically emitted photons. R_{rec} is defined after (5.13). The photons hit the antenna orthogonally, of course. The energy which the observer receives in his rest frame on his antenna at $(t'_{rec}, x' = 0)$ during the interval $\Delta t'_{rec}$, cf. (5.17), is

$$\hat{E}_{rec}^{\prime(antenna)} = \hat{E}_{em}^{rad}\frac{1}{1+z}\frac{r_0^2\pi}{4\pi R_{rec}^2}, \tag{5.20}$$

where we used (5.15). The apparent luminosity L_{app} is defined as the energy per unit time and unit area absorbed by the antenna in its rest frame. Combining (5.19) and (5.20), we obtain

$$L_{app} := \frac{\hat{E}_{rec}^{\prime(antenna)}}{r_0^2\pi\,\hat{\Delta}t'_{rec}} = \frac{\hat{E}_{em}^{rad}}{4\pi(1+z)^2\hat{\Delta}t_{em}R_{rec}^2}. \tag{5.21}$$

The intrinsic luminosity of the source is $L = \hat{E}_{em}^{rad}/\hat{\Delta}t_{em}$, i.e., the total energy emitted by the source in its rest frame per unit time. Thus we may write

$$\frac{L_{app}}{L} = \frac{1}{4\pi d_L^2}, \quad d_L = (1+z)R_{rec}. \tag{5.22}$$

We have $R_{rec} = |\mathbf{v}|\gamma t'_{rec}$, see after (5.13) and (5.11). Inverting (5.16), we obtain

$$\mathbf{v} = \tanh(\hat{A}t_{rec}^{\prime-\beta}z), \quad \gamma = \cosh(\hat{A}t_{rec}^{\prime-\beta}z), \tag{5.23}$$

so that the luminosity-distance in the forward light cone reads

$$d_L = c(1+z)t'_{rec} \sinh\left(\frac{|\hat{A}|z}{(\Lambda t'_{rec})^{\beta}}\right). \tag{5.24}$$

We have restored here the units c and $\Lambda = cR^{-1}$. Since t'_{rec} is identified with τ_{rec}, cf. the Remark following (5.17), this coincides with d_L as calculated in (3.15). Clearly,

$$H_0 = \beta/t'_{rec}, \quad c_0 = |c(t'_{rec}, x' = 0)| = c|\hat{A}|\beta(\Lambda t'_{rec})^{-\beta}, \tag{5.25}$$

see (5.7).

Remark: In Minkowski space there is no obvious analogue to the metric distance $d(\tau_{rec})$ between source and observer at the time of absorption, cf. (3.16). As metrical distance at the time of absorption we may either take R_{rec} or R'_{rec}, see after (5.13). The relation between metrical and luminosity distance then reads

$$d_L = (1+z)R_{rec} = (1+z)R'_{rec} \cosh(\hat{A}t'^{-\beta}_{rec} z). \tag{5.26}$$

Finally we discuss the appearance of the photon defined by the trajectory (5.3) in the rest frame of the observer. In the (t', x')-frame the trajectory (5.3) is given by the same equation, apart from κ being replaced by $\kappa' = \kappa(1-\mathbf{v})^{1/2}(1+\mathbf{v})^{-1/2}$. We introduce a parameter representation, writing $\lambda = (t'^2 - x'^2)^{-\beta/2}$,

$$t'(\lambda) = \lambda^{-1/\beta} \cosh(\hat{A}\lambda - \log\kappa'), \quad x'(\lambda) = -\lambda^{-1/\beta} \sinh(\hat{A}\lambda - \log\kappa'),$$

$$\kappa' := (1-\mathbf{v})^{1/2}(1+\mathbf{v})^{-1/2} \exp(\hat{A}t^{-\beta}_{em}),$$

$$\lambda_{init} = t^{-\beta}_{em} = \gamma^{\beta} t'^{-\beta}_{em}, \quad \lambda_{term} = t'^{-\beta}_{rec} = \gamma^{\beta} t^{-\beta}_{rec}. \tag{5.27}$$

We have $\lambda_{init} > \lambda_{term}$, cf. (5.17). The time coordinate $t'(\lambda)$ may have a minimum in the interval $[\lambda_{init}, \lambda_{term}]$; the condition for an extremum reads

$$\coth\left(\hat{A}(\lambda - t^{-\beta}_{em}) + \frac{1}{2}\log\frac{1+\mathbf{v}}{1-\mathbf{v}}\right) = \beta\hat{A}\lambda, \tag{5.28}$$

and from (5.9) we obtain the identity

$$\hat{A}(\gamma^{\beta} t^{-\beta}_{rec} - t^{-\beta}_{em}) + \frac{1}{2}\log\frac{1+\mathbf{v}}{1-\mathbf{v}} = 0. \tag{5.29}$$

Note that $\beta > 0$, and in the following discussion we also assume $\hat{A} > 0$, $\mathbf{v} > 0$, see after (5.9). The case $\hat{A} < 0$, $\mathbf{v} < 0$ can be treated analogously. It is geometrically evident that (5.28) has always a unique solution for positive λ, which corresponds to a minimum of $t'(\lambda)$. However, the solution lies in the relevant range $[\lambda_{init}, \lambda_{term}]$ only if

$$\frac{1}{\mathbf{v}} \equiv \coth\left(\frac{1}{2}\log\frac{1+\mathbf{v}}{1-\mathbf{v}}\right) < \beta\hat{A}t^{-\beta}_{em} \tag{5.30}$$

holds. We denote this solution by $\lambda_{\min}$.

Thus, if $\mathbf{v} > (\beta\hat{A})^{-1}t_{em}^{\beta}$, there appear two photons in the primed frame. One parametrized in the interval $[\lambda_{\min}, \lambda_{term}]$, and another corresponding to the parameter range $[\lambda_{\min}, \lambda_{init}]$. The observer sees in his frame of rest (t', x') two tachyons emerging at $(t'(\lambda_{\min}), x'(\lambda_{\min}))$, one reaches him at $t'_{rec} = t'(\lambda_{term})$, whereas the other reaches the source at $t'_{em} = t'(\lambda_{init})$. During the time interval $[t'(\lambda_{\min}), \min(t'_{em}, t'_{rec})]$ there appear in this frame two images of the photon that connects source and observer in the comoving frame. Multiple images of photons which appear at different places at the same time are extensively discussed in Tomaschitz (1998b). A similar phenomenon occurs if one considers superluminal particles (Feinberg, 1967) in individual geodesic rest frames of galactic observers (Tomaschitz, 1997a, 1998a).

It is easy to see that $t'_{rec} < t'_{em}$ if $\mathbf{v}$ is sufficiently close to one, cf. (5.9) and (5.11). [In the rest frame (t, x) of the source we always have $t_{em} < t_{rec}$, of course.] If, however, $\mathbf{v} < (\beta\hat{A})^{-1}t_{em}^{\beta}$, then there is no minimum in the range $[\lambda_{init}, \lambda_{term}]$, and we have $t'_{em} < t'_{rec}$; there is only one ray moving straight from the source to the observer. This can be seen as follows. The inequality $t'_{em} < t'_{rec}$ is via (5.9) and (5.11) equivalent to

$$t_{em}^{\prime-\beta} < \frac{1}{2\hat{A}(\gamma^{\beta}-1)} \log \frac{1+\mathbf{v}}{1-\mathbf{v}}, \tag{5.31}$$

and $\mathbf{v} < (\beta\hat{A})^{-1}t_{em}^{\beta}$ is equivalent to $t_{em}^{\prime-\beta} < (\mathbf{v}\beta\hat{A})^{-1}\gamma^{-\beta}$, cf. (5.11). The assertion then follows from

$$\frac{1}{\mathbf{v}\beta\gamma^{\beta}} < \frac{1}{2(\gamma^{\beta}-1)} \log \frac{1+\mathbf{v}}{1-\mathbf{v}}. \tag{5.32}$$

It is easy to see that this inequality holds, if we write it as

$$\frac{\beta}{2}(\log(1+\mathbf{v}) - \log(1-\mathbf{v})) - \frac{1}{\mathbf{v}} + \frac{(1-\mathbf{v}^2)^{\beta/2}}{\mathbf{v}} > 0. \tag{5.33}$$

This is indeed valid for $\beta > 0$, which can be shown by differentiation and by inspecting the limits $\mathbf{v} \to 0, 1$. The appearance of the photon in the individual geodesic rest frames of galactic observers may be quite different compared to the photon trajectory in the comoving frame, but it always arrives at the observer at $x' = 0$ with positive energy, see (5.8) and after (5.14).

6. Effects of the Ether on Source Counts in the Forward Light Cone

We start with the line elements (2.5), (2.6), in comoving coordinates and use the Poincaré ball representation B^3 for the hyperbolic 3-space, with $d\sigma^2$ defined after (2.12); Cartesian B^3-coordinates are denoted by $\hat{\mathbf{x}}$ in this section. Introducing polar coordinates $(\hat{r}, \vartheta, \varphi)$ in B^3, we have

$$dN(\tau, \hat{\mathbf{x}}) = \hat{\rho}(\tau)\frac{8\sin\vartheta\, d\vartheta\, d\varphi\, \hat{r}^2 d\hat{r}}{(1-\hat{r}^2)^3} \tag{6.1}$$

as the number of galaxies in the solid angle $\sin\vartheta\, d\vartheta\, d\varphi$ between $\hat{r}$ and $\hat{r} + d\hat{r}$, cf. (4.1). The line element ds^2 in (2.5) with $a(\tau) = \tau$ and $d\sigma^2$ as defined after (2.12) (with $\mathbf{x}$ replaced by $\hat{\mathbf{x}}$) is mapped onto $ds^2 = -dt^2 + d\mathbf{x}^2$ in the forward light cone by the transformation (Infeld and Schild, 1945)

$$\tau = \sqrt{t^2 - |\mathbf{x}|^2}, \quad \hat{x}^i = \frac{x^i}{|\mathbf{x}|^2}\left(t - \sqrt{t^2 - |\mathbf{x}|^2}\right);$$

$$t = \tau\frac{1 + |\hat{\mathbf{x}}|^2}{1 - |\hat{\mathbf{x}}|^2}, \quad x^i = \frac{2\tau\hat{x}^i}{1 - |\hat{\mathbf{x}}|^2}. \tag{6.2}$$

Introducing polar coordinates (r, ϑ, φ) also in the forward light cone, we obtain

$$\frac{d\hat{r}}{\hat{r}} = \frac{1}{\sqrt{t^2 - r^2}}\left(\frac{t}{r}dr - dt\right), \tag{6.3}$$

and therefore

$$dN(\tau, \hat{\mathbf{x}}) = \hat{\rho}\left(\sqrt{t^2 - |\mathbf{x}|^2}\right)\frac{\sin\vartheta\, d\vartheta\, d\varphi}{(t^2 - r^2)^2} r^2(t\,dr - r\,dt). \tag{6.4}$$

To understand the meaning of this formula, we replace $d\sigma^2$ by the isometric line element

$$d\sigma^2 = \frac{1}{1 - \mathbf{v}^2}\left(\delta_{ij} + \frac{v_i v_j}{1 - \mathbf{v}^2}\right) dv^i dv^j, \tag{6.5}$$

$|\mathbf{v}| < 1$. This is the projective model K^3 of hyperbolic geometry, (Magnus, 1974; Fock, 1959) obtained from B^3 by the isomorphism

$$\hat{x}^i = \frac{v^i}{|\mathbf{v}|^2}\left(1 - \sqrt{1 - |\mathbf{v}|^2}\right); \quad v^i = \frac{2\hat{x}^i}{1 + |\hat{\mathbf{x}}|^2}. \tag{6.6}$$

Therefore we may write (6.1) as

$$dN(\tau, \hat{\mathbf{x}}) = \hat{\rho}(\tau)\sin\vartheta\, d\vartheta\, d\varphi \frac{|\mathbf{v}|^2 d|\mathbf{v}|}{(1 - |\mathbf{v}|^2)^2}. \tag{6.7}$$

The isometry which maps the line element ds^2 in (2.5) with $a(\tau) = \tau$ and $d\sigma^2$ as in (6.5) onto the forward light cone reads

$$\tau = \sqrt{t^2 - |\mathbf{x}|^2}, \quad \mathbf{v} = \frac{\mathbf{x}}{t}; \quad t = \frac{\tau}{\sqrt{1 - |\mathbf{v}|^2}}, \quad \mathbf{x} = \frac{\tau\mathbf{v}}{\sqrt{1 - |\mathbf{v}|^2}}. \tag{6.8}$$

Thus, $\mathbf{v}$ is just the velocity of a galaxy at $(t, \mathbf{x})$. By means of the third equation in (6.8) we write (6.7) as

$$dN(\tau, \hat{\mathbf{x}}) = \hat{\rho}\left(t\sqrt{1-|\mathbf{v}|^2}\right)\frac{d^3\mathbf{v}}{(1-|\mathbf{v}|^2)^2} =: dN(t, \mathbf{v}). \tag{6.9}$$

$dN(t, \mathbf{v})$ is the number of galaxies with velocities in the range $(\mathbf{v}, \mathbf{v} + d\mathbf{v})$ at time t. Equation (6.4) may therefore be written as

$$dN(t, \mathbf{v}) = dN(t, \mathbf{x}) - dn(t, r, \vartheta, \varphi), \tag{6.10}$$

where

$$dN(t, \mathbf{x}) := \hat{\rho}\left(\sqrt{t^2 - |\mathbf{x}|^2}\right)\frac{t d^3\mathbf{x}}{(t^2-|\mathbf{x}|^2)^2} \tag{6.11}$$

is the number of galaxies in the range $(\mathbf{x}, \mathbf{x} + d\mathbf{x})$ at time t, and

$$dn(t, r, \vartheta, \varphi) := \hat{\rho}\left(\sqrt{t^2 - r^2}\right)\sin\vartheta\, d\vartheta\, d\varphi \frac{r^3}{(t^2-r^2)^2} dt \tag{6.12}$$

is the number of galaxies pouring through the surface element $r^2 \sin\vartheta\, d\vartheta\, d\varphi$ of a sphere of radius r during the interval $(t, t + dt)$.

Remark: The line element in (6.5) is diagonal in polar coordinates $(|\mathbf{v}|, \vartheta, \varphi)$, and its volume element is $d\mathrm{vol}(K^3) = (1 - |\mathbf{v}|^2)^{-2} d^3\mathbf{v}$. To obtain the symmetry group of (6.5) we consider a Lorentz boost along the x-axis of the forward light cone, $x' = \gamma(x - ut)$, $t' = \gamma(t - ux)$, $y' = y$, $z' = z$, $\gamma = (1 - u^2)^{-1/2}$, which relates the rest frame (t, x) of the galaxy at $x = 0$ to the rest frame (t', x') of a galaxy moving with speed u $(x = ut)$ along the x-axis. If we write $\mathbf{v} = \mathbf{x}/t$, $\mathbf{v}' = \mathbf{x}'/t'$, this transformation reads

$$v_1' = \frac{v_1 - u}{1 - v_1 u}, \quad v_{2,3}' = \frac{1}{\gamma}\frac{v_{2,3}}{1 - v_1 u}. \tag{6.13}$$

The symmetry group of K^3 is generated by rotations and the transformations (6.13). The curious fact that the special relativistic addition law for velocities can be realized as symmetry transformations of K^3 was pointed out by Fock (1959), and is much less curious in this cosmological context. Since the volume element $d\mathrm{vol}(K^3)$ is invariant under (6.13), it follows that $dN(t', \mathbf{v}') = dN(t, \mathbf{v})$.

With $\hat{\rho}(\tau)$ as in (4.6) we can calculate from (6.11) the number of galaxies in a ball of radius r, $r < t$, as

$$N(t, r) = 4\pi\hat{\rho}(t)t^{-2\lambda+1}\int_0^r dr r^2 (t^2 - r^2)^{\lambda-2}$$

$$= \frac{4\pi}{3}\hat{\rho}(t)(r/t)^3 \,{}_2F_1(2 - \lambda, 3/2; 5/2; (r/t)^2). \tag{6.14}$$

Also note that

$$N(t, r = |\mathbf{v}|t) = \int_{|\tilde{\mathbf{v}}|<|\mathbf{v}|} dN(t, \tilde{\mathbf{v}}) \tag{6.15}$$

is the number of galaxies with velocity smaller than $|\mathbf{v}|$ at a given time t. If $\lambda > 1$, the integral (6.14) converges also for $r = t$,

$$N(t, r = t) = \pi^{3/2} \frac{\Gamma(\lambda - 1)}{\Gamma(\lambda + 1/2)} \hat{\rho}(t). \tag{6.16}$$

We obtain from (6.14), writing $v = r/t$,

$$N(t, r, \lambda = 0) = 2\pi \left(\frac{v}{1 - v^2} - \frac{1}{2} \log \frac{1 + v}{1 - v} \right) \hat{\rho}, \quad \hat{\rho} = \text{const.}, \tag{6.17}$$

$$N(t, r, \lambda = 2) = \frac{4}{3} \pi \hat{\rho}(t) v^3. \tag{6.18}$$

Integrating (6.12), we obtain

$$n(t, r) = 4\pi r^3 \int_t^\infty \hat{\rho} \left(\sqrt{t^2 - |\mathbf{x}|^2} \right) (t^2 - r^2)^{-2} dt \tag{6.19}$$

as the number of galaxies which pass through the sphere $|\mathbf{x}| = r$ within the interval (t, ∞). This integral converges if $\lambda < 3/2$ and $t > r$. Note that $n(t, r, \lambda = 0) = N(t, r, \lambda = 0)$, which means that the number of galaxies is conserved; all galaxies which lie at time t within a sphere of radius r will later pour through this sphere, and only these. If $\lambda > 0$ new galaxies are formed in this sphere, and if $\lambda < 0$ the galactic density decreases in the cosmic evolution. As pointed out at the end of Section 4, we do not consider luminosity evolution here.

Finally, we derive the source counting function (4.7) in the forward light cone. From (6.9) we obtain

$$dN(t_{em}, |\mathbf{v}|) = 4\pi \hat{\rho} \left(t_{em} \sqrt{1 - |\mathbf{v}|^2} \right) \frac{|\mathbf{v}|^2 d|\mathbf{v}|}{(1 - |\mathbf{v}|^2)^2} \tag{6.20}$$

as the number of galaxies with velocities ranging in $(|\mathbf{v}|, |\mathbf{v}| + d|\mathbf{v}|)$ at time t_{em}. Note that observer and source are interchanged compared to Section 5; the primed coordinates of Section 5 correspond to the (t, x)-frame here. With (5.23), (5.17) and (5.11) we may write (6.20) as

$$dN(t_{rec}, z) = 4\pi |\hat{A}| t_{rec}^{-\beta} \hat{\rho}(t_{rec}) (1 + z)^{-2\lambda/\beta} \sinh^2(|\hat{A}| t_{rec}^{-\beta} z) dz, \tag{6.21}$$

with $\beta = 4/3$. By integration we arrive at (4.7), since $t_{rec} = \tau_{rec}$ as pointed out in the Remark following (5.17), and $|\hat{A}| t_{rec}^{-\beta} = D(t_{rec})$, cf. (3.6).

7. Conclusion

The purpose of this paper is to demonstrate that the cosmological redshift and all that goes with it may be a consequence of a cosmic ether rather than a space expansion. What is locally perceived as vacuum speed of light is actually varying in cosmic time, as is the Hubble constant. However, this does not necessarily mean that the 3-space is static. In Section 5 we consider a flat spacetime and geodesic coordinates in which the redshift is a Doppler shift, a combined effect of the galactic recession and the permeability of the ether. The ether as introduced here does not induce dispersion that could lead to a dimming of remote sources (Sandage, 1988).

The cosmology developed here is based on two symmetric tensor fields, a spacetime metric, and a symmetric permeability tensor representing the world ether. This tensor we assume as homogeneous and isotropic; it is determined by two scale factors $h(\tau)$ and $b(\tau)$, both functions of cosmic time like the expansion factor $a(\tau)$ in the RW metric. Electromagnetic fields are coupled to the permeability tensor as in a dielectric medium, cf. Section 2. Classical mechanics in the ether is defined by replacing in the Hamilton-Jacobi equation the spacetime metric by the permeability tensor. Hamiltonian mechanics in Minkowski space works because in this local limit we can accommodate the effects of the ether in the fundamental constants, which become so adiabatically varying functions of cosmic time.

The ether is introduced in a completely phenomenological way, in terms of a macroscopic permeability tensor defined by two scaling functions. If the ether really exists, it must be regarded as the carrier of electromagnetic and quantum fields, and as the physical substance of cosmic space, whose microscopic structure makes wave propagation at all possible. Minkowski space and vacuum electrodynamics as well as the constancy of the speed of light are local geometric idealizations.

Varying fundamental constants are not compatible with Einstein's equations; a gravitational theory in which perihelion shifts are generated by a scalar gravitational potential and the permeability tensor of the ether has recently been proposed in Tomaschitz (1998c). As for cosmology, the possibilities of evolution in an open universe go far beyond what is predictable by Einstein's equations (Dyson, 1979; Tomaschitz, 1996; 1997c).

In Sections 5 and 6 we consider a RW cosmology that is flat and isometric to the forward light cone. In this cosmology globally geodesic coordinates can be introduced for individual galactic observers. In a general RW cosmology a similar reasoning holds in locally geodesic coordinate frames. We then have to restrict ourselves to infinitesimal Lorentz boosts. Calculations then get even simpler, because infinitesimal neighborhoods are a good excuse for linearization, though this is against the spirit of cosmology, which deals, after all, with the global structure of the Universe. For the same reason we did not introduce in Section 3 systematic power series expansions for the cosmic scale factors to derive the luminosity-distance, as is usually done in RW cosmology. By assuming power laws for the cosmic scale factors, we can obtain the distance-redshift relation for large distances and redshifts, where local power series expansions are meaningless.

The ether has a substantial impact on the source counting function $N(z)$, as the exponents of the scale factors significantly enter in the location of the peak of $dN(z)/dz$. In the case of linear expansion, the scaling exponent of the galactic density is determined by this peak.

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