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Heavy Quark Theory

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Abstract

These lectures describe the most important theoretical methods in b -physics. We discuss the formalism of effective weak Hamiltonians, heavy quark effective theory, the heavy quark expansion for inclusive decays of b -hadrons and, finally, the more recent ideas of QCD factorization for exclusive nonleptonic B decays. While the main emphasis is put on introducing the basic theoretical concepts, some key applications in phenomenology are also presented for illustration.

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1 Preface

The dedicated study of b -flavoured hadrons has developed into one of the most active and most promising areas of experimental high-energy physics. The detailed investigation of b decays at the B -meson factories and at hadron colliders will probe the flavour physics of quarks with unprecedented precision. To fully exploit this rich source of data a systematic theoretical approach is necessary. The required field theoretical tools are the subject of these lectures.

We shall discuss the construction of effective weak Hamiltonians, introducing the operator product expansion (OPE) and the renormalization group (RG) and presenting the effective Hamiltonians for nonleptonic $\Delta B = 1$ and $\Delta B = 2$ transitions as examples. The subsequent chapter is devoted to heavy-quark effective theory (HQET). It explains the basic formalism as well as the application to heavy-light and heavy-heavy currents, discussing the B -meson decay constant f_B and the exclusive semileptonic decay $B \rightarrow D^* l \nu$, respectively. Inclusive b decays and the heavy-quark expansion (HQE) are treated next, in particular the general formalism, the issue of quark-hadron duality, the theory of b -hadron lifetimes and of inclusive semileptonic decays. Finally, we discuss QCD factorization for exclusive hadronic B decays, focussing on $B \rightarrow D\pi$ and $B \rightarrow \pi\pi$. We conclude with a short summary.

The effective-Hamiltonian framework is the oldest and most general of the methods we shall discuss. It dates back, more or less, to the beginnings of the standard model itself. HQET and HQE are later developments that have been established in the second half of the eighties and at the beginning of the nineties. The last topic, QCD factorization for exclusive B decays is the most recent. It is the least well established among these methods and it continues to be studied and developed in further detail.

We would like to mention a very short selection of literature, which we hope will be helpful to obtain further information on the various subjects related to the contents of these lectures. Very useful resources are the BaBar Physics Book (Harrison & Quinn 1998) and the Fermilab B Physics Report (Anikeev et al. 2002). They collect nice reviews on both theoretical and experimental topics in B physics. A textbook more

specifically directed towards theoretical heavy quark physics is the work of Manohar & Wise (2000). Review articles on particular subjects are (Buchalla, Buras & Lautenbacher 1996) on effective weak Hamiltonians, (Neubert 1994) on HQET and (Bigi et al. 1994; Bigi, Shifman and Uraltsev 1997) on HQE.

2 Introduction and overview

2.1 Motivation

In the following chapters we will study the theoretical tools to compute weak decay properties of heavy hadrons. To put the formalism into perspective, we start by recalling the main motivation for this subject.

The central goal is the investigation of flavour physics, the most complicated sector in our understanding of fundamental interactions. A good example is give by particle-antiparticle mixing, as first studied with neutral kaons. The strong interaction eigenstates K^0 ($\bar{s}d$) and \bar{K}^0 ($d\bar{s}$) can be transformed into each other through second order weak interactions, which leads to a tiny off-diagonal entry M_{12} in the mass matrix (Fig. 1). The

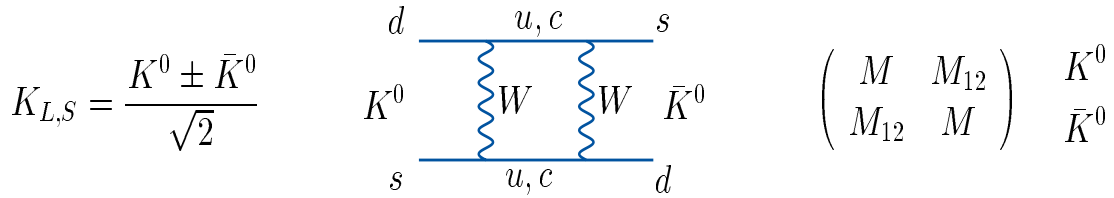


Figure 1. $K^0 - \bar{K}^0$ mixing.

physical eigenstates are $K_{L,S} = (K^0 \pm \bar{K}^0)/\sqrt{2}$. Their mass difference $\Delta m_K = m_L - m_S$ is given by $2|M_{12}|$ and reads

$$\frac{\Delta m_K}{m_K} \approx \frac{G_F^2 f_K^2 B_K}{6\pi^2} |V_{cs} V_{cd}|^2 m_c^2 = 7 \cdot 10^{-15} \quad (1)$$

where the number on the right-hand side is the experimental value. The theoretical expression is derived neglecting the third generation of quarks and CP violation, which is a valid approximation for Δm_K . The factors $f_K^2 B_K$ ($B_K \approx 1$ for the purpose of a first estimate) account for the binding of the quarks into mesons. A crucial feature of (1) is the Glashow-Iliopoulos-Maiani (GIM) cancellation mechanism: The orthogonality of the quark mixing matrix V_{ij} ($i = u, c$; $j = d, s$) leads to a cancellation among the various contributions with intermediate up and charm quarks (symbolically the amplitude has the form $(uu) - (uc) - (cu) + (cc)$). For $m_u = m_c$ this cancellation would be complete, giving $\Delta m_K = 0$. Still, even for $m_u \neq m_c$, the contributions from virtual momenta $k \gg m_c$, m_u cancel since both m_u and m_c are negligible in this case. What is left is a characteristic effect proportional to m_c^2 , the up-quark contribution being subleading for $m_u \ll \Lambda_{QCD} \ll m_c$. This circumstance allowed Gaillard and Lee in 1974 to correctly estimate the charm-quark mass $m_c \approx 1.5$ GeV, before charm was eventually discovered in the Fall of the same year.

In a similar way the discovery of $B_d - \bar{B}_d$ mixing by the ARGUS collaboration (Albrecht et al. 1987) proved to be another milestone in flavour physics. In full analogy to $K - \bar{K}$ mixing we have

$$\frac{\Delta m_B}{m_B} \approx \frac{G_F^2 f_B^2 B_B}{6\pi^2} |V_{tb}V_{td}|^2 M_W^2 S\left(\frac{m_t^2}{M_W^2}\right) = 6 \cdot 10^{-14} \quad (2)$$

where now the top-quark contribution is completely dominant. The unexpectedly large value observed by ARGUS provided the first evidence that the top-quark mass ($m_{t,pole} = 176$ GeV) was comparable to the weak scale and in any case much heavier than anticipated at the time.

These examples show very nicely the “flavour” of flavour physics: Precision observables are sensitive probes of high-energy scales and yield crucial insights into the fundamental structure of weak interactions. At the same time we see that hadronic effects manifest in quantities such as f_B , B_B , and strong interactions of the participating quarks in general, play an important role. Their understanding is necessary to reveal the underlying flavour dynamics and is the main subject of heavy quark theory.

$B - \bar{B}$ mixing, CP violation in B decays and other loop-induced reactions of b -flavoured hadrons are of great interest and continue to be pursued by numerous experiments. A central target is the Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \varrho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} \quad (3)$$

which parametrizes charged-current weak interactions in the standard model. The second equality in (3) introduces the approximate form of the Wolfenstein parametrization, where the four independent CKM quantities are λ , A , ϱ and η . The unitarity triangle, as defined in terms of ϱ and η is shown in Fig. 2, indicating the CP violating angles α , β and γ .

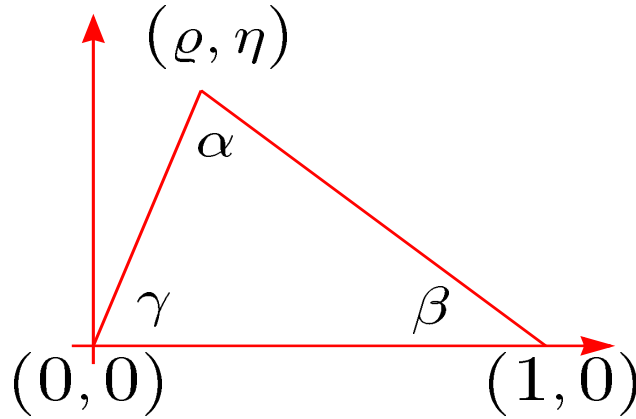


Figure 2. Definition of the unitarity triangle.

The status of the unitarity triangle (in terms of $\bar{\varrho} = \varrho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$) is displayed in Fig. 3, with input from CP violation in the kaon sector (ε_K) and from B decays ($|V_{ub}/V_{cb}|$, Δm_{B_d} , Δm_{B_s} and $\sin 2\beta$).

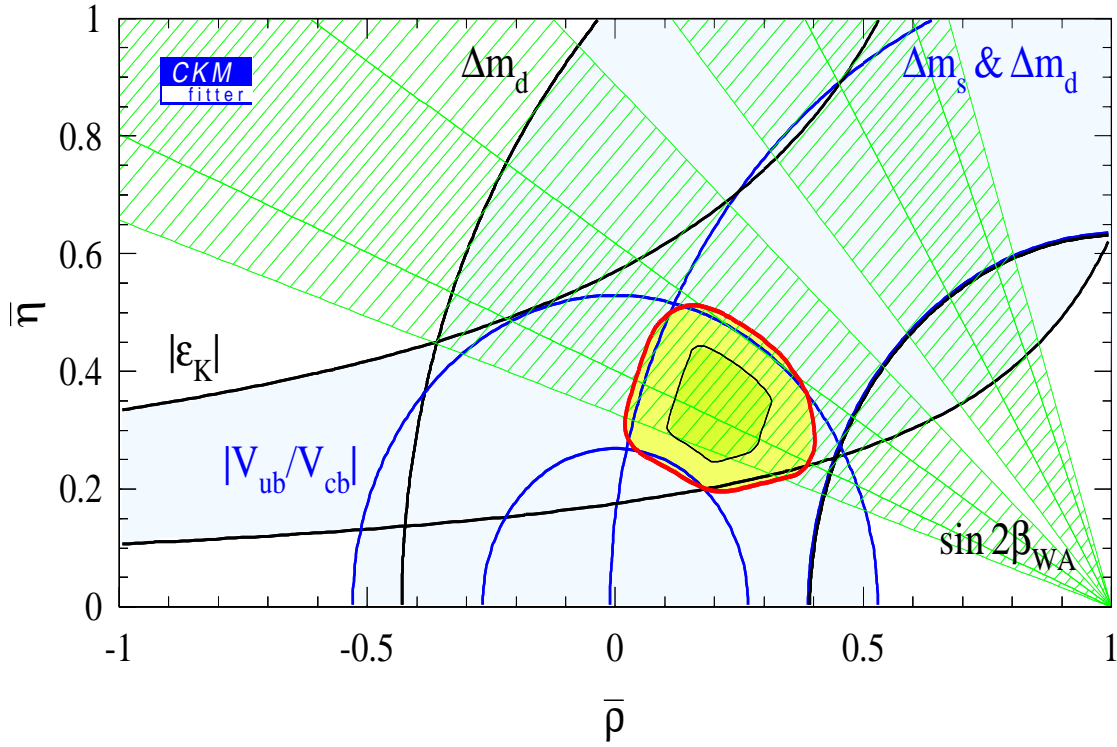


Figure 3. Global fit of the unitarity triangle (darker shaded region), without $\sin 2\beta$ from CP violation in $B \rightarrow J/\Psi K_S$. The constraint from the world average $\sin 2\beta$ (hatched areas) is overlaid (Höcker et al. 2001).

2.2 B decays – overview

A vast number of different B -decay observables is available to further test the standard scenario of flavour dynamics. One may distinguish the following broad classes.

- dominant decays
 $b \rightarrow c\bar{u}d$, $b \rightarrow c\bar{c}s$, $b \rightarrow cl\bar{\nu}$
 $\bar{B} \rightarrow D\pi$, $\bar{B} \rightarrow \Psi K$, $\bar{B} \rightarrow D^{(*)}l\bar{\nu}$
- rare decays
 $b \rightarrow u\bar{u}d$, $b \rightarrow u\bar{u}s$, $b \rightarrow ul\bar{\nu}$
 $\bar{B} \rightarrow \pi\pi$, $\bar{B} \rightarrow \pi K$, $\bar{B} \rightarrow \pi l\bar{\nu}$, $\bar{B} \rightarrow l\bar{\nu}$
- rare and radiative (loop induced) decays
 $b \rightarrow s(d)\gamma$, $b \rightarrow s(d)l^+l^-$, $b \rightarrow s(d)\nu\bar{\nu}$
 $\bar{B} \rightarrow K^{(*)}\gamma$, $\bar{B} \rightarrow \rho\gamma$, ...
- $\Delta B = 2$ oscillations
 $B_d - \bar{B}_d$, $B_s - \bar{B}_s$ mixing

Other obvious classifications are between inclusive and exclusive processes or between hadronic and (semi)leptonic decays.

A few key properties of b -hadrons enhance considerably the possibilities in B physics both experimentally and theoretically. First, the smallness of $V_{cb} = 0.04$ leads to a long

lifetime of $\tau_B \approx 1.6$ ps. In addition the b -quark mass is large compared with the QCD scale

$$m_b \gg \Lambda_{QCD} \approx 0.3\text{GeV} \quad (4)$$

The exact value of m_b depends on the definition. In particular, the running \overline{MS} mass $\bar{m}_b(\bar{m}_b) = 4.2 \pm 0.1\text{GeV}$, the pole mass $m_{b,pole} \approx 4.8\text{GeV}$, whereas the mass of the lightest b -hadron is $m_B = 5.28\text{GeV}$. The smallness of Λ_{QCD}/m_b provides us with a useful expansion parameter. Together with the property of asymptotic freedom of QCD and $\alpha_s(m_b) \ll 1$, this opens the possibility of systematic approximations, which are exploited in the various applications of heavy quark theory.

3 Effective weak Hamiltonians

The task of computing weak decays of hadrons represents a complicated problem in quantum field theory. Two typical cases, the first-order nonleptonic process $\bar{B}^0 \rightarrow \pi^+\pi^-$, and the loop-induced, second-order weak transition $B^- \rightarrow K^-\nu\bar{\nu}$ are illustrated in Fig. 4. The

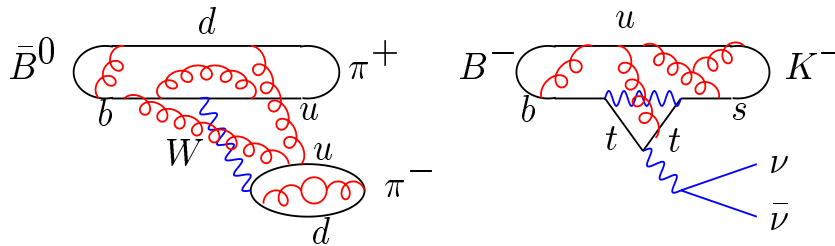


Figure 4. *QCD effects in weak decays.*

dynamics of the decays is determined by a nontrivial interplay of strong and electroweak forces, which is characterized by several energy scales of very different magnitude, the W mass, the various quark masses and the QCD scale: $m_t, M_W \gg m_b, m_c \gg \Lambda_{QCD} \gg m_u, m_d, (m_s)$. While it is usually sufficient to treat electroweak interactions to lowest nonvanishing order in perturbation theory, it is necessary to consider all orders in QCD. Asymptotic freedom still allows us to compute the effect of strong interactions at short distances perturbatively. However, since the participating hadrons are bound states with light quarks, confined inside the hadron by long-distance dynamics, it is clear that also nonperturbative QCD interactions enter the decay process in an essential way.

To deal with this situation, we need a method to disentangle long- and short-distance contributions to the decay amplitude in a systematic fashion. A basic tool for this purpose is provided by the operator product expansion (OPE).

3.1 Operator product expansion

We will now discuss the basic concepts of the OPE for B meson decay amplitudes. These concepts are of crucial importance for the theory of weak decay processes, not only in the case of B mesons, but also for kaons, mesons with charm, light or heavy baryons and weakly decaying hadrons in general. Consider, for instance, the basic W -boson exchange

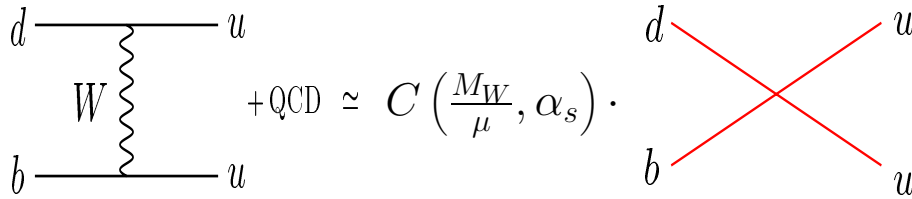


Figure 5. *OPE for weak decays.*

process shown on the left-hand side of Fig. 5. This diagram mediates the decay of a b quark and triggers the nonleptonic decay of a B meson such as $\bar{B}^0 \rightarrow \pi^+ \pi^-$. The quark-level transition shown is understood to be dressed with QCD interactions of all kinds, including the binding of the quarks into the mesons. To simplify this problem, we may look for a suitable expansion parameter, as we are used to do in theoretical physics. Here, a key feature is provided by the fact that the W mass M_W is very much heavier than the other momentum scales p in the problem ($m_b, \Lambda_{QCD}, m_u, m_d, m_s$). We can therefore expand the full amplitude A , schematically, as follows

$$A = C \left(\frac{M_W}{\mu}, \alpha_s \right) \cdot \langle Q \rangle + \mathcal{O} \left(\frac{p^2}{M_W^2} \right) \quad (5)$$

which is sketched in Fig. 5. Up to negligible power corrections of $\mathcal{O}(p^2/M_W^2)$, the full amplitude on the left-hand side is written as the matrix element of a local four-quark operator Q , multiplied by a Wilson coefficient C . This expansion in $1/M_W$ is called a (short-distance) operator product expansion because the nonlocal product of two bilinear quark-current operators $(\bar{d}u)$ and $(\bar{u}b)$ that interact via W exchange, is expanded into a series of local operators. Physically, the expansion in Fig. 5 means that the exchange of the very heavy W boson can be approximated by a point-like four-quark interaction. With this picture the formal terminology of the OPE can be expressed in a more intuitive language by interpreting the local four-quark operator as a four-quark interaction vertex and the Wilson coefficient as the corresponding coupling constant. Together they define an effective Hamiltonian $\mathcal{H}_{eff} = C \cdot Q$, describing weak interactions of light quarks at low energies. Ignoring QCD the OPE reads explicitly (in momentum space)

$$\begin{aligned} A &= \frac{g_W^2}{8} V_{ud}^* V_{ub} \frac{i}{k^2 - M_W^2} (\bar{d}u)_{V-A} (\bar{u}b)_{V-A} \\ &= -i \frac{G_F}{\sqrt{2}} V_{ud}^* V_{ub} C \cdot \langle Q \rangle + \mathcal{O} \left(\frac{k^2}{M_W^2} \right) \end{aligned} \quad (6)$$

with $C = 1$, $Q = (\bar{d}u)_{V-A} (\bar{u}b)_{V-A}$ and

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{ub} (\bar{d}u)_{V-A} (\bar{u}b)_{V-A} \quad (7)$$

As we will demonstrate in more detail below after including QCD effects, the most important property of the OPE in (5) is the *factorization* of long- and short-distance contributions: All effects of QCD interactions above some factorization scale μ (short distances) are contained in the Wilson coefficient C . All the low-energy contributions

below μ (long distances) are collected into the matrix elements of local operators $\langle Q \rangle$. In this way the short-distance part of the amplitude can be systematically extracted and calculated in perturbation theory. The problem to evaluate the matrix elements of local operators between hadron states remains. This task requires in general nonperturbative techniques, as for example lattice QCD or QCD sum rules, but it is considerably simpler than the original problem of the full standard-model amplitude. In some cases also the approximate flavour symmetries of QCD (isospin, $SU(3)$) can help to determine the nonperturbative input. This is true in general for hadronic weak decays. A decisive advantage of *heavy* hadrons is the fact that the heavy-quark mass itself is still large in comparison to Λ_{QCD} . The limit $\Lambda_{QCD}/m_b \ll 1$ can then be exploited, which is achieved, depending on the application, by using heavy quark effective theory, heavy quark expansion or QCD factorization for exclusive nonleptonic decays.

The short-distance OPE that we have described, the resulting effective Hamiltonian, and the factorization property are fundamental for the theory of B decays. However, the concept of factorization of long- and short-distance contributions reaches far beyond these applications. In fact, the idea of factorization, in various forms and generalizations, is the key to essentially all applications of perturbative QCD, including the important areas of deep-inelastic scattering and jet or lepton pair production in hadron-hadron collisions. The reason is the same in all cases: Perturbative QCD is a theory of quarks and gluons, but those never appear in isolation and are always bound inside hadrons. Nonperturbative dynamics is therefore always relevant to some extent in hadronic reactions, even if these occur at very high energy or with a large intrinsic mass scale. Thus, before perturbation theory can be applied, nonperturbative input has to be isolated in a systematic way, and this is achieved by establishing the property of factorization. It turns out that the weak effective Hamiltonian for nonleptonic B decays provides a nice example to demonstrate the general idea of factorization in simple and explicit terms.

We would therefore like to discuss the OPE for B decays in more detail, including the effects of QCD, and illustrate the calculation of the Wilson coefficients. A diagrammatic representation for the OPE is shown in Fig. 6. The key to calculating the coefficients

$$\text{Tree} + \text{Loop} + \dots = C_i \cdot \left(\text{Tree} + \text{Loop} + \dots \right)$$

Figure 6. Calculation of Wilson coefficients of the OPE.

C_i is again the property of factorization. Since factorization implies the separation of all long-distance sensitive features of the amplitude into the matrix elements of $\langle Q_i \rangle$, the short-distance quantities C_i are, in particular, independent of the external states. This means that the C_i are always the same, no matter whether we consider the actual physical amplitude where the quarks are bound inside mesons, or any other, unphysical amplitude with on-shell or even off-shell external quark lines. Thus, even though we are ultimately interested in, e.g., $B \rightarrow \pi\pi$ amplitudes, for the perturbative evaluation of C_i we are free to choose any treatment of the external quarks according to our calculational convenience. A convenient choice that we will use below is to take all external quarks massless and with the same off-shell momentum p ($p^2 \neq 0$).

The computation of the C_i in perturbation theory then proceeds in the following steps:

- Compute the amplitude A in the full theory (with W propagator) for arbitrary external states.
- Compute the matrix elements $\langle Q_i \rangle$ with the same treatment of external states.
- Extract the C_i from $A = C_i \langle Q_i \rangle$.

We remark that with the off-shell momenta p for the quark lines the amplitude is even gauge dependent and clearly unphysical. However, this dependence is identical for A and $\langle Q_i \rangle$ and drops out in the coefficients. The actual calculation is most easily performed in Feynman gauge. To $\mathcal{O}(\alpha_s)$ there are four relevant diagrams, the one shown in Fig. 6 together with the remaining three possibilities to connect the two quark lines with a gluon. Gluon corrections to either of these quark currents need not be considered, they are the same on both sides of the OPE and drop out in the C_i . The operators that appear on the right-hand side follow from the actual calculations. Without QCD corrections there is only one operator of dimension 6

$$Q_1 = (\bar{d}_i u_i)_{V-A} (\bar{u}_j b_j)_{V-A} \quad (8)$$

where the colour indices have been made explicit. To $\mathcal{O}(\alpha_s)$ QCD generates another operator

$$Q_2 = (\bar{d}_i u_j)_{V-A} (\bar{u}_j b_i)_{V-A} \quad (9)$$

which has the same Dirac and flavour structure, but a different colour form. Its origin is illustrated in Fig. 7, where we recall the useful identity for $SU(N)$ Gell-Mann matrices

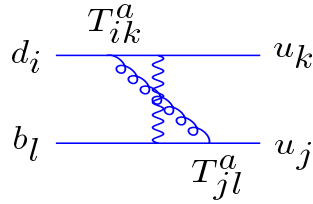


Figure 7. *QCD correction with colour assignment.*

$$(\bar{d}_i T_{ik}^a u_k)(\bar{u}_j T_{jl}^a b_l) = -\frac{1}{2N}(\bar{d}_i u_i)(\bar{u}_j b_j) + \frac{1}{2}(\bar{d}_i u_j)(\bar{u}_j b_i) \quad (10)$$

It is convenient to employ a different operator basis, defining

$$Q_{\pm} = \frac{Q_1 \pm Q_2}{2} \quad (11)$$

The corresponding coefficients are then given by

$$C_{\pm} = C_1 \pm C_2 \quad (12)$$

If we denote by S_{\pm} the spinor expressions that correspond to the operators Q_{\pm} (in other words: the tree-level matrix elements of Q_{\pm}), the full amplitude can be written as

$$A = \left(1 + \gamma_+ \alpha_s \ln \frac{M_W^2}{-p^2}\right) S_+ + \left(1 + \gamma_- \alpha_s \ln \frac{M_W^2}{-p^2}\right) S_- \quad (13)$$

Here we have focused on the logarithmic terms and dropped a constant contribution (of order α_s , but nonlogarithmic). Further, p^2 is the virtuality of the quarks and γ_{\pm} are numbers that we will specify later on. We next compute the matrix elements of the operators in the effective theory, using the same approximations, and find

$$\langle Q_{\pm} \rangle = \left(1 + \gamma_{\pm} \alpha_s \left(\frac{1}{\varepsilon} + \ln \frac{\mu^2}{-p^2} \right) \right) S_{\pm} \quad (14)$$

The divergence that appears in this case has been regulated in dimensional regularization ($D = 4 - 2\varepsilon$ dimensions). Requiring

$$A = C_+ \langle Q_+ \rangle + C_- \langle Q_- \rangle \quad (15)$$

we obtain

$$C_{\pm} = 1 + \gamma_{\pm} \alpha_s \ln \frac{M_W^2}{\mu^2} \quad (16)$$

where the divergence has been subtracted in the minimal subtraction scheme. The effective Hamiltonian we have been looking for then reads

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{ub} (C_+(\mu) Q_+ + C_-(\mu) Q_-) \quad (17)$$

with the coefficients C_{\pm} determined in (16) to $\mathcal{O}(\alpha_s \log)$ in perturbation theory. The following points are worth noting:

- The $1/\varepsilon$ (ultraviolet) divergence in the effective theory (14) reflects the $M_W \rightarrow \infty$ limit. This can be seen from the amplitude in the full theory (13), which is finite, but develops a logarithmic singularity in this limit. Consequently, the renormalization in the effective theory is directly linked to the $\ln M_W$ dependence of the decay amplitude.
- We observe that although A and $\langle Q_{\pm} \rangle$ both depend on the long-distance properties of the external states (through p^2), this dependence has dropped out in C_{\pm} . Here we see explicitly how factorization is realized. Technically, to $\mathcal{O}(\alpha_s \log)$, factorization is equivalent to splitting the logarithm of the full amplitude according to

$$\ln \frac{M_W^2}{-p^2} = \ln \frac{M_W^2}{\mu^2} + \ln \frac{\mu^2}{-p^2} \quad (18)$$

Ultimately the logarithms stem from loop momentum integrations and the range of large momenta, between M_W and the factorization scale μ , is indeed separated into the Wilson coefficients.

- To obtain a decay amplitude from \mathcal{H}_{eff} in (17), the matrix elements $\langle f | Q_{\pm} | \bar{B} \rangle(\mu)$ have to be taken, normalized at a scale μ . An appropriate value for μ is close to the b -quark mass scale in order not to introduce an unnaturally large scale into the calculation of $\langle Q \rangle$.
- The factorization scale μ is unphysical. It cancels between Wilson coefficient and hadronic matrix element, to a given order in α_s , to yield a scale independent decay amplitude. The mechanism of this cancellation to $\mathcal{O}(\alpha_s)$ is clear from the above example (13) – (16).

- In the construction of \mathcal{H}_{eff} the W -boson is said to be “integrated out”, that is, removed from the effective theory as an explicit degree of freedom. Its effect is still implicitly contained in the Wilson coefficients. The extraction of these coefficients is often called a “matching calculation”, matching the full to the effective theory by “adjusting” the couplings C_{\pm} .
- If we go beyond the leading logarithmic approximation $\mathcal{O}(\alpha_s \log)$ and include the finite corrections of $\mathcal{O}(\alpha_s)$ in (13), (14), an ambiguity arises when renormalizing the divergence in (14) (or, equivalently, in the Wilson coefficients C_{\pm}). This ambiguity consists in what part of the full (non-logarithmic) $\mathcal{O}(\alpha_s)$ term is attributed to the matrix elements, and what part to the Wilson coefficients. In other words, coefficients and matrix elements become *scheme dependent*, that is, dependent on the renormalization scheme, beyond the leading logarithmic approximation. The scheme dependence is unphysical and cancels in the product of coefficients and matrix elements. Of course, both quantities have to be evaluated in the same scheme to obtain a consistent result. The renormalization scheme is determined in particular by the subtraction constants (minimal or non-minimal subtraction of $1/\varepsilon$ poles), and also by the definition of γ_5 used in $D \neq 4$ dimensions in the context of dimensional regularization.
- Finally, the effective Hamiltonian (17) can be considered as a modern version of the old Fermi theory for weak interactions. It is a systematic low-energy approximation to the standard model for b -hadron decays and provides the basis for any further analysis.

3.2 Renormalization group

Let us have a closer look at the Wilson coefficients, which read explicitly

$$C_{\pm} = 1 + \frac{\alpha_s(\mu)}{4\pi} \frac{\gamma_{\pm}^{(0)}}{2} \ln \frac{\mu^2}{M_W^2} \quad \gamma_{\pm}^{(0)} = \begin{cases} 4 \\ -8 \end{cases} \quad (19)$$

where we have now specified the exact form of the $\mathcal{O}(\alpha_s \log)$ correction. Numerically the factor $\alpha_s(m_b)\gamma_{\pm}^{(0)}/(8\pi)$ is about $+3.5\%$ (-7%), a reasonable size for a perturbative correction (we used $\alpha_s(\mu = 4.2 \text{ GeV}) = 0.22$). However, this term comes with a large logarithmic factor of $\ln(\mu^2/M_W^2) = -6$, for an appropriate scale of $\mu = 4.2 \text{ GeV}$. The total correction to $C_{\pm} = 1$ in (19) is then -21% (42%)! The presence of the large logarithm spoils the validity of a straightforward perturbative expansion, despite the fact that the coupling constant itself is still reasonably small. This situation is quite common in renormalizable quantum field theories. Logarithms appear naturally and can become very large when the problem involves very different scales. The general situation is indicated in the following table, where we display the form of the correction terms in higher orders,

denoting $\ell \equiv \ln(\mu/M_W)$

$$\begin{array}{cccc}
& \text{LL} & \text{NLL} & \\
\alpha_s \ell & & \alpha_s & \\
\alpha_s^2 \ell^2 & \alpha_s^2 \ell & \alpha_s^2 & \\
\alpha_s^3 \ell^3 & \alpha_s^3 \ell^2 & \alpha_s^3 \ell & \alpha_s^3 \\
\downarrow & \downarrow & & \\
\mathcal{O}(1) & \mathcal{O}(\alpha_s) & &
\end{array} \tag{20}$$

In ordinary perturbation theory the expansion is organized according to powers of α_s alone, corresponding to the rows in the above scheme. This approach is invalidated by the large logarithms since $\alpha_s \ell$, in contrast to α_s , is no longer a small parameter, but a quantity of order 1. The problem can be resolved by resumming the terms $(\alpha_s \ell)^n$ to all orders n . The expansion is then reorganized in terms of columns of the above table. The first column is of $\mathcal{O}(1)$ and yields the leading logarithmic approximation, the second column gives a correction of relative order α_s , and so forth. Technically the reorganization is achieved by solving the renormalization group equation (RGE) for the Wilson coefficients. The RGE is a differential equation describing the change of $C_{\pm}(\mu)$ under a change of scale. To leading order this equation can be read off from (19)

$$\frac{d}{d \ln \mu} C_{\pm}(\mu) = \frac{\alpha_s}{4\pi} \gamma_{\pm}^{(0)} \cdot C_{\pm}(\mu) \tag{21}$$

$(\alpha_s/4\pi)\gamma_{\pm}^{(0)}$ are called the anomalous dimensions of C_{\pm} . To understand the term “dimension”, compare with the following relation for the quantity μ^n , which has (energy) dimension n :

$$\frac{d}{d \ln \mu} \mu^n = n \cdot \mu^n \tag{22}$$

The analogy is obvious. Of course, the $C_{\pm}(\mu)$ are dimensionless numbers in the usual sense; they can depend on the energy scale μ only because there is another scale, M_W , present under the logarithm in (19). Their “dimension” is therefore more precisely called a scaling dimension, measuring the rate of change of C_{\pm} with a changing scale μ . The nontrivial scaling dimension derives from $\mathcal{O}(\alpha_s)$ loop corrections and is thus a genuine quantum effect. Classically the coefficients are scale invariant, $C_{\pm} \equiv 1$. Whenever a symmetry that holds at the classical level is broken by quantum effects, we speak of an “anomaly”. Hence, the $\gamma_{\pm}^{(0)}$ represent the anomalous (scaling) dimensions of the Wilson coefficients.

We can solve (21), using

$$\frac{d\alpha_s}{d \ln \mu} = -2\beta_0 \frac{\alpha_s^2}{4\pi} \quad \beta_0 = \frac{33-2f}{3} \quad C_{\pm}(M_W) = 1 \tag{23}$$

and find

$$C_{\pm}(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{\frac{\gamma_{\pm}^{(0)}}{2\beta_0}} = \left[\frac{1}{1 + \beta_0 \frac{\alpha_s(\mu)}{4\pi} \ln \frac{M_W^2}{\mu^2}} \right]^{\frac{\gamma_{\pm}^{(0)}}{2\beta_0}} \tag{24}$$

This is the solution for the Wilson coefficients C_{\pm} in leading logarithmic approximation, that is to leading order in RG improved perturbation theory. The all-orders resummation of $\alpha_s \log$ terms is apparent in the final expression in (24).

3.3 $\Delta B = 1$ effective Hamiltonian

In this section we will complete the discussion of the $\Delta B = 1$ effective Hamiltonian. So far we have considered the operators

$$Q_1^p = (\bar{d}_i p_i)_{V-A} (\bar{p}_j b_j)_{V-A} \quad (25)$$

$$Q_2^p = (\bar{d}_i p_j)_{V-A} (\bar{p}_j b_i)_{V-A} \quad (26)$$

which come from the simple W -exchange graph and the corresponding QCD corrections (Fig. 8). We have slightly generalized our previous notation, allowing for the cases $p = u, c$. In addition, there is a further type of diagram at $\mathcal{O}(\alpha_s)$, which we have omitted until

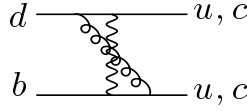


Figure 8. *QCD correction to W exchange.*

now: the QCD-penguin diagram shown in Fig. 9. It gives rise to the four new operators

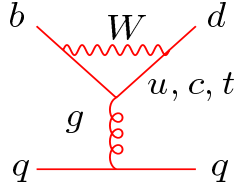


Figure 9. *QCD-penguin diagram.*

$$Q_3 = (\bar{d}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A} \quad (27)$$

$$Q_4 = (\bar{d}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A} \quad (28)$$

$$Q_5 = (\bar{d}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A} \quad (29)$$

$$Q_6 = (\bar{d}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A} \quad (30)$$

Two structures appear when the light-quark current $(\bar{q}q)_V$ from the bottom end of the diagram is split into $V - A$ and $V + A$ parts. In turn, each of those comes in two colour forms in a way similar to Q_1 and Q_2 . Finally, one further gauge-invariant operator of dimension six appears in the matching procedure, the chromomagnetic penguin operator

$$Q_{8g} = -\frac{g}{8\pi^2} m_b \bar{d}_i \sigma^{\mu\nu} (1 + \gamma_5) T_{ij}^a b_j G_{\mu\nu}^a \quad (31)$$

This operator corresponds to the diagrams in Fig. 9 with the lower quark line omitted. The gluon is thus an external field, represented in (31) by the field-strength tensor $G_{\mu\nu}^a$. Note that the characteristic tensor current necessitates a helicity flip in the $b \rightarrow d$ transition, which is accompanied by a factor of the quark mass m_b (the effect of m_d is

neglected). The contribution of Q_{8g} would be very small for the Hamiltonian of K decays, which only involves light external quarks, but it is unsuppressed for b decays.

The operators Q_1, \dots, Q_6, Q_{8g} mix under renormalization, that is the RGE for their Wilson coefficients is governed by a matrix of anomalous dimensions, generalizing (21). In this way the RG evolution of $C_{1,2}$ affects the evolution of C_3, \dots, C_6, C_{8g} . On the other hand $C_{1,2}$ remain unchanged in the presence of the penguin operators Q_3, \dots, Q_6, Q_{8g} , so that the results for $C_{1,2}$ derived above are still valid.

The construction of the effective Hamiltonian follows the principles we have discussed in the previous sections. First the Wilson coefficients $C_i(\mu_W)$, $i = 1, \dots, 6, 8g$, are determined at a large scale $\mu_W = \mathcal{O}(M_W, m_t)$ to a given order in perturbation theory. In this step both the W boson and the heavy top quark are integrated out. Since the renormalization scale is chosen to be $\mu_W = \mathcal{O}(M_W, m_t)$, no large logarithms appear and straightforward perturbation theory can be used for the matching calculation. The anomalous dimensions are computed from the divergent parts of the operator matrix elements, which correspond to the UV-renormalization of the Wilson coefficients. Solving the RGE the C_i are evolved from μ_W to a scale $\mu = \mathcal{O}(m_b)$ in a theory with $f = 5$ active flavours $q = u, d, s, c, b$. The terms taken into account in the RG improved perturbative evaluation of $C_i(\mu)$ are, schematically:

$$\text{LO: } \left(\alpha_s \ln \frac{M_W}{\mu}\right)^n, \quad \text{NLO: } \alpha_s \left(\alpha_s \ln \frac{M_W}{\mu}\right)^n,$$

at leading and next-to-leading order, respectively.

The final result for the $\Delta B = 1$ effective Hamiltonian can be written as

$$\mathcal{H}_{eff}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\dots,6,8g} C_i Q_i \right] + \text{h.c.} \quad (32)$$

where $\lambda_p \equiv V_{pd}^* V_{pb}$. In principle there are three different CKM factors, λ_u , λ_c and λ_t , corresponding to the different flavours of up-type quarks that can participate in the charged-current weak interaction. Using CKM unitarity, one of them can be eliminated. If we eliminate λ_t , we arrive at the CKM structure of (32).

The Hamiltonian in (32) is the basis for computing nonleptonic b -hadron decays within the standard model (to lowest order in electroweak interactions) with $\Delta B = 1$ and $\Delta S, \Delta C = 0$. The Hamiltonian for b -decays with different flavour quantum numbers of the light quarks has a completely analogous form. For instance $\Delta B = 1$ transitions with a simultaneous change in strangeness, $\Delta S = 1$, are simply described by (32) after the replacement $d \rightarrow s$. When new physics is present at some higher energy scale, the effective Hamiltonian can be derived in an analogous way. The matching calculation at the high scale μ_W will give new contributions to the coefficients $C_i(\mu_W)$, the initial conditions for the RG evolution. In general, new operators may also be induced. The Wilson coefficients C_i are known in the standard model at NLO. A more detailed account of $\mathcal{H}_{eff}^{\Delta B=1}$ and information on the technical aspects of the necessary calculations can be found in (Buchalla, Buras & Lautenbacher 1996) and (Buras 1998).

3.4 $B - \bar{B}$ mixing at NLO

In the following section we present the effective Hamiltonian for $\Delta B = 2$ transition, which is relevant for $B - \bar{B}$ mixing. In this case only a single operator contributes. The form of the Hamiltonian is therefore particularly simple. We use this example to illustrate the structure of Wilson coefficients at next-to-leading order. The mass difference Δm_B in the $B - \bar{B}$ system is related to the effective Hamiltonian $\mathcal{H}_{eff}^{\Delta B=2}$ by

$$\Delta m_B = 2|M_{12}|_B = \frac{1}{m_B} |\langle \bar{B} | \mathcal{H}_{eff}^{\Delta B=2} | B \rangle| \quad (33)$$

In order to construct $\mathcal{H}_{eff}^{\Delta B=2}$, the full standard model amplitude for $\Delta B = 2$ transitions is matched onto the effective theory amplitude at the matching scale $\mu_t = \mathcal{O}(m_t) = \mathcal{O}(M_W)$. This is sketched in Fig. 10.

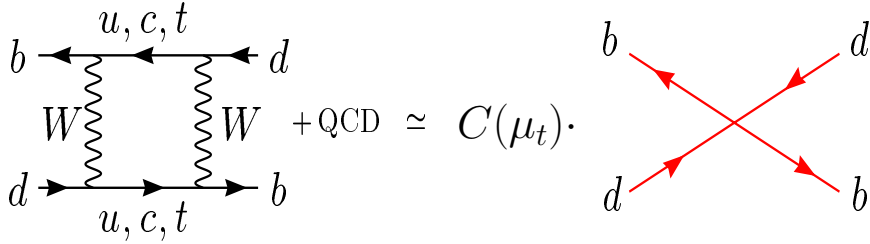


Figure 10. OPE for $B - \bar{B}$ mixing.

There is only one local operator

$$Q = (\bar{b}d)_{V-A}(\bar{b}d)_{V-A} \quad (34)$$

The Wilson coefficient, up to next-to-leading order, can be written as

$$C(\mu_t) = C^{(0)}(\mu_t) + \frac{\alpha_s}{4\pi} C^{(1)}(\mu_t) \quad (35)$$

where $C^{(0)}$ is the lowest order result and $C^{(1)}$ comes from the corrections with one-gluon exchange. The RG evolution from the high scale μ_t down to a scale $\mu = \mathcal{O}(m_b)$ has the form

$$C(\mu) = \left[1 + \frac{\alpha_s(\mu) - \alpha_s(\mu_t)}{4\pi} J_5 \right] \cdot \left[\frac{\alpha_s(\mu_t)}{\alpha_s(\mu)} \right]^{6/23} \cdot C(\mu_t) \quad (36)$$

The second factor on the right-hand side is familiar from the leading logarithmic approximation (the only difference is that at NLO the two-loop expression for $\alpha_s(\mu)$ has to be used). The first factor represents the next-to-leading order correction. Here J_5 is a scheme-dependent constant, which in the usual, so-called NDR scheme reads $J_5 = 5165/3174$.

We now have the ingredients to write the effective Hamiltonian up to NLO precision

$$\begin{aligned} \mathcal{H}_{eff}^{\Delta B=2} &= \frac{G_F^2 M_W^2}{16\pi^2} (V_{tb}^* V_{td})^2 \cdot C(\mu) Q \\ &= \frac{G_F^2 M_W^2}{16\pi^2} (V_{tb}^* V_{td})^2 S_0(x_t) \eta_B[\alpha_s(\mu)]^{\frac{-6}{23}} \left[1 + \frac{\alpha_s(\mu)}{4\pi} J_5 \right] Q \end{aligned} \quad (37)$$

The result is entirely dominated by the top-quark contribution. It is common practice to separate the coefficient $C(\mu)$ into the function $S_0(x_t)$ ($x_t = m_t^2/M_W^2$), which would be the coefficient in the absence of QCD effects, into the terms that depend on $\alpha_s(\mu)$, and the remainder, which is defined as the QCD-correction factor η_B . This has been done in the second equation in (37). Taking the matrix element of $\mathcal{H}_{eff}^{\Delta B=2}$ between the B and the \bar{B} state and using (33) gives

$$\Delta m_B = \frac{G_F^2 M_W^2}{6\pi^2} |V_{tb}^* V_{td}|^2 S_0(x_t) \eta_B B_B f_B^2 m_B \quad (38)$$

One encounters the hadronic matrix element of Q , which is written as

$$\langle \bar{B} | Q | B \rangle(\mu) \equiv \frac{8}{3} f_B^2 m_B^2 B_B(\mu) \quad (39)$$

defining the (scale and scheme dependent) hadronic parameter $B_B(\mu)$. The combination

$$B_B \equiv B_B(\mu) [\alpha_s(\mu)]^{\frac{-6}{23}} \left[1 + \frac{\alpha_s(\mu)}{4\pi} J_5 \right] \quad (40)$$

is formally scale and scheme independent and has been used in (38). The parameter $B_B(\mu)$ is a nonperturbative quantity and has to be determined e.g. by lattice calculations. At present the value of B_B is still very uncertain, in contrast to the short-distance QCD corrections, which are precisely known. A numerical illustration is given in Table 1, where we have put $B_B(\mu) = 0.9$ as an example.

Table 1.

0.846			0.9	= 0.761
η_B	$[\alpha_s(\mu)]^{\frac{-6}{23}}$	$\left[1 + \frac{\alpha_s(\mu)}{4\pi} J_5 \right]$	$B_B(\mu)$	
0.551	1.38			= 0.761

Two different definitions of a short-distance QCD factor can be considered, depending on where the terms with $\alpha_s(\mu)$ are included. One possibility is to include them with η_B into a Wilson coefficient (=0.846), which is to be multiplied by the hadronic matrix element $B_B(\mu) = 0.9$. The other possibility is the formally scheme independent separation into $\eta_B = 0.551$ and $B_B = 1.38$ (for η_B this is the precise result; $B_B = 1.38$ is only true in our example). The purpose of this exercise is to remind us that different definitions are sometimes employed for the parameter B_B and care has to be taken which one is being used, in order to combine it with the appropriate short-distance corrections. We can also see that the large deviation of the QCD correction factor η_B from 1 is merely a consequence of pulling out the large factor $[\alpha_s(\mu)]^{-6/23}$. It is somewhat artificial and does certainly not indicate a problem for perturbation theory. In fact, the coefficient 0.846 is the one that has the proper limit, approaching 1 as $\alpha_s \rightarrow 0$. It is indeed much closer to unity in accordance with the expectation for a perturbative correction factor. Still, the use of $\eta_B = 0.551$ is often adopted due to its formally scheme invariant definition.

An important application is the ratio of the mass differences for B_d and B_s mesons, for which (38) implies

$$\frac{\Delta m_{B_d}}{\Delta m_{B_s}} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{m_{B_d} f_{B_d}^2 B_{B_d}}{m_{B_s} f_{B_s}^2 B_{B_s}} \quad (41)$$

This quantity is a very useful measure of $|V_{td}/V_{ts}|$. All other short-distance physics (top-dependence, η_B) has dropped out. Hadronic uncertainties are reduced in the ratio of matrix elements, which is 1 in the limit of unbroken $SU(3)$ flavour symmetry. The cancellation of the short-distance contribution is a direct consequence of the factorization property of the OPE. Lattice calculations give for the ratio of matrix elements (Höcker et al. 2001, and refs. therein)

$$\frac{f_{B_s} \sqrt{B_{B_s}}}{f_{B_d} \sqrt{B_{B_d}}} = 1.16 \pm 0.06 \quad (42)$$

The ratio $\Delta m_{B_d}/\Delta m_{B_s}$ is a very powerful constraint for the unitarity triangle, as can be seen in Fig. 3.

4 Heavy quark effective theory

4.1 Basic formalism

Heavy quark effective theory is an effective field theory designed to systematically exploit the simplifications of QCD interactions in the heavy-quark limit for the case of hadrons containing a single heavy quark. The HQET Lagrangian can be derived as follows. We start from the usual QCD Lagrangian for a heavy-quark field Ψ with mass m

$$\mathcal{L} = \bar{\Psi} i \not{D} \Psi - m \bar{\Psi} \Psi \quad (43)$$

with the covariant derivative

$$D_\mu = \partial_\mu - ig T^a A_\mu^a \quad (44)$$

The heavy-quark momentum can be decomposed as

$$p = mv + k \quad (45)$$

where v is the 4-velocity of the heavy *hadron*. Once mv , the large kinematical part of the momentum is singled out, the remaining component k is determined by soft QCD bound state interactions, and thus $k = \mathcal{O}(\Lambda_{QCD}) \ll m$. We next decompose the quark field Ψ into

$$h_v(x) \equiv e^{imv \cdot x} \frac{1 + \not{v}}{2} \Psi(x) \quad (46)$$

$$H_v(x) \equiv e^{imv \cdot x} \frac{1 - \not{v}}{2} \Psi(x) \quad (47)$$

which implies

$$\Psi(x) = e^{-imv \cdot x} (h_v(x) + H_v(x)) \quad (48)$$

The expressions $(1 \pm \not{v})/2$ are projection operators. Their action represents the covariant generalization of decomposing Ψ into upper and lower components. Using the standard

representation for γ -matrices, this is evident in the rest frame where $\not{v} = \gamma^0$. Note also that the equation of motion with respect to the large momentum components, $m(\not{v} - 1)h_v = 0$, is manifest for h_v .

The exponential factor $\exp(imv \cdot x)$ in (46), (47) removes the large-frequency part of the x -dependence in $\Psi(x)$ resulting from the large momentum mv . Consequently, the x -dependence of h_v , H_v is only governed by the small residual momentum and derivatives acting on h_v and H_v count as $\mathcal{O}(\Lambda_{QCD})$. (Our sign conventions are appropriate for a heavy *quark*. To describe the case of a heavy *anti-quark*, similar definitions are valid with the sign of v reversed.)

Multiplying the QCD e.o.m. $(i \not{D} - m)\Psi = 0$ with the projectors $(1 - \not{v})/2$ and $(1 + \not{v})/2$, and using (46) – (48) and the definition

$$D_\perp^\mu \equiv D^\mu - v^\mu v \cdot D \quad (49)$$

we obtain the coupled system of equations

$$iv \cdot D h_v = -i \not{D}_\perp H_v \quad (50)$$

$$(iv \cdot D + 2m)H_v = i \not{D}_\perp h_v \quad (51)$$

They represent the e.o.m. in terms of h_v and H_v . The second equation implies that $H_v = \mathcal{O}(\Lambda_{QCD}/m)h_v$ by power counting. Hence H_v is suppressed with respect to h_v in the heavy-quark limit. In other words, h_v contains the large components, H_v the small components of Ψ .

The HQET Lagrangian is obtained starting from (43), expressing Ψ in terms of h_v , H_v and eliminating H_v using (51). We find

$$\mathcal{L} = \bar{h}_v iv \cdot D h_v + \bar{h}_v i \not{D}_\perp \frac{1}{iv \cdot D + 2m} i \not{D}_\perp h_v \quad (52)$$

Alternatively, H_v as obtained from (51) in terms of h_v can be inserted into (50) to yield the e.o.m. for h_v . This equation is just the e.o.m. implied by (52) (upon variation with respect to \bar{h}_v , i.e. $\delta\mathcal{L}/\delta\bar{h}_v = 0$). The Lagrangian may thus be written down immediately given the e.o.m. for the field h_v .

The second term in (52) contains the nonlocal operator $(iv \cdot D + 2m)^{-1}$. It can be expanded in powers of Λ_{QCD}/m to yield a series of local operators. Keeping only the leading-power correction we can simply replace $(iv \cdot D + 2m)^{-1}$ by $(2m)^{-1}$ and get

$$\mathcal{L} = \bar{h}_v iv \cdot D h_v + \frac{1}{2m} \bar{h}_v (iD_\perp)^2 h_v + \frac{g}{4m} \bar{h}_v \sigma^{\mu\nu} G_{\mu\nu} h_v \quad (53)$$

Let us discuss some important aspects of this result.

- The first term on the r.h.s. of (53) is the basic, lowest-order Lagrangian of HQET. It describes the “residual” QCD dynamics of the heavy quark once the kinematic dependence on m is separated out. Since there is no longer any reference to the mass m , the only parameter to distinguish quark flavours, this term is flavour symmetric: The dynamics is the same for b and c quarks in the static limit. Since the operator $v \cdot D$ contains no γ -matrices, which would act on the spin degrees of freedom, the leading HQET Lagrangian also exhibits a spin symmetry. This corresponds to the

decoupling of the heavy-quark spin in the $m \rightarrow \infty$ limit. Together, we have the famous spin – flavour symmetries of HQET (Isgur & Wise 1989). They lead to relations among different heavy-hadron form factors.

- From the Lagrangian $\bar{h}_v i v \cdot D h_v$ the Feynman rules for HQET can be read off. The propagator is

$$\frac{i}{v \cdot k} \frac{1 + \not{v}}{2} \quad (54)$$

where the projector $(1 + \not{v})/2$ appears since h_v is a constrained spinor (see (46)). The interaction of the heavy-quark field h_v with gluons is given by the vertex

$$ig v^\mu T^a \quad (55)$$

These Feynman rules enter in the computation of QCD quantum corrections.

- The remaining terms in (53) are the leading power corrections. They have an intuitive interpretation. In the first term one recognizes the operator for the nonrelativistic kinetic energy $\vec{p}^2/(2m)$, which describes the residual motion of the heavy quark recoiling against the light degrees of freedom inside the heavy hadron. The last term represents the chromomagnetic interaction of the heavy-quark spin with the gluon cloud. Both effects violate flavour symmetry, the chromomagnetic term also spin symmetry, but they are power suppressed.
- So far we have only considered QCD interactions. Weak interactions introduce external currents, which can also be incorporated in HQET. A generic heavy-light transition current $\bar{q} \Gamma \Psi$, arising for instance in semileptonic decays, can be represented as

$$\bar{q} \Gamma \Psi = \bar{q} \Gamma h_v + \mathcal{O}\left(\frac{1}{m}\right) \quad (56)$$

replacing the heavy-quark field Ψ by the HQET field h_v using (48).

4.2 Theory of heavy-hadron masses

Before considering HQET in the context of weak decays, let us discuss a first application of the basic HQET Lagrangian (53) in the spectroscopy of heavy hadrons. To be specific, we shall analyze the masses of the ground-state mesons B and B^* . These mesons constitute a doublet that arises because the spin 1/2 of the heavy quark couples with the *total spin* 1/2 of the light degrees of freedom in their ground state to form a spin-0 and a spin-1 meson, the pseudoscalar B and the vector B^* , respectively. Because the b -quark spin decouples in the heavy-quark limit, the state of the light cloud is identical for B and B^* to leading order, and the angular-momentum coupling described above is the appropriate scheme. If we neglect the power corrections in (53), we can immediately write down the composition of the meson masses

$$m_B^{(0)} = m_{B^*}^{(0)} = m_b + \bar{\Lambda} \quad (57)$$

Evidently the meson mass has a component m_b from the heavy quark. In addition it has a term $\bar{\Lambda} = \mathcal{O}(\Lambda_{QCD})$ from the energy of the light constituents. The latter is determined only by the interactions among the light degrees of freedom and their interaction with

the static b -quark (h_v) through the first term in (53). It is therefore independent of m_b . The sum of m_b and $\bar{\Lambda}$ is a physical quantity, however, separately both parameters are dependent on the scheme used to define them.

In order to include the first power corrections, we treat the $1/m$ terms in (53) as perturbations to the lowest-order HQET dynamics. To first order in perturbation theory the corrections to (57) are then simply given by the expectation values of the $1/m$ terms. The proper normalization is obtained as follows. If $\mathcal{H} = -\mathcal{L}_{1/m}$ is the Hamiltonian (density) corresponding to the correction term $\mathcal{L}_{1/m}$ in (53), and $H = \int d^3x \mathcal{H}$ is the Hamilton operator, the mass correction due to \mathcal{H} is just

$$\delta m_B = \langle B_1 | H | B_1 \rangle \quad (58)$$

where $|B_1\rangle$ is the B -meson state normalized to one, $\langle B_1 | B_1 \rangle = 1$. Using the conventionally normalized states with $\langle B | B \rangle = 2m_B V$, we can write

$$\delta m_B = \frac{1}{2m_B V} \langle B | \int d^3x \mathcal{H}(\vec{x}) | B \rangle = \frac{1}{2m_B V} \int d^3x \langle B | \mathcal{H}(0) | B \rangle = \frac{\langle B | \mathcal{H}(0) | B \rangle}{2m_B} \quad (59)$$

where we have used the translation invariance of \mathcal{H} and $\int d^3x = V$. Defining

$$\lambda_1 \equiv \frac{\langle B | \bar{h}(iD)^2 h | B \rangle}{2m_B} \quad \lambda_2 \equiv \frac{1}{6} \frac{\langle B | \bar{h} g \sigma \cdot G h | B \rangle}{2m_B} \quad (60)$$

we obtain

$$\delta m_B = -\frac{\lambda_1 + 3\lambda_2}{2m_b} \quad (61)$$

Note that we may replace D^2 by D_\perp^2 in the definition of λ_1 , up to higher order corrections (see (49), (50)). The parameter λ_1 corresponds to (minus) the expectation value of the momentum squared of the heavy quark, $\lambda_1 = -\langle \vec{p}_h^2 \rangle = \mathcal{O}(\Lambda_{QCD}^2)$. This gives a positive correction in (61) representing the (small) kinetic energy of the heavy-quark. The λ_2 -correction to the mass reflects the interaction energy of the heavy-quark spin with its hadronic environment, as already discussed in the previous section. While the λ_1 -term is independent of the heavy-quark spin and identical for B and B^* , the chromomagnetic correction $\sim \lambda_2 = \mathcal{O}(\Lambda_{QCD}^2)$ is different for B^* . We have

$$\delta m_{B^*} = -\frac{\lambda_1 - \lambda_2}{2m_b} \quad (62)$$

Including (57) we arrive at the following expansion for the meson masses

$$m_B = m_b + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_b} \quad (63)$$

$$m_{B^*} = m_b + \bar{\Lambda} - \frac{\lambda_1 - \lambda_2}{2m_b} \quad (64)$$

where the dependence on m_b is explicit order by order.

If we apply the heavy-quark limit to D mesons, we obtain analogous relations

$$m_D = m_c + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_c} \quad (65)$$

$$m_{D^*} = m_c + \bar{\Lambda} - \frac{\lambda_1 - \lambda_2}{2m_c} \quad (66)$$

with the same $\bar{\Lambda}$, λ_1 and λ_2 as before.

These results have a few interesting consequences. First, λ_2 parametrizes the spin-splitting between the pseudoscalar and the vector mesons:

$$m_{B^*} - m_B = \frac{2\lambda_2}{m_b} = 46 \text{ MeV} \quad (67)$$

$$m_{D^*} - m_D = \frac{2\lambda_2}{m_c} = 141 \text{ MeV} \quad (68)$$

HQET predicts that the spin-splitting scales inversely proportional to the heavy-quark mass. This is seen to be quite well fulfilled given that $m_b \approx 3m_c$. Relation (67) can be used to determine the nonperturbative quantity λ_2 from experiment

$$\lambda_2 = \frac{1}{4}(m_{B^*}^2 - m_B^2) = 0.12 \text{ GeV}^2 \quad (69)$$

On the other hand, the quantity λ_1 has to be estimated theoretically. Finally one may introduce the spin-averaged masses

$$\bar{m}_B \equiv \frac{m_B + 3m_{B^*}}{4} = m_b + \bar{\Lambda} - \frac{\lambda_1}{2m_b} \quad (70)$$

$$\bar{m}_D \equiv \frac{m_D + 3m_{D^*}}{4} = m_c + \bar{\Lambda} - \frac{\lambda_1}{2m_c} \quad (71)$$

This eliminates λ_2 and yields the useful result

$$m_b - m_c = (\bar{m}_B - \bar{m}_D) \left(1 - \frac{\lambda_1}{2\bar{m}_B \bar{m}_D} \right) \quad (72)$$

Since the λ_1 -correction is fairly small, the quark-mass difference is rather well determined, much better than individual quark masses.

Exercise

Derive the relative factor between the chromomagnetic correction to the mass of the B and the B^ meson.*

Solution: Denote the heavy-quark spin by \vec{s} , the total spin of the light degrees of freedom by \vec{j} and the total spin of the meson by $\vec{J} = \vec{s} + \vec{j}$. The chromomagnetic field of the light cloud has to be proportional to \vec{j} . Hence the energy of the interaction between this field and \vec{s} is proportional to $\langle \vec{s} \cdot \vec{j} \rangle$. Angular momentum algebra gives $\langle 2\vec{s} \cdot \vec{j} \rangle = J(J+1) - s(s+1) - j(j+1)$, which is $(-3/2)$ for B and $(1/2)$ for B^* , hence the relative factor $(-1/3)$ of the λ_2 -term in (62) with respect to (61).

4.3 Heavy-light currents and f_B

The B -meson decay constant f_B is defined by the matrix element

$$\langle 0 | A_\mu | B(p) \rangle = -if_B m_B v_\mu \quad (73)$$

of the heavy-light axial vector current

$$A_\mu \equiv \bar{q}\gamma_\mu\gamma_5\Psi \quad (74)$$

Here q is the light-quark, Ψ the heavy-quark field in full QCD, with $\Psi = b$ in the present case. The B -meson momentum is $p = m_B v$.

Let us analyze A_μ in HQET, including QCD corrections. The expansion of A_μ in HQET to leading order in $1/m$, but allowing for QCD effects, has the form

$$A = C_1(\mu)\tilde{A}_1 + C_2(\mu)\tilde{A}_2 + \mathcal{O}\left(\frac{1}{m}\right) \quad (75)$$

$$\tilde{A}_1 = \bar{q}\gamma_\mu\gamma_5 h_v \quad \tilde{A}_2 = \bar{q}v_\mu\gamma_5 h_v \quad (76)$$

The matching conditions at the b -quark mass scale $\mu = m_b$ are

$$C_1(m_b) = 1 + \mathcal{O}(\alpha_s) \quad C_2(m_b) = \mathcal{O}(\alpha_s) \quad (77)$$

To leading order in QCD only \tilde{A}_1 is present in HQET, with coefficient one. Radiative corrections at $\mathcal{O}(\alpha_s)$ modify C_1 and generate a new operator \tilde{A}_2 . Note that the matching calculation of the full-QCD current A onto HQET, leading to (75), is completely analogous to the OPE procedure of constructing the effective weak Hamiltonian from the W -exchange amplitude in the full standard model, which we have discussed in sec. 3. The difference is only that a $1/M_W$ expansion is performed in the latter case, and a $1/m_b$ expansion in the case of HQET. The basic philosophy is essentially the same. In particular, a factorization of long and short-distance contributions is obtained: Contributions from large scales $> \mu$, including the m_b -dependence, is again contained in the coefficient functions $C_{1,2}$. Soft scales $< \mu$ are factorized into the hadronic matrix elements of $\tilde{A}_{1,2}$.

In contrast to the full-QCD current A , the HQET currents do have an anomalous dimension, reflecting a logarithmic dependence of f_B on the heavy-quark mass at $\mathcal{O}(\alpha_s)$. The logarithms can be resummed by renormalization group methods, again in full analogy to the procedure in sec. 3. In leading logarithmic approximation (LLA) C_2 can be neglected and C_1 acquires the familiar form

$$C_1(\mu) = \left[\frac{\alpha_s(m_b)}{\alpha_s(\mu)} \right]^{-2/\beta_0} \quad (78)$$

Here the LLA assumes the hierarchy $\alpha_s(m_b) \ll 1$, $\alpha_s \ln(m_b/\mu) = \mathcal{O}(1)$, which holds in the heavy-quark limit (m_b large, $\mu = \mathcal{O}(1 \text{ GeV})$).

To express f_B in HQET via (73), (75) and (78), we need the matrix element of \tilde{A}_1

$$\langle 0 | \tilde{A}_1 | B(p) \rangle = -i\tilde{f}(\mu)\sqrt{m_B}v_\mu \quad (79)$$

Since the dynamics of HQET is independent of m_b , the reduced decay constant $\tilde{f}(\mu)$ is m_b -independent. The only m_b -dependence in (79) enters through a trivial factor $\sqrt{m_B}$ from the normalization of the B -meson state, which in the usual convention is given by

$$\langle B | B \rangle = 2m_B V \quad (80)$$

Collecting the ingredients, (75) yields

$$f_B = \frac{\tilde{f}(\mu)}{\sqrt{m_B}} \left[\frac{\alpha_s(m_b)}{\alpha_s(\mu)} \right]^{-2/\beta_0} \quad (81)$$

This expression for f_B is true to leading order in the HQET expansion in Λ_{QCD}/m_b and in leading logarithmic approximation in QCD. $\tilde{f}(\mu)$ in (81) is still a nonperturbative quantity to be determined by other methods. However, the dependence of f_B on the heavy-quark mass is now explicit. Eq. (81) implies the scaling behaviour $f_B \sim 1/\sqrt{m_B}$, up to a calculable logarithmic dependence on m_b . In principle such a relation can be used to relate f_B to the analogous quantity f_D for heavy mesons with charm. In practice, it turns out that the leading order scaling result for f_B is not very well fulfilled even for the b -mass scale and that subleading power corrections are important in this case. Nevertheless the result in (81) is of conceptual interest and can serve as a simple example of an application of HQET.

4.4 Heavy-heavy currents: $\bar{B} \rightarrow D^{(*)} l \bar{\nu}$ and V_{cb}

One of the most important results of HQET is the extraction of V_{cb} from exclusive semileptonic $\bar{B} \rightarrow D^* l \bar{\nu}$ decay. We will here give a short outline of the main steps in this analysis. Starting point is the differential decay rate

$$\frac{d\Gamma(\bar{B} \rightarrow D^* l \bar{\nu})}{dw} = |V_{cb}|^2 \mathcal{K}(w) \mathcal{F}^2(w) \quad (82)$$

in the kinematical variable $w = v \cdot v'$, where v and v' are the 4-velocities of \bar{B} and D^* , respectively. The dependence of (82) on $|V_{cb}|$, the quantity of interest, is obvious, and $\mathcal{K}(w)$ is a known kinematical function. Finally, $\mathcal{F}(w)$ contains the nontrivial QCD dynamics encoded in the $\bar{B} \rightarrow D^*$ transition form factors. The corresponding matrix elements of the weak currents can be written in the heavy-quark limit as

$$\frac{1}{\sqrt{m_{D^*} m_B}} \langle D^*(v', \epsilon) | \bar{c} \gamma_\mu b | \bar{B}(v) \rangle = i \xi(w) \epsilon(\mu, \epsilon, v', v) \quad (83)$$

$$\frac{1}{\sqrt{m_{D^*} m_B}} \langle D^*(v', \epsilon) | \bar{c} \gamma_\mu \gamma_5 b | \bar{B}(v) \rangle = \xi(w) \left[(1+w) \epsilon_\mu - (\epsilon \cdot v) v'_\mu \right] \quad (84)$$

In the heavy-quark limit, that is to lowest order in HQET, all hadronic dynamics is expressed in a single function $\xi(w)$, the Isgur–Wise function (Isgur & Wise 1989). In this limit we further have

$$\mathcal{F}(w) = \xi(w) \quad (85)$$

Moreover, ξ is absolutely normalized at the no-recoil point

$$\xi(1) = 1 \quad (86)$$

The no-recoil point $w = 1$ corresponds to the kinematical situation where the D^* meson stays at rest in the rest frame of the decaying \bar{B} ($v' = v \Rightarrow w = 1$). Measuring $d\Gamma/dw$ at $w = 1$, $|V_{cb}|$ can then be determined from (82) since all ingredients are known. Because $w = 1$ is at the edge of phase space, an extrapolation is necessary to find $d\Gamma/dw|_{w=1}$ from the measured spectrum.

For a realistic analysis corrections to the heavy-quark limit need to be considered. An important property of $\bar{B} \rightarrow D^* l \bar{\nu}$ is that linear power corrections in HQET are absent, $\delta_{1/m} = 0$, where m can be either m_c or m_b . Consequently the leading corrections enter only at second order and are thus greatly reduced. This result is known as Luke's theorem. The absence of linear corrections does not hold for $\bar{B} \rightarrow D l \bar{\nu}$ decays, hence the particular importance of $\bar{B} \rightarrow D^* l \bar{\nu}$. Including corrections, the lowest order approximation $\mathcal{F}(1) = \xi(1) = 1$ is modified to

$$\mathcal{F}(1) = \eta_A(1 + \delta_{1/m^2}) \quad (87)$$

where δ_{1/m^2} are the second order power corrections and η_A is a correction from perturbative QCD. To first order in α_s it reads

$$\eta_A = 1 + \frac{\alpha_s}{\pi} \left(\frac{m_b + m_c}{m_b - m_c} \ln \frac{m_b}{m_c} - \frac{8}{3} \right) \quad (88)$$

A detailed numerical analysis yields (Harrison & Quinn 1998)

$$\mathcal{F}(1) = 0.913 \pm 0.042 \quad (89)$$

which gives (Höcker et al. 2001)

$$V_{cb} = 0.0409 \pm 0.0014_{exp} \pm 0.0019_{th} \quad (90)$$

To summarize the crucial points for the extraction of V_{cb} from $\bar{B} \rightarrow D^* l \bar{\nu}$ decay:

- Heavy-quark symmetry relates the various semileptonic form factors (four different functions V , A_0 , A_1 , A_2 in full QCD) to a single quantity $\xi(w)$, the Isgur–Wise function.
- The function ξ is absolutely normalized, $\xi(1) = 1$. This property has an intuitive reason: At the kinematical point $w = 1$ the decaying b -quark at rest is transformed into a c -quark, also at rest. Since both quarks are treated in the static approximation ($m_b, m_c \rightarrow \infty$, m_b/m_c fixed), the light hadronic cloud doesn't notice the flavour change $b \rightarrow c$ and is transferred from the \bar{B} to a D meson with probability one. The function ξ is identical for $\bar{B} \rightarrow D$ and $\bar{B} \rightarrow D^*$ transitions, because these are related by heavy-quark spin symmetry.
- HQET provides a framework for systematic corrections to the strict heavy-quark limit governed by $\xi(w)$. Luke's theorem guarantees the absence of first-order corrections in $1/m$ for $\bar{B} \rightarrow D^* l \bar{\nu}$.

4.5 HQET – conclusions

We would finally like to summarize the basic ideas and virtues of HQET, and to re-emphasize the salient points.

- HQET describes the static approximation for a heavy quark, covariantly formulated as an effective field theory and allowing for a systematic inclusion of power corrections.

- Order by order in the expansion in Λ_{QCD}/m HQET achieves a *factorization* of hard, perturbative contributions (momentum scales between m and a factorization scale μ) and soft, nonperturbative contributions (scales below μ). The former are contained in Wilson coefficients, the latter in the matrix elements of HQET operators.
- The procedure of matching full QCD onto HQET is analogous to the construction of the effective weak Hamiltonian \mathcal{H}_{eff} . The difference lies in the massive degrees of freedom that are being integrated out: the W boson (mass M_W) for \mathcal{H}_{eff} , the lower-component spinor field H_v (mass $2m$) for HQET. The perturbative matching can be supplemented by RG resummation of logarithms, $\ln(M_W/\mu)$ in the former case, $\ln(m/\mu)$ in the latter.
- The usefulness of HQET is based on two important features: The spin-flavour symmetry of HQET relates form factors in the heavy-quark limit and thus reduces the number of unknown hadronic quantities. The dependence on the heavy-quark masses is made explicit (scaling, power corrections).

We conclude with briefly mentioning another field, called large energy effective theory (LEET), which has some similarities with HQET. LEET is needed for matrix elements of the form $\langle M|\bar{q}\Gamma b|\bar{B}\rangle$ at *large recoil* of the light meson $M = \pi, \rho, K^{(*)}, \dots$. HQET is not sufficient in this situation because not only soft but also collinear infrared singularities need to be factorized. The latter occur due to the light-like kinematics of the fast and energetic light quark emitted from the weak current. To define LEET the usual heavy-quark limit can be considered for the B meson with velocity v . The large-energy limit is taken for the light meson M with light-like momentum vector En . Here $E = \mathcal{O}(m_b)$ is the energy of M and n is a light-like 4-vector with $n^2 = 0$ and $v \cdot n = 1$. The momentum of the energetic light quark q is written as $p_q = En + k$, with a residual momentum $k = \mathcal{O}(\Lambda_{QCD})$. In formal analogy to the fields h_v and H_v in HQET, the new light-quark fields

$$q_n(x) = e^{iEn \cdot x} \frac{\not{n}}{2} q(x) \quad Q_n(x) = e^{iEn \cdot x} \frac{\not{n}}{2} q(x) \quad (91)$$

can be defined and used in the construction of LEET (Dugan & Grinstein 1991, Charles et al. 1999, Beneke & Feldmann 2001, Bauer et al. 2001a). As a consequence of the LEET limit the ten form factors needed to describe all matrix elements $\langle M|\bar{q}\Gamma b|\bar{B}\rangle$ of bilinear heavy-light currents can be reduced to only three independent functions. LEET has received increasing interest quite recently and is still under active development.

5 Inclusive decays and the heavy quark expansion

5.1 Basic formalism and theory of lifetimes

The heavy-quark limit, $m \gg \Lambda_{QCD}$, proves to be extremely useful also for the computation of *inclusive* decay rates of heavy hadrons (Chay et al. 1990, Bigi et al. 1992, 1997). The specific technique appropriate for this application is distinct from HQET and goes by the name of heavy quark expansion (HQE). Consider the total decay rate Γ_H of a heavy hadron H . Starting point for the HQE is the following representation of Γ_H

$$\Gamma_H = \frac{1}{2m_H} \langle H|\mathcal{T}|H\rangle \equiv \langle \mathcal{T} \rangle \quad (92)$$

where the transition operator \mathcal{T} is defined as

$$\mathcal{T} = \text{Im } i \int d^4x T \mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0) \quad (93)$$

with \mathcal{H}_{eff} the effective weak Hamiltonian. Eqs. (92), (93) express the total decay rate as the absorptive part of the forward scattering amplitude $H \rightarrow H$ under the action of \mathcal{H}_{eff} . This expression is referred to as the optical theorem by analogy to a similar relation in optics. One may rewrite (92), (93) in a more directly understandable form by inserting a complete set of states $|X\rangle\langle X|$ between the two factors of \mathcal{H}_{eff} in (93) and removing the T -product by explicitly taking the absorptive part. This yields

$$\Gamma_H \sim \langle H | \mathcal{H}_{eff} | X \rangle \langle X | \mathcal{H}_{eff} | H \rangle \quad (94)$$

where one immediately recognizes the decay rate as the modulus squared of the decay amplitude (summed over all final states X). The reason to introduce (93) is that the T -product, by means of Wick's theorem, allows for a direct evaluation in terms of Feynman diagrams.

In order to compute Γ_H an operator product expansion is applied to (93), resulting in a series of local operators of increasing dimension. The coefficients of these operators are correspondingly suppressed by increasing powers of $1/m_b$. The series has the form

$$\mathcal{T} = \Gamma_b \bar{b}b + \frac{z_G}{m_b^2} \bar{b}g\sigma \cdot Gb + \sum \frac{z_{qi}}{m_b^3} \bar{b}\Gamma_i q \bar{q}\Gamma_i b + \dots \quad (95)$$

where we have written the first few operators of dimension three ($\bar{b}b$), five ($\bar{b}g\sigma \cdot Gb$) and six ($\bar{b}\Gamma_i q \bar{q}\Gamma_i b$). The matrix elements of the operators contain the soft, nonperturbative physics, their Wilson coefficients Γ_b , z_k the hard contributions, which are calculable in perturbation theory. Again, the coefficients are determined by an appropriate matching calculation between (93) and the r.h.s. of (95). The Feynman diagrams for the three terms in (95) are shown in Fig. 11. The two weak-interaction vertices in these diagrams

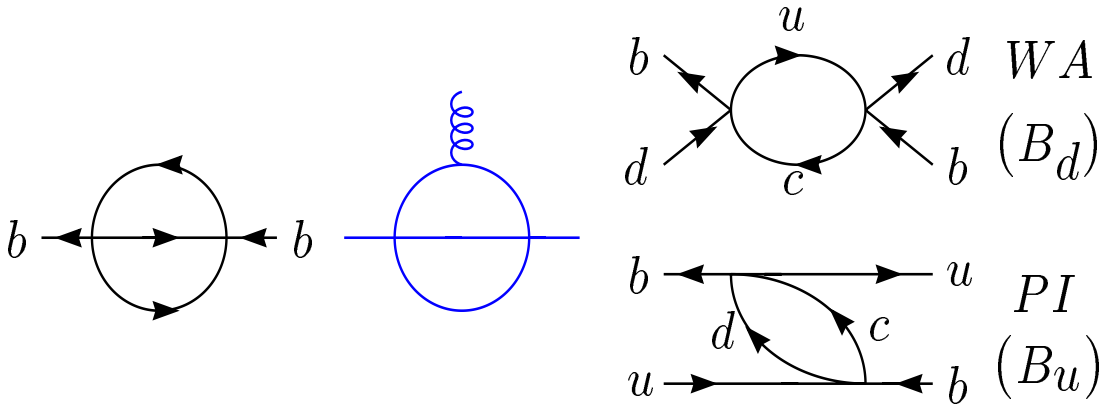


Figure 11. Heavy quark expansion for the total decay rate of b -hadrons.

correspond to the two factors of \mathcal{H}_{eff} in the definition of \mathcal{T} in (93) (the absorptive part of the diagrams is understood).

Obviously, the heavy quark expansion is different from HQET. However, we may still use HQET in conjunction with (95) in order to further analyse the hadronic matrix elements. An important example is the leading dimension-three operator $\bar{b}b$. Its matrix element between heavy-hadron states H can be expanded in HQET as

$$\langle \bar{b}b \rangle = 1 + \frac{1}{2m_b^2} \langle \bar{h}(iD)^2 h \rangle + \frac{1}{4m_b^2} \langle \bar{h}g\sigma Gh \rangle \quad (96)$$

where $\langle \dots \rangle \equiv \langle H | \dots | H \rangle / (2m_H)$.

- Eqs. (92), (95) and (96) imply that to leading order in the HQE $\Gamma_H = \Gamma_b$, that is the total decay rate of all b -flavoured hadrons is equal to the rate of free b -quark decay. Pictorially this can be seen from the first diagram in Fig. 11, which represents essentially the amplitude squared for the partonic decay of a b -quark. Note also that perturbative QCD corrections to Γ_b can consistently be taken into account. The gluonic corrections to inclusive b -quark decay are infrared safe, as required for Γ_b in its role as a Wilson coefficient of the HQE. Also, corrections proportional to powers of $\alpha_s(m_b) \sim 1/\ln(m_b/\Lambda)$ are only suppressed by inverse powers of $\ln m_b$ in the heavy-quark limit, and hence formally leading in comparison to higher corrections in the HQE, which are suppressed by powers of Λ/m_b . The calculation of heavy-quark decay in the parton picture has been used since the beginnings of heavy-quark physics as an approximation for inclusive decays of the corresponding heavy hadrons. As we have seen, the HQE gives a formal justification for this approach and provides us with a theoretical framework to compute nonperturbative corrections.
- The first correction term in (96) depends on the expectation value of the momentum squared $\langle \vec{p}^2 \rangle$ of the heavy quark inside the hadron. This matrix element is non-zero because the heavy quark is recoiling against the light degrees of freedom through gluonic interactions in the hadronic bound state. This term has a very intuitive interpretation. It corresponds to a correction factor $1 - \langle \vec{p}^2 \rangle / (2m_b^2) = 1 - \langle \vec{v}_b^2 \rangle / 2$, which is just the reduction of the free decay rate from time dilatation due to the recoil motion of the heavy quark. The second correction comes from interactions of the light hadronic cloud with the heavy-quark spin. We have

$$\langle \bar{h}g\sigma Gh \rangle = \begin{cases} \frac{3}{2}(m_{B^*}^2 - m_B^2) & H = B \\ 0 & H = \Lambda_b \end{cases} \quad (97)$$

The result is zero for the Λ_b baryon since the light degrees of freedom are in a state of zero total angular momentum. Note that the spin interaction enters twice in (95), explicitly with coefficient z_G and via the expansion of $\langle \bar{b}b \rangle$.

- The leading nonperturbative corrections start only at second order. There is no correction linear in $1/m_b$. This is because there is no gauge-invariant operator of dimension four that could appear in the HQE.
- At order $1/m_b^3$ contributions appear where the spectator quark participates directly in the weak interactions. For b -mesons they can be interpreted as the effect of weak annihilation of the b -quark with the valence \bar{d} -quark (for \bar{B}_d) and as the effect of Pauli interference (for \bar{B}_u). The latter phenomenon occurs because in the nonleptonic decay of a \bar{B}_u , $b(\bar{u}) \rightarrow c\bar{u}d(\bar{u})$, two identical \bar{u} -quarks are present in the final

state. These corrections distinguish in particular among B_d and B_u mesons and are responsible for their lifetime difference. Despite the suppression by three powers of m_b these effects can be relatively important due to their two-body kinematics, which brings a phase-space enhancement factor of $16\pi^2$ in comparison to the leading three-body decay.

As one of the possible applications, the HQE provides us with a theory of heavy-hadron lifetimes. The deviations of lifetime ratios from unity probes the power corrections. At present there are still sizeable theoretical uncertainties due to the hadronic matrix elements $\langle \bar{b} \Gamma q \bar{q} \Gamma b \rangle$. They can in principle be computed with the help of lattice gauge theory. Table 2 shows a comparison of theoretical predictions and experimental results (see for instance (Ligeti 2001)).

Table 2.

	exp.	th.
$\tau(B^+)/\tau(B_d^0)$	1.068 ± 0.016	$1 - 1.1$
$\bar{\tau}(B_s)/\tau(B_d)$	0.947 ± 0.038	$0.99 - 1.01$
$\tau(\Lambda_b)/\tau(B_d)$	0.795 ± 0.053	$0.9 - 1.0$

5.2 Local quark-hadron duality

A systematic uncertainty within the HQE framework, which is often debated in the literature, arises from the issue of quark-hadron duality. In this paragraph we give a brief and heuristic discussion of the basic idea behind this topic.

The theoretical prediction for an inclusive decay rate obtained from the HQE has the form

$$\Gamma/\Gamma_0 = 1 + \sum_{n=2}^{\infty} z_n \left(\frac{\Lambda}{m_b} \right)^n \quad (98)$$

where we have denoted the leading, free-quark result by Γ_0 . Let us consider the decay rate as a function of m_b , keeping $\Lambda = \Lambda_{QCD}$ constant. Then the quantity Γ/Γ_0 , to any finite order in (Λ/m) , is a simple polynomial expression in this variable. This is sketched as the monotonous curve in Fig. 12 showing Γ/Γ_0 as function of m_b (in arbitrary units). Now, since by construction the HQE for Γ/Γ_0 yields a power expansion in (Λ/m) , any term of the form

$$\exp \left(- \left(\frac{m_b}{\Lambda} \right)^k \right) \sin \left(\frac{m_b}{\Lambda} \right)^k \quad (99)$$

for example, present in the true result for Γ/Γ_0 would be missed by the HQE. This is due to the exponential suppression in the expansion parameter. In fact, the function $\exp(-1/x)$ is non-analytic. Its power expansion around $x = 0$ gives identically zero. However, such (or similar) terms are expected to be part of the true Γ/Γ_0 on general grounds. The corresponding complete result for Γ/Γ_0 , including such a term, is sketched as the oscillating graph in Fig. 12. This true curve represents the physical result for

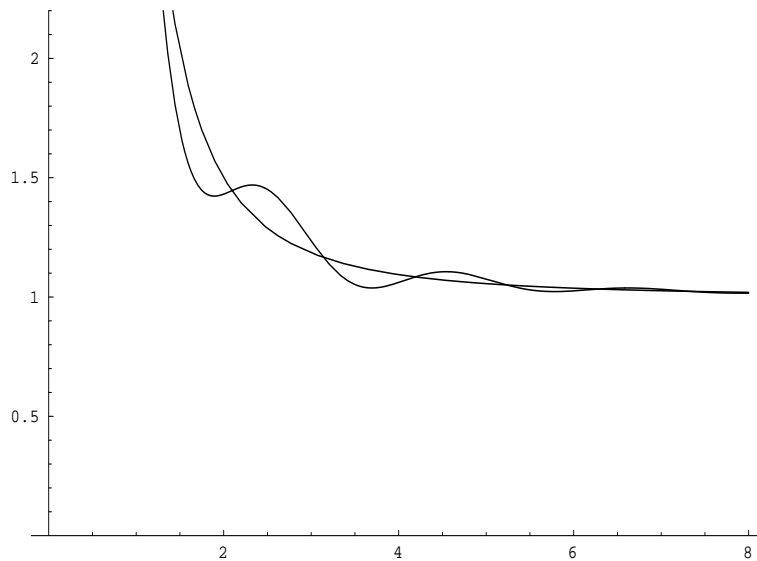


Figure 12. Γ/Γ_0 as function of m_b .

the decay rate Γ/Γ_0 , which consists of the inclusive sum over all the different exclusive decay channels. It is intuitively understandable that the true m_b -dependence will have such a damped oscillating behaviour: If we imagine to continually increase m_b , Γ/Γ_0 will undergo a small jump whenever it reaches a value at which the presence of a further higher hadronic resonances in the final state becomes kinematically allowed. Since the excited hadrons have finite widths, the threshold behaviour will be smoothed out, resulting in the pattern of damped oscillations.

The term *quark-hadron duality* refers to the idea that the inclusive rate as the sum over all exclusive *hadronic* decay channels and the inclusive rate as predicted by the heavy *quark* expansion are dual to each other. This means they are both valid representations of the same quantity using different descriptions, the hadron level or the quark level. The term *local* refers to the fact that the energy scale m_b is a fixed quantity, as opposed to e.g. the centre-of-mass energy in e^+e^- annihilation, which can be averaged to obtain suitably defined “global” quantities. In principle, the hadronic description gives the true result, measured in experiment. The problem is, however, that we would have to compute all exclusive rates first, which is far beyond our current control of nonperturbative QCD. On the other hand, the HQE calculation can be performed, within some uncertainties, but it is clear that the result need not be identical to the true answer. A deviation between the latter and the HQE (including power corrections) is referred to as a violation of quark-hadron duality. Indeed, contributions violating quark-hadron duality are expected (see(99)), but the numerical size of these terms cannot be strictly computed at present. Conceptually this is no problem because they are formally subleading in comparison to power corrections, so that the HQE still makes sense even at higher orders. The remaining question is how large can violations of quark-hadron duality be numerically. While there

are at the moment, within the uncertainties intrinsic to HQE, no established cases in inclusive B decays where duality is violated, the issue clearly needs further investigation, both theoretically and phenomenologically.

A more detailed account of the status of quark-hadron duality can be found in the papers by Blok, Shifman & Zhang (1998), Shifman (2000) and Bigi & Uraltsev (2001).

5.3 Inclusive semileptonic decays – V_{ub} , V_{cb}

The HQE cannot only be applied to the total decay rates, but also to inclusive rates with specific flavour quantum numbers in the final state, such as semileptonic processes. Furthermore one can analyze differential decay rates.

An example of special interest is the inclusive decay $\bar{B} \rightarrow X_c l \bar{\nu}$, which can be used to extract V_{cb} . The HQE for the integrated rate has the form

$$\Gamma(B \rightarrow X_c l \nu) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left[z_3 \left(1 + \frac{\lambda_1 + 3\lambda_2}{2m_b^2} \right) + z_5 \frac{6\lambda_2}{m_b^2} + \dots \right] \quad (100)$$

with

$$\langle \bar{b}b \rangle = 1 + \frac{\lambda_1 + 3\lambda_2}{2m_b^2} \quad \langle \bar{b}\sigma G b \rangle = 6\lambda_2 = \frac{3}{2}(m_{B^*}^2 - m_B^2) \quad (101)$$

The Wilson coefficients read

$$z_3 = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x + \mathcal{O}(\alpha_s) \quad (102)$$

$$z_5 = -(1 - x)^4 \quad (103)$$

Where $x = (m_c/m_b)^2$.

A major source of theoretical uncertainty for the determination of $|V_{cb}|$ using (100) is the b -quark mass. This appears to be especially problematic since m_b comes with the fifth power in (100). Fortunately, however, the actual situation is not as bad. Taking into account the phase-space function z_3 , one finds that the combined dependence on m_b and m_c shows the approximate behaviour

$$\Gamma(B \rightarrow X_c l \nu) \sim m_b^{2.3} (m_b - m_c)^{2.7} \quad (104)$$

Since the difference $m_b - m_c$ is better known than the individual quark masses, the corresponding uncertainty is reduced. The quark-mass difference is in fact constrained by HQET, which gives (72)

$$\begin{aligned} m_b - m_c &= (\bar{m}_B - \bar{m}_D) \left(1 - \frac{\lambda_1}{2\bar{m}_B \bar{m}_D} \right) \\ &= 3.40 \pm 0.03 \pm 0.03 \text{ GeV} \end{aligned} \quad (105)$$

where $\bar{m}_B \equiv (m_B + 3m_{B^*})/4$.

The QCD corrections to z_3 are known to $\mathcal{O}(\alpha_s)$ and partly at $\mathcal{O}(\alpha_s^2)$. The special class of corrections $\mathcal{O}(\beta_0^{n-1} \alpha_s^n)$ has been calculated to all orders n .

Numerically the inclusive method gives (Höcker et al. 2001)

$$V_{cb} = 0.04076 \pm 0.00050_{exp} \pm 0.00204_{th} \quad (106)$$

which can be compared with the result from the exclusive determination via $\bar{B} \rightarrow D^* l \bar{\nu}$.

One can also try to extract $|V_{ub}|$ from $B \rightarrow X_u l \nu$ decays. This is more difficult since the very large background from semileptonic $b \rightarrow c$ transitions requires kinematical cuts (in the lepton energy, the hadronic or the dilepton invariant mass), which renders the HQE less reliable and introduces larger uncertainties. A recent discussion has been given by Bauer et al. (2001b). The HQE has further useful applications, for instance in the case of the inclusive rare decays $B \rightarrow X_{s,d} \gamma$, $B \rightarrow X_{s,d} l^+ l^-$, or $B \rightarrow X_{s,d} \nu \bar{\nu}$.

Exercise

Show that quark-hadron duality is exactly fulfilled for the semileptonic $b \rightarrow c$ transition rate in the Shifman-Voloshin (small-velocity, or SV) limit $m_b, m_c \gg m_b - m_c \gg \Lambda_{QCD}$. This holds with only two exclusive channels on the hadronic side of the duality relation, that is the inclusive rate is saturated as $\Gamma(B \rightarrow X_c l \nu) \equiv \Gamma(B \rightarrow D l \nu) + \Gamma(B \rightarrow D^ l \nu)$ in this limit.*

Solution: We start from the exclusive differential decay rates in the *heavy-quark limit*. They read (see e.g. Harrison & Quinn 1998):

$$\frac{d\Gamma(B \rightarrow D l \nu)}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (m_B + m_D)^2 m_D^3 (w^2 - 1)^{3/2} \xi^2(w) \quad (107)$$

$$\begin{aligned} \frac{d\Gamma(B \rightarrow D^* l \nu)}{dw} &= \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (m_B - m_{D^*})^2 m_{D^*}^3 \sqrt{w^2 - 1} (w + 1)^2 \\ &\cdot \left(1 + \frac{4w}{w + 1} \frac{m_B^2 - 2w m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2} \right) \xi^2(w) \end{aligned} \quad (108)$$

In the strict SV limit we have

$$m_B = m_b \quad m_{D^*} = m_D = m_c \quad m_c = m_b(1 - \epsilon) \quad (109)$$

where $\epsilon \equiv (m_b - m_c)/m_b$ is a small parameter.

The variable w is related to the dilepton invariant mass q^2 through

$$q^2 = m_B^2 + m_D^2 - 2m_B m_D w \quad (110)$$

The kinematic limits of q^2 are easily identified as

$$q_{max}^2 = (m_b - m_c)^2 \quad q_{min}^2 = 0 \quad (111)$$

The corresponding limits of w are

$$w_{min} = 1 \quad w_{max} = \frac{m_b^2 + m_c^2}{2m_b m_c} \quad (112)$$

Defining $s \equiv w - 1$ we have $0 \leq s \leq \epsilon^2/2$, where the upper limit is valid to leading order in ϵ . Expanded to leading order in ϵ , (107) gives

$$\Gamma(B \rightarrow D l \nu) = \frac{G_F^2 |V_{cb}|^2}{6\pi^3} m_b^5 \sqrt{2} \int_0^{\epsilon^2/2} s^{3/2} ds = \frac{G_F^2 |V_{cb}|^2}{60\pi^3} (m_b - m_c)^5 \quad (113)$$

which is the decay rate in the SV limit. In this derivation we have made use of the fact that $\xi(w) = \xi(1) + \mathcal{O}(\epsilon^2)$, which can be approximated by $\xi(1) = 1$. In this way any dependence on nontrivial hadronic input has disappeared. Similarly we can expand the integral over (108) in ϵ to extract the leading contribution in the SV limit. We obtain

$$\Gamma(B \rightarrow D^* l \nu) = \frac{G_F^2 |V_{cb}|^2}{20\pi^3} (m_b - m_c)^5 \quad (114)$$

We also observe that higher D -meson resonances and hadronic multiparticle states have wave functions of the light degrees of freedom that are orthogonal to the ground state wave function of the light cloud (identical for D , D^* and B) in the SV limit. There is therefore no overlap of those higher excitations with the initial B and the corresponding rates vanish.

Finally, we need to take the SV limit of the inclusive rate as obtained from the heavy quark expansion in (100). In this limit the second-order power corrections and perturbative QCD corrections disappear, and we only have to expand the phase space function z_3 in the small- ϵ limit. We find $z_3 = 64\epsilon^5/5 + \mathcal{O}(\epsilon^6)$ and

$$\Gamma(B \rightarrow X_c l \nu) = \frac{G_F^2 |V_{cb}|^2}{15\pi^3} (m_b - m_c)^5 \quad (115)$$

We see that indeed the inclusive HQE result (115) is saturated by the sum of just the two exclusive rates (113) and (114). Clearly, the SV limit is a very special situation. Nevertheless, it is an interesting example of exact (local) quark-hadron duality. Moreover, the semileptonic rates into D and D^* measured in experiment account for roughly two thirds of the inclusive rate, indicating that the SV limit is not even entirely unrealistic.

6 QCD factorization in exclusive hadronic B decays

6.1 Introduction

Decay amplitudes for exclusive nonleptonic B decays, such as $B \rightarrow \pi\pi$, can be computed starting from the effective weak Hamiltonian discussed in sec. 3.3. Whereas the Wilson coefficients C_i are well understood, the main problem is posed by the hadronic matrix elements of the operators Q_i . In some cases this problem can be circumvented (CP asymmetry in $B \rightarrow J/\Psi K_S$), or at least reduced using SU(2) or SU(3) flavour symmetries and an appropriate combination of various channels. However, an improved understanding of the QCD dynamics in exclusive hadronic B decays would greatly enhance our capability to extract from these processes the underlying flavour physics.

Indeed, it turns out that the heavy-quark limit leads to substantial simplifications also in the problem of hadronic two-body decays of heavy hadrons. Again the main feature is the factorization of short-distance and long-distance contributions. In the case of the matrix elements of four-quark operators Q_i the factorization takes the form

$$\begin{aligned} \langle \pi(p') \pi(q) | Q_i | \bar{B}(p) \rangle &= f^{B \rightarrow \pi}(q^2) \int_0^1 du T_i^I(u) \Phi_\pi(u) \\ &+ \int_0^1 d\xi du dv T_i^{II}(\xi, u, v) \Phi_B(\xi) \Phi_\pi(u) \Phi_\pi(v) \end{aligned} \quad (116)$$

This *factorization formula* is valid up to corrections of relative order Λ_{QCD}/m_b . Here $f^{B \rightarrow \pi}(q^2)$ is a $B \rightarrow \pi$ form factor evaluated at $q^2 = m_\pi^2 \approx 0$, and Φ_π (Φ_B) are leading-twist light-cone distribution amplitudes “wave functions”) of the pion (B meson). These objects contain the long-distance dynamics. The short-distance physics, dominated by scales of order m_b , is described by the hard-scattering kernels $T_i^{I,II}$, which are calculable in perturbation theory. T_i^I starts at $\mathcal{O}(\alpha_s^0)$, T_i^{II} at $\mathcal{O}(\alpha_s^1)$ (see Fig. 13). In (116) long-

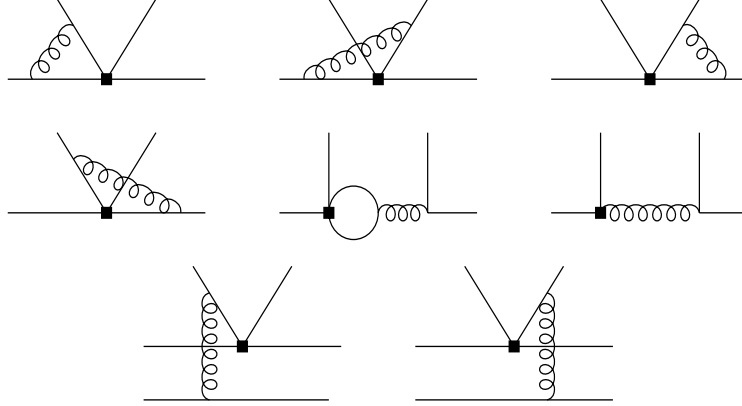


Figure 13. Order α_s corrections to the hard scattering kernels T_i^I (first two rows) and T_i^{II} (last row). In the case of T_i^I , the spectator quark does not participate in the hard interaction and is not drawn. The two lines directed upwards represent the two quarks forming the emitted pion.

and short-distance contributions are thus systematically disentangled, that is *factorized*. The long-distance sensitive quantities (form factors and wave functions) still need to be determined by other means, but they are universal quantities and much simpler than the original full $B \rightarrow \pi\pi$ matrix elements we started with. They could in principle be calculated by nonperturbative methods or extracted experimentally from other observables. In any case (116) represents a substantial simplification of our problem.

The general expression (116) further simplifies when we neglect perturbative α_s -corrections. The T^{II} term is then absent and the kernel T^I becomes a constant in u , such that the pion distribution amplitude integrates to the pion decay constant. The matrix element of operator Q_1^u , for instance, reduces to

$$\langle \pi^+ \pi^- | (\bar{u}b)_{V-A} (\bar{d}u)_{V-A} | \bar{B} \rangle \rightarrow \langle \pi^+ | (\bar{u}b)_{V-A} | \bar{B} \rangle \cdot \langle \pi^- | (\bar{d}u)_{V-A} | 0 \rangle = im_B^2 f^{B \rightarrow \pi}(0) f_\pi \quad (117)$$

This procedure, termed “naive factorization” has long been used in phenomenological application, but the justification had been less clear. An obvious issue is the scheme and scale dependence of the matrix elements of four-quark operators, which is needed to cancel the corresponding dependence in the Wilson coefficients. This dependence is lost in naive factorization as the factorized currents are scheme independent objects. In QCD factorization (116) the proper scale and scheme dependence is recovered by the inclusion of $\mathcal{O}(\alpha_s)$ corrections as we will see explicitly below.

A qualitative justification for (117) had been given by Bjorken (1989). It is based on the *colour transparency* of the hadronic environment for the highly energetic pion emitted in B decay (the π^- in the above example, which is being created from the vacuum). This is related to the decoupling of soft gluons from the small-size colour-singlet object that the emitted pion represents. The QCD factorization approach as encoded in (116) may be viewed as a consistent formalization and generalization of Bjorken’s colour transparency argument. This treatment of hadronic B decays is based on the analysis of Feynman diagrams in the heavy quark limit, utilizing consistent power counting to identify the leading contributions. The framework is very similar in spirit to more conventional applications of perturbative QCD in exclusive hadronic processes with a large momentum transfer, as the pion electromagnetic form factor (see the article by Sterman and Stoler (1997) for a recent review). It justifies and extends the ansatz of naive factorization. In particular the method includes, for $B \rightarrow \pi\pi$, the hard nonfactorizable spectator interactions, penguin contributions and rescattering effects (Fig. 13). As a corollary, one finds that strong rescattering phases are either of $\mathcal{O}(\alpha_s)$, and calculable, or power suppressed. In any case they vanish therefore in the heavy quark limit. QCD factorization is valid for cases where the emitted particle (the meson created from the vacuum in the weak process, as opposed to the one that absorbs the b -quark spectator) is a small size colour-singlet object, e.g. either a fast light meson (π , ρ , K , K^*) or a J/Ψ .

Note that the term *factorization* is used here for two a priori entirely different things: In the case of QCD factorization (116), it refers to the factorization of short-distance and long-distance contributions. In the sense of the phenomenological approach of naive factorization (117), it simply denotes the separation of the hadronic matrix element of a four-quark operator into two factors of matrix elements of bilinear currents. It is a nontrivial result that the latter, naive factorization is obtained as the lowest order approximation of QCD factorization. To avoid confusion, it is useful to keep the distinction in mind. For example, the hard gluon exchange corrections between the two quark currents in Fig. 13 are “nonfactorizable” in the sense of naive factorization, although they are a consistent ingredient of (116), hence “factorizable” in the sense of QCD.

In the following we shall discuss QCD factorization in some detail using the example of $B \rightarrow D\pi$ decays. In this case the $b \rightarrow u$ transition current is replaced by a heavy-heavy $b \rightarrow c$ current. This case is somewhat simpler than $B \rightarrow \pi\pi$ since the spectator interaction (the T^{II} term, bottom line of Fig. 13) does not contribute to leading power. This is because for a heavy-to-heavy transition the spectator quark, and hence the gluon attached to it, is always soft. This leads to a suppression, according to the colour transparency argument, when this gluon couples to the emitted pion. Also penguin contributions are absent for $B \rightarrow D\pi$. We shall illustrate explicitly how factorization emerges at the one-loop order in this specific case, and in the heavy-quark limit, defined as $m_b, m_c \gg \Lambda_{\text{QCD}}$ with m_c/m_b fixed.

Further details on QCD factorization in B decays and additional literature can be found in the articles by Beneke et al. (1999, 2000 & 2001).

6.2 $B \rightarrow D\pi$: Factorization to one-loop order

6.2.1 Preliminaries

The effective Hamiltonian relevant for $B \rightarrow D\pi$ can be written as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} (C_0 O_0 + C_8 O_8), \quad (118)$$

with the operators

$$O_0 = \bar{c}\gamma^\mu(1 - \gamma_5)b \bar{d}\gamma_\mu(1 - \gamma_5)u, \quad (119)$$

$$O_8 = \bar{c}\gamma^\mu(1 - \gamma_5)T^a b \bar{d}\gamma_\mu(1 - \gamma_5)T^a u. \quad (120)$$

Here we have chosen to write the two independent operators in the singlet-octet basis, which is most convenient for our purposes, rather than in the more conventional bases of Q_1 , Q_2 or Q_+ , Q_- (see the discussion in sec. 3; because all four quark flavours are different in (118), penguin operators are absent). The Wilson coefficients C_0 , C_8 have been calculated at next-to-leading order in renormalization-group improved perturbation theory (Altarelli et al. 1981, Buras & Weisz 1990) and are given by

$$C_0 = \frac{N_c + 1}{2N_c} C_+ + \frac{N_c - 1}{2N_c} C_-, \quad C_8 = C_+ - C_-, \quad (121)$$

where

$$C_\pm(\mu) = \left(1 + \frac{\alpha_s(\mu)}{4\pi} B_\pm\right) \bar{C}_\pm(\mu), \quad (122)$$

$$\bar{C}_\pm(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{d_\pm} \left[1 + \frac{\alpha_s(M_W) - \alpha_s(\mu)}{4\pi} (B_\pm - J_\pm)\right]. \quad (123)$$

(The coefficients C_0 , C_8 are related to the ones of the standard basis by $C_0 = C_1 + C_2/3$ and $C_8 = 2C_2$.) We employ the next-to-leading order expression for the running coupling,

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda_{\text{QCD}}^2)} \left[1 - \frac{\beta_1}{\beta_0^2} \frac{\ln \ln(\mu^2/\Lambda_{\text{QCD}}^2)}{\ln(\mu^2/\Lambda_{\text{QCD}}^2)}\right], \quad (124)$$

$$\beta_0 = \frac{11N_c - 2f}{3}, \quad \beta_1 = \frac{34}{3}N_c^2 - \frac{10}{3}N_c f - 2C_F f, \quad C_F = \frac{N_c^2 - 1}{2N_c}, \quad (125)$$

where N_c is the number of colours, and f the number of light flavours. $\Lambda_{\text{QCD}} \equiv \Lambda_{\overline{\text{MS}}}^{(f)}$ is the QCD scale in the $\overline{\text{MS}}$ scheme with f flavours. Next we have

$$d_\pm = \frac{\gamma_\pm^{(0)}}{2\beta_0}, \quad \gamma_\pm^{(0)} = \pm 12 \frac{N_c \mp 1}{2N_c}, \quad B_\pm = \pm \frac{N_c \mp 1}{2N_c} B. \quad (126)$$

The general definition of J_\pm may be found in (Buchalla, Buras & Lautenbacher 1996). Numerically, for $N_c = 3$ and $f = 5$

$$d_\pm = \begin{cases} \frac{6}{23}, \\ -\frac{12}{23}, \end{cases} \quad B_\pm - J_\pm = \begin{cases} \frac{6473}{3174}, \\ -\frac{9371}{1587}. \end{cases} \quad (127)$$

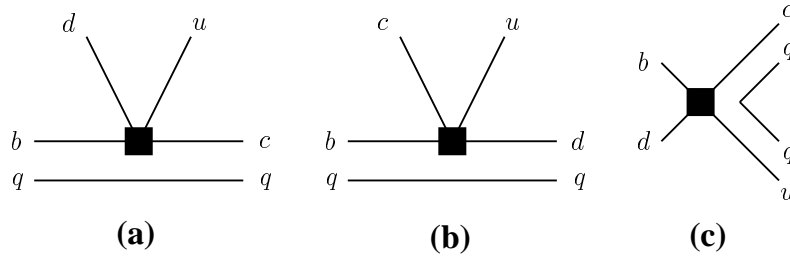


Figure 14. Basic quark-level topologies for $B \rightarrow D\pi$ decays ($q = u, d$): (a) class-I, (b) class-II, (c) weak annihilation. $\bar{B}_d \rightarrow D^+\pi^-$ receives contributions from (a) and (c), $\bar{B}_d \rightarrow D^0\pi^0$ from (b) and (c), and $B^- \rightarrow D^0\pi^-$ from (a) and (b). Only (a) contributes in the heavy-quark limit.

The quantities $\beta_0, \beta_1, d_\pm, B_\pm - J_\pm$ are scheme independent. The scheme dependence of the coefficients at next-to-leading order is parametrized by B_\pm in (122). In the naive dimensional regularization (NDR) and ‘t Hooft-Veltman (HV) schemes, this scheme dependence is expressed in a single number B with $B_{\text{NDR}} = 11$ and $B_{\text{HV}} = 7$. The dependence of the Wilson coefficients on the renormalization scheme and scale is cancelled by a corresponding scale and scheme dependence of the hadronic matrix elements of the operators O_0 and O_8 .

Before continuing with a discussion of these matrix elements, it is useful to consider the flavour structure for the various contributions to $B \rightarrow D\pi$ decays. The possible quark-level topologies are depicted in Fig. 14. In the terminology generally adopted for two-body non-leptonic decays, the decays $\bar{B}_d \rightarrow D^+\pi^-$, $\bar{B}_d \rightarrow D^0\pi^0$ and $B^- \rightarrow D^0\pi^-$ are referred to as class-I, class-II and class-III, respectively. In both $\bar{B}_d \rightarrow D^+\pi^-$ and $B^- \rightarrow D^0\pi^-$ decays the pion can be directly created from the weak current. We may call this a class-I contribution, following the above terminology. In addition, in the case of $\bar{B}_d \rightarrow D^+\pi^-$ there is a contribution from weak annihilation and a class-II amplitude contributes to $B^- \rightarrow D^0\pi^-$, see Fig. 14. The important point is that the spectator quark goes into the light meson in the case of the class-II amplitude. This amplitude is suppressed in the heavy-quark limit, as is the annihilation amplitude. The amplitude for $\bar{B}_d \rightarrow D^0\pi^0$, receiving only class-II and annihilation contributions, is therefore subleading compared with $\bar{B}_d \rightarrow D^+\pi^-$ and $B^- \rightarrow D^0\pi^-$, which are dominated by the class-I topology. The treatment of this leading class-I mechanism will be the main subject of the following sections. We shall use the one-loop analysis for $\bar{B}_d \rightarrow D^+\pi^-$ as a concrete example on which we will illustrate explicitly how the factorization formula can be derived.

6.2.2 Soft and collinear cancellations at one-loop

In order to demonstrate the property of factorization for $\bar{B}_d \rightarrow D^+\pi^-$, we have to analyze the “non-factorizable” one-gluon exchange contributions (Fig. 15) to the $b \rightarrow c\bar{u}d$ transition. We consider the leading, valence Fock state of the emitted pion. This is justified since higher Fock components only give power-suppressed contributions to the decay amplitude in the heavy-quark limit. The valence Fock state of the pion can be written as

$$|\pi(q)\rangle = \int \frac{du}{\sqrt{u\bar{u}}} \frac{d^2l_\perp}{16\pi^3} \frac{1}{\sqrt{2N_c}} \left(a_\uparrow^\dagger(l_q) b_\downarrow^\dagger(l_{\bar{q}}) - a_\downarrow^\dagger(l_q) b_\uparrow^\dagger(l_{\bar{q}}) \right) |0\rangle \Psi(u, \vec{l}_\perp), \quad (128)$$

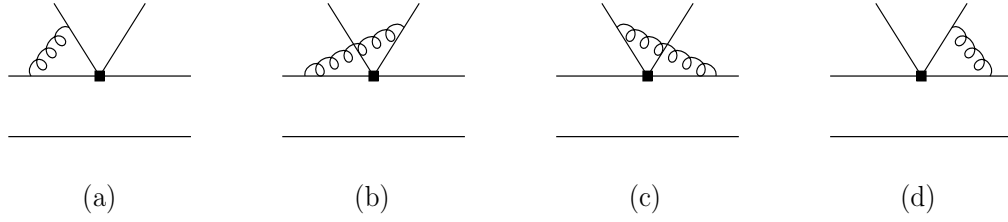


Figure 15. “Non-factorizable” vertex corrections.

where a_s^\dagger (b_s^\dagger) denotes the creation operator for a quark (antiquark) in a state with spin $s = \uparrow$ or $s = \downarrow$, and we have suppressed colour indices. This representation of the pion state is adequate for a leading-power analysis. The wave function $\Psi(u, \vec{l}_\perp)$ is defined as the amplitude for the pion to be composed of two on-shell quarks, characterized by longitudinal momentum fraction u and transverse momentum l_\perp . The on-shell momenta ($l_{q,\bar{q}}^2 = 0$) of the quark (l_q) and the antiquark ($l_{\bar{q}}$) are given by

$$l_q = uq + l_\perp + \frac{\vec{l}_\perp^2}{4uE}n_-, \quad l_{\bar{q}} = \bar{u}q - l_\perp + \frac{\vec{l}_\perp^2}{4\bar{u}E}n_-. \quad (129)$$

Here $q = E(1, 0, 0, 1)$ is the pion momentum, $E = p_B \cdot q/m_B$ the pion energy and $n_- = (1, 0, 0, -1)$. Furthermore $l_\perp \cdot q = l_\perp \cdot n_- = 0$. For the purpose of power counting $l_\perp \sim \Lambda_{\text{QCD}} \ll E \sim m_b$. Note that the invariant mass of the valence state is $(l_q + l_{\bar{q}})^2 = \vec{l}_\perp^2/(u\bar{u})$, which is of order Λ_{QCD}^2 and hence negligible in the heavy-quark limit, unless u is in the vicinity of the endpoints (0 or 1). In this case the invariant mass of the quark-antiquark pair becomes large and the valence Fock state is no longer a valid representation of the pion. However, in the heavy-quark limit the dominant contributions to the decay amplitude come from configurations where both partons are hard (u and \bar{u} both of order 1) and the two-particle Fock state yields a consistent description. The suppression of the soft regions (u or $\bar{u} \ll 1$) is related to the endpoint behaviour of the pion wave function. We will provide an explicit consistency check of this important feature later on.

As a next step we write down the amplitude

$$\langle \pi(q) | u(0)_\alpha \bar{d}(y)_\beta | 0 \rangle = \int du \frac{d^2 l_\perp}{16\pi^3} \frac{1}{\sqrt{2N_c}} \Psi^*(u, \vec{l}_\perp) (\gamma_5 \not{q})_{\alpha\beta} e^{il_q \cdot y}, \quad (130)$$

which appears as an ingredient of the $B \rightarrow D\pi$ matrix element. The right-hand side of (130) follows directly from (128). Using (130) it is straightforward to write down the one-gluon exchange contribution to the $B \rightarrow D\pi$ matrix element of the operator O_8 (Fig. 15). We have

$$\begin{aligned} \langle D^+ \pi^- | O_8 | \bar{B}_d \rangle_{1\text{-gluon}} &= \\ ig_s^2 \frac{C_F}{2} \int \frac{d^4 k}{(2\pi)^4} \langle D^+ | \bar{c} A_1(k) b | \bar{B}_d \rangle \frac{1}{k^2} \int_0^1 du \frac{d^2 l_\perp}{16\pi^3} \frac{\Psi^*(u, \vec{l}_\perp)}{\sqrt{2N_c}} \text{tr}[\gamma_5 \not{q} A_2(l_q, l_{\bar{q}}, k)], \end{aligned} \quad (131)$$

where

$$A_1(k) = \frac{\gamma^\lambda (\not{p}_c - \not{k} + m_c) \Gamma}{2p_c \cdot k - k^2} - \frac{\Gamma (\not{p}_b + \not{k} + m_b) \gamma^\lambda}{2p_b \cdot k + k^2}, \quad (132)$$

$$A_2(l_q, l_{\bar{q}}, k) = \frac{\Gamma(\not{l}_{\bar{q}} + \not{k})\gamma_\lambda}{2l_{\bar{q}} \cdot k + k^2} - \frac{\gamma_\lambda(\not{l}_q + \not{k})\Gamma}{2l_q \cdot k + k^2}. \quad (133)$$

Here $\Gamma = \gamma^\mu(1 - \gamma_5)$ and p_b, p_c are the momenta of the b quark and the c quark, respectively. Note that this expression holds in an arbitrary covariant gauge. The gauge-parameter dependent part of the gluon propagator gives no contribution to (131), as can be seen from (132) and (133). There is no correction to the matrix element of O_0 at order α_s , because in this case the $(d\bar{u})$ pair is necessarily in a colour-octet configuration and cannot form a pion.

In (131) the pion wave function $\Psi(u, l_\perp)$ appears separated from the $B \rightarrow D$ transition. This is merely a reflection of the fact that we have represented the pion state by (128). It does not, by itself, imply factorization, since the right-hand side of (131) involves still nontrivial integrations over \vec{l}_\perp and gluon momentum k , and long- and short-distance contributions are not yet disentangled. In order for (131) to make sense we need to show that the integral over k receives only subdominant contributions from the region of small k^2 . This is equivalent to showing that the integral over k does not contain infrared divergences at leading power in $1/m_b$.

To demonstrate infrared finiteness of the one-loop integral

$$J \equiv \int d^4k \frac{1}{k^2} A_1(k) \otimes A_2(l_q, l_{\bar{q}}, k) \quad (134)$$

at leading power, the heavy-quark limit and the corresponding large light-cone momentum of the pion are again essential. First note that when k is of order m_b , $J \sim 1$ for dimensional reasons. Potential infrared divergences could arise when k is soft or when k is collinear to the pion momentum q . We need to show that the contributions from these regions are power suppressed in m_b . (Note that we do not need to show that J is infrared finite. It is enough that logarithmic divergences have coefficients that are power suppressed.)

We treat the soft region of integration first. Here all components of k become small simultaneously, which we describe by scaling $k \sim \lambda$. Counting powers of λ ($d^4k \sim \lambda^4$, $1/k^2 \sim \lambda^{-2}$, $1/p \cdot k \sim \lambda^{-1}$) reveals that each of the four diagrams (corresponding to the four terms in the product in (134)) is logarithmically divergent. However, because k is small, the integrand can be simplified. For instance, the second term in A_2 can be approximated as

$$\frac{\gamma_\lambda(\not{l}_q + \not{k})\Gamma}{2l_q \cdot k + k^2} = \frac{\gamma_\lambda(u \not{q} + \not{l}_\perp + \frac{\vec{l}_\perp^2}{4uE} \not{n}_- + \not{k})\Gamma}{2uq \cdot k + 2l_\perp \cdot k + \frac{\vec{l}_\perp^2}{2uE} n_- \cdot k + k^2} \simeq \frac{q_\lambda}{q \cdot k} \Gamma, \quad (135)$$

where we used that \not{q} to the extreme left or right of an expression gives zero due to the on-shell condition for the external quark lines. We get exactly the same expression but with an opposite sign from the other term in A_2 and hence the soft divergence cancels out when adding the two terms in A_2 . More precisely, we find that the integral is infrared finite in the soft region when l_\perp is neglected. When l_\perp is not neglected, there is a divergence from soft k which is proportional to $l_\perp^2/m_b^2 \sim \Lambda_{\text{QCD}}^2/m_b^2$. In either case, the soft contribution to J is of order Λ_{QCD}/m_b or smaller and hence suppressed relative to the hard contribution. This corresponds to the standard soft cancellation mechanism, which is a technical manifestation of colour transparency.

Each of the four terms in (134) is also divergent when k becomes collinear with the light-cone momentum q . It is convenient to introduce, for any four-vector v , the light-cone components

$$v_{\pm} = \frac{v^0 \pm v^3}{\sqrt{2}} \quad (136)$$

The collinear region is then described by the scaling

$$k^+ \sim \lambda^0, \quad k_{\perp} \sim \lambda, \quad k^- \sim \lambda^2. \quad (137)$$

Then $d^4k \sim dk^+ dk^- d^2k_{\perp} \sim \lambda^4$ and $q \cdot k = q^+ k^- \sim \lambda^2$, $k^2 = 2k^+ k^- + k_{\perp}^2 \sim \lambda^2$. The divergence is again logarithmic and it is thus sufficient to consider the leading behaviour in the collinear limit. Writing $k = \alpha q + \dots$ we can now simplify the second term of A_2 as

$$\frac{\gamma_{\lambda}(\not{l}_q + \not{k})\Gamma}{2l_q \cdot k + k^2} \simeq q_{\lambda} \frac{2(u + \alpha)\Gamma}{2l_q \cdot k + k^2}. \quad (138)$$

No simplification occurs in the denominator (in particular l_{\perp} cannot be neglected), but the important point is that the leading-power contribution is proportional to q_{λ} . Therefore, substituting $k = \alpha q$ into A_1 and using $q^2 = 0$, we obtain

$$q_{\lambda} A_1 \simeq \frac{\not{q}(\not{p}_c + m_c)\Gamma}{2\alpha p_c \cdot q} - \frac{\Gamma(\not{p}_b + m_b)\not{q}}{2\alpha p_b \cdot q} = 0, \quad (139)$$

employing the equations of motion for the heavy quarks. Hence the collinearly divergent region is seen to cancel out via the standard collinear Ward identity. This completes the proof of the absence of infrared divergences at leading power in the hard-scattering kernel for $\bar{B}_d \rightarrow D^+ \pi^-$ to one-loop order. In other words, we have shown that the “non-factorizable” diagrams of Fig. 15 are dominated by hard gluon exchange. The proof at two loops has been given by Beneke et al. (2000) and a proof to all orders by Bauer et al. (2001c).

Since we have now established that the leading contribution to J arises from k of order m_b (“hard” k), and since $|\vec{l}_{\perp}| \ll E$, we may expand A_2 in $|\vec{l}_{\perp}|/E$. To leading power the expansion simply reduces to neglecting l_{\perp} altogether, which implies $l_q = uq$ and $l_{\bar{q}} = \bar{u}q$ in (133). As a consequence, we may perform the l_{\perp} integration in (131) over the pion wave function. Defining

$$\int \frac{d^2 l_{\perp}}{16\pi^3} \frac{\Psi^*(u, \vec{l}_{\perp})}{\sqrt{2N_c}} \equiv \frac{if_{\pi}}{4N_c} \Phi_{\pi}(u), \quad (140)$$

the matrix element of O_8 in (131) becomes

$$\begin{aligned} \langle D^+ \pi^- | O_8 | \bar{B}_d \rangle_{1\text{-gluon}} &= \\ -g_s^2 \frac{C_F}{8N_c} \int \frac{d^4 k}{(2\pi)^4} \langle D^+ | \bar{c} A_1(k) b | \bar{B}_d \rangle \frac{1}{k^2} f_{\pi} \int_0^1 du \Phi_{\pi}(u) \text{tr}[\gamma_5 \not{q} A_2(uq, \bar{u}q, k)]. \end{aligned} \quad (141)$$

Putting y on the light-cone in (130), $y^+ = y_{\perp} = 0$, hence $l_q \cdot y = l_q^+ y^- = uqy$, we see that the l_{\perp} -integrated wave function $\Phi_{\pi}(u)$ in (140) is precisely the light-cone distribution amplitude of the pion in the usual definition. Indeed, the leading-twist light-cone distribution amplitude for pseudoscalar mesons (P) reads

$$\langle P(q) | \bar{q}(y)_{\alpha} q'(x)_{\beta} | 0 \rangle \Big|_{(x-y)^2=0} = \frac{if_P}{4} (\not{q} \gamma_5)_{\beta\alpha} \int_0^1 du e^{i(\bar{u}qx + uqy)} \Phi_P(u, \mu) \quad (142)$$

Here it is understood that the operator on the left-hand side is a colour singlet. We use the “bar”-notation, i.e. $\bar{v} \equiv 1 - v$ for any longitudinal momentum fraction variable. The parameter μ is the renormalization scale of the light-cone operators on the left-hand side. The distribution amplitude is normalized as $\int_0^1 du \Phi_P(u, \mu) = 1$. One defines the asymptotic distribution amplitude as the limit in which the renormalization scale is sent to infinity. The asymptotic form is

$$\Phi_P(u, \mu) \xrightarrow{\mu \rightarrow \infty} 6u\bar{u}. \quad (143)$$

The decay constant appearing in (142) refers to the normalization in which $f_\pi = 131$ MeV. There is a path-ordered exponential that connects the two quark fields at different positions and makes the light-cone operators gauge invariant. In (142) we have suppressed this standard factor.

This derivation demonstrates the relevance of the light-cone wave function for the factorization formula. Note that the collinear approximation for the quark-antiquark momenta emerges automatically in the heavy-quark limit.

After the k integral is performed, the expression (141) can be cast into the form

$$\langle D^+ \pi^- | O_8 | \bar{B}_d \rangle_{1\text{-gluon}} \sim F_{B \rightarrow D}(0) \int_0^1 du T_8(u, z) \Phi_\pi(u), \quad (144)$$

where $z = m_c/m_b$, $T_8(u, z)$ is the hard-scattering kernel, and $F_{B \rightarrow D}(0)$ the form factor that parametrizes the $\langle D^+ | \bar{c}[\dots] b | \bar{B}_d \rangle$ matrix element. The result for $T_8(u, z)$ will be given in the following section.

6.2.3 Matrix elements at next-to-leading order

As we have seen above, the $\bar{B}_d \rightarrow D^+ \pi^-$ amplitude factorizes in the heavy-quark limit into a matrix element of the form $\langle D^+ | \bar{c}[\dots] b | \bar{B}_d \rangle$ for the $B \rightarrow D$ transition and a matrix element $\langle \pi^- | \bar{d}(x)[\dots] u(0) | 0 \rangle$ with $x^2 = 0$ that gives rise to the pion light-cone distribution amplitude. Leaving aside power-suppressed contributions, the essential requirement for this conclusion was the absence of both soft and collinear infrared divergences in the gluon exchange between the $(\bar{c}b)$ and $(\bar{d}u)$ currents. This gluon exchange is therefore calculable in QCD perturbation theory. We now present these corrections explicitly to order α_s .

The effective Hamiltonian (118) can be written as

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} \left\{ \left[\frac{N_c + 1}{2N_c} \bar{C}_+(\mu) + \frac{N_c - 1}{2N_c} \bar{C}_-(\mu) + \frac{\alpha_s(\mu)}{4\pi} \frac{C_F}{2N_c} BC_8(\mu) \right] O_0 \right. \\ & \left. + C_8(\mu) O_8 \right\}, \end{aligned} \quad (145)$$

where the scheme-dependent terms in the coefficient of the operator O_0 , proportional to the constant B defined after (127), have been written explicitly.

Schematically, the matrix elements of both O_0 and O_8 can be expressed in a form analogous to (144). Because the light-quark pair has to be in a colour singlet to produce the pion in the leading Fock state, only O_0 gives a contribution to zeroth order in α_s . Similarly, to first order in α_s only O_8 can contribute. The result of computing the diagrams

in Fig. 15 with an insertion of O_8 can be written in a form that holds simultaneously for $H = D, D^*$ and $L = \pi, \rho$, using only that the $(\bar{u}d)$ pair is a colour singlet and that the external quarks can be taken on-shell. One obtains ($z = m_c/m_b$)

$$\begin{aligned} \langle H(p')L(q)|O_8|\bar{B}_d(p)\rangle &= \frac{\alpha_s C_F}{4\pi 2N_c} if_L \int_0^1 du \Phi_L(u) \\ &\times \left[- \left(6 \ln \frac{\mu^2}{m_b^2} + B \right) (\langle J_V \rangle - \langle J_A \rangle) + F(u, z) \langle J_V \rangle - F(u, -z) \langle J_A \rangle \right], \end{aligned} \quad (146)$$

where

$$\langle J_V \rangle = \langle H(p')|\bar{c} \not{q} b|\bar{B}_d(p)\rangle, \quad \langle J_A \rangle = \langle H(p')|\bar{c} \not{q} \gamma_5 b|\bar{B}_d(p)\rangle. \quad (147)$$

In obtaining (146) we have used the equations of motion for the quarks to reduce the operator basis to J_V and J_A . The form of (146) is identical for pions and longitudinally polarized ρ mesons. The production of transversely polarized ρ mesons is power suppressed in Λ_{QCD}/m_b .

In the case of a distribution amplitude $\Phi_L(u)$ that is symmetric under $u \leftrightarrow \bar{u}$, which is relevant for $L = \pi, \rho$, the function $F(u, z)$ appearing in (146) can be compactly written as

$$F(u, z) = 3 \ln z^2 - 7 + f(u, z) + f(u, 1/z), \quad (148)$$

with

$$f(u, z) = - \frac{u(1-z^2)[3(1-u(1-z^2)) + z]}{[1-u(1-z^2)]^2} \ln[u(1-z^2)] - \frac{z}{1-u(1-z^2)}. \quad (149)$$

The contribution of $f(u, z)$ in (148) comes from the first two diagrams in Fig. 15 with the gluon coupling to the b quark, whereas $f(u, 1/z)$ arises from the last two diagrams with the gluon coupling to the charm quark. The absorptive part of the amplitude, which is responsible for the occurrence of strong rescattering phases, arises from $f(u, 1/z)$ and can be obtained by recalling that z^2 is $z^2 - i\epsilon$ with $\epsilon > 0$ infinitesimal. We then have

$$\frac{1}{\pi} \text{Im} F(u, z) = - \frac{(1-u)(1-z^2)[3(1-u(1-z^2)) + z]}{[1-u(1-z^2)]^2} \quad (150)$$

As mentioned above, (146) is applicable to all decays of the type $\bar{B}_d \rightarrow D^{(*)+}L^-$, where L is a light hadron such as a pion or a (longitudinally polarized) ρ meson. Only the operator J_V contributes to $\bar{B}_d \rightarrow D^+L^-$, and only J_A contributes to $\bar{B}_d \rightarrow D^{*+}L^-$. (Due to helicity conservation the vector current $B \rightarrow D^*$ matrix element contributes only in conjunction with a transversely polarized ρ meson and hence is power suppressed in the heavy-quark limit.) Our final result can therefore be written as

$$\langle D^+L^-|O_{0,8}|\bar{B}_d\rangle = \langle D^+|\bar{c}\gamma^\mu(1-\gamma_5)b|\bar{B}_d\rangle \cdot if_L q_\mu \int_0^1 du T_{0,8}(u, z) \Phi_L(u), \quad (151)$$

where $L = \pi, \rho$, and the hard-scattering kernels are

$$T_0(u, z) = 1 + O(\alpha_s^2), \quad (152)$$

$$T_8(u, z) = \frac{\alpha_s C_F}{4\pi 2N_c} \left[-6 \ln \frac{\mu^2}{m_b^2} - B + F(u, z) \right] + O(\alpha_s^2). \quad (153)$$

When the D meson is replaced by a D^* meson, the result is identical except that $F(u, z)$ in (153) must be replaced by $F(u, -z)$. Since no order α_s corrections exist for O_0 , the matrix element retains its leading-order factorized form

$$\langle D^+ L^- | O_0 | \bar{B}_d \rangle = i f_L q_\mu \langle D^+ | \bar{c} \gamma^\mu (1 - \gamma_5) b | \bar{B}_d \rangle \quad (154)$$

to this accuracy. From (149) it follows that $T_8(u, z)$ tends to a constant as u approaches the endpoints ($u \rightarrow 0, 1$). Therefore the contribution to (151) from the endpoint region is suppressed, both by phase space and by the endpoint suppression intrinsic to $\Phi_L(u)$, which can be represented as

$$\Phi_L(u) = 6u(1-u) \left[1 + \sum_{n=1}^{\infty} \alpha_n^L(\mu) C_n^{3/2}(2u-1) \right], \quad (155)$$

vanishing $\sim u$ ($\sim (1-u)$) at the endpoints. Here $C_1^{3/2}(x) = 3x$, $C_2^{3/2}(x) = \frac{3}{2}(5x^2 - 1)$, etc. are Gegenbauer polynomials. The Gegenbauer moments $\alpha_n^L(\mu)$ are nonperturbative quantities. They are multiplicatively renormalized and approach zero as $\mu \rightarrow \infty$. (The scale dependence of these quantities enters the results for the coefficients only at order α_s^2 , which is beyond the accuracy of a NLO calculation.)

As a consequence of the endpoint suppression the emitted light meson is indeed dominated by energetic constituents, as required for the self-consistency of the factorization formula (151).

Combining (145), (151), (152) and (153), we obtain our final result for the class-I, non-leptonic $\bar{B}_d \rightarrow D^{(*)+} L^-$ decay amplitudes in the heavy-quark limit, and at next-to-leading order in α_s . The results can be compactly expressed in terms of the matrix elements of a “transition operator”

$$\mathcal{T} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} [a_1(DL) Q_V - a_1(D^*L) Q_A], \quad (156)$$

where

$$Q_V = \bar{c} \gamma^\mu b \otimes \bar{d} \gamma_\mu (1 - \gamma_5) u, \quad Q_A = \bar{c} \gamma^\mu \gamma_5 b \otimes \bar{d} \gamma_\mu (1 - \gamma_5) u, \quad (157)$$

and hadronic matrix elements of $Q_{V,A}$ are understood to be evaluated in factorized form, i.e.

$$\langle DL | j_1 \otimes j_2 | \bar{B} \rangle \equiv \langle D | j_1 | \bar{B} \rangle \langle L | j_2 | 0 \rangle. \quad (158)$$

Eq. (156) defines the quantities $a_1(D^{(*)}L)$, which include the leading “non-factorizable” corrections, in a renormalization-scale and -scheme independent way. To leading power in Λ_{QCD}/m_b these quantities should not be interpreted as phenomenological parameters (as is usually done), because they are dominated by hard gluon exchange and thus calculable in QCD. At next-to-leading order we get

$$\begin{aligned} a_1(DL) &= \frac{N_c + 1}{2N_c} \bar{C}_+(\mu) + \frac{N_c - 1}{2N_c} \bar{C}_-(\mu) \\ &\quad + \frac{\alpha_s}{4\pi} \frac{C_F}{2N_c} C_8(\mu) \left[-6 \ln \frac{\mu^2}{m_b^2} + \int_0^1 du F(u, z) \Phi_L(u) \right], \end{aligned} \quad (159)$$

$$\begin{aligned} a_1(D^*L) &= \frac{N_c + 1}{2N_c} \bar{C}_+(\mu) + \frac{N_c - 1}{2N_c} \bar{C}_-(\mu) \\ &\quad + \frac{\alpha_s}{4\pi} \frac{C_F}{2N_c} C_8(\mu) \left[-6 \ln \frac{\mu^2}{m_b^2} + \int_0^1 du F(u, -z) \Phi_L(u) \right]. \end{aligned} \quad (160)$$

These expressions generalize the well-known leading-order formula

$$a_1^{\text{LO}} = \frac{N_c + 1}{2N_c} C_+^{\text{LO}}(\mu) + \frac{N_c - 1}{2N_c} C_-^{\text{LO}}(\mu). \quad (161)$$

We observe that the scheme dependence, parametrized by B , is cancelled between the coefficient of O_0 in (145) and the matrix element of O_8 in (151). Likewise, the μ dependence of the terms in brackets in (159) and (160) cancels against the scale dependence of the coefficients $\bar{C}_\pm(\mu)$, ensuring a consistent physical result at next-to-leading order in QCD.

The coefficients $a_1(DL)$ and $a_1(D^*L)$ are seen to be non-universal, i.e. they are explicitly dependent on the nature of the final-state mesons. This dependence enters via the light-cone distribution amplitude $\Phi_L(u)$ of the light emission meson and via the analytic form of the hard-scattering kernel ($F(u, z)$ vs. $F(u, -z)$). However, the non-universality enters only at next-to-leading order.

Exercise

Verify that the coefficients $a_1(D^{()}L)$ in (159) and (160) are independent of the unphysical renormalization scale μ up to terms of $\mathcal{O}(\alpha_s^2)$.*

6.2.4 Phenomenological applications for $B \rightarrow D\pi$

An important test of QCD factorization can be performed by comparing the hadronic decays $\bar{B}_d \rightarrow D^{(*)+}L^-$ with the semileptonic processes $\bar{B}_d \rightarrow D^{(*)+}l^-\nu$. It is useful to introduce the ratios

$$R_L^{(*)} = \frac{\Gamma(\bar{B}_d \rightarrow D^{(*)+}L^-)}{d\Gamma(\bar{B}_d \rightarrow D^{(*)+}l^-\bar{\nu})/dq^2|_{q^2=m_L^2}} = 6\pi^2 |V_{ud}|^2 f_L^2 |a_1(D^{(*)}L)|^2 X_L^{(*)}, \quad (162)$$

where $X_\rho = X_\rho^* = 1$ for a vector meson (because the production of the lepton pair via a $V - A$ current in semi-leptonic decays is kinematically equivalent to that of a vector meson with momentum q), whereas X_π and X_π^* deviate from 1 only by (calculable) terms of order m_π^2/m_B^2 , which numerically are below 1%. The main virtue of (162) is that the $B \rightarrow D^{(*)}$ form factors cancel in the ratio. The theoretical prediction for the QCD coefficients, based on QCD factorization to leading power and at NLO in perturbative QCD, is $|a_1(D^{(*)}L)| = 1.05$. The uncertainty of this leading-power result is small, about ± 0.01 . The prediction is to be compared with the experimental results, extracted from (162), which read $|a_1(D^*\pi)| = 1.08 \pm 0.07$, $|a_1(D^*\rho)| = 1.09 \pm 0.10$ and $|a_1(D^*a_1)| = 1.08 \pm 0.11$. These values show fair agreement with the theoretical number, albeit within experimental uncertainties that are still large.

Another interesting consideration concerns the comparison of class-I modes with those of class II and III. For $B \rightarrow D^{(*)}\pi$, all three decay modes, which are related by isospin, have been measured. A nice discussion of the present experimental status and its interpretation in the context of QCD factorization has been given by Neubert & Petrov (2001). Let us briefly discuss here a few important aspects. The experimental status is summarized in Table 3. Denoting the basic topologies from Figs. 14 (a), (b) and (c), by T , C

Table 3. *Experimental data for $\bar{B} \rightarrow D^{(*)}\pi$ branching ratios (in units of 10^{-3}) and extracted parameters $|C - A|/|T + A|$, δ_{TC} (see Neubert & Petrov (2001)).*

	$\bar{B} \rightarrow D\pi$	$\bar{B} \rightarrow D^*\pi$
$\bar{B}^0 \rightarrow D^{(*)+}\pi^-$	3.0 ± 0.4	2.76 ± 0.21
$\bar{B}^0 \rightarrow D^{(*)0}\pi^0$	0.27 ± 0.05	0.17 ± 0.05
$B^- \rightarrow D^{(*)0}\pi^-$	5.3 ± 0.5	4.6 ± 0.4
$ C - A / T + A $	0.42 ± 0.05	0.35 ± 0.05
δ_{TC}	$(56 \pm 20)^\circ$	$(51 \pm 20)^\circ$

and A , respectively (where the notation refers to “tree”, “colour-suppressed tree” and “annihilation”), we have

$$\mathcal{A}(\bar{B}_d \rightarrow D^+\pi^-) = T + A \quad (163)$$

$$\sqrt{2}\mathcal{A}(\bar{B}_d \rightarrow D^0\pi^0) = C - A \quad (164)$$

$$\mathcal{A}(B^- \rightarrow D^0\pi^-) = T + C \quad (165)$$

For later use we may further define the spectator-interaction contribution to T , T_{spec} (see bottom row of Fig. 13). A similar decomposition holds for the $\bar{B} \rightarrow D^*\pi$ modes. Isospin symmetry is reflected in the amplitude relation $\mathcal{A}(\bar{B}_d \rightarrow D^+\pi^-) + \sqrt{2}\mathcal{A}(\bar{B}_d \rightarrow D^0\pi^0) = \mathcal{A}(B^- \rightarrow D^0\pi^-)$, which is manifest in the parametrization of (163) – (165). This means that there are only two independent amplitudes, which we can take to be $(T + A)$ and $(C - A)$. These amplitudes are complex due to strong phases from final-state interactions. Only the relative phase of the two independent amplitudes is an observable and we define δ_{TC} to be the relative phase of $(T + A)$ and $(C - A)$. The phase can then be extracted from the data through the relation

$$\cos \delta_{TC} = \frac{\frac{\tau(\bar{B}^0)}{\tau(B^-)}B(B^- \rightarrow D^0\pi^-) - B(\bar{B}^0 \rightarrow D^+\pi^-) - 2B(\bar{B}^0 \rightarrow D^0\pi^0)}{2\sqrt{2}\sqrt{B(\bar{B}^0 \rightarrow D^+\pi^-)B(\bar{B}^0 \rightarrow D^0\pi^0)}} \quad (166)$$

In the usual heavy-quark limit, $m_b \sim m_c \gg \Lambda_{QCD}$, only T is calculable. T_{spec} , C and A are not, but they are power suppressed. It is instructive to consider the alternative limit $m_b \gg m_c \gg \Lambda_{QCD}$, which allows us to distinguish between m_b and m_c (Beneke et al. 2000). In this case, due to $m_c \ll m_b$, also the D becomes a “light” meson and in that respect the process is similar to $B \rightarrow \pi\pi$, where both T_{spec} and C are calculable. However, since $m_c \gg \Lambda_{QCD} \equiv \Lambda$, the D -meson wave function $\Phi_D(u)$ is highly asymmetric and strongly peaks at $\bar{u} \equiv (1 - u) \sim \Lambda/m_c$, where u is the c -quark momentum fraction. These properties can be used to derive the scaling rules

$$\frac{A}{T} \sim \frac{\Lambda}{m_b} \quad \frac{T_{spec}}{T} \sim \frac{\Lambda}{m_c} \quad \frac{C}{T} \sim \frac{\Lambda}{m_c} \quad (167)$$

The amplitude A is still not calculable in this scheme, while T , T_{spec} and C are. Note that from (167) we can recover the two standard scenarios we have been discussing: In

the heavy-quark limit $m_c \sim m_b$ (167) reduces to a simple power suppression $\sim \Lambda/m_b$ for A , T_{spec} and C compared to T . On the other hand, if m_c becomes a truly light quark, corresponding to the case of $B \rightarrow \pi\pi$, we count $m_c \sim \Lambda$ and see that both T_{spec} and C are of the same order as T , while A is still power suppressed.

The general scenario $m_b \gg m_c \gg \Lambda_{QCD}$ allows us to interpret the experimental results in Table 3. We can even make numerical estimates for T , T_{spec} and C based on the factorization formula for light-light final states (116). These are somewhat model dependent because Φ_D is not known at present. It is not too difficult to accomodate substantial values $|C - A|/|T + A| \sim 0.2 - 0.3$ and $\delta_{TC} \sim 40^\circ$, in qualitative agreement with Table 3. Given the special role of the charm quark (not light, but also not too heavy), the current experimental situation is not in contradiction with QCD factorization in the large- m_b limit. For a comparison with experiment it is useful to keep in mind that, according to (167), the suppression of C over T is only $\sim \Lambda/m_c$ (not Λ/m_b) and that δ_{TC} can also be substantial.

Exercise

Derive the relation (166).

6.3 CP violation in $B \rightarrow \pi^+\pi^-$ decay

Hadronic B decays into a pair of light mesons, such as $B \rightarrow \pi K$ or $B \rightarrow \pi\pi$, have a very rich phenomenology. Their main interest lies in their sensitivity to short-distance flavour physics, including CKM parameters, CP violation and the search for new physics. By way of an outlook we mention here the important example of CP violation in $B \rightarrow \pi^+\pi^-$ decay. The starting point for computing the required decay amplitudes is the effective Hamiltonian in (32). The needed matrix elements of the operators Q_i can be analyzed within QCD factorization using (116). We will not go into the technical details of such an analysis and the discussion of limitations of the approach, in particular from power corrections in Λ_{QCD}/m_b . These can be found in (Beneke et al. 2001). Here we just want to present the phenomenological motivation and to illustrate that a theoretical approach towards a direct dynamical calculation of hadronic matrix elements will be very valuable, even if it necessitates some approximations.

The observable of interest is the time-dependent CP asymmetry in the decays $B^0, \bar{B}^0 \rightarrow \pi^+\pi^-$, which is sensitive to the $B_d - \bar{B}_d$ mixing phase $e^{-2i\beta}$. We define

$$\begin{aligned} A_{CP}^{\pi\pi}(t) &= \frac{B(B^0(t) \rightarrow \pi^+\pi^-) - B(\bar{B}^0(t) \rightarrow \pi^+\pi^-)}{B(B^0(t) \rightarrow \pi^+\pi^-) + B(\bar{B}^0(t) \rightarrow \pi^+\pi^-)} \\ &= -S_{\pi\pi} \sin(\Delta m_B t) + C_{\pi\pi} \cos(\Delta m_B t) \end{aligned} \quad (168)$$

where

$$S_{\pi\pi} = \frac{2 \operatorname{Im} \lambda_{\pi\pi}}{1 + |\lambda_{\pi\pi}|^2}, \quad C_{\pi\pi} = \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2}, \quad \lambda_{\pi\pi} = e^{-2i\beta} \frac{e^{-i\gamma} + P_{\pi\pi}/T_{\pi\pi}}{e^{i\gamma} + P_{\pi\pi}/T_{\pi\pi}} \quad (169)$$

The amplitudes $T_{\pi\pi}$ (“tree”) and $P_{\pi\pi}$ (“penguin”) are the components of the $B \rightarrow \pi^+\pi^-$ amplitude corresponding to the terms in (32) involving λ_u and λ_c , respectively. In the

standard phase conventions λ_c is real and λ_u has a weak phase $-\gamma$, which has been factored out above. The coefficient $C_{\pi\pi}$, which is a function of γ , represents direct CP violation and is expected to be small. We shall not discuss it further here. The mixing-induced asymmetry $S_{\pi\pi}$ depends on γ and β . In fact, in the limit where $P_{\pi\pi}/T_{\pi\pi}$ is set to zero it follows that $\lambda_{\pi\pi} = e^{-2i(\beta+\gamma)} = e^{2i\alpha}$, and hence $S_{\pi\pi} = \sin 2\alpha$. In this limit $\lambda_{\pi\pi}$ is just the relative weak phase between the direct amplitude $B \rightarrow \pi^+\pi^-$ and the one with mixing $B \rightarrow \bar{B} \rightarrow \pi^+\pi^-$. All dependence on hadronic input has canceled in this situation. In practice, however, $P_{\pi\pi}/T_{\pi\pi}$ is not fully negligible. It is here that information on hadronic dynamics becomes crucial. QCD factorization predicts that $P_{\pi\pi}/T_{\pi\pi}$ is suppressed (either by α_s or by powers of Λ_{QCD}/m_b), because $T_{\pi\pi}$ can arise at tree level, $P_{\pi\pi}$ only through loops. Estimates within this framework give values of about $0.25 - 0.3$.

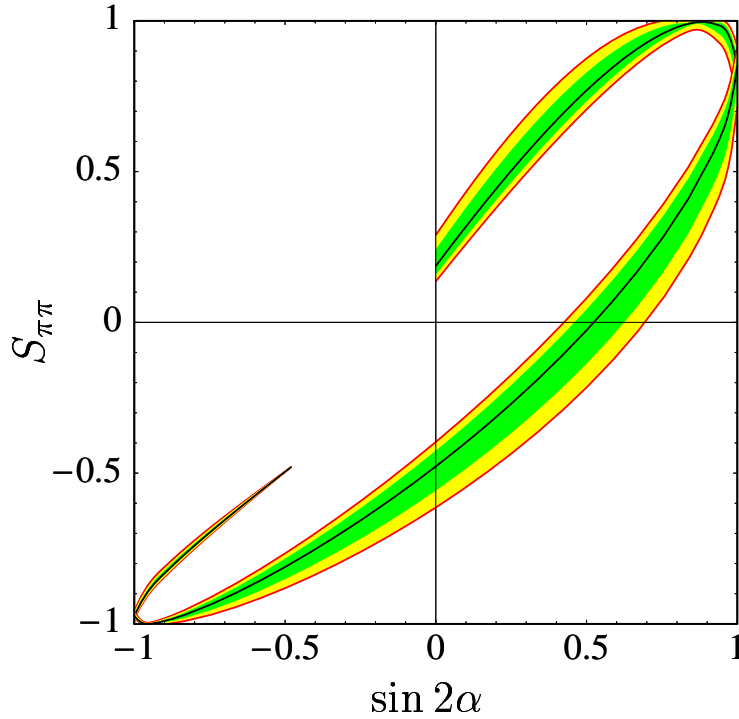


Figure 16. Relation between $\sin 2\alpha$ and the mixing-induced CP asymmetry $S_{\pi\pi}$, assuming $\sin 2\beta = 0.48$. The dark band reflects parameter variations of the first kind, the light band shows the total theoretical uncertainty. The lower portion of the band refers to values $45^\circ < \alpha < 135^\circ$, the upper one to $0 < \alpha < 45^\circ$ (right branch) or $135^\circ < \alpha < 180^\circ - \beta$ (left branch).

To illustrate the effect of penguins, we first assume that $|V_{ub}/V_{cb}|$ and the weak phase β have been determined accurately. Then using $\gamma = 180^\circ - \alpha - \beta$ the expression for $\lambda_{\pi\pi}$ in (169) becomes a function of α and our prediction for the penguin-to-tree ratio $P_{\pi\pi}/T_{\pi\pi}$. If we further assume that the unitarity triangle lies in the upper half of the $(\bar{\rho}, \bar{\eta})$ plane, then a measurement of $S_{\pi\pi}$ determines $\sin 2\alpha$ with at most a two-fold discrete ambiguity. Figure 16 shows the relation between the two quantities for the particular case where $|V_{ub}/V_{cb}| = 0.085$ and $\beta = 14.3^\circ$, corresponding to $\sin 2\beta = 0.48$. The dark band shows the theoretical uncertainty due to input parameter variations, whereas the light band indicates the total theoretical uncertainty including estimates of the effect of power

corrections. We observe that for negative values $\sin 2\alpha$ as preferred by the global analysis of the unitarity triangle, a measurement of the coefficient $S_{\pi\pi}$ could be used to determine $\sin 2\alpha$ with a theoretical uncertainty of about ± 0.1 . Interestingly, for such values of $\sin 2\alpha$ the “penguin pollution” effect enhances the value of the mixing-induced CP asymmetry, yielding values of $S_{\pi\pi}$ between -0.5 and -1 . Such a large asymmetry should be relatively easy to observe experimentally.

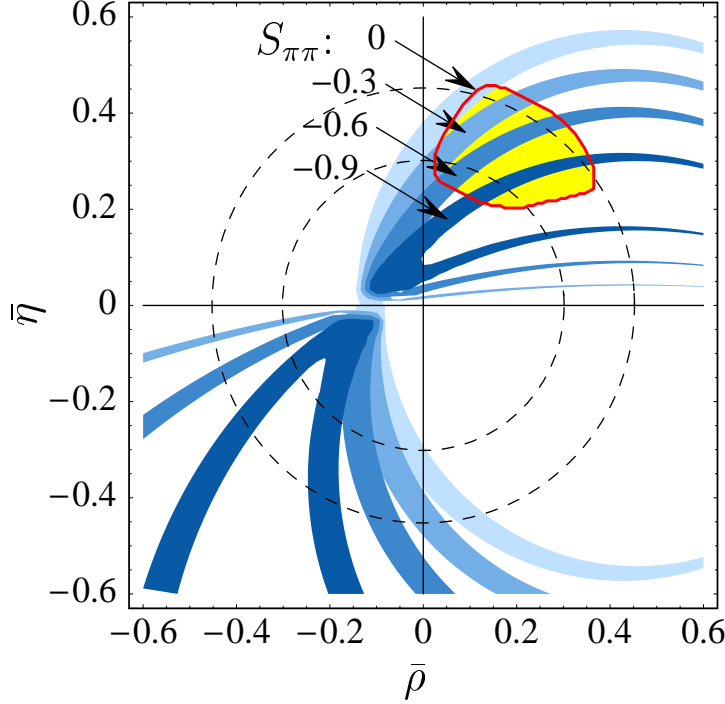


Figure 17. Allowed regions in the $(\bar{\rho}, \bar{\eta})$ plane corresponding to constant values of the mixing-induced asymmetry $S_{\pi\pi}$ assuming the Standard Model. The widths of the bands reflect the total theoretical uncertainty. The corresponding bands for positive values of $S_{\pi\pi}$ are obtained by a reflection about the $\bar{\rho}$ axis. The light circled area in the left-hand plot shows the allowed region obtained from the standard global fit of the unitarity triangle (Höcker et al. 2001).

Although it illustrates nicely the effect of “penguin pollution” on the determination of $\sin 2\alpha$, Figure 16 is not the most appropriate way to display the constraint on the unitarity triangle implied by a measurement of $S_{\pi\pi}$. In general, there is a four-fold discrete ambiguity in the determination of $\sin 2\alpha$, which we have reduced to a two-fold ambiguity by assuming that the triangle lies in the upper half-plane. Next, and more importantly, we have assumed that $|V_{ub}/V_{cb}|$ and β are known with precision, whereas α is undetermined. However, in the Standard Model $|V_{ub}/V_{cb}|$ and the angles α, β, γ are all functions of the Wolfenstein parameters $\bar{\rho}$ and $\bar{\eta}$. It is thus more appropriate to represent the constraint implied by a measurement of $S_{\pi\pi}$ as a band in the $(\bar{\rho}, \bar{\eta})$ plane. To this end, we write

$$e^{\mp i\gamma} = \frac{\bar{\rho} \mp i\bar{\eta}}{\sqrt{\bar{\rho}^2 + \bar{\eta}^2}} \quad e^{-2i\beta} = \frac{(1 - \bar{\rho})^2 - \bar{\eta}^2 - 2i\bar{\eta}(1 - \bar{\rho})}{(1 - \bar{\rho})^2 + \bar{\eta}^2} \quad \frac{P_{\pi\pi}}{T_{\pi\pi}} = \frac{r_{\pi\pi} e^{i\phi_{\pi\pi}}}{\sqrt{\bar{\rho}^2 + \bar{\eta}^2}} \quad (170)$$

where $r_{\pi\pi} e^{i\phi_{\pi\pi}}$ is independent of $\bar{\rho}$ and $\bar{\eta}$. We now insert these relations into (169) and

draw contours of constant $S_{\pi\pi}$ in the $(\bar{\rho}, \bar{\eta})$ plane. The result is shown by the bands in Figure 17. The widths of the bands reflect the total theoretical uncertainty (including power corrections). For clarity we show only bands for negative values of $S_{\pi\pi}$; those corresponding to positive $S_{\pi\pi}$ values can be obtained by a reflection about the $\bar{\rho}$ axis (i.e., $\bar{\eta} \rightarrow -\bar{\eta}$). Note that even a rough measurement of $S_{\pi\pi}$ would translate into a rather narrow band in the $(\bar{\rho}, \bar{\eta})$ plane, which intersects the ring representing the $|V_{ub}/V_{cb}|$ constraint at almost right angle. In a similar way, the constraint is also quite robust against hadronic uncertainties. Even the approximate knowledge of hadronic matrix elements, as provided by QCD factorization, will therefore be very valuable and can lead to powerful constraints on the Wolfenstein parameters.

7 Summary

In these lectures we have discussed the theory of heavy quarks, focussing on the important case of B physics. (All methods relying on the heavy-quark limit could in principle be applied to charmed hadrons as well, but they are in most cases much less reliable due to the smaller value of the charm mass.) We shall conclude by summarizing the key points.

- A crucial and general idea for dealing with QCD effects is the *factorization of short-distance and long-distance dynamics*. We have encountered this principle in many different forms and applications:
 - The OPE to construct the effective weak Hamiltonians ($\mathcal{H}_{eff}^{\Delta B=1,2}$) factorizes the short-distance scales of order M_W , m_t from the scales of order m_b .
 - The heavy-quark scale m treated as a short-distance scale can be factorized further from the intrinsic long-distance scale of QCD, Λ_{QCD} . This leads to a systematic expansion of observables simultaneously in $1/m$ and $\alpha_s(m)$ with often very important simplifications. The precise formulation of this class of factorization depends on the physical situation and can take the form of HQET, LEET, HQE or QCD factorization in exclusive hadronic B decays.
- HQET exhibits the spin-flavour symmetry of QCD in the heavy-quark limit, which allows us to relate different form factors, and makes the m_Q dependence explicit. Examples of typical applications are $B \rightarrow D^{(*)}l\nu$ or f_B .
- HQE is a theory for inclusive B decays. It justifies the “parton model” and allows us to study the nonperturbative power corrections. This is of great use for processes as $B \rightarrow X_{c,u}l\nu$, $B \rightarrow X_s\gamma$, $B \rightarrow X_sl^+l^-$, and the lifetimes of b -flavoured hadrons.
- QCD factorization, finally, refers to a framework for analyzing exclusive hadronic B decays with a fast light meson as for instance $B \rightarrow D\pi$, $B \rightarrow \pi\pi$, $B \rightarrow \pi K$ and $B \rightarrow V\gamma$.

With these tools at hand we are in a good position to make full use of the rich experimental results in the physics of heavy flavours. We can determine fundamental parameters of the flavour sector, such as V_{ub} , V_{cb} , V_{td}/V_{ts} , η and $\sin 2\alpha$, and probe electroweak dynamics at the quantum level through $b \rightarrow s\gamma$ or $B - \bar{B}$ mixing. This will enable us to

thoroughly test the standard model and to learn about new structures and phenomena that are yet to be discovered.

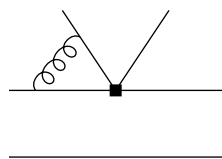
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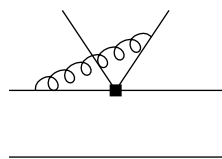
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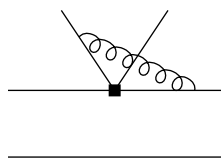
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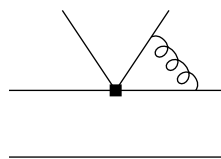
(a)



(b)



(c)



(d)

