

# Developments in Neutrino Physics

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## Abstract

Measurements of various features of the fluxes of atmospheric and solar neutrinos have provided evidence for neutrino oscillations and therefore for neutrino masses and mixing. We review the phenomenology of neutrino oscillations in vacuum and in matter. We present the existing evidence from solar and atmospheric neutrinos as well as the results from laboratory searches, including the final status of the LSND experiment. We describe the theoretical inputs that are used to interpret the experimental results in terms of neutrino oscillations. We derive the allowed ranges for the mass and mixing parameters in three frameworks: First, each set of observations is analyzed separately in a two-neutrino framework; Second, the data from solar and atmospheric neutrinos are analyzed in a three active neutrino framework; Third, the LSND results are added, and the status of accommodating all three signals in the framework of three active and one sterile light neutrinos is presented. We review the theoretical implications of these results: the existence of new physics, the estimate of the scale of this new physics and the lessons for grand unified theories, for supersymmetric models with  $R$ -parity violation, for models of extra dimensions and singlet fermions in the bulk, and for flavor models.

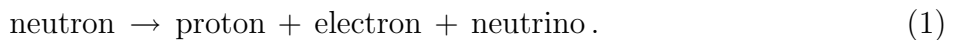
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## I. INTRODUCTION

In 1930 Wolfgang Pauli postulated the existence of the neutrino in order to reconcile data on the radioactive decay of nuclei with energy conservation. In radioactive decays, nuclei of atoms mutate into different nuclei when neutrons are transformed into slightly lighter protons with the emission of electrons:



Without the neutrino, energy conservation requires that the electron and proton share the neutron's energy. Each electron is therefore produced with a fixed energy while experiments indicated conclusively that the electrons were not mono-energetic but observed with a range of energies. This energy range corresponded exactly to the many ways the three particles in the final state of the reaction above can share energy while satisfying its conservation law. The postulated neutrino had no mass, no electric charge and, for all practical purposes, does not interact with matter; it just serves as an agent to balance energy and momentum in above reaction. In fact, Pauli pointed out that for the neutrino to do the job, it had to weigh less than one percent of the proton mass, thus establishing the first limit on neutrino mass.

Observing neutrinos is straightforward — in principle. Pauli had to wait a quarter of a century before Fred Reines and Clyde Cowan Jr. observed neutrinos produced by a nuclear reactor. In the presence of protons, neutrinos occasionally initiate the inverse reaction of radioactive decay:



Experimentally, one exposes a material rich in protons to a neutrino beam and simply looks for the coincident appearance of an electron and a neutron. In the alternative possibility where the incident neutrino carries muon flavour, a muon appears in the final state instead.

By the 1960's neutrino beams passed from a futuristic dream to one of the most important tools of particle physics. The technique contributed in important ways to the discovery of quarks, the constituents of protons and neutrons. The technique is conceptually simple, though technologically very challenging. A very intense beam of accelerated protons is shot into a *beam-dump* which typically consists of a kilometer-long mound of earth or a 100 meter-long block of stainless steel. Particle physics does the rest. Protons interact with nuclei in the dump and produce tens of pions in each collision. Charged pions decay into a muon and a neutrino. The material in the dump will eventually absorb the muons, photons and any

other charged particles, so that only neutrinos exit at the opposite end, forming an intense and controlled beam.

Neutrinos are also produced in natural sources. Starting at the 1960's, neutrinos produced in the sun and in the atmosphere have been observed. As we will see in this review, these observations play an important role in understanding the detailed features of the neutrinos. In 1987, neutrinos from a supernova in the Large Magellanic Cloud were also detected.

The properties of the neutrino and in particular the question of its mass have intrigued physicists' minds ever since it was proposed. In the laboratory neutrino masses have been searched for in two types of experiments: (i) direct kinematic searches of neutrino mass, of which the most sensitive is the study of tritium beta decay, and (ii) neutrinoless double- $\beta$  decay experiments. Experiments achieved higher and higher precision, reaching upper limits for the electron-neutrino mass of  $10^{-9}$  the proton mass, rather than the  $10^{-2}$  originally obtained by Pauli. This raised the question of whether neutrinos are truly massless like photons.

Can one go further below the eV scale (that is  $10^{-9}$  the proton mass) in the search for neutrino masses? This is a very difficult task in direct measurements. In 1957, however, Bruno Pontecorvo realized that the existence of neutrino masses implies the possibility of neutrino oscillations. This phenomenon is similar to what happens in the quark sector, where neutral kaons oscillate. Flavor oscillations of neutrinos have been searched for using either neutrino beams from reactors or accelerators, or natural neutrinos generated at astrophysical sources (the Sun giving the largest flux) or in the atmosphere. The longer the distance that the neutrinos travel from their production point to the detector, the smaller masses that can be signalled by their oscillation. Indeed, the solar neutrinos allow us to search for masses that are as small as  $10^{-5}$  eV, that is  $10^{-14}$  of the proton mass!

In fact, in recent years, experiments studying natural neutrino fluxes have provided us with the strongest evidence of neutrino masses and mixing. Experiments that measure the flux of atmospheric neutrinos have found results that suggest the disappearance of muon-neutrinos when propagating over distances of order hundreds (or more) kilometers. Experiments that measure the flux of solar neutrinos found results that suggest the disappearance of electron-neutrinos while propagating within the Sun or between the Sun and the Earth. The disappearance of both atmospheric  $\nu_\mu$ 's and solar  $\nu_e$ 's is most easily explained in terms of neutrino oscillations. As concerns experiments performed with laboratory beams, most have given no evidence of oscillations. One exception is the LSND experiment which has observed the appearance of electron anti-neutrinos in a muon anti-neutrino beam. This signal has not been confirmed so far by any other experiment.

What can we learn from measurements of neutrino masses about our theories of particle physics? The Standard Model (SM) of particle physics is a mathematical description of the strong, weak and electromagnetic interactions. Since it was conceived in the 1960's by Glashow, Salam and Weinberg, it has successfully passed numerous experimental tests. In the absence of any direct evidence for their mass, neutrinos were introduced in the SM as truly massless fermions for which no gauge invariant renormalizable mass term can be constructed. Consequently, in the SM there is neither mixing nor CP violation in the lepton sector. Therefore, experimental evidence for neutrino masses or mixing or leptonic CP violation provides an unambiguous signal for *new physics* (NP).

The SM prediction of massless neutrinos is an accidental fact: unlike photons, no profound principle protects them from having a mass. On the contrary, modern elementary particle theories anticipate ways in which they have small, but definitely non-vanishing masses. In fact, there are good theoretical reasons to expect that neutrinos are massive but much lighter than all the charged fermions of the SM. Specifically, it is very likely that neutrino masses are inversely proportional to the scale of NP. Consequently, if neutrino masses are measured, we can estimate the relevant *new scale*.

All dimensionless flavor-blind parameters of the SM, that is, the three gauge couplings and the quartic Higgs coupling, are of order one. In contrast, most of the flavor parameters – quark and charged lepton masses (except for the mass of the top quark), and the three CKM mixing angles – are small and hierarchical. This situation constitutes *the flavor puzzle*: Within the SM, the hierarchical structure can be accommodated but is not explained. If neutrinos have masses, then there are nine flavor parameters beyond the thirteen of the SM. If some of these extra parameters are measured, we will be able to test and refine theories that try to solve the flavor puzzle.

In *grand unified theories* (GUTs), lepton masses are often related to quark masses. Measurements of neutrino parameters provide further tests of these theories.

The values of neutrino parameters that explain the anomalies observed in atmospheric and solar neutrino fluxes can be used to address the many theoretical questions described above: they imply NP, they suggest an energy scale at which this NP takes place, they provide stringent tests of flavor models, and they probe GUTs.

In this review we first present the low energy formalism for adding neutrino masses to the SM and the induced leptonic mixing (Sec. II) and then describe the phenomenology associated with neutrino oscillations in vacuum and in matter (Sec. III). In Secs. IV and V we discuss the evidence from, respectively, solar and atmospheric neutrinos. We review the theoretical modeling that is involved in interpreting the experimental results in terms of neutrino oscillations. We briefly describe the techniques used in the derivation of the allowed ranges for the neutrino flavor parameters which, in these sections, is performed in the framework of mixing between two neutrinos. Sec. VI is devoted to the results from searches at laboratory experiments including the final status of the LSND experiment. The two more robust pieces of evidence, from solar and atmospheric neutrinos, can be accommodated assuming masses and mixing of three standard neutrinos: in Sec. VII.A we derive the allowed ranges of parameters in this case. The present phenomenological status of the possibility of mixing between the three standard neutrinos with a light sterile one, needed to accommodate also the LSND result, is discussed in Sec. VII.B. In Secs. VIII and IX we explain the various lessons for theory which can be drawn on the basis of these interpretations, focusing on models that explain the atmospheric and solar neutrino data through mixing among three active neutrinos. In particular, we discuss flavor models, GUTs, supersymmetric models with  $R$ -parity violation, and models of extra dimensions. Our conclusions are summarized in Sec. X.

## II. THE STANDARD MODEL AND NEUTRINO MASSES

One of the most beautiful aspects of modern theories of particles physics is the relation between forces mediated by spin-1 particles and local (gauge) symmetries. Within the

Standard Model, the strong, weak and electromagnetic interactions are related to, respectively,  $SU(3)$ ,  $SU(2)$  and  $U(1)$  gauge groups. Many features of the various interactions are then explained by the symmetry to which they are related. In particular, the way that the various fermions are affected by the different types of interactions is determined by their representations (or simply their charges in the case of Abelian gauge symmetries) under the corresponding symmetry groups.

Neutrinos are fermions that have neither strong nor electromagnetic interactions. In group theory language, they are singlets of  $SU(3)_C \times U(1)_{EM}$ . Active neutrinos have weak interactions, that is, they are not singlets of  $SU(2)_L$ . Sterile neutrinos have none of the SM gauge interactions and they are singlets of the SM gauge group.

The SM has three active neutrinos. They reside in lepton doublets,

$$L_\ell = \begin{pmatrix} \nu_{L\ell} \\ \ell_L^- \end{pmatrix}, \quad \ell = e, \mu, \tau. \quad (3)$$

Here  $e$ ,  $\mu$  and  $\tau$  are the charged lepton mass eigenstates. The three neutrino interaction eigenstates,  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$ , are defined as the  $SU(2)_L$ -partners of these mass eigenstates. In other words, the charged current interaction terms for leptons read

$$-\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \sum_\ell \bar{\nu}_{L\ell} \gamma^\mu \ell_L^- W_\mu^+ + \text{h.c.} \quad (4)$$

In addition, the SM neutrinos have neutral current interactions,

$$-\mathcal{L}_{NC} = \frac{g}{2 \cos \theta_W} \sum_\ell \bar{\nu}_{L\ell} \gamma^\mu \nu_{L\ell} Z_\mu^0. \quad (5)$$

Eqs. (4) and (5) give all the neutrino interactions within the SM.

The measurement of the decay width of the  $Z^0$  boson into neutrinos makes the existence of three, and only three, light (that is,  $m_\nu \lesssim m_Z/2$ ) active neutrinos an experimental fact. When expressed in units of the SM prediction for a single neutrino generation, one gets (Groom *et al.*, 2000)

$$\begin{aligned} N_\nu &= 2.994 \pm 0.012 \quad (\text{Standard Model fits to LEP data}), \\ N_\nu &= 3.00 \pm 0.06 \quad (\text{Direct measurement of invisible Z width}). \end{aligned} \quad (6)$$

### A. The Standard Model Implies $m_\nu = 0$

The SM is based on the gauge group

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y, \quad (7)$$

with three fermion generations, where a single generation consists of five different representations of the gauge group,

$$Q_L(3, 2)_{+1/6}, \quad U_R(3, 1)_{+2/3}, \quad D_R(3, 1)_{-1/3}, \quad L_L(1, 2)_{-1/2}, \quad E_R(1, 1)_{-1}. \quad (8)$$

Our notation here means that, for example, a left-handed lepton field  $L_L$  is a singlet (1) of the  $SU(3)_C$  group, a doublet (2) of the  $SU(2)_L$  group, and carries hypercharge  $-1/2$  under the  $U(1)_Y$  group.

The vacuum expectations value (VEV) of the single Higgs doublet  $\phi(1, 2)_{+1/2}$  breaks the symmetry,

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \implies G_{\text{SM}} \rightarrow SU(3)_C \times U(1)_{\text{EM}}. \quad (9)$$

Since the fermions reside in chiral representations of the gauge group, there can be no bare mass terms. Fermions masses arise from the Yukawa interactions,

$$-\mathcal{L}_{\text{Yukawa}} = Y_{ij}^d \overline{Q_{Li}} \phi D_{Rj} + Y_{ij}^u \overline{Q_{Li}} \tilde{\phi} U_{Rj} + Y_{ij}^\ell \overline{L_{Li}} \phi E_{Rj} + \text{h.c.}, \quad (10)$$

(where  $\tilde{\phi} = i\tau_2 \phi^*$ ) after spontaneous symmetry breaking. The Yukawa interactions of Eq. (10) lead to charged fermion masses but leave the neutrinos massless. One could think that neutrino masses would arise from loop corrections if these corrections induced effective terms

$$\frac{Y_{ij}^\nu}{v} \phi \phi L_{Li} L_{Lj}. \quad (11)$$

This however cannot happen, as can be easily understood by examining the accidental symmetries of the Standard Model. (It often happens that, as a consequence of the symmetries that define a model and of its particle content, all renormalizable Lagrangian terms obey additional symmetries, that are not a-priori imposed on the model. These are called accidental symmetries.) Within the SM, with the gauge symmetry of Eq. (7) and the particle content of Eq. (8), the following accidental global symmetry arises:

$$G_{\text{SM}}^{\text{global}} = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau. \quad (12)$$

Here  $U(1)_B$  is the baryon number symmetry, and  $U(1)_{e,\mu,\tau}$  are the three lepton flavor symmetries, with total lepton number given by  $L = L_e + L_\mu + L_\tau$ . Terms of the form (11) violate  $G_{\text{SM}}^{\text{global}}$  and therefore cannot be induced by loop corrections. Furthermore, the  $U(1)_{B-L}$  subgroup of  $G_{\text{SM}}^{\text{global}}$  is non-anomalous. Terms of the form (11) have  $B - L = -2$  and therefore cannot be induced even by nonperturbative corrections.

It follows that the SM predicts that neutrinos are precisely massless. Consequently, there is neither mixing nor CP violation in the leptonic sector.

## B. Extensions of the Standard Model Allow $m_\nu \neq 0$

There are many good reasons to think that the SM is not a complete picture of Nature. For example, the fine-tuning problem of the Higgs mass can be solved by Supersymmetry; gauge coupling unification and the variety of gauge representations may find an explanation in GUTs; baryogenesis can be initiated by decays of heavy singlet fermions (leptogenesis); and the existence of gravity suggests that string theories are relevant to Nature. If any of these (or many other proposed) extensions is indeed realized in Nature, the SM must be

thought of as an effective low energy theory. That means that it is a valid approximation up to the scale  $\Lambda_{\text{NP}}$  which characterizes the NP.

By thinking of the SM as an effective low energy theory, we still retain the gauge group (7), the fermionic spectrum (8), and the pattern of spontaneous symmetry breaking (9) as valid ingredients to describe Nature at energies  $E \ll \Lambda_{\text{NP}}$ . The SM predictions are, however, modified by small effects that are proportional to powers of  $E/\Lambda_{\text{NP}}$ . In other words, the difference between the SM as a complete description of Nature and as a low energy effective theory is that in the latter case we must consider also nonrenormalizable terms.

There is no reason for generic NP to respect the accidental symmetries of the SM (12). Indeed, there is a single set of dimension-five terms that is made of SM fields and is consistent with the gauge symmetry, and this set violates (12). It is given by

$$\frac{Z_{ij}^\nu}{\Lambda_{\text{NP}}} \phi \phi L_{Li} L_{Lj}. \quad (13)$$

(While these terms have the same form as (11) we are thinking now not about SM radiative corrections as their source but about some heavy fields, related to NP, which can induce such terms by tree or loop diagrams.) In particular, (13) violates  $L$  (and  $B - L$ ) by two units and leads, upon spontaneous symmetry breaking, to neutrino masses:

$$(M_\nu)_{ij} = \frac{Z_{ij}^\nu}{2} \frac{v^2}{\Lambda_{\text{NP}}}. \quad (14)$$

A few comments are in order, regarding Eq. (14):

1) Since Eq. (14) would arise in a generic extension of the SM, we learn that neutrino masses are very likely to appear if there is NP.

2) If neutrino masses arise effectively from nonrenormalizable terms, we gain an understanding not only for the existence of neutrino masses but also for their smallness. The scale of neutrino masses is suppressed, compared to the scale of charged fermion masses, by  $v/\Lambda_{\text{NP}}$ .

3) The terms (14) break not only total lepton number but also the lepton flavor symmetry  $U(1)_e \times U(1)_\mu \times U(1)_\tau$ . Therefore we should expect lepton mixing and CP violation.

The best known scenario that leads to (13) is the *see-saw mechanism* (Gell-Mann *et al.*, 1979; Yanagida, 1979). Here one assumes the existence of heavy sterile neutrinos  $N_i$ . Such fermions have, in general, bare mass terms and Yukawa interactions:

$$-\mathcal{L}_N = \frac{1}{2} M_{Nij} \overline{N}_i^c N_j + Y_{ij}^\nu \overline{L}_{Li} \tilde{\phi} N_j + \text{h.c.}. \quad (15)$$

The resulting mass matrix (see Sec. II.C for details) in the basis  $\begin{pmatrix} \nu_{Li} \\ N_j \end{pmatrix}$  has the following form:

$$M_\nu = \begin{pmatrix} 0 & Y^\nu \frac{v}{\sqrt{2}} \\ (Y^\nu)^T \frac{v}{\sqrt{2}} & M_N \end{pmatrix}. \quad (16)$$

If the eigenvalues of  $M_N$  are all well above the electroweak breaking scale  $v$ , then the diagonalization of  $M_\nu$  leads to three light mass eigenstates with a mass matrix of the form



(14). In particular, the scale  $\Lambda_{\text{NP}}$  is identified with the mass scale of the heavy sterile neutrinos, that is the typical scale of the eigenvalues of  $M_N$ .

Two well-known examples of extensions of the SM that lead to a see-saw mechanism for neutrino masses are SO(10) GUTs (Gell-Mann *et al.*, 1979; Yanagida, 1979) and left-right symmetry (Mohapatra and Senjanovic, 1980).

### C. Dirac and Majorana Neutrino Mass Terms

If the only modification that we make to the SM is to assume that it is a low energy effective theory, that is, allowing for non-renormalizable terms that are consistent with the gauge symmetry and the fermionic content of the SM, then the only way that neutrinos can gain masses is through terms of the form (13). These are Majorana mass terms which, in particular, violate lepton number by two units.

One can, however, open up other possibilities by adding new fields. The most relevant extension is that where an arbitrary number  $m$  of sterile neutrinos  $\nu_{si}(1, 1)_0$  is added to the three standard generations of Eq. (8). Now there are, in general, two types of mass terms that arise from *renormalizable* terms:

$$-L_{M\nu} = M_{Dij}\overline{\nu_{Li}}\nu_{sj} + \frac{1}{2}M_{Nij}\overline{\nu_{si}^c}\nu_{sj} + \text{h.c.} \quad (17)$$

Here  $\nu^c$  indicates a charged conjugated field,  $\nu^c = C\overline{\nu}^T$  and  $C$  is the charged conjugation matrix.

The first term is a Dirac mass term. It has the following properties:

(i) Since it transforms as the doublet representation of  $SU(2)_L$ , it is generated after spontaneous electroweak symmetry breaking from the Yukawa interactions  $Y_{ij}^{\nu}\overline{L_{Li}}\tilde{\phi}\nu_{sj}$ , similarly to the charged fermion masses discussed in Sec. II.A.

(ii) Since it has a neutrino field and an antineutrino field, it conserves total lepton number [though it breaks the lepton flavor number symmetries of Eq. (12)].

(iii)  $M_D$  is a complex  $3 \times m$  matrix.

The second term in Eq. (17) is a Majorana mass term. It is different from the Dirac mass terms in many important aspects:

(i) It is a singlet of the SM gauge group. Therefore, it can appear as a bare mass terms. [Had we written a similar term for the active neutrinos, it would transform as a triplet of  $SU(2)_L$ . In the absence of Higgs triplet, it cannot be generated by renormalizable Yukawa interactions. Such terms are generated for active neutrinos from the non-renormalizable Yukawa interactions of Eq. (13).]

(ii) Since it involves two neutrino fields, it breaks lepton number conservation by two units. More generally, such a term is allowed only if the neutrinos carry no additive conserved charge. This is the reason that such terms are not allowed for any charged fermions which, by definition, carry  $U(1)_{\text{EM}}$  charges.

(iii)  $M_N$  is a symmetric matrix (as follows from simple Dirac algebra) of dimension  $m \times m$ .

It is convenient to define a  $(3 + m)$ -dimensional neutrino vector  $\vec{\nu}$ ,

$$\vec{\nu} = \begin{pmatrix} \nu_{Li} \\ \nu_{sj} \end{pmatrix}. \quad (18)$$

That allows us to rewrite Eq. (17) in a unified way:

$$-L_{M_\nu} = \frac{1}{2} \overline{\nu^c} M_\nu \nu + \text{h.c.} , \quad (19)$$

where

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_N \end{pmatrix}. \quad (20)$$

The matrix  $M_\nu$  is complex and symmetric. It can be diagonalized by a unitary matrix of dimension  $(3 + m)$ . The resulting mass eigenstates,  $\nu_k$ , obey the Majorana condition,  $\nu_k^c = \nu_k$ .

There are three interesting cases, differing in the hierarchy of scales between  $M_N$  and  $M_D$ :

(i) One can assume that the scale of the mass eigenvalues of  $M_N$  is much higher than the scale of electroweak symmetry breaking  $\langle \phi \rangle$ . This is the natural situation in various extensions of the SM that are characterized by a high energy scale. We are back in the framework of the see-saw mechanism discussed in the previous section. If we simply integrate out the sterile neutrinos, we get a low energy effective theory with three light, active neutrinos of the Majorana-type.

(ii) One can assume that the scale of some eigenvalues of  $M_N$  is not higher than the electroweak scale. Now the SM is not even a good low energy effective theory: there are more than three light neutrinos, and they are mixtures of doublet and singlet fields. These light fields are all of the Majorana-type.

(iii) One can assume that  $M_N = 0$ . This is equivalent to imposing lepton number symmetry on this model. Again, the SM is not even a good low energy theory: both the fermionic content and the assumed symmetries are different. (Recall that within the SM lepton number is an accidental symmetry.) Now only the first term in Eq. (17) is allowed, which is a Dirac mass term. It is generated by the Higgs mechanism in the same way that charged fermions masses are generated. If indeed it is the only neutrino mass term present and  $m = 3$ , we can identify the three sterile neutrinos with the right handed component of a four-component spinor neutrino field (actually with its charge conjugate). In this way, the six massive Majorana neutrinos combine to form three massive neutrino Dirac states, equivalently to the charged fermions. In this particular case the  $6 \times 6$  diagonalizing matrix is block diagonal and it can be written in terms of a  $3 \times 3$  unitary matrix.

From the phenomenological point of view, it will make little difference for our purposes whether the light neutrinos are of the Majorana- or Dirac-type. In particular, the analysis of neutrino oscillations is the same in both cases. Only in the discussion of neutrinoless double beta decay will the question of Majorana versus Dirac neutrinos be crucial.

From the theoretical model building point of view, however, the two cases are very different. In particular, the see-saw mechanism provides a natural explanation for the lightness of Majorana neutrinos, while for Dirac neutrinos there is no such generic mechanism.

## D. Lepton Mixing

We briefly review here our notation for lepton mixing. We denote the neutrino mass eigenstates by  $(\nu_1, \nu_2, \nu_3, \dots, \nu_n)$  where  $n = 3 + m$ , and the charged lepton mass eigenstates

by  $(e, \mu, \tau)$ . The corresponding interaction eigenstates are denoted by  $(e^I, \mu^I, \tau^I)$  and  $\vec{\nu} = (\nu_{Le}, \nu_{L\mu}, \nu_{L\tau}, \nu_{s1}, \dots, \nu_{sm})$ . In the mass basis, leptonic charged current interactions are given by

$$-\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}} (\overline{e_L} \ \overline{\mu_L} \ \overline{\tau_L}) \gamma^\mu U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \vdots \\ \nu_n \end{pmatrix} W_\mu^+ - \text{h.c.} \quad (21)$$

Here  $U$  is a  $3 \times n$  matrix (Schechter and Valle; 1980a, 1980b).

Given the charged lepton mass matrix  $M_\ell$  and the neutrino mass matrix  $M_\nu$  in some interaction basis,

$$-\mathcal{L}_M = (\overline{e_L^I} \ \overline{\mu_L^I} \ \overline{\tau_L^I}) M_\ell \begin{pmatrix} e_R^I \\ \mu_R^I \\ \tau_R^I \end{pmatrix} + \frac{1}{2} \overline{\vec{\nu}^c} M_\nu \vec{\nu} + \text{h.c.} \ , \quad (22)$$

we can find the diagonalizing matrices  $V^\ell$  and  $V^\nu$ :

$$V^{\ell\dagger} M_\ell M_\ell^\dagger V^\ell = \text{diag}(m_e^2, m_\mu^2, m_\tau^2), \quad V^{\nu\dagger} M_\nu^\dagger M_\nu V^\nu = \text{diag}(m_1^2, m_2^2, m_3^2, \dots, m_n^2). \quad (23)$$

Here  $V^\ell$  is a unitary  $3 \times 3$  matrix while  $V^\nu$  is a unitary  $n \times n$  matrix. The  $3 \times n$  mixing matrix  $U$  can be found from these diagonalizing matrices:

$$U_{ij} = P_{\ell,ii} V_{ik}^{\ell\dagger} V_{kj}^\nu (P_{\nu,jj}). \quad (24)$$

A few comments are in order:

(i) Note that the indices  $i$  and  $k$  run from 1 to 3, while  $j$  runs from 1 to  $n$ . In particular, only the first three lines of  $V^\nu$  play a role in Eq. (24).

(ii)  $P_\ell$  is a diagonal  $3 \times 3$  phase matrix, that is conventionally used to reduce by three the number of phases in  $U$ .

(iii)  $P_\nu$  is a diagonal matrix with additional arbitrary phases (chosen to reduce the number of phases in  $U$ ) only for Dirac states. For Majorana neutrinos, this matrix is simply a unit matrix. The reason for that is that if one rotates a Majorana neutrino by a phase, this phase will appear in its mass term which will no longer be real.

We conclude that the number of phases that can be absorbed by redefining the mass eigenstates depends on whether the neutrinos are Dirac or Majorana particles. In particular, if there are only three Majorana neutrinos,  $U$  is a  $3 \times 3$  matrix analogous to the CKM matrix for the quarks (Maki, Nakagawa and Sakata, 1962; Kobayashi and Maskawa, 1973) but due to the Majorana nature of the neutrinos it depends on six independent parameters: three mixing angles and three phases. [CP conservation implies that the three lepton phases are either zero or  $\pi$  (Schechter and Valle, 1981; Wolfenstein 1981).] This is to be compared to the case of three Dirac neutrinos, where the number of physical phases is one, similarly to the CKM matrix.

If no new interactions for the charged leptons are present we can identify their interaction eigenstates with the corresponding mass eigenstates after phase redefinitions. In this case the charged current lepton mixing matrix  $U$  is simply given by a  $3 \times n$  sub-matrix of the unitary matrix  $V^\nu$ .

### III. NEUTRINO OSCILLATIONS

#### A. Neutrino Oscillations in Vacuum

Neutrino oscillations in vacuum would arise if neutrinos were massive and mixed. In other words, the neutrino state that is produced by electroweak interactions is not a mass eigenstate. This phenomenon was first pointed out by Pontecorvo in 1957 (Pontecorvo, 1957) while the possibility of arbitrary mixing between two massive neutrino states was first introduced in Maki, Nakagawa and Sakata (1962).

From Eq. (21) we see that if neutrinos have masses, the weak eigenstates,  $\nu_\alpha$ , produced in a weak interaction (*i.e.*, an inverse beta reaction or a weak decay) are, in general, linear combinations of the mass eigenstates  $\nu_i$

$$|\nu_\alpha\rangle = \sum_{i=1}^n U_{\alpha i}^* |\nu_i\rangle \quad (25)$$

where  $n$  is the number of light neutrino species and  $U$  is the the mixing matrix in Eq.(24). (Implicit in our definition of the state  $|\nu\rangle$  is its energy-momentum and space-time dependence). After traveling a distance  $L$  (or, equivalently for relativistic neutrinos, time  $t$ ), a neutrino originally produced with a flavor  $\alpha$  evolves as follows:

$$|\nu_\alpha(t)\rangle = \sum_{i=1}^n U_{\alpha i}^* |\nu_i(t)\rangle . \quad (26)$$

It can be detected in the charged-current (CC) interaction  $\nu_\alpha(t)N' \rightarrow \ell_\beta N$  with a probability

$$P_{\alpha\beta} = |\langle\nu_\beta|\nu_\alpha(t)\rangle|^2 = \left| \sum_{i=1}^n \sum_{j=1}^n U_{\alpha i}^* U_{\beta j} \langle\nu_j(0)|\nu_i(t)\rangle \right|^2 , \quad (27)$$

where  $E_i$  and  $m_i$  are, respectively, the energy and the mass of the neutrino mass eigenstate  $\nu_i$ .

We use the standard approximation that  $|\nu\rangle$  is a plane wave (for a pedagogical discussion of the possible quantum mechanical problems in this naive description of neutrino oscillations we refer the reader to Lipkin, 1999, and Kim and Pevsner, 1993),  $|\nu_i(t)\rangle = e^{-iE_i t} |\nu_i(0)\rangle$ . In all cases of interest to us, the neutrinos are relativistic:

$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2E_i} . \quad (28)$$

Furthermore, we can assume that  $p_i \simeq p_j \equiv p \simeq E$ . Then we obtain the following transition probability (we only include here the CP conserving piece):

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{i=1}^{n-1} \sum_{j=i+1}^n \text{Re}[U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}] \sin^2 x_{ij} , \quad (29)$$

where

$$x_{ij} \equiv \Delta_{ij} L/2, \quad \Delta_{ij} \equiv \Delta m_{ij}^2/(2E), \quad \Delta m_{ij}^2 \equiv m_i^2 - m_j^2, \quad (30)$$

and  $L = t$  is the distance between the source (that is, the production point of  $\nu_\alpha$ ) and the detector (that is, the detection point of  $\nu_\beta$ ). In deriving Eq. (29) we used the orthogonality relation  $\langle \nu_j(0) | \nu_i(0) \rangle = \delta_{ij}$ . It is convenient to use the following units:

$$x_{ij} = 1.27 \frac{\Delta m_{ij}^2}{\text{eV}^2} \frac{L/E}{m/\text{MeV}}. \quad (31)$$

The transition probability [Eq. (29)] has an oscillatory behavior, with oscillation length

$$L_{0,ij}^{\text{osc}} = \frac{4\pi E}{\Delta m_{ij}^2} \quad (32)$$

and amplitude that is proportional to elements in the mixing matrix. Thus, in order to have oscillations, neutrinos must have different masses ( $\Delta m_{ij}^2 \neq 0$ ) and they must mix ( $U_{\alpha i} U_{\beta i} \neq 0$ ).

An experiment is characterized by the typical neutrino energy  $E$  and by the source-detector distance  $L$ . In order to be sensitive to a given value of  $\Delta m_{ij}^2$ , the experiment has to be set up with  $E/L \approx \Delta m_{ij}^2$  ( $L \sim L_{0,ij}^{\text{osc}}$ ). The typical values of  $L/E$  for different types of neutrino sources and experiments are summarized in Table I.

If  $(E/L) \gg \Delta m_{ij}^2$  ( $L \ll L_{0,ij}^{\text{osc}}$ ), the oscillation does not have time to give an appreciable effect because  $\sin^2 x_{ij} \ll 1$ . The case of  $(E/L) \ll \Delta m_{ij}^2$  ( $L \gg L_{0,ij}^{\text{osc}}$ ) requires more careful consideration. One must take into account that, in general, neutrino beams are not monochromatic. Thus, rather than measuring  $P_{\alpha\beta}$ , the experiments are sensitive to the average probability

$$\begin{aligned} \langle P_{\alpha\beta} \rangle &= \frac{\int dE_\nu \frac{d\Phi}{dE_\nu} \sigma_{CC}(E_\nu) P_{\alpha\beta}(E_\nu) \epsilon(E_\nu)}{\int dE_\nu \frac{d\Phi}{dE_\nu} \sigma_{CC}(E_\nu) \epsilon(E_\nu)} \\ &= \delta_{\alpha\beta} - 4 \sum_{i=1}^{n-1} \sum_{j=i+1}^n \text{Re}[U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}] \langle \sin^2 x_{ij} \rangle, \end{aligned} \quad (33)$$

where  $\Phi$  is the neutrino energy spectrum,  $\sigma_{CC}$  is the cross section for the process in which the neutrino is detected (in general, a CC interaction), and  $\epsilon(E_\nu)$  is the detection efficiency. For  $L \gg L_{0,ij}^{\text{osc}}$ , the oscillating phase goes through many cycles before the detection and is averaged to  $\langle \sin^2 x_{ij} \rangle = 1/2$ .

For a two-neutrino case, the mixing matrix depends on a single parameter,

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \quad (34)$$

and there is a single mass-squared difference  $\Delta m^2$ . Then  $P_{\alpha\beta}$  of Eq. (29) takes the well known form

$$P_{\alpha\beta} = \delta_{\alpha\beta} - (2\delta_{\alpha\beta} - 1) \sin^2 2\theta \sin^2 x. \quad (35)$$

The physical parameter space is covered with  $\Delta m^2 \geq 0$  and  $0 \leq \theta \leq \frac{\pi}{2}$  (or, alternatively,  $0 \leq \theta \leq \frac{\pi}{4}$  and either sign for  $\Delta m^2$ ).

Changing the sign of the mass difference,  $\Delta m^2 \rightarrow -\Delta m^2$ , and changing the octant of the mixing angle,  $\theta \rightarrow \frac{\pi}{2} - \theta$ , amounts to redefining the mass eigenstates,  $\nu_1 \leftrightarrow \nu_2$ :  $P_{\alpha\beta}$  must be invariant under such transformation. Eq. (35) reveals, however, that  $P_{\alpha\beta}$  is actually invariant under each of these transformations separately. This situation implies that there is a two-fold discrete ambiguity in the interpretation of  $P_{\alpha\beta}$  in terms of two-neutrino mixing: the two different sets of physical parameters,  $(\Delta m^2, \theta)$  and  $(\Delta m^2, \frac{\pi}{2} - \theta)$ , give the same transition probability in vacuum. One cannot tell from a measurement of, say,  $P_{e\mu}$  in vacuum whether the larger component of  $\nu_e$  resides in the heavier or in the lighter neutrino mass eigenstate.

Neutrino oscillation experiments measure  $P_{\alpha\beta}$ . It is common practice for the experiments to interpret their results in the two-neutrino framework. In other words, the constraints on  $P_{\alpha\beta}$  are translated into allowed or excluded regions in the plane  $(\Delta m^2, \sin^2 2\theta)$  by using Eq. (35). An example is given in Fig. 1. We now explain some of the typical features of these constraints.

When an experiment is taking data at fixed  $\langle L \rangle$  and  $\langle E \rangle$ , as is the case for most laboratory searches, its result can always be accounted for by  $\Delta m^2$  that is large enough to be in the region of averaged oscillations,  $\langle \sin^2 x_{ij} \rangle = 1/2$ . Consequently, no upper bound on  $\Delta m^2$  can be achieved by such experiment. So, for negative searches that set an upper bound on the oscillation probability,  $\langle P_{\alpha\beta} \rangle \leq P_L$ , the excluded region lies always on the upper-right side of the  $(\Delta m^2, \sin^2 2\theta)$  plane, limited by the following asymptotic lines:

- For  $\Delta m^2 \gg 1/\langle L/E \rangle$ , a vertical line at  $\sin^2 2\theta = 2 P_L$ .
- For  $\Delta m^2 \ll 1/\langle L/E \rangle$ , the oscillating phase can be expanded and the limiting curve takes the form  $\Delta m^2 \sin 2\theta = 4\sqrt{P_L}/\langle L/E \rangle$ , which in a log-log plot gives a straight line of slope  $-1/2$ .

If, instead, data are taken at several values of  $\langle L \rangle$  and/or  $\langle E \rangle$ , the corresponding region may be closed as it is possible to have direct information on the characteristic oscillation wavelength.

## B. Neutrinos in Matter: Effective Potentials

When neutrinos propagate in dense matter, the interactions with the medium affect their properties. These effects are either coherent or incoherent. For purely incoherent inelastic  $\nu$ -p scattering, the characteristic cross section is very small:

$$\sigma \sim \frac{G_F^2 s}{\pi} \sim 10^{-43} \text{cm}^2 \left( \frac{E}{1 \text{MeV}} \right)^2. \quad (36)$$

The smallness of this cross section is demonstrated by the fact that if a beam of  $10^{10}$  neutrinos with  $E \sim 1$  MeV was aimed at the Earth, only one would be deflected by the Earth matter. It may seem then that for neutrinos matter is irrelevant. However, one must take into account that Eq. (36) does not contain the contribution from forward elastic coherent interactions. In coherent interactions, the medium remains unchanged and it is possible to have interference of scattered and unscattered neutrino waves which enhances the effect. Coherence further allows one to decouple the evolution equation of the neutrinos

from the equations of the medium. In this approximation, the effect of the medium is described by an effective potential which depends on the density and composition of the matter (Wolfenstein, 1978).

As an example we derive the effective potential for the evolution of  $\nu_e$  in a medium with electrons, protons and neutrons. The effective low-energy Hamiltonian describing the relevant neutrino interactions is given by

$$H_W = \frac{G_F}{\sqrt{2}} [J^{(+)\alpha}(x)J_\alpha^{(-)}(x) + \frac{1}{4}J^{(N)\alpha}(x)J_\alpha^{(N)}(x)] , \quad (37)$$

where the  $J_\alpha$ 's are the standard fermionic currents,

$$J_\alpha^{(+)}(x) = \bar{\nu}_e(x)\gamma_\alpha(1 - \gamma_5)e(x) , \quad (38)$$

$$J_\alpha^{(-)}(x) = \bar{e}(x)\gamma_\alpha(1 - \gamma_5)\nu_e(x) , \quad (39)$$

$$J_\alpha^{(N)}(x) = \bar{\nu}_e(x)\gamma_\alpha(1 - \gamma_5)\nu_e(x) - \bar{e}(x)[\gamma_\alpha(1 - \gamma_5) - 4\sin^2\theta_W\gamma_\alpha]e(x) \\ + \bar{p}(x)[\gamma_\alpha(1 - g_A^{(p)}\gamma_5) - 4\sin^2\theta_W\gamma_\alpha]p(x) - \bar{n}(x)\gamma_\alpha(1 - g_A^{(n)}\gamma_5)n(x) . \quad (40)$$

$g_A^{(n,p)}$  are the axial couplings for neutrons and protons, respectively. For the sake of simplicity we concentrate on the effect of the charged current interactions. The effective CC Hamiltonian due to electrons in the medium is

$$H_C^{(e)} = \frac{G_F}{\sqrt{2}} \int d^3p_e f(E_e, T) \\ \times \left\langle \langle e(s, p_e) | \bar{e}(x)\gamma^\alpha(1 - \gamma_5)\nu_e(x)\bar{\nu}_e(x)\gamma_\alpha(1 - \gamma_5)e(x) | e(s, p_e) \rangle \right\rangle \\ = \frac{G_F}{\sqrt{2}} \bar{\nu}_e(x)\gamma_\alpha(1 - \gamma_5)\nu_e(x) \int d^3p_e f(E_e, T) \left\langle \langle e(s, p_e) | \bar{e}(x)\gamma_\alpha(1 - \gamma_5)e(x) | e(s, p_e) \rangle \right\rangle , \quad (41)$$

where  $s$  is the electron spin and  $p_e$  its momentum. The energy distribution function of the electrons in the medium,  $f(E_e, T)$ , is assumed to be homogeneous and isotropic and is normalized as

$$\int d^3p_e f(E_e, T) = 1 . \quad (42)$$

By  $\langle \dots \rangle$  we denote the averaging over electron spinors and summing over all electrons in the medium. Notice that coherence implies that  $s, p_e$  are the same for initial and final electrons.

Expanding the electron fields  $e(x)$  in plane waves we find

$$\langle e(s, p_e) | \bar{e}(x)\gamma_\alpha(1 - \gamma_5)e(x) | e(s, p_e) \rangle \\ = \frac{1}{V} \langle e(s, p_e) | \bar{u}_s(p_e)a_s^\dagger(p_e)\gamma_\alpha(1 - \gamma_5)a_s(p_e)u_s(p_e) | e(s, p_e) \rangle , \quad (43)$$

where  $V$  is a volume normalization factor. The averaging gives

$$\frac{1}{V} \left\langle \langle e(s, p_e) | a_s^\dagger(p_e)a_s(p_e) | e(s, p_e) \rangle \right\rangle = N_e(p_e) \frac{1}{2} \sum_s , \quad (44)$$

where  $N_e(p_e)$  is the number density of electrons with momentum  $p_e$ . We assumed here that the medium has equal numbers of spin  $+1/2$  and spin  $-1/2$  electrons, and we used the fact that  $a_s^\dagger(p_e)a_s(p_e) = \mathcal{N}_e^{(s)}(p_e)$  is the number operator. We thus obtain:

$$\begin{aligned} \left\langle \langle e(s, p_e) | \bar{e}(x) \gamma_\alpha (1 - \gamma_5) e(x) | e(s, p_e) \rangle \right\rangle &= N_e(p_e) \frac{1}{2} \sum_s \overline{u_{(s)}}(p_e) \gamma_\alpha (1 - \gamma_5) u_{(s)}(p_e) \\ &= \frac{N_e(p_e)}{2} \text{Tr} \left[ \frac{m_e + \not{p}}{2E_e} \gamma_\alpha (1 - \gamma_5) \right] = N_e(p_e) \frac{p_e^\alpha}{E_e}. \end{aligned} \quad (45)$$

Isotropy implies that  $\int d^3 p_e \vec{p}_e f(E_e, T) = 0$ . Thus, only the  $p^0$  term contributes upon integration, with  $\int d^3 p_e f(E_e, T) N_e(p_e) = N_e$  (the electron number density). Substituting Eq. (45) in Eq. (41) we obtain:

$$H_C^{(e)} = \frac{G_F N_e}{\sqrt{2}} \bar{\nu}_e(x) \gamma_0 (1 - \gamma_5) \nu_e(x). \quad (46)$$

The effective potential for  $\nu_e$  induced by its charged current interactions with electrons in matter is then given by

$$V_C = \langle \nu_e | \int d^3 x H_C^{(e)} | \nu_e \rangle = \frac{G_F N_e}{\sqrt{2}} \frac{2}{V} \int d^3 x u_\nu^\dagger u_\nu = \sqrt{2} G_F N_e. \quad (47)$$

For  $\bar{\nu}_e$  the sign of  $V$  is reversed. This potential can also be expressed in terms of the matter density  $\rho$ :

$$V_C = \sqrt{2} G_F N_e \simeq 7.6 Y_e \frac{\rho}{10^{14} \text{g/cm}^3} \text{ eV}, \quad (48)$$

where  $Y_e = \frac{N_e}{N_p + N_n}$  is the relative number density. Three examples that are relevant to observations are the following:

- At the Earth core  $\rho \sim 10 \text{ g/cm}^3$  and  $V_C \sim 10^{-13} \text{ eV}$ ;
- At the solar core  $\rho \sim 100 \text{ g/cm}^3$  and  $V_C \sim 10^{-12} \text{ eV}$ ;
- At a supernova  $\rho \sim 10^{14} \text{ g/cm}^3$  and  $V_C \sim \text{eV}$ .

Following the same procedure we can obtain the effective potentials for  $\nu_e$  due to interactions with different particles in the medium. The results are listed in Table II (Kim and Pevsner, 1993). For  $\nu_\mu$  and  $\nu_\tau$ ,  $V_C = 0$  for any of these media while  $V_N$  is the same as for  $\nu_e$ . One can further generalize this analysis to other types of interactions (Bergmann, Grossman and Nardi, 1999).

### C. Evolution Equation in Matter: Effective Mass and Mixing

There are several derivations in the literature of the evolution equation of a neutrino system in matter (see, for instance, Halprin 1986, Mannheim 1988). We follow here the discussion in Baltz and Weneser, 1988. Consider a state which is an admixture of two neutrino species  $|\nu_e\rangle$  and  $|\nu_X\rangle$  or, equivalently, of  $|\nu_1\rangle$  and  $|\nu_2\rangle$ :



$$\Phi(x) = \Phi_e(x)|\nu_e\rangle + \Phi_X(x)|\nu_X\rangle = \Phi_1(x)|\nu_1\rangle + \Phi_2(x)|\nu_2\rangle \quad (49)$$

The evolution of  $\Phi$  in a medium is described by a system of coupled Dirac equations:

$$\begin{aligned} E\Phi_1 &= \left[ \frac{\hbar}{i}\alpha_x \frac{\partial}{\partial x} + \beta m_1 + V_{11} \right] \Phi_1 + V_{12}\Phi_2, \\ E\Phi_2 &= \left[ \frac{\hbar}{i}\alpha_x \frac{\partial}{\partial x} + \beta m_2 + V_{22} \right] \Phi_2 + V_{12}\Phi_1, \end{aligned} \quad (50)$$

where  $\beta = \gamma_0$  and  $\alpha_x = \gamma_0\gamma_1$ . The  $V_{ij}$  terms give the effective potential for neutrino mass eigenstates. They can be simply derived from the effective potential for interaction eigenstates [such as  $V_{ee}$  of Eq. (47)]:

$$V_{ij} = \langle \nu_i | \int d^3x H_{int}^{\text{medium}} | \nu_j \rangle = U_{i\alpha} V_{\alpha\alpha} U_{j\alpha}^*. \quad (51)$$

We decompose the neutrino state:  $\Phi_i(x) = C_i(x)\phi_i(x)$ . Here  $\phi_i(x)$  is the Dirac spinor part satisfying:

$$\left( \alpha_x \{ [E - V_{ii}(x)]^2 - m_i^2 \}^{1/2} + \beta m_i + V_{ii} \right) \phi_i(x) = E\phi_i(x). \quad (52)$$

So  $\phi_i(x)$  has the form of free particle solutions with local energy  $\mathcal{E}_i(x) = E - V_{ii}(x)$ :

$$\phi_i(x) = \left[ \frac{\mathcal{E}_i + m_i}{2\mathcal{E}_i} \right]^{1/2} \times \left[ \frac{\chi}{\frac{\sqrt{\mathcal{E}_i^2 - m_i^2}}{\mathcal{E}_i + m_i} \sigma_x \chi} \right], \quad (53)$$

where  $\chi$  is the Pauli spinor. We make the following approximations:

- (i) The scale over which  $V$  changes is much larger than the microscopic wavelength of the neutrino:  $\frac{\partial V}{\partial x}/V \ll \hbar m/E^2$ .
- (ii) Expanding to first order in  $V$  implies that  $V_{12}\alpha_x\phi_2 \simeq \phi_1$ ,  $V_{12}\alpha_x\phi_1 \simeq \phi_2$  and  $\{ [E - V_{ii}(x)]^2 - m_i^2 \}^{1/2} \simeq E - V_{ii}(x) - \frac{m_i^2}{2E}$ .

From (i) we find that the Dirac equations take the form:

$$\begin{aligned} EC_1\phi_1 &= \frac{\hbar}{i}\alpha_x \frac{\partial C_1}{\partial x} \phi_1 + (\beta m_1 + V_{11})C_1\phi_1 + V_{12}C_2\phi_2, \\ EC_2\phi_2 &= \frac{\hbar}{i}\alpha_x \frac{\partial C_2}{\partial x} \phi_2 + (\beta m_2 + V_{22})C_2\phi_2 + V_{12}C_1\phi_1. \end{aligned} \quad (54)$$

Then multiplying by  $\alpha_x$  and using the equation of motion of  $\phi_i$  and (ii), we can drop the dependence on the spinor  $\phi$  and obtain

$$\begin{aligned} \frac{\hbar}{i} \frac{\partial C_1}{\partial x} &= [E - V_{11}(x) - \frac{m_1^2}{2E}]C_1 - V_{12}C_2, \\ \frac{\hbar}{i} \frac{\partial C_2}{\partial x} &= [E - V_{22}(x) - \frac{m_2^2}{2E}]C_2 - V_{12}C_1. \end{aligned} \quad (55)$$

Changing notations  $C_{i,\alpha}(x) \rightarrow \nu_{i,\alpha}(x)$ , we rewrite Eq. (55) in a matrix form :

$$\frac{\hbar}{i} \frac{\partial}{\partial x} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} E - V_{11} - \frac{m_1^2}{2E} & -V_{12} \\ -V_{12} & E - V_{22} - \frac{m_2^2}{2E} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}. \quad (56)$$

After removing the diagonal piece that is proportional to  $E$ , we can rotate Eq. (56) to the flavor basis ( $\hbar = 1$ ) (Wolfenstein, 1978):

$$-i \frac{\partial}{\partial x} \begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix} = \left( -\frac{M_w^2}{2E} \right) \begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix}, \quad (57)$$

where we have defined an effective mass matrix in matter:

$$M_w^2 = \begin{pmatrix} \frac{m_1^2 + m_2^2}{2} + 2EV_e - \frac{\Delta m^2}{2} \cos 2\theta & \frac{\Delta m^2}{2} \sin 2\theta \\ \frac{\Delta m^2}{2} \sin 2\theta & \frac{m_1^2 + m_2^2}{2} + 2EV_X + \frac{\Delta m^2}{2} \cos 2\theta \end{pmatrix}. \quad (58)$$

Here  $\Delta m^2 = m_2^2 - m_1^2$ .

We define the instantaneous mass eigenstates in matter,  $\nu_i^m$ , as the eigenstates of  $M_w$  for a fixed value of  $x$  (or  $t$ ). They are related to the interaction eigenstates through a unitary rotation,

$$\begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix} = U(\theta_m) \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} = \begin{pmatrix} \cos \theta_m & \sin \theta_m \\ -\sin \theta_m & \cos \theta_m \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}. \quad (59)$$

The eigenvalues of  $M_w$ , that is, the effective masses in matter are given by (Wolfenstein, 1978; Mikheyev and Smirnov, 1985):

$$\mu_{1,2}^2(x) = \frac{m_1^2 + m_2^2}{2} + E(V_e + V_X) \mp \frac{1}{2} \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}, \quad (60)$$

while the mixing angle in matter is given by

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A}. \quad (61)$$

The quantity  $A$  is defined by

$$A \equiv 2E(V_e - V_X). \quad (62)$$

In Figs. 2 and 3 we plot, respectively, the effective masses and the mixing angle in matter as functions of the potential  $A$ , for  $A > 0$  and  $\Delta m^2 \cos 2\theta > 0$ . Notice that even massless neutrinos acquire non-vanishing effective masses in matter.

The resonant density (or potential)  $A_R$  is defined as the value of  $A$  for which the difference between the effective masses is minimal:

$$A_R = \Delta m^2 \cos 2\theta. \quad (63)$$

Notice that once the sign of  $V_e - V_X$  (which depends on the composition of the medium and on the state  $X$ ) is known, this resonance condition can only be achieved for a given sign of  $\Delta m^2 \cos 2\theta$ , *i.e.* for mixing angles in only one of the two possible octants. We learn that the symmetry present in vacuum oscillations is broken by matter potentials. Also if the

resonant condition is achieved for two neutrinos it cannot be achieved for antineutrinos of the same flavor and viceversa. The mixing angle  $\tan \theta_m$  changes sign at  $A_R$ . As can be seen in Fig. 3, for  $A > A_R$  we have  $\theta_m \gg \theta$ .

We define an oscillation length in matter:

$$L^{\text{osc}} = \frac{L_0^{\text{osc}} \Delta m^2}{\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}}, \quad (64)$$

where the oscillation length in vacuum,  $L_0^{\text{osc}}$ , was defined in Eq. (32). The oscillation length in matter presents a resonant behaviour. At the resonance point the oscillation length is

$$L_R^{\text{osc}} = \frac{L_0^{\text{osc}}}{\sin 2\theta}. \quad (65)$$

The width (in distance) of the resonance,  $\delta r_R$ , corresponding to  $\delta A_R = 2\Delta m^2 \sin^2 2\theta$ , is given by

$$\delta r_R = \frac{\delta A_R}{\left| \frac{dA}{dr} \right|_R} = \frac{2 \tan 2\theta}{h_R}, \quad (66)$$

where we have defined the resonance height:

$$h_R \equiv \left| \frac{1}{A} \frac{dA}{dr} \right|_R \quad (67)$$

For constant  $A$ , *i.e.*, for constant matter density, the evolution of the neutrino system is described just in terms of the masses and mixing in matter. But for varying  $A$ , this is in general not the case.

#### D. Adiabatic versus Non-adiabatic Transitions

Taking time derivative of Eq. (59), we find:

$$\frac{\partial}{\partial t} \begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix} = \dot{U}(\theta_m) \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} + U(\theta_m) \begin{pmatrix} \dot{\nu}_1^m \\ \dot{\nu}_2^m \end{pmatrix}. \quad (68)$$

Using the evolution equation in the flavor basis, Eq. (57), we get

$$i \begin{pmatrix} \dot{\nu}_1^m \\ \dot{\nu}_2^m \end{pmatrix} = \frac{1}{2E} U^\dagger(\theta_m) M_w^2 U(\theta_m) \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} - i U^\dagger \dot{U}(\theta_m) \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}. \quad (69)$$

For constant matter density,  $\theta_m$  is constant and the second term vanishes. In general, using the definition of the effective masses  $\mu_i(t)$  in Eq. (60), and subtracting a diagonal piece  $(\mu_1^2 + \mu_2^2)/2E \times I$ , we can rewrite the evolution equation as:

$$i \begin{pmatrix} \dot{\nu}_1^m \\ \dot{\nu}_2^m \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta(t) & -4iE\dot{\theta}_m(t) \\ 4iE\dot{\theta}_m(t) & \Delta(t) \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} \quad (70)$$

where we defined  $\Delta(t) \equiv \mu_2^2(t) - \mu_1^2(t)$ .

The evolution equations (70) constitute a system of coupled equations: the instantaneous mass eigenstates,  $\nu_i^m$ , mix in the evolution and are not energy eigenstates. The importance of this effect is controlled by the relative size of the off-diagonal piece  $4 E \dot{\theta}_m(t)$  with respect to the diagonal one  $\Delta(t)$ . When  $\Delta(t) \gg 4 E \dot{\theta}_m(t)$ , the instantaneous mass eigenstates,  $\nu_i^m$ , behave approximately as energy eigenstates and they do not mix in the evolution. This is the *adiabatic* transition approximation. From the definition of  $\theta_m$  in Eq. (61) we find:

$$\dot{\theta}_m = \frac{\Delta m^2 \sin 2\theta}{2\Delta(t)^2} \dot{A}. \quad (71)$$

The adiabaticity condition reads then

$$\Delta(t) \gg \frac{2EA\Delta m^2 \sin 2\theta}{\Delta(t)^2} \left| \frac{\dot{A}}{A} \right|. \quad (72)$$

Since the maximum of  $\dot{\theta}_m$  occurs at the resonance point (as seen in Fig. 3), the strongest adiabaticity condition is obtained when Eq. (72) is evaluated at the resonance. We define the adiabaticity parameter  $Q$  at the resonance as follows:

$$Q = \frac{\Delta m^2 \sin^2 2\theta}{E \cos 2\theta h_R} = \frac{4 \pi \delta r_R}{L_R^{osc}}, \quad (73)$$

where we used the definitions of  $A_R$ ,  $\delta r_R$ , and  $h_R$  in Eqs. (63), (66), and (67). Written in this form, we see that the adiabaticity condition,  $Q \gg 1$ , implies that many oscillations take place in the resonant region. Conversely, when  $Q \lesssim 1$  the transition is non-adiabatic.

The survival amplitude of a  $\nu_e$  produced in matter at  $t_0$  and exiting the matter at  $t > t_0$  can be written as follows:

$$A(\nu_e \rightarrow \nu_e; t) = \sum_{i,j} \langle \nu_e(t) | \nu_j(t) \rangle \langle \nu_j(t) | \nu_j(t_R) \rangle \langle \nu_j(t_R) | \nu_i(t_R) \rangle \langle \nu_i(t_R) | \nu_i(t_0) \rangle \langle \nu_i(t_0) | \nu_e(t_0) \rangle, \quad (74)$$

where  $\langle \nu_i(t_0) | \nu_e(t_0) \rangle = U_{ei}^*(\theta_{m,0})$  is the  $(ei)$  element of the mixing matrix in matter at the production point and  $\langle \nu_e(t) | \nu_j(t) \rangle = U_{ej}(\theta)$  is the  $(ej)$  element of the mixing matrix in vacuum.

In the adiabatic approximation the mass eigenstates do not mix so

$$\langle \nu_j(t) | \nu_j(t_R) \rangle \langle \nu_j(t_R) | \nu_i(t_R) \rangle \langle \nu_i(t_R) | \nu_i(t_0) \rangle = \delta_{ij} \langle \nu_i(t) | \nu_i(t_0) \rangle = \delta_{ij} \exp \left\{ i \int_{t_0}^t E_i(t') dt' \right\}. \quad (75)$$

Note that  $E_i$  is a function of time because the effective mass  $\mu_i$  is a function of time,

$$E_i(t') \simeq p + \frac{\mu_i^2(t')}{2p}. \quad (76)$$

Thus the transition probability for the adiabatic case is given by

$$P(\nu_e \rightarrow \nu_e; t) = \left| \sum_i U_{ei}(\theta) U_{ei}^*(\theta_{m,0}) \exp \left( -\frac{i}{2E} \int_{t_0}^t \mu_i^2(t') dt' \right) \right|^2. \quad (77)$$

For the case of two-neutrino mixing, Eq. (77) takes the form

$$P(\nu_e \rightarrow \nu_e; t) = \cos^2 \theta_m \cos^2 \theta + \sin^2 \theta_m \sin^2 \theta + \frac{1}{2} \sin 2\theta_m \sin 2\theta \cos \left( \frac{\delta(t)}{2E} \right), \quad (78)$$

where

$$\delta(t) = \int_{t_0}^t \Delta(t') dt' = \int_{t_0}^t \sqrt{(\Delta m^2 \cos 2\theta - A(t'))^2 + (\Delta m^2 \sin 2\theta)^2} dt', \quad (79)$$

which, in general, has to be evaluated numerically. There are some analytical approximations for specific forms of  $A(t')$ : exponential, linear ... (see for instance, Kuo and Pantaleone, 1989). For  $\delta(t) \gg E$  the last term in Eq. (78) is averaged out and the survival probability takes the form

$$P(\nu_e \rightarrow \nu_e; t) = \frac{1}{2} [1 + \cos 2\theta_m \cos 2\theta] \quad (80)$$

In Fig. 4 we plot isocontours of constant survival probability in the parameter plane  $(\Delta m^2, \tan^2 \theta)$  for the particular case of the sun density for which  $A > 0$ . Notice that, unlike  $\sin^2 2\theta$ ,  $\tan^2 \theta$  is a single valued function in the full parameter range  $0 \leq \theta \leq \pi/2$ . Therefore it is a more appropriate variable once matter effects are included and the symmetry of the survival probability with respect to the change of octant for the mixing angle is lost. As seen in the figure, for  $\theta < \pi/4$ ,  $P(\nu_e \rightarrow \nu_e)$  in matter can be larger or smaller than  $1/2$ , in contrast to the case of vacuum oscillations where, in the averaged regime,  $P_{ee}^{vac} = 1 - \frac{1}{2} \sin^2 2\theta > \frac{1}{2}$ .

In Fig. 4 we also plot the limiting curve for  $Q = 1$ . To the left and below of this curve the adiabatic approximation breaks down and the isocontours in Fig. 4 deviate from the expression in Eq. (80). In this region, the off-diagonal term  $\dot{\theta}_m$  cannot be neglected and the mixing between instantaneous mass eigenstates is important. The possibility of this *level crossing* can be described in terms of the Landau-Zener probability (Landau, 1932; Zener, 1932):

$$P_{LZ} = |\langle \nu_j(t_R) | \nu_i(t_R) \rangle|^2 \quad (i \neq j). \quad (81)$$

Introducing this transition probability in Eq. (74) we find that in the non-adiabatic regime (after averaging out the oscillatory term), the survival probability can be written as

$$P(\nu_e \rightarrow \nu_e; t) = \frac{1}{2} [1 + (1 - 2P_{LZ}) \cos 2\theta_m \cos 2\theta]. \quad (82)$$

The physical interpretation of this expression is straightforward. An electron neutrino produced at  $A > A_R$  consists of an admixture of  $\nu_1$  with fraction  $\cos^2 \theta_m$  and  $\nu_2$  with fraction  $\sin^2 \theta_m$ . In particular, for very small mixing angles in vacuum,  $\theta_m \sim \pi/2$  (see Fig. 3) so  $\nu_e$  is almost a pure  $\nu_2(t_0)$  state. When the neutrino state reaches the resonance,  $\nu_2$  ( $\nu_1$ ) can become  $\nu_2$  ( $\nu_1$ ) with probability  $[1 - P_{LZ}]$  or  $\nu_1$  ( $\nu_2$ ) with probability  $P_{LZ}$ . So after passing the resonance, the  $\nu_e$  flux contains a fraction of  $\nu_1$ :  $P_{e1} = \sin^2 \theta_m P_{LZ} + \cos^2 \theta_m (1 - P_{LZ})$ , and a fraction of  $\nu_2$ :  $P_{e2} = \cos^2 \theta_m P_{LZ} + \sin^2 \theta_m (1 - P_{LZ})$ . At the exit  $\nu_1$  consists of  $\nu_e$  with fraction  $\cos^2 \theta$  and  $\nu_2$  consists of  $\nu_e$  with fraction  $\sin^2 \theta$  so (Parke, 1986, Haxton, 1986, Petcov, 1987)

$$P_{ee} = \cos^2 \theta P_{e1} + \sin^2 \theta P_{e2}, \quad (83)$$

which reproduces Eq. (82).

The Landau-Zener probability can be evaluated in the WKB approximation (this derivation follows Kim and Pevsner, 1993):

$$P_{LZ} = \exp \left[ -2\mathcal{I}m \int_{t_R}^{t_0} \frac{\mu_2^2(t) - \mu_1^2(t)}{2E} dt \right] = \exp \left[ -2\mathcal{I}m \int_{A_R}^{A_0} \frac{\mu_2^2(A) - \mu_1^2(A)}{2E|\dot{A}|} dA \right], \quad (84)$$

where  $t_0$  (or  $A_0$ ) is a root of  $\mu_2^2 - \mu_1^2 = 0$  lying in the upper half plane of the complex variable  $t$  (or  $A$ ),  $A_0 = \Delta m^2 (\cos 2\theta + i \sin 2\theta)$ . Assuming that  $|\dot{A}|$  is slowly varying near the resonance, we get

$$\mathcal{I}m \int_{A_R}^{A_0} \frac{\mu_2^2(A) - \mu_1^2(A)}{2E|\dot{A}|} dA \sim \frac{1}{2E|\dot{A}|_R} \mathcal{I}m \int_{A_R}^{A_0} [\mu_2^2(A) - \mu_1^2(A)] dA. \quad (85)$$

Shifting the integral  $A \rightarrow A - \Delta m^2 \cos 2\theta$ , we obtain:

$$\mathcal{I}m \int_{A_R}^{A_0} [\mu_2^2(A) - \mu_1^2(A)] dA = \mathcal{I}m \int_0^{i\Delta m^2 \sin 2\theta} dA \sqrt{A^2 + (\Delta m^2 \sin 2\theta)^2} = \frac{\pi}{4} (\Delta m^2 \sin 2\theta)^2. \quad (86)$$

So we finally find

$$P_{LZ} = \exp \left( -\frac{\pi}{4} Q \right) \quad (87)$$

where  $Q$  is the adiabaticity parameter defined in Eq. (73). In this derivation we assumed that  $|\dot{A}|$  is almost constant. This assumption is strictly valid only for linearly growing densities  $A \propto r$ . For other forms of  $A$ , the adiabaticity parameter  $Q$  has to be multiplied by some factor (for a list of some of these factors see Kuo and Pantaleone, 1989). For instance, for an exponential density,  $A \sim \exp(-r)$ , the factor is  $(1 - \tan^2 \theta)$ . Moreover this approximate derivation is not valid for  $Q \ll 1$ . The general form of the Landau-Zener probability for an exponential density can be written as (Petcov, 1988; Krastev and Petcov 1988):

$$P_{LZ} = \frac{\exp(-\gamma \sin^2 \theta) - \exp(-\gamma)}{1 - \exp(-\gamma)}, \quad \gamma \equiv \pi \frac{\Delta m^2}{E|\dot{A}/A|_R}. \quad (88)$$

When  $\nu_e$  is produced at  $A \gg A_R$  and  $\theta$  is small,  $\theta_m \sim 90^\circ$ . In this case the survival probability is simply given by

$$P(\nu_e \rightarrow \nu_e; t) \simeq P_{LZ} \simeq \exp \left( -\frac{\pi}{4} Q \right)$$

Since  $Q \sim (\Delta m^2)^2 \sin^2 2\theta / E$ , the isocontours of constant probability in this regime correspond to diagonal lines in the  $(\Delta m^2, \tan^2 \theta)$  plane in a log-log plot, as illustrated in Fig. 4.

## E. Propagation in the Sun: MSW Effect

As an illustration of the matter effects discussed in the previous section we describe now the propagation of a  $\nu_e - \nu_X$  neutrino system in the matter density of the sun. For the sake of concreteness we assume that  $X$  is some superposition of  $\mu$  and  $\tau$ .

The solar density distribution decreases monotonically (see Fig. 5). For  $R < 0.9R_\odot$  it can be approximated by an exponential,

$$N_e(R) = N_e(0) \exp(-R/r_0), \quad (89)$$

with  $r_0 = R_\odot/10.54 = 6.6 \times 10^7 \text{ m} = 3.3 \times 10^{14} \text{ eV}^{-1}$ .

After traversing this density the dominant component of the exiting neutrino state depends on the value of the mixing angle in vacuum and the relative size of  $\Delta m^2 \cos 2\theta$  versus  $A_0 = 2 E G_F N_{e,0}$  (at the neutrino production point):

(i)  $\Delta m^2 \cos 2\theta \gg A_0$ : matter effects are negligible and the propagation occurs as in vacuum. The survival probability at the sunny surface of the Earth is

$$P_{ee}(\Delta m^2 \cos 2\theta \gg A_0) = 1 - \frac{1}{2} \sin^2 2\theta > \frac{1}{2}. \quad (90)$$

(ii)  $\Delta m^2 \cos 2\theta \gtrsim A_0$ : the neutrino does not pass the resonance but its mixing is affected by the matter. This effect is well described by an adiabatic propagation:

$$P_{ee}(\Delta m^2 \cos 2\theta \gtrsim A_0) = \frac{1}{2} [1 + \cos 2\theta_m \cos 2\theta]. \quad (91)$$

Since the resonance is not crossed,  $\cos 2\theta_m$  has the same sign as  $\cos 2\theta$  and the corresponding survival probability is also larger than  $1/2$ .

(iii)  $\Delta m^2 \cos 2\theta < A_0$ : the neutrino crosses the resonance on its way out. In this case, as discussed in the previous section, for small mixing angle in vacuum,  $\nu_e \sim \nu_2^m$  at the production point and remains  $\nu_2^m$  till the resonance point. It is important in this case to find whether the transition is adiabatic. For the solar density,  $Q \sim 1$  corresponds to

$$\frac{(\Delta m^2/\text{eV}^2) \sin^2 2\theta}{(E/\text{MeV}) \cos 2\theta} \sim 3 \times 10^{-9}. \quad (92)$$

For  $Q \gg 1$  the transition is adiabatic and the neutrino state remains  $\nu_2$  after the resonance. As seen in Fig. 3,  $\theta_m$  decreases dramatically after the resonance so the  $\nu_e$  component of  $\nu_2$  decreases. For small mixing angle in vacuum, at the exit point  $\nu_2$  is almost a pure  $\nu_X$ . Consequently,  $P_{ee}$  can be very small. More explicitly,

$$P_{ee}(\Delta m^2 \cos 2\theta < A_0, Q \gg 1) = \frac{1}{2} [1 + \cos 2\theta_{m,0} \cos 2\theta] \quad (93)$$

can be much smaller than  $1/2$  because  $\cos 2\theta_{m,0}$  and  $\cos 2\theta$  have opposite signs. Note that the smaller the mixing angle in vacuum the larger is the deficit of electron neutrinos in the outgoing state. This is the MSW effect (Wolfenstein, 1978; Mikheyev and Smirnov, 1985). This behaviour is illustrated in Fig. 6 where we plot the electron survival probability as a function of  $\Delta m^2/E$  for different values of the mixing angle.

For smaller values of  $\Delta m^2/E$  (right side of Fig. 6) we approach the regime where  $Q < 1$  and non-adiabatic effects start playing a role. In this case, at the resonance the state can jump from  $\nu_2$  into  $\nu_1$  with probability  $P_{LZ}$  and remain  $\nu_1$  until it exits the sun. At the surface,  $\nu_1 \sim \nu_e$  (for  $\theta$  small) and the  $\nu_e$  component of the exiting neutrino increases. This can be seen from the expression for  $P_{ee}$ ,

$$P_{ee}(\Delta m^2 \cos 2\theta < A_0, Q \ll 1) = \frac{1}{2} [1 + (1 - 2P_{LZ}) \cos 2\theta_m \cos 2\theta], \quad (94)$$

and from Fig. 6.

#### IV. SOLAR NEUTRINOS

Solar neutrinos are electron neutrinos produced in the thermonuclear reactions which generate the solar energy. These reactions occur via two main chains, the  $pp$ -chain and the CNO cycle, shown in Figs. 7 and 8, respectively. There are five reactions which produce  $\nu_e$  in the  $pp$  chain and three in the CNO cycle. Both chains result in the overall fusion of protons into  ${}^4\text{He}$ :



where the energy released in the reaction,  $Q = 4m_p - m_{{}^4\text{He}} - 2m_e \simeq 26$  MeV, is mostly radiated through the photons and only a small fraction is carried by the neutrinos,  $\langle E_{2\nu_e} \rangle = 0.59$  MeV.

In order to precisely determine the rates of the different reactions in the two chains which would give us the final neutrino fluxes and their energy spectrum, a detailed knowledge of the sun and its evolution is needed. Solar Models (Bahcall and Ulrich, 1988; Bahcall and Pinsonneault, 1992 and 1995; Bahcall, Basu, and Pinsonneault, 1998, 2001; Turck-Chiez *et al.*, 1988) describe the properties of the sun and its evolution after entering the main sequence. The models are based on a set of observational parameters: the surface luminosity ( $3.844 \times 10^{26}$  W), the age ( $4.5 \times 10^9$  years), the radius ( $6.961 \times 10^8$  m) and the mass ( $1.989 \times 10^{30}$  kg), and on several basic assumptions: spherical symmetry, hydrostatic and thermal equilibrium, equation of state of an ideal gas, and present surface abundances of elements similar to the primordial composition. Over the past four decades, the solar models have been steadily refined as the result of increased observational and experimental information about the input parameters (such as nuclear reaction rates and the surface abundances of different elements), more accurate calculations of constituent quantities (such as radiative opacity and equation of state), the inclusion of new physical effects (such as element diffusion), and the development of faster computers and more precise stellar evolution codes.

We use as Solar Standard Model (SSM) the most updated version of the model developed by Bahcall, Pinsonneault and Basu, (2001) (BP00). In Fig. 9 we show the energy spectrum of the fluxes from the five  $pp$  chain reactions. In what follows we refer to the neutrino fluxes by the corresponding source reaction, so, for instance, the neutrinos produced from  ${}^8\text{B}$  decay are called  ${}^8\text{B}$  neutrinos. Most reactions produce a neutrino spectrum characteristic of  $\beta$  decay. For  ${}^8\text{B}$  neutrinos the energy distribution presents deviations with respect to the maximum allowed energy because the final state,  ${}^8\text{Be}$ , is a wide resonance. On the other hand, the  ${}^7\text{Be}$  neutrinos are almost monochromatic, with an energy width of about  $\sim 2$  keV



which is characteristic of the temperature in the core of the sun. After the publication of BP00, a new measurement of the low energy cross section for the fusion reaction  ${}^7\text{Be}(p, \gamma){}^8\text{B}$  was presented by Junghans *et al.* (2002),  $S_{17}(0) = 22.3 \pm 0.7$  (expt)  $\pm 0.5$  (theor) eV barn, which implies a new value for the  ${}^8\text{B}$  flux (that is larger and more precise than the original BP00 value):  $\phi(\text{New } {}^8\text{B}) = 5.93 \times (1_{-0.15}^{+0.14}) \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$  (Bahcall, Gonzalez-Garcia and Peña-Garay, 2001b). Most of the results presented here include the impact of this new value of the  ${}^8\text{B}$  flux.

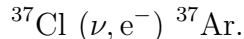
As discussed in Sec. III, to describe the evolution of neutrinos in the solar matter, one needs to know other quantities that are predicted by the SSM, such as the density and composition of solar matter, which give the solar electron number density. As discussed in Sec. III and shown in Fig. 5, the solar matter density decreases monotonically and can be approximated by the exponential profile given in Eq. (89). Furthermore, in order to precisely determine the evolution of the neutrino system one also needs to know the production point distribution for the different neutrino fluxes. In Fig. 10 we show the BP00 prediction for the distribution probability for the  $pp$  chain.

The standard solar models have had notable successes. In particular, the comparison between the observed and the theoretically predicted sound speeds and pressures has tested the SSM and provided accurate information on the solar interior. The solar quantities that have been determined by helioseismology include the sound velocity and matter density as a function of solar radius, the depth of the convective zone, the interior rotation rate, and the surface helium abundance. The excellent agreement between the helioseismological observations and the solar model calculations has made a convincing case that the large discrepancies between solar neutrino measurements and solar model predictions cannot be due to errors in the solar models.

## A. Solar Neutrino Experiments

### 1. Chlorine experiment: Homestake

The first result on the detection of solar neutrinos was announced by Ray Davis Jr and his collaborators from Brookhaven in 1968 (Davis, Harmer and Hoffman, 1968). In the gold mine of Homestake in Lead, South Dakota, they installed a detector consisting of  $\sim 615$  Tons of  $\text{C}_2\text{Cl}_4$ . Solar  $\nu_e$ 's are captured via



The energy threshold for this reaction is 0.814 MeV, so the relevant fluxes are the  ${}^7\text{Be}$  and  ${}^8\text{B}$  neutrinos.  ${}^7\text{Be}$  neutrinos excite the Gamow-Teller (GT) transition to the ground state with strength known from the lifetime of the electronic capture of  ${}^{37}\text{Ar}$ .  ${}^8\text{B}$  neutrinos can excite higher states of  ${}^{37}\text{Ar}$ , including the dominant transition to the isobaric state with transition energy 4.99 MeV. The strength of the GT transitions to these excited states are determined from the isospin related  $\beta$  decay,  ${}^{37}\text{Ca}(\beta^+){}^{37}\text{K}$ . The final result is that, for the SSM fluxes, 78% of the expected number of events are due to  ${}^8\text{B}$  neutrinos while 13% arise from  ${}^7\text{Be}$  neutrinos. The produced  ${}^{37}\text{Ar}$  is extracted radiochemically every three months approximately and the number of  ${}^{37}\text{Ar}$  decays ( $t_{\frac{1}{2}}=34.8$  days) is measured in a proportional counter.

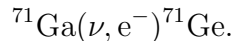
The average event rate measured during the more than 20 years of operation is

$$R_{\text{Cl}} = 2.56 \pm 0.16 \pm 0.16 \text{ SNU} ,$$

(1 SNU =  $10^{-36}$  captures/atom/sec) which corresponds to approximately one third of the SSM prediction (for the latest publications see Cleveland *et al.*, 1998 and Lande *et al.*, 1999).

## 2. Gallium experiments: SAGE and GALLEX/GNO

In January 1990 and May 1991, two new radiochemical experiments using a  $^{71}\text{Ga}$  target started taking data, SAGE and GALLEX. The SAGE detector is located in Baksan, Kabardino-Balkaria, Russia, with 30 Tons (increased to 57 Tons from July 1991) of liquid metallic Ga. GALLEX is located in Gran Sasso, Italy, and consists of 30 Tons of  $\text{GaCl}_3\text{-HCl}$ . In these experiments the solar neutrinos are captured via



The special properties of this target include a low threshold (0.233 MeV) and a strong transition to the ground level of  $^{71}\text{Ge}$ , which gives a large cross section for the lower energy  $pp$  neutrinos. According to the SSM, approximately 54% of the events are due to  $pp$  neutrinos, while 26% and 11% arise from  $^7\text{Be}$  and  $^8\text{B}$  neutrinos, respectively. The extraction of  $^{71}\text{Ge}$  takes place every 3–4 weeks and the number of  $^{71}\text{Ge}$  decays ( $t_{1/2}=11.4$  days) is measured in a proportional counter.

The event rates measured by SAGE (Abdurashitov *et al.*, 1999; Gavrin *et al.*, 2000) and GALLEX (Hampel *et al.*, 1999) are

$$R_{\text{SAGE}} = 75.4_{-6.8}^{+7.0} \text{ }_{-3.0}^{+3.5} \text{ SNU} ,$$

$$R_{\text{GALLEX}} = 77.5 \pm 6.2_{-4.7}^{+4.3} \text{ SNU} .$$

while the prediction of the SSM is 130 SNU.

The GALLEX program was completed in fall 1997 and its successor GNO started taking data in spring 1998. The latest combined GALLEX/GNO result is (Altmann *et al.*, 2000)

$$R_{\text{GALLEX+GNO}} = 74.1_{-6.8}^{+6.7} \text{ SNU} .$$

Since the  $pp$  flux is directly constrained by the solar luminosity, in all stationary solar models there is a theoretical minimum of the expected number of events of 79 SNU. Furthermore, the largest uncertainties on the capture cross section for  $^7\text{Be}$  neutrinos were considerably reduced after direct calibration using neutrinos from  $^{51}\text{Cr}$  decay as a source.

## 3. Water Cerenkov experiments: Kamiokande and SuperKamiokande

Kamiokande and its successor SuperKamiokande (SK) in Japan are water Cerenkov detectors that are able to detect in real time the electrons scattered from the water by elastic interaction of the solar neutrinos,

$$\nu_a + e^- \rightarrow \nu_a + e^-, \quad (96)$$

The scattered electrons produce Cerenkov light which is detected by photomultipliers. Notice that, in contrast to the radiochemical experiments, the detection process of Eq. (96) is sensitive to all active neutrino flavors, although  $\nu_e$ 's give the largest contribution since their scattering cross section is approximately 6 times larger than that of  $\nu_\mu$ 's or  $\nu_\tau$ 's.

Kamiokande, with 2140 tons of water, started taking data in January 1987 and was terminated in February 1995. SK, with 45000 tons of water (of which 22500 are usable in solar neutrino measurements) started in May 1996 and it has analyzed so far the events corresponding to 1258 days. The detection threshold in Kamiokande was 7.5 MeV while SK started at a 6.5 MeV threshold and is currently running at 5 MeV. This means that these experiments are able to measure only the  $^8\text{B}$  neutrinos (and the very small hep neutrino flux). Their results are presented in terms of measured  $^8\text{B}$  flux.

The final result of Kamiokande (Fukuda *et al.*, 1996) and the latest result of SK (Fukuda *et al.*, 2001) are

$$\Phi_{\text{Kam}} = (2.80 \pm 0.19 \pm 0.33) \times 10^6 \text{ cm}^{-2}\text{s}^{-1}.$$

and

$$\Phi_{\text{SK}} = (2.32 \pm 0.03 \pm 0.08) \times 10^6 \text{ cm}^{-2}\text{s}^{-1},$$

corresponding to about 40–50% of the SSM prediction.

There are three features unique to the water Cerenkov detectors. First, they are real time experiments. Each event is individually recorded. Second, for each event the scattered electron keeps the neutrino direction within an angular interval which depends on the neutrino energy as  $\sqrt{2m_e/E_\nu}$ . Thus, it is possible, for example, to correlate the neutrino detection with the position of the Sun. Third, the amount of Cerenkov light produced by the scattered electron allows a measurement of its energy. In summary, the experiment provides information on the time, direction and energy for each event. As we discuss below, signatures of neutrino oscillations might include distortion of the recoil electron energy spectrum, difference between the night-time solar neutrino flux and the day-time flux, or a seasonal variation in the neutrino flux. Observation of these effects would be strong evidence in support of solar neutrino oscillations independent of absolute flux calculations. Conversely, non-observation of these effects can constrain oscillation solutions to the solar neutrino problem.

Along the years the SK collaboration has presented the information on the energy and time dependence of their event rates in different forms. In the last published results in Fukuda *et al.* (2001), they show their event rates in 19 electron energy bins of 0.5 MeV width from 5 MeV, with a last bin corresponding to energy above 14 MeV. In Fig. 11 we show their spectrum, corresponding to 1258 days of data, relative to the predicted spectrum (Ortiz *et al.*, 2000) normalized to BP00. As seen from the figure, no significant distortion is observed and the data is compatible with a horizontal straight line.

The SK Collaboration has also measured the event rates as a function of the time of day or night or, in other words, of the position of the sun in the sky. The results are presented as a nadir angle (the angle which the sun forms with the vertical) distribution of events. In Fig. 12 we show their nadir angle distribution, corresponding to 1258 days of data (Fukuda

*et al.*, 2001). SK has also presented their results on the day-night variation in the form of a day-night asymmetry,

$$A_{N-D} \equiv 2 \frac{N - D}{D + N} = 0.033 \pm 0.022(\text{stat.}) \pm 0.013(\text{syst.}) \quad (97)$$

where  $D$  ( $N$ ) is the event rate during the day (night) period. The SK results show a small excess of events during the night but only at the  $1.3 \sigma$  level.

In order to simultaneously account for the energy and day-night variation, SK present the observed energy spectrum during the day and during the night separately. This is the most convenient observable for including both time and energy dependence simultaneously in the analysis. The reason for that is that the separate distributions – energy spectrum averaged in time and time dependence averaged in energy – do not correspond to independent data samples and therefore have non-trivial correlations.

Finally, SK has also measured the seasonal dependence of the solar neutrino flux. Their results after 1258 days of data are shown in Fig. 13. The points represent the measured flux, and the curve shows the expected variation due to the orbital eccentricity of the Earth (assuming no new physics, and normalized to the measured total flux). As seen from the figure, the data are consistent with the expected annual variation.

#### 4. SNO

The Sudbury Neutrino Observatory (SNO) was first proposed in 1987 and it started taking data in November 1999 (McDonald *et al.*, 2000). The detector, a great sphere surrounded by photomultipliers, contains approximately 1000 Tons of heavy water,  $D_2O$ , and is located at the Creighton mine, near Sudbury in Canada. SNO was designed to give a model independent test of the possible explanations of the observed deficit in the solar neutrino flux by having sensitivity to all flavors of active neutrinos and not just to  $\nu_e$ .

This sensitivity is achieved because energetic neutrinos can interact in the  $D_2O$  of SNO via three different reactions. Electron neutrinos may interact via the Charged Current (CC) reaction

$$\nu_e + d \rightarrow p + p + e^- , \quad (98)$$

and can be detected above an energy threshold of a few MeV (presently  $T_e > 6.75$  MeV). All active neutrinos ( $\nu_a = \nu_e, \nu_\mu, \nu_\tau$ ) interact via the Neutral Current (NC) reaction

$$\nu_a + d \rightarrow n + p + \nu_a , \quad (99)$$

with an energy threshold of 2.225 MeV. The non-sterile neutrinos can also interact via Elastic Scattering (ES),  $\nu_a + e^- \rightarrow \nu_a + e^-$ , but with smaller cross section.

The experimental plan of SNO consists of three phases of approximately one year duration each. In its first year of operation, SNO has concentrated on the measurement of the CC reaction rate while in a following phase, after the addition of  $MgCl_2$  salt to enhance the NC signal, it will also perform a precise measurement of the NC rate. In the third phase, the salt will be eliminated and a network of proportional counters filled with  $^3He$  will be added with the purpose of directly measuring the neutral current rate  $^3He(n,p)^3H$ .

SNO can also perform measurements of the energy spectrum and time variation of the event rates. But the uniqueness of SNO lies in its ability to directly test if the deficit of solar  $\nu_e$  is due to changes in the flavor composition of the solar neutrino beam, since the ratio CC/NC compares the number of  $\nu_e$  interactions with those from all active flavors. This comparison is independent of the overall flux normalization.

In June 2001, SNO published their first results (Ahmad, 2001)

$$\Phi_{\text{SNO}}^{\text{CC}} = (1.75 \pm 0.07_{-0.11}^{+0.12}) \times 10^6 \text{ cm}^{-2}\text{s}^{-1},$$

which is about a third of the expected flux in the BP00 model, and

$$\Phi_{\text{SNO}}^{\text{ES}} = (2.39 \pm 0.34_{-0.14}^{+0.16}) \times 10^6 \text{ cm}^{-2}\text{s}^{-1},$$

both with threshold energy of 6.75 MeV. The ES result is compatible with the result measured at SK but with much lower statistical significance. SNO have also presented a preliminary result on the energy dependence of the CC event rate which is still not precise enough to lead to any conclusion.

## 5. Future: Borexino and Low Energy experiments

The Borexino experiment (Oberauer, 1999) is designed to detect low-energy solar neutrinos in real-time through the observation of the ES process  $\nu_a + e^- \rightarrow \nu_a + e^-$ . The energy threshold for the recoil electrons is 250 keV. It will use 300 tons of liquid scintillator in an unsegmented detector with 2000 photomultiplier tubes. The event rate predicted by the SSM for a fiducial volume of about 100 tons is about 50 events per day, mostly generated by the 0.86 MeV monoenergetic line of  ${}^7\text{Be}$  solar neutrinos. Since this line gives a characteristic spectral signature in the ES process, the flux on the earth of  ${}^7\text{Be}$  solar neutrinos will be determined and it will be possible to check if it is suppressed with respect to the one predicted by the SSM as suggested by the results of current experiments. The Borexino experiment is under construction in the Laboratori Nazionali del Gran Sasso in Italy and is scheduled to start data taking around the year 2002.

A new generation of experiments aiming at a high precision real time measurement of the low energy solar neutrino spectrum is now under study (de Bellfon, 1999; Lanou *et al.*, 1993; Arpesella, Brogini, and Cattadori, 1996; Raghavan, 1997). Some of them, such as HELLAZ, HERON and SUPER-MuNu, intend to detect the elastic scattering of the electron neutrinos with the electrons of a gas and measure the recoil electron energy and its direction of emission. The proposed experiment LENS plans to detect the electron neutrino via its absorption in a heavy nuclear target with the subsequent emission of an electron and a delayed gamma emission. The expected rates at these experiments for the proposed detector sizes are of the order of  $\sim 1 - 10$  *pp* neutrinos a day. Consequently, with a running time of two years, they can reach a sensitivity of a few percent in the total neutrino rate at low energy, provided that they can achieve sufficient background rejection.

## B. The Solar Neutrino Problem

Table III summarizes our present knowledge of the solar neutrino fluxes, their contribution to the expected rates and the data from measurements of solar neutrino experiments.

The predicted event rates are linear functions of the seven important neutrino fluxes:  $p$ - $p$ ,  $pep$ ,  $hep$ ,  ${}^7\text{Be}$ ,  ${}^8\text{B}$ ,  ${}^{13}\text{N}$ , and  ${}^{15}\text{O}$ . We can make the following statements (see the last row of the table):

- All experiments observe a flux that is smaller by a factor  $\sim 0.3 - 0.6$  compared to the SSM predictions.
- The deficit is not the same for the various experiments, which may indicate that the effect is energy dependent.

These two statements constitute the solar neutrino problem (Bahcall, Bahcall and Shaviv, 1968; Bahcall and Davis, 1976). More quantitatively, a fit to the data using BP00 shows a disagreement of  $6.7 \sigma$ , which means that the probability of explaining this results as a consequence of a statistical fluctuation of standard particle physics is  $P = 3 \times 10^{-11}$  (we follow here the latest discussion by Bahcall, 2002a).

One may wonder about the solar model dependence of the solar neutrino problem. To answer this question one can allow all the solar neutrino fluxes, with undistorted energy spectra, to be free parameters in fitting the measured solar neutrino event rates, subject only to the condition that the total observed luminosity of the sun is produced by nuclear fusion. This luminosity constraint can be written as a convenient linear equation in the neutrino fluxes:

$$1 = \sum_i \left( \frac{\alpha_i}{10 \text{ MeV}} \right) \left( \frac{\phi_i}{8.532 \times 10^{10} \text{ cm}^{-2}\text{s}^{-1}} \right), \quad (100)$$

where  $\phi_i$  are the individual neutrino fluxes ( $i = pp, pep, hep, {}^7\text{Be}, {}^8\text{B}, {}^{13}\text{N}$ , and  ${}^{15}\text{O}$ ) and  $\alpha_i$  is the luminous energy released in the corresponding reaction (the most recent determination of these parameters can be found in Bahcall, 2002b). Since there are at present six experiments, one cannot use all seven of the neutrino fluxes as free parameters and some additional constraints must be used. For instance, the ratio of the  $pep$  to  $p$ - $p$  fluxes is usually taken to be the same as in the SSM. Also the CNO nuclear reactions are assumed to be in equilibrium and the  ${}^{13}\text{N}$  and  ${}^{15}\text{O}$  neutrino fluxes are taken to be exactly equal (or, in some analyses, both are taken to be zero). Finally the contribution from  $hep$  neutrinos is usually neglected.

A fit of this type (Bahcall, 2002b) shows that, previous to the SNO measurement, the best fit corresponded always to an unphysical, negative  ${}^7\text{Be}$  flux. Forcing all fluxes to be positive, the hypothesis of no-new-physics was rejected at the effective  $2.5\sigma$  level (99% C.L.). Furthermore, it required that  ${}^7\text{Be}$  neutrinos be entirely missing, a result which many authors have argued is not physically or astrophysically reasonable (for instance, Bahcall and Bethe, 1990; Hata, Bludman, and Langacker, 1994; Parke, 1995; Heeger and Robertson, 1996; Bahcall, Krastev and Smirnov, 1998).

The published results of the SNO CC measurement have provided further model independent evidence of the problem. Both SNO and SK are sensitive mainly to the  ${}^8\text{B}$  flux. Without NP, the measured fluxes in these two experiments should be equal. In reality, the ratio

$$\frac{\phi_{8B}^{\text{SNO}}}{\phi_{8B}^{\text{SK}}} = 0.754 \pm 0.069 \quad (101)$$

is inconsistent with unity at the level of  $3.5\sigma$  (Berezinsky, 2001). Villante, Fiorentini and Lisi (1999) have discussed how one can choose the energy thresholds for the SK and SNO experiments in such a way that the response functions for the two experiments are made approximately equal (see also Fogli *et al*, 2001 for the latest analysis). The advantage of this method is that some of the systematic errors are reduced, but there is a slight loss of statistical power (Bahcall, 2002a, Berezinski, 2001).

Combining all the experimental data results in the rejection of the no-new-physics hypothesis at the  $4\sigma$  level with the best solution corresponding to the same unphysical case of zero  ${}^7\text{Be}$  flux.

### C. Solar Neutrino Oscillation Probabilities

The most generic and popular explanation of the solar neutrino anomaly is given by the introduction of neutrino masses and mixing, leading to oscillations of  $\nu_e$  into an active ( $\nu_\mu$  and/or  $\nu_\tau$ ) or a sterile ( $\nu_s$ ) neutrino. We now discuss some issues that have been raised in the literature concerning the computation of the corresponding neutrino survival probabilities in the full range of mass and mixing relevant to the solar neutrino problem.

#### 1. Quasivacuum oscillations and the *Dark Side*

The presence of neutrino mass and mixing implies the possibility of neutrino oscillations. Solar electron neutrinos can undergo oscillations either in vacuum or via the matter-enhanced *MSW mechanism*, depending on the actual values of mass-squared differences and mixing angles. However, this simplified picture of solar neutrino oscillations contains a set of approximations which are not always valid in the context of solar neutrinos. In order to clarify this issue let us first review the calculation of the solar neutrino survival probability in the two-neutrino case.

The survival amplitude for a solar  $\nu_e$  of energy  $E$  at a terrestrial detector can be written as follows:

$$A_{ee} = \sum_{i=1}^2 A_{ei}^S A_{ie}^E \exp[-im_i^2(L-r)/(2E)] . \quad (102)$$

Here  $A_{ei}^S$  is the amplitude of the transition  $\nu_e \rightarrow \nu_i$  from the production point to the surface of the Sun,  $A_{ie}^E$  is the amplitude of the transition  $\nu_i \rightarrow \nu_e$  from the surface of the Earth to the detector. The propagation amplitude in vacuum from the surface of the Sun to the surface of the Earth is given by the exponential term, where  $L$  is the distance between the center of the Sun and the surface of the Earth, and  $r$  is the radius of the Sun. The corresponding survival probability  $P_{ee}$  is then given by:

$$P_{ee} = P_1 P_{1e} + P_2 P_{2e} + 2\sqrt{P_1 P_2 P_{1e} P_{2e}} \cos \xi . \quad (103)$$

Here  $P_i \equiv |A_{ei}^S|^2$  is the probability that the solar neutrinos reach the surface of the Sun as  $|\nu_i\rangle$ , while  $P_{ie} \equiv |A_{ie}^E|^2$  is the probability of  $\nu_i$  arriving at the surface of the Earth to be detected as a  $\nu_e$ . Unitarity implies  $P_1 + P_2 = 1$  and  $P_{1e} + P_{2e} = 1$ . The phase  $\xi$  is given by

$$\xi = \frac{\Delta m^2(L-r)}{2E} + \delta, \quad (104)$$

where  $\delta$  contains the phases due to propagation in the Sun and in the Earth and can be safely neglected. In the evaluation of both  $P_i$  and  $P_{ie}$  the effect of coherent forward interaction with the matter of the Sun and the Earth must be taken into account.

From Eq. (103) one can recover more familiar expressions for  $P_{ee}$  that hold in certain limits:

(1) For  $\Delta m^2/E \lesssim 5 \times 10^{-17}$  eV, the matter effect suppresses flavor transitions in both the Sun and the Earth. Consequently, the probabilities  $P_1$  and  $P_{2e}$  are simply the projections of the  $\nu_e$  state onto the mass eigenstates:  $P_1 = \cos^2 \theta$ ,  $P_{2e} = \sin^2 \theta$ . In this case we are left with the standard vacuum oscillation formula,

$$P_{ee}^{\text{vac}} = 1 - \sin^2 2\theta \sin^2[\Delta m^2(L-r)/4E], \quad (105)$$

which describes the oscillations on the way from the surface of the Sun to the surface of the Earth. The probability is symmetric under the change of octant,  $\theta \leftrightarrow \frac{\pi}{2} - \theta$ , and change of sign  $\Delta m^2 \leftrightarrow -\Delta m^2$  (see Sec. III). This symmetry implies that each point in the  $(\Delta m^2, \sin^2 2\theta)$  parameter space corresponds to two physically independent solutions, one in each octant.

Averaging Eq. (105) over the Earth Orbit,  $L(t) = L_0[1 - \varepsilon \cos \frac{2\pi t}{T}]$ , one gets :

$$\langle P_{ee}^{\text{vac}} \rangle = 1 - \frac{1}{2} \sin^2 2\theta \left[ 1 - \cos \left( \frac{\Delta m^2 L_0}{2E} \right) J_0 \left( \frac{\varepsilon \Delta m^2 L_0}{2E} \right) \right], \quad (106)$$

where  $\varepsilon = 0.0167$  is the orbit eccentricity and  $J_0$  is the Bessel function. In Fig. 14.a we display the value of  $\langle P_{ee}^{\text{vac}} \rangle$  as a function of  $4E/\Delta m^2$ . As seen in the figure, for large values of  $\Delta m^2$  the probability averages out to a constant value  $1 - \frac{1}{2} \sin^2(2\theta)$ .

(2) For  $\Delta m^2/E \gtrsim 10^{-14}$  eV, the last term in Eq. (103) vanishes and we recover the incoherent MSW survival probability. In this case  $P_1$  and  $P_{2e}$  must be obtained by solving the evolution equation of the neutrino states in the Sun and the Earth respectively. As discussed in Sec. III.D, the approximate solution for the evolution in the Sun takes the well-known form

$$P_1 = \frac{1}{2} + \left( \frac{1}{2} - P_{LZ} \right) \cos 2\theta_{m,0}. \quad (107)$$

Here  $P_{LZ}$  denotes the standard Landau-Zener probability of Eq. (81) which, for the exponential profile, takes the form shown in Eq. (88), and  $\theta_{m,0}$  is the mixing angle in matter at the neutrino production point given in Eq. (61). For the approximation of exponential density profile,  $\gamma = \pi \frac{\Delta m^2}{E} r_0$ , which is independent of the point in the Sun where the resonance takes place. Improvement over this constant-slope exponential density profile approximation can be obtained by numerically deriving the exact  $N_e(r)$  profile at the resonant point. In this case  $\gamma = \pi \frac{\Delta m^2}{E} r_0(r_{res})$ .

During the day there is no Earth matter effect. The survival probabilities  $P_{ie}$  are obtained by simple projection of the  $\nu_e$  state onto the mass eigenstates:  $P_{2e,D} = 1 - P_{1e,D} = \sin^2 \theta$ . One obtains the expression in Eq. (82)



$$P_{ee,D}^{\text{MSW}} = \frac{1}{2} + \left( \frac{1}{2} - P_{LZ} \right) \cos 2\theta_{m,0} \cos 2\theta. \quad (108)$$

In Fig. 14.b (14.c) we plot the this survival probability as a function of  $4E/\Delta m^2$  for a large (small) mixing angle.

Let us make some remarks concerning Eq. (108):

(i) In both the limits of large and very small  $E/\Delta m^2$ ,  $P_{ee}^{\text{MSW}} \rightarrow 1 - \frac{1}{2} \sin^2 2\theta$  (see Fig. 14.b) which is the same expression as for averaged vacuum oscillations. This result, however, comes from very different reasons in the two regimes. For large  $E/\Delta m^2$ ,  $P_{LZ} = \cos^2 \theta$  and  $\cos 2\theta_{m,0} = -1$ . For small  $E/\Delta m^2$ ,  $P_{LZ} = 0$  and  $\cos 2\theta_{m,0} = \cos 2\theta$ .

(ii) Due to matter effects,  $P_{ee}^{\text{MSW}}$  is only symmetric under the simultaneous transformation  $(\Delta m^2, \theta) \rightarrow (-\Delta m^2, \frac{\pi}{2} - \theta)$ . For  $\Delta m^2 > 0$ , the resonance is only possible for  $\theta < \frac{\pi}{4}$  and MSW solutions are usually plotted in the  $(\Delta m^2, \sin^2 2\theta)$  plane assuming that now each point on this plane represents only one physical solution with  $\theta$  in the first octant. But in principle non-resonant solutions are also possible for  $\theta > \frac{\pi}{4}$ , the so called *dark side* (de Gouvea, Friedland and Murayama, 2000; Gonzalez-Garcia and Peña-Garay, 2000).

(3) The analysis of the survival probability of solar neutrinos would be simplified if there were a region where  $E/\Delta m^2$  is small enough that vacuum oscillations are averaged out and large enough to be in the extremely non-adiabatic MSW regime. This, however, is not the case, as can be seen from comparison of panels (a) and (b) of Fig.14. In the intermediate range,  $2 \times 10^{14} \lesssim 4E/\Delta m^2 \lesssim 10^{16} \text{ eV}^{-1}$ , adiabaticity is violated and the  $\cos \xi$  coherent term should be taken into account. The result is similar to vacuum oscillations but with small matter corrections. We define this case as quasi-vacuum oscillations (QVO) (Petcov, 1988; Pantaleone, 1990; Pakvasa and Pantaleone, 1990; Friedland 2000). The range of  $E/\Delta m^2$  for the QVO regime depends on the value of  $E/\Delta m^2$  for which the MSW probability in Eq. (108) acquires the asymptotic value  $1 - \frac{1}{2} \sin^2 2\theta$ : the smaller  $E/\Delta m^2$  the more separated the MSW and vacuum regimes are, and the narrower the QVO region is.

It is clear from these considerations that in order to compute the survival probability for solar neutrinos that would be valid for any experiment and any value of the neutrino mass and mixing, the full expression (103) has to be evaluated. The results that we show here were obtained using the general expression for the survival probability in Eq. (103) with  $P_1$  and  $P_{2e}$  found by numerically solving the evolution equation in the Sun and the Earth. In the Sun the evolution equation has to be solved from the neutrino production point to the edge of the Sun and averaged over the production point distribution shown in Fig. 10.

## 2. Evolution in the Earth

In this section we discuss matter effects in the Earth (for some of the original works see J. Bouchez *et. al.* 1986, Mikheyev and Smirnov, 1987, A. J. Baltz and J. Weneser 1987 and 1988). Unlike the Sun, the Earth's matter density is not strongly varying. It consists mainly of two layers, the lower-density mantle and the higher-density core, each with approximately constant density. The core radius is  $L = 2896 \text{ km}$ .

When a neutrino crosses only the mantle, the Earth matter effects are well approximated by evolution with a constant potential. In this case, the resulting probability simplifies to:

$$P_{2e} = \sin^2 \theta + \frac{4EV_e}{\Delta m^2} \sin^2 2\theta_E \sin^2 \frac{\pi L}{L_m}, \quad (109)$$

where  $\theta_E$  is the mixing angle in the Earth [Eq. (61)],  $L$  is the distance traveled by the neutrino within the Earth and  $L_m$  is the oscillation length in matter [Eq (64)].

The right hand side of Eq. (109) consists of two terms. The first term gives a simple projection of the mass to flavor state in vacuum (which corresponds to the probability during the day time). The second is the *regeneration term* and will be denoted by  $f_{\text{reg}}$  (for details see Gonzalez-Garcia, Peña-Garay and Smirnov, 2001). It contains the Earth matter effects and it is always positive. Averaging out the distance dependent term, we can now rewrite Eq. (103):

$$P_{ee}^N = P_{ee}^D - (1 - 2P_{LZ}) \cos 2\theta_{m,0} f_{\text{reg}}, \quad (110)$$

where  $P_{ee}^D$  ( $P_{ee}^N$ ) is the  $\nu_e$  survival probability during the day (night). Since  $f_{\text{reg}} > 0$  and, for the interesting cases of MSW transitions in the Sun,  $\cos 2\theta_{m,0} < 0$ , the relative magnitude of  $P_{ee}^D$  and  $P_{ee}^N$  depends on the sign of  $(1 - 2P_{LZ})$ . For  $P_{LZ} < 0.5$  (and, in particular, for adiabatic MSW transitions,  $P_{LZ} = 0$ ) the survival probability during the night is larger than during the day. The opposite holds for large adiabaticity violations,  $P_{LZ} > 0.5$ .

When neutrinos cross the core, adiabaticity may be violated in their evolution. The abrupt density change between the mantle and the core may induce a new form of resonance, different from the MSW resonance (Petcov, 1998; Akhmedov, 1999) This *parametric resonance* (Ermilova, Tsarev and Chechin, 1986; Akhmedov, 1987) is relevant mainly for small mixing angles.

## D. Two-neutrino Oscillation Analysis

### 1. Predictions

The expected event rate in the presence of oscillations in the experiment  $i$ ,  $R_i^{\text{th}}$ , can be written as follows:

$$R_i^{\text{th}} = \sum_{k=1,8} \phi_k \int dE_\nu \lambda_k(E_\nu) \times [\sigma_{e,i}(E_\nu) \langle P_{ee}(E_\nu, t) \rangle + \sigma_{x,i}(E_\nu) (1 - \langle P_{ee}(E_\nu, t) \rangle)], \quad (111)$$

where  $E_\nu$  is the neutrino energy,  $\phi_k$  is the total neutrino flux and  $\lambda_k$  is the neutrino energy spectrum (normalized to 1) from the solar nuclear reaction  $k$ .  $\sigma_{e,i}$  ( $\sigma_{x,i}$ ) is the  $\nu_e$  ( $\nu_x$ ,  $x = \mu, \tau$ ) interaction cross section in the SM (Bahcall, Kamionkowski and Sirlin, 1995), with the target corresponding to experiment  $i$ , and  $\langle P_{ee}(E_\nu, t) \rangle$  is the time-averaged  $\nu_e$  survival probability. The expected signal in the absence of oscillations,  $R_i^{\text{BP00}}$ , can be obtained from Eq. (111) by substituting  $P_{ee} = 1$ . In Table III we give the expected rates at the different experiments which we obtain using the fluxes of Bahcall, Pinsonneault, and Basu (2001).

For the Chlorine, Gallium and SNO(CC) measurements, only the electron neutrino contributes and the  $\sigma_{x,i}$ -term in Eq. (111) vanishes. For SK there is a possible contribution from the NC interaction of the other active neutrino flavors present in the beam. For absorption in chlorine and gallium, the cross sections  $\sigma_{e,i}(E)$  can be found in (Bahcall, 2001). The cross sections for SNO(CC) and SK(ES) can be obtained from the differential cross sections of (Nakamura *et al.*, 2001) and (Bahcall, Kamionkowski, and Sirlin, 1995), respectively, by integrating with the corresponding detector resolutions and for the given detection thresholds. In particular, for the SK energy spectrum data, to obtain  $R^{\text{th}}$  in a given energy bin

one has to integrate within the corresponding electron recoil energy bin and to take into account that the finite energy resolution implies that the *measured* kinetic energy  $T$  of the scattered electron is distributed around the *true* kinetic energy  $T'$  according to a resolution function  $Res(T, T')$  of the form (from Bahcall, Krastev and Lisi, 1997):

$$Res(T, T') = \frac{1}{\sqrt{2\pi}s} \exp \left[ -\frac{(T - T')^2}{2s^2} \right], \quad (112)$$

where  $s = s_0 \sqrt{T'/\text{MeV}}$ , and  $s_0 = 0.47$  MeV for SK (from Faid *et al.*, 1997). On the other hand, the distribution of the true kinetic energy  $T'$  for an interacting neutrino of energy  $E_\nu$  is dictated by the differential cross section  $d\sigma_\alpha(E_\nu, T')/dT'$  and the kinematic limits are:

$$0 \leq T' \leq \bar{T}'(E_\nu) = \frac{E_\nu}{1 + m_e/2E_\nu}. \quad (113)$$

For assigned values of  $s_0$ ,  $T_{\min}$ , and  $T_{\max}$ , the corrected cross section  $\sigma_a(E)$  ( $a = e, x$ ) is given as

$$\sigma_a(E_\nu) = \int_{T_{\min}}^{T_{\max}} dT \int_0^{\bar{T}'(E_\nu)} dT' Res(T, T') \frac{d\sigma_a(E_\nu, T')}{dT'}. \quad (114)$$

For data taken within a given zenith angle period  $i$ , the expected number of events in the presence of oscillations is

$$R_i^{\text{th}} = \frac{1}{\Delta\tau_i} \int_{\tau(\cos\Phi_{\min,i})}^{\tau(\cos\Phi_{\max,i})} d\tau \sum_{k=1,8} \phi_k \int dE_\nu \lambda_k(E_\nu) \times [\sigma_{e,i}(E_\nu) \langle P_{ee}(E_\nu, \tau) \rangle + \sigma_{x,i}(E_\nu) (1 - \langle P_{ee}(E_\nu, \tau) \rangle)], \quad (115)$$

where  $\tau$  measures the annual averaged length of the period  $i$  normalized to 1:  $\Delta\tau_i = \tau(\cos\Phi_{\max,i}) - \tau(\cos\Phi_{\min,i})$ . For instance, if the period  $i$  corresponds to the entire day ( $i = D$ ) or night ( $i = N$ ) then  $\Delta\tau_{D(N)} = 0.5$ . From these predictions one can easily build the corresponding expected day-night asymmetry. In Fig. 15 we show the isocontours of expected  $A_{N-D}$  in the  $(\Delta m^2, \tan^2\theta)$  plane for active neutrino oscillations (the results are very similar for sterile neutrinos). As discussed in Sec. IV.C, in most of the parameter space, the effect of traveling across the Earth is the regeneration of the  $\nu_e$  component, resulting in a positive day-night asymmetry with the exception of the region where the non-adiabaticity of the oscillations in the Sun is important ( $\tan^2\theta \sim 10^{-3}$ ,  $\Delta m^2 \sim 3 \times 10^{-6}$  eV<sup>2</sup>). As discussed above, SK has observed a very small day-night asymmetry. This observation implies that some of the oscillation parameters space can be excluded as we discuss below.

In the same fashion, integrating Eq. (115) for the different electron recoil energy bins and for the day and night periods separately, one can obtain the predictions for SK day-night spectrum.

## 2. Analysis of total event rates: allowed masses and mixing

The goal of the analysis of the solar neutrino data in terms of neutrino oscillation is to determine which range of mass-squared difference and mixing angle can be responsible for the

observed deficit (see for instance Hata and Langacker, 1997; Fogli, Lisi and Montanino, 1998; Bahcall, Krastev and Smirnov, 1998; Gonzalez-Garcia *et al.*, 2000). In order to answer this question in a statistically meaningful way one must compare the predictions in the different oscillation regimes with the observations, including all the sources of uncertainties and their correlations, by building, for instance, the  $\chi^2$  function

$$\chi_R^2 = \sum_i (R_i^{\text{th}} - R_i^{\text{exp}}) \sigma_{ij}^{-2} (R_j^{\text{th}} - R_j^{\text{exp}}). \quad (116)$$

Here  $R_i^{\text{th}}$  is the theoretical expectation (111),  $R_i^{\text{exp}}$  is the observed number of events in the experiment  $i$ , and  $\sigma_{ij}$  is the error matrix which contains both the theoretical uncertainties and the experimental statistical and systematic errors. The main sources of uncertainty are the theoretical errors in the prediction of the solar neutrino fluxes for the different reactions. These errors are due to uncertainties in the twelve basic ingredients of the solar model, which include the nuclear reaction rates (parametrized in terms of the astrophysical factors  $S_{11}$ ,  $S_{33}$ ,  $S_{34}$ ,  $S_{1,14}$  and  $S_{17}$ ), the solar luminosity, the metalicity  $Z/X$ , the sun age, the opacity, the diffusion, and the electronic capture of  ${}^7\text{Be}$ ,  $C_{\text{Be}}$ . These errors are strongly correlated as the same astrophysical factor can affect the different neutrino production rates. Another source of theoretical error arises from the uncertainties in the neutrino interaction cross section for the different detection processes. For a detailed description of the way to include all these uncertainties and correlations in the construction of  $\sigma_{ij}$  we refer to the work of Fogli and Lisi (1995a). Updated uncertainties can be found in Fogli *et al.* (2000a) and Gonzalez-Garcia and Peña-Garay (2001).

The results of the analysis of the total event rates are shown in Fig. 16 and Table IV (from Bahcall, Gonzalez-Garcia and Peña-Garay, 2001b). In Fig. 16 we show the allowed regions which correspond to 90%, 95%, 99% and 99.73% ( $3\sigma$ ) CL for  $\nu_e$  oscillations into active or sterile neutrinos and in the table we list the local minima in each of the allowed regions and the value of  $\chi_{\text{min}}^2$  in each local minimum.

As seen in the figure, at present, for oscillations into active neutrinos there are several oscillation regimes which are compatible within errors with the experimental data. These allowed parameter regions are denoted as *MSW small mixing angle* (SMA), *MSW large mixing angle* (LMA), *MSW low mass* (LOW) and *vacuum oscillations* (VAC). Before including the SNO(CC) data, the best fit corresponded to the SMA solution, but after SNO the best fit corresponds to the LMA solution.

For oscillations into sterile neutrinos the parameter space is considerably smaller. Oscillations into sterile neutrinos would imply very similar neutrino fluxes in SNO and SK. Schematically, in presence of oscillations, the ratio given in Eq. (101) is given by:

$$R_{\text{SNO/SK}} \equiv \frac{\phi_{8B}^{\text{SNO}}}{\phi_{8B}^{\text{SK}}} = \frac{\langle P_{ee} \rangle_{\text{SNO}}}{\langle P_{ee} \rangle_{\text{SK}} + r \langle P_{ea} \rangle_{\text{SK}}} \quad (117)$$

which has to be compared with the measured value  $R_{\text{SNO/SK}} = 0.754 \pm 0.069$ .  $P_{ee}$  is the  $\nu_e$  survival probability while  $P_{ea}$  is the oscillation probability into active neutrinos. By  $\langle P \rangle_{\text{SNO(SK)}}$  we denote the corresponding averaged oscillation probability at SNO (SK) taking into account the detector response function which accounts for energy threshold and resolution and the neutrino interaction cross section (see Eq. (111)).  $r \equiv \sigma_{\mu-e} / \sigma_{e-e} \simeq 0.15$  is the ratio of the the  $\nu_e - e$  and  $\nu_{\mu} - e$  elastic scattering cross-sections.

Naively, ignoring the difference between the detectors response functions we see that oscillations into sterile neutrinos predict that Eq. (117) would be equal to  $\sim 1$  which is ruled out at  $\sim 3\sigma$ . Eq. (117) also shows us that the difference between the solar neutrino fluxes measured in SNO(CC) and SK(ES) must come from transitions into active neutrinos, which would contribute only to SK(ES) through NC interactions. In summary, the combination of SNO(CC) and SK(ES) results strongly disfavors the hypothesis of pure  $\nu_e \rightarrow \nu_s$  oscillations.

### 3. Day-Night spectrum: excluded masses and mixing

Further information on the different oscillation regimes can be obtained from the analysis of the energy and time dependence data from SK which is currently presented in the form of the observed day-night spectrum (Fukuda *et al.*, 2001). The way to statistically treat these data has been discussed for instance in Fukuda *et al.* (2001); Bahcall, Krastev, and Smirnov (2000); Gonzalez-Garcia *et al.* (2000); Gonzalez-Garcia and Peña-Garay (2001).

The observed day-night spectrum is essentially undistorted in comparison to the SSM expectation and shows no significant differences between the day and the night periods. Consequently, a large region of the oscillation parameter space where these variations are expected to be large can be excluded. In Fig. 11 we show the SK spectrum corresponding to 1258 days of data relative to the Ortiz *et al.* (2000) spectrum normalized to BP00, together with the expectations from the best fit points for the LMA, SMA and LOW solutions in table IV.

The various solutions give different predictions for the day night spectrum. For LMA and LOW, the expected spectrum is very little distorted. For SMA, a positive slope is expected, with larger slope for larger mixing angle within SMA. For VAC, large distortions are expected. The details are strongly dependent on the precise values of the oscillation parameters.

In Fig. 17 we show the excluded regions at the 99% CL from the analysis of SK day-night spectrum data, together with the contours corresponding to the 95% and 99.73% ( $3\sigma$ ) CL. In particular, the central region ( $2 \times 10^{-5} < \Delta m^2 < 3 \times 10^{-7}$ ,  $\tan^2 \theta > 3 \times 10^{-3}$ ) is excluded due to the small observed day-night variation (compare to Fig. 15). The rest of the excluded region is due to the absence of any observed distortion of the energy spectrum.

Superimposing Fig. 17 and Fig. 16 we can deduce the main consequences of adding the day-night spectrum information to the analysis of the total event rates:

- Active SMA: within this region, the part with larger mixing angle fails to comply with the observed energy spectrum, while the part with smaller mixing angles gives a bad fit to the total rates.
- VAC (either active or sterile): the observed flat spectrum cannot be accommodated.
- Active LMA and active LOW: the small  $\Delta m^2$  part of LMA and the large  $\Delta m^2$  part of LOW are reduced because they predict a day-night variation that is larger than observed. Both active LMA and active LOW solutions predict a flat spectrum in agreement with the observation.

## 4. Global analysis

In order to quantitatively discuss the combined analysis of the total event rates and the day-night spectrum, one must define a global statistical function. How to implement such a program is a question that is being discussed intensively in the literature (Fogli *et al.*, 2001; Bandyopadhyay *et al.*, 2001; Bahcall, Gonzalez-Garcia, and Peña-Garay, 2001a and 2001b; Garzelli and Giunti, 2001; Krastev and Smirnov, 2001; Creminelli, Signorelli and Strumia, 2001). The situation at present is that, although one finds slightly different globally allowed regions, depending on the particular prescription used in the combination, the main conclusions are independent of the details of the analysis.

We show in Fig. 18 and Table V the results from the global analysis of Bahcall, Gonzalez-Garcia and Peña-Garay, (2001b). In Fig. 18 we show the allowed regions which correspond to 90%, 95%, 99% and 99.73% ( $3\sigma$ ) CL for  $\nu_e$  oscillations into active or sterile neutrinos. In the table we list the local minima of the allowed regions, the value of  $\chi^2_{\min}$  in each local minimum. The CHOOZ reactor data is also included in the analysis (see Sec. VI.C) and leads to the cut at  $\Delta m^2 \sim 8 \times 10^{-4}$  on the LMA region.

The results show that at present the most favoured solution is the active LMA oscillation while the active LOW solution provides a slightly worse fit. There is a small allowed *island* for active VAC oscillations. The active SMA solution does not appear at  $3\sigma$  as a consequence of the incompatibility between the observed small CC rate at SNO, which would favour larger mixing, and the flat SK spectrum which prefers smaller mixing. Oscillations of solar neutrinos into pure sterile state are also disfavoured at the  $3\sigma$  level. As explained below, in Sec. VII, this difference between active and sterile oscillations has important implications on  $4\nu$  mixing schemes.

## V. ATMOSPHERIC NEUTRINOS

Atmospheric neutrinos are produced in cascades initiated by collisions of cosmic rays with the Earth's atmosphere. Some of the mesons produced in these cascades, mostly pions and some kaons, decay into electron and muon neutrinos and anti-neutrinos. The predicted absolute fluxes of neutrinos produced by cosmic-ray interactions in the atmosphere are uncertain at the 20% level. The ratios of neutrinos of different flavor are, however, expected to be accurate to better than 5%. Since  $\nu_e$  is produced mainly from the decay chain  $\pi \rightarrow \mu\nu_\mu$  followed by  $\mu \rightarrow e\nu_\mu\nu_e$ , one naively expects a 2 : 1 ratio of  $\nu_\mu$  to  $\nu_e$ . (For higher energy events the expected ratio is smaller because some of the muons arrive to Earth before they had time to decay.) In practice, however, the theoretical calculation of the ratio of muon-like interactions to electron-like interactions in each experiment is more complicated.

Atmospheric neutrinos are observed in underground experiments using different techniques and leading to different type of events, which we briefly summarize here. They can be detected by the direct observation of their charged current interaction inside the detector. These are the *contained* events. Contained events can be further classified into *fully contained* events, when the charged lepton (either electron or muon) that is produced in the neutrino interaction does not escape the detector, and *partially contained* muons, when the produced muon exits the detector. For fully contained events the flavor, kinetic energy and direction of the charged lepton can be best determined. As discussed later, some ex-

periments further divide the contained data sample into sub-GeV and multi-GeV events, according to whether the visible energy is below or above 1.2 GeV. On average, sub-GeV events arise from neutrinos of several hundreds of MeV while multi-GeV events are originated by neutrinos with energies of the order of several GeV. Higher energy muon neutrinos and antineutrinos can also be detected indirectly by observing the muons produced in their charged current interactions in the vicinity of the detector. These are the so called *upgoing muons*. Should the muon stop inside the detector, it is classified as a *stopping muon* (which arises from neutrinos  $E_\nu \sim 10$  GeV), while if the muon track crosses the full detector the event is classified as a *through-going muon* (which is originated by neutrinos with energies of the order of hundreds of GeV). Downgoing muons from  $\nu_\mu$  interactions above the detector cannot be distinguished from the background of cosmic ray muons. Higher energy  $\nu_e$ 's cannot be detected this way as the produced  $e$  showers immediately in the rock. In Fig. 19 we display the characteristic neutrino energy distribution for these different type of events (taken from Engel, Gaisser and Stanev, 2000).

Atmospheric neutrinos were first detected in the 1960's by the underground experiments in South Africa (Reines *et al.*, 1965) and the Kolar Gold Field experiment in India (Achar *et al.*, 1965). These experiments measured the flux of horizontal muons (they could not discriminate between downgoing and upgoing directions) and although the observed total rate was not in full agreement with theoretical predictions (Zatsepin and Kuzmin, 1962; Cowsik *et al.*, 1965; Osborne, Said and Wolfendale, 1965), the effect was not statistically significant.

A set of modern experiments were proposed and built in the 1970's and 1980's. The original purpose was to search for nucleon decay, for which atmospheric neutrinos constitute background, although the possibility of using them to search for oscillation was also known (Ayres *et al.*, 1984). Two different detection techniques were employed. In water Cerenkov detectors the target is a large volume of water surrounded by photomultipliers which detect the Cerenkov-ring produced by the charged leptons. The event is classified as an electron-like (muon-like) event if the ring is diffuse (sharp). In iron calorimeters, the detector is composed of a set of alternating layers of iron which act as a target and some tracking element (such as plastic drift tubes) which allows the reconstruction of the shower produced by the electrons or the tracks produced by muons. Both types of detectors allow for flavor classification of the events.

The two oldest iron calorimeter experiments, Fréjus (Daum *et al.*, 1995) and NUSEX (Aglietta *et al.*, 1989), found atmospheric neutrino fluxes in agreement with the theoretical predictions. On the other hand, two water Cerenkov detectors, IMB (Becker-Szendy *et al.*, 1992) and Kamiokande, detected a ratio of  $\nu_\mu$ -induced events to  $\nu_e$ -induced events smaller than the expected one by a factor of about 0.6. Kamiokande performed separate analyses for both sub-GeV neutrinos (Hirata *et al.*, 1992) and multi-GeV neutrinos (Fukuda *et al.*, 1994), which showed the same deficit. This was the original formulation of the atmospheric neutrino anomaly. In Fig. 20 we show the values of the ratio  $R_{\mu/e}/R_{\mu/e}^{MC}$ , which denotes the double ratio of experimental-to-expected ratio of muon-like to electron-like events. Whether  $R_{\mu/e}/R_{\mu/e}^{MC}$  is small because there is  $\nu_\mu$  disappearance or  $\nu_e$  appearance or a combination of both could not be determined. Furthermore, the fact that the anomaly appeared only in water Cerenkov and not in iron calorimeters left the window open for the suspicion of a possible systematic problem as the origin of the effect.

Kamiokande also presented the zenith angular dependence of the deficit for the multi-GeV neutrinos (Fukuda *et al.*, 1994). The zenith angle, parametrized in terms of  $\cos\theta$ , measures the direction of the reconstructed charged lepton with respect to the vertical of the detector. Vertically downgoing (upgoing) particles correspond to  $\cos\theta = +1(-1)$ . Horizontally arriving particles come at  $\cos\theta = 0$ . Kamiokande results seemed to indicate that the deficit was mainly due to the neutrinos coming from below the horizon. Atmospheric neutrinos are produced isotropically at a distance of about 15 km above the surface of the Earth. Therefore neutrinos coming from the top of the detector have traveled approximately those 15 kilometers before interacting while those coming from the bottom of the detector have traversed the full diameter of the Earth,  $\sim 10^4$  Km before reaching the detector. The Kamiokande distribution suggested that the deficit increases with the distance between the neutrino production and interaction points.

In the last five years, the case for the atmospheric neutrino anomaly has become much stronger with the high precision and large statistics data from SK and it has received important confirmation from the iron calorimeter detectors Soudan2 and MACRO.

SK is a 50 kiloton water Cerenkov detector constructed under Mt. Ikenoyama located in the central part of Japan, giving it a rock over-burden of 2,700 m water-equivalent. The fiducial mass of the detector for atmospheric neutrino analysis is 22.5 kilotons. In June 1998, in the Neutrino98 conference, SK presented *evidence* of  $\nu_\mu$  oscillations (Fukuda *et al.*, 1998) based on the angular distribution for their contained event data sample. Since then SK accumulated more statistics and has also studied the angular dependence of the upgoing muon sample (Fukuda *et al.*, 1999a and 1999b). In Fig. 21 (from Toshito *et al.*, 2001) we show their data at summer 2001, corresponding to 79 kiloton year (1289 days) exposure for their contained sub-GeV (2864 1-ring  $e$ -like events and 2788 1-ring  $\mu$ -like events) and multi-GeV (626 1-ring  $e$ -like events, 558 1-ring  $\mu$ -like events and 754 PC events), as well as the stopping-muons and through-going muon samples (1416 events).

In the figure we show the angular zenith distribution of the different samples. Comparing the observed and the expected (MC) distributions, we can make the following statements:

(i)  $\nu_e$  distributions are well described by the MC while  $\nu_\mu$  presents a deficit. Thus the atmospheric neutrino deficit is mainly due to disappearance of  $\nu_\mu$  and not the appearance of  $\nu_e$ .

(ii) The suppression of contained  $\mu$ -like events is stronger for larger  $\cos\theta$ , which implies that the deficit grows with the distance traveled by the neutrino from its production point to the detector. This effect is more obvious for multi-GeV events because at higher energy the direction of the charged lepton is more aligned with the direction of the neutrino. It can also be described in terms of an up-down asymmetry:

$$A_\mu \equiv \frac{U - D}{U + D} = -0.316 \pm 0.042(\text{stat.}) \pm 0.005(\text{syst.}) \quad (118)$$

where  $U$  ( $D$ ) are the contained  $\mu$ -like events with zenith angle in the range  $-1 < \cos\theta < -0.2$  ( $0.2 < \cos\theta < 1$ ). It deviates from the SM value,  $A_\mu = 0$ , by 7.5 standard deviations.

(iii) The overall suppression of the flux of stopping-muons,  $\Phi_{\text{ST}}$ , is by a factor of about 0.6, similar to contained events. However, for the flux of through-going muons,  $\Phi_{\text{TH}}$ , the suppression is weaker, which implies that the effect is smaller at larger neutrino energy. This effect is also parametrized in terms of the double flux ratio:



$$\frac{\Phi_{\text{ST}}/\Phi_{\text{TH}}|_{\text{obs}}}{\Phi_{\text{ST}}/\Phi_{\text{TH}}|_{\text{MC}}} = 0.635 \pm 0.049(\text{stat.}) \pm 0.035(\text{syst.}) \pm 0.084(\text{theo.}) \quad (119)$$

which deviates from the SM value of 1 by about 3 standard deviations.

These effects have been confirmed by the results of the iron calorimeters Soudan2 and MACRO which removed the suspicion that the atmospheric neutrino anomaly is simply a systematic effect in the water detectors. In particular Soudan2, which has mainly analyzed contained events (Allison *et al.*, 1999), have measured a ratio  $R_{\mu/e}/R_{\mu/e}^{\text{MC}} = 0.68 \pm 0.11 \pm 0.06$ , in good agreement with the results from the water Cerenkov experiments. The main results from MACRO concern the angular distribution for through-going muons (see Ambrosio *et al.* (2001) for their latest data) and shows good agreement with the results from SK. The Baksan experiment (Boliev *et al.*, 1999) has also reported results on the angular distribution of through-going muons but their data is less precise.

To analyze the atmospheric neutrino data in terms of oscillations one needs to have a good understanding of the different elements entering into the theoretical predictions of the event rates: the atmospheric neutrino fluxes and their interaction cross section, which we describe next.

## A. Atmospheric Neutrino Fluxes

Modern calculations of atmospheric neutrino fluxes consist of a Monte Carlo procedure that combines the measured energy spectra and chemical composition of the cosmic ray flux at the top of the atmosphere with the properties of their hadronic interaction with the light atmospheric nuclei followed by the neutrino production from secondary  $\pi$ ,  $K$  and  $\mu$  decay.

Present experiments use the neutrino flux calculations from mainly Honda *et al.* (1990,1995,1996), the Bartol group (Gaisser, Stanev and Barr, 1988; Barr, Gaisser and Stanev, 1989; Agraval *et al.*, 1996; Lipari, Gaisser, and Stanev, 1998) and Fiorentini, Naumov and Villante (2001). These calculations have in common a one-dimensional picture, in which all secondary particles in the showers, neutrinos included, are considered collinear with the primary cosmic rays (see also Bugaev and Naumov, 1989). The three-dimensional picture was first considered by Lee and Koh (1990) and most recently by Batistoni *et al.* (2000) and Tserknyav *et al.* (1999). [For detailed comparisons between the various simulations, see Gaisser *et al.* (1996), Lipari (2000) and Battistoni (2001)].

The flux of neutrinos of flavor  $i$  coming from the direction  $\Omega$  can be schematically written as

$$\phi_{\nu_i}(\Omega) = \sum_A \Phi_A \otimes R_A \otimes Y_{p,n \rightarrow \nu_i}. \quad (120)$$

Here  $\Phi_A$  is the primary cosmic-ray spectrum,  $R_A$  is the geomagnetic cutoff for the protons or light nuclei incident on the atmosphere from the direction  $\Omega$ , and  $Y_{p,n \rightarrow \nu_i}$  is the yield per nucleon of  $\nu_i$ . The separation to different nuclear species of the cosmic ray spectrum is necessary because the neutrino yield depends on the energy-per-nucleon of the incident cosmic rays but the geomagnetic cutoff depends on the magnetic rigidity ( $\mathcal{R} = pc/Ze$ ) which, at the same energy per nucleon, depends on  $A/Z$ .

We now briefly summarize the uncertainties in each of these three ingredients of the calculation, namely the primary cosmic ray flux, the geomagnetic effects and the hadronic interactions on light nuclei.

## 1. Cosmic ray spectrum

The cosmic radiation incident at the top of the atmosphere includes all stable charged particles and nuclei. Apart from particles associated with solar flares, the cosmic rays are assumed to come from outside the solar system. In Fig. 22 (from Simpson, 1983) we show a compilation of the major components of the cosmic rays as a function of the energy per nucleon. The following features of the cosmic ray spectrum are relevant to the atmospheric neutrino fluxes:

(i) Composition: Most of the primaries are protons although the chemical composition varies with energy at low energies. Above a few GeV of energy-per-nucleon, about 79% of the primaries are protons while helium, the next most abundant component, is about 3%. This fraction remains constant till beyond 100 TeV.

(ii) Energy Dependence: For energy above a few GeV, the cosmic ray flux is a steeply falling function of the energy:  $d\phi/dE \propto E^{-3.7}$ .

(iii) Solar modulation: At energies below  $\sim 10$  GeV, the incoming charged particles are modulated by the solar wind which prevents the lower energy cosmic rays from the inner solar system to reach the Earth. Because of this effect there is an anticorrelation between the eleven-year-cycle solar activity and the intensity of cosmic rays below 10 GeV, which makes the cosmic ray fluxes time dependent. The flux difference at solar maximum and solar minimum is more than a factor of two for 1 GeV cosmic rays, and it decreases to  $\sim 10\%$  for 10 GeV cosmic rays. The fluxes shown in Fig. 22 correspond to a particular epoch of the solar cycle.

## 2. Geomagnetic effects

Lower energy cosmic rays are affected by the geomagnetic field which they must penetrate to reach the top of the atmosphere. This effect depends on the point of the Earth where the cosmic rays arrive (being stronger near the geomagnetic equator), and on their direction. Near the geomagnetic poles almost all primary particles can reach the atmosphere moving along field lines. In contrast, close to the geomagnetic equator the field restricts the flux at the top of the atmosphere to particles with energy larger than a few GeV, the exact value depending on the direction of the particle trajectory. The relevant quantity is the magnetic rigidity,  $\mathcal{R} = pc/Ze$ , which parameterizes the characteristic radius of curvature of the particle trajectory in the presence of a magnetic field: particles with  $\mathcal{R}$  below a local cutoff are bent away by the Earth magnetic field and do not reach the atmosphere.

Given a map of the magnetic field around the Earth, the value of the rigidity cutoff can be obtained from a computer simulation of cosmic ray trajectories using a backtracking technique in which one determines if a given primary particle three-momentum and position correspond to an allowed or a forbidden trajectory. To do so, one integrates back the equation of motion of the cosmic ray particle (actually one integrates the anti-particle equation) in the geomagnetic field, and finds if the past trajectory remains confined to a finite distance

from the Earth, in which case it is a forbidden trajectory, or it originates from large enough ( $\gtrsim 10R_{\oplus}$ ) distances. The rigidity cutoff is calculated as the minimum  $\mathcal{R}$  for which the trajectory is allowed or, in other words, the backtracked trajectory escapes from the magnetic field of the Earth.

The following features of the geomagnetic effects are relevant to the atmospheric neutrino fluxes:

(i) For a fixed direction, the cutoff rigidity grows monotonically from zero at the magnetic pole to a maximum value at the magnetic equator.

(ii) The cutoff rigidities for particles traveling toward the magnetic west are higher than those for particles traveling toward the magnetic east.

(iii) Thus, the highest cutoff corresponds to westward going, horizontal particles reaching the surface of the Earth at the magnetic equator and it is approximately 60 GV.

### 3. The neutrino yield

As cosmic rays propagate in the atmosphere and interact with air nuclei, they create  $\pi$ - and  $K$ -mesons, which in turn decay and create atmospheric  $\nu$ 's:

$$\begin{aligned} A_{\text{cr}} + A_{\text{air}} &\rightarrow \pi^{\pm}, K^{\pm}, K^0, \dots; \\ \pi^{\pm}, K^{\pm} &\rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu}) \\ \mu^{\pm} &\rightarrow e^{\pm} + \nu_e(\bar{\nu}_e) + \bar{\nu}_{\mu}(\nu_{\mu}). \end{aligned} \quad (121)$$

We have only displayed the dominant channels. Since the decay distributions of the secondary mesons and muons are extremely well known, the largest source of differences between the various calculations for the neutrino yield is the use of different models for the hadronic interactions (Gaisser *et al.*, 1996). Since neutrinos produced by the decays of pions, kaons and muons have different energy spectra, the main features of the interaction model which affect the  $\nu_{\mu}/\nu_e$  composition and the energy and angular dependence of the neutrino fluxes are the  $K/\pi$  ratio and their momentum distribution.

### 4. The neutrino fluxes

In Figs. 23 and 24 we show the atmospheric neutrino fluxes from the calculation of the Bartol group (Gaisser, Stanev and Barr, 1988; Barr, Gaisser, and Stanev, 1989; Agraval *et al.*, 1996; Lipari, Gaisser, and Stanev, 1998) for the location of SK and for maximum solar activity. In the first figure we show the flux as a function of the neutrino energy averaged over the arrival direction. In the second figure we show the fluxes as a function of the zenith angle for various neutrino energies. We emphasize the following points:

(i) Energy dependence: For  $E_{\nu} \gtrsim 1$  GeV, the fluxes obey an approximate power law,  $d\Phi/dE \propto E^{-\gamma}$  where  $\gamma \sim 3(3.5)$  for muon (electron) neutrinos and antineutrinos. For  $E_{\nu} \lesssim 1$  GeV, the dependence on energy is weaker as a consequence of the *bending* of the cosmic ray spectrum by geomagnetic effects and solar modulation.

(ii)  $\bar{\nu}/\nu$  ratio: For  $E \lesssim 1$  GeV, when all pions and subsequent muons decay before reaching the Earth, the ratio  $\bar{\nu}_e/\nu_e = \pi^{-}/\pi^{+} < 1$ . At these energies, the cosmic rays are mainly protons. The positively charged protons produce, on average, more  $\pi^{+}$ 's than  $\pi^{-}$ 's

in their interactions. On the other hand, since  $\pi^+$  ( $\mu^+$ ) produces a  $\nu_\mu$  ( $\bar{\nu}_\mu$ ) in its decay, we expect  $\bar{\nu}_\mu/\nu_\mu = 1$ .

As energy increases, the secondary  $\mu^\pm$ 's do not have time to decay before reaching the surface of the Earth and consequently  $\bar{\nu}_\mu/\nu_\mu$  decreases.

(iii)  $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$  ratio: At  $E \lesssim 1$  GeV, the ratio is very close to 2 as expected from the chain decays in Eq. (121). At higher energies, the ratio decreases because, as mentioned above, the  $\mu$ 's do not have time to decay before reaching the Earth and less  $\nu_e$ 's are produced.

(iv) Up-down asymmetry: At  $E \lesssim$  a few GeV, the fluxes are up-down asymmetric due to geomagnetic effects. Geomagnetic effects are very small already at  $E = 2$  GeV (see Fig. 24) and the flux becomes up-down symmetric above that energy.

(v) Horizontal-vertical ratio: For  $E \gtrsim$  a few GeV, the fluxes are maximal for neutrinos arriving horizontally and minimal for neutrinos arriving vertically (see Fig. 24). This is due to the difference in the atmosphere density which determines whether the pions decay before reinteracting with the air (thereby producing neutrinos). Pions arriving horizontally travel longer time in less dense atmosphere and are more likely to decay before reinteracting.

## B. Interaction Cross Sections

In order to determine the expected event rates at the experiment, one needs to know the neutrino-nucleon interaction cross sections in the detector. The standard approach is to consider separately the contributions to this cross section from the exclusive channels of lower multiplicity: quasi-elastic scattering and single pion production, and include all additional channels as part of the deep inelastic (DIS) cross section:

$$\sigma_{CC} = \sigma_{QE} + \sigma_{1\pi} + \sigma_{DIS} . \quad (122)$$

In Fig. 25 we plot the cross sections for these processes.

The cross section for the quasi-elastic interaction is given by (Smith, 1972):

$$\frac{d\sigma_{QE}}{d|q^2|}(\nu n \rightarrow \ell^- p) = \frac{M^2 G_F^2 \cos^2 \theta_c}{8\pi E_\nu^2} \left[ A_1(q^2) - A_2(q^2) \frac{s-u}{M^2} + A_3(q^2) \frac{(s-u)^2}{M^4} \right], \quad (123)$$

where  $s - u = 4ME_\nu + q^2 - m_\ell^2$ ,  $M$  is the proton mass,  $m_\ell$  is the charged lepton mass, and  $q^2$  is the momentum transfer. For  $\bar{\nu} p \rightarrow \ell^+ n$ , a similar formula applies with the only change  $A_2 \rightarrow -A_2$ . The functions  $A_1$ ,  $A_2$ , and  $A_3$  can be written in terms of axial and vector form factors:

$$\begin{aligned} A_1 &= \frac{m_\ell^2 - q^2}{4M^2} \left[ \left(4 - \frac{q^2}{M^2}\right) |F_A|^2 - \left(4 + \frac{q^2}{M^2}\right) |F_V^1|^2 - \frac{q^2}{M^2} |\xi F_V^2|^2 - \frac{4q^2}{M^2} \text{Re}(F_V^{1*} \xi F_V^2) \right] \\ &\quad - \frac{m_\ell^2}{M^2} (|F_V^1 + \xi F_V^2|^2 + |F_A|^2), \\ A_2 &= -\frac{q^2}{M^2} \text{Re}[F_A^* (F_V^1 + \xi F_V^2)], \\ A_3 &= -\frac{1}{4} \left( |F_A|^2 + |F_V^1|^2 - \frac{q^2}{4M^2} |\xi F_V^2|^2 \right), \end{aligned} \quad (124)$$

where we neglected second class currents and assumed CVC. With this assumption all form factors are real and can be written as follows:

$$\begin{aligned}
F_V^1(q^2) &= \left(1 - \frac{q^2}{4M^2}\right)^{-1} \left(1 - \frac{q^2}{M_V^2}\right)^{-2} \left[1 - \frac{q^2}{4M^2}(1 + \mu_p - \mu_n)\right], \\
\xi F_V^2(q^2) &= \left(1 - \frac{q^2}{4M^2}\right)^{-1} \left(1 - \frac{q^2}{M_V^2}\right)^{-2} (\mu_n - \mu_p), \\
F_A &= F_A(0) \left(1 - \frac{q^2}{4M_A^2}\right)^{-2}.
\end{aligned} \tag{125}$$

Here  $\mu_p$  and  $\mu_n$  are the proton and neutron anomalous magnetic moments and  $M_V^2 = 0.71 \text{ GeV}^2$  is the vector mass which is measured with high precision in electron scattering experiments. The largest uncertainties in this calculation are associated with the axial form factor. For example, the axial mass used by various collaborations varies in the range  $M_A^2 = 0.71 - 1.06 \text{ GeV}^2$ .

So far we have neglected nuclear effects. The most important of these effects is related to the Pauli principle and can be included by using a simple Fermi gas model (Smith, 1972). In this approximation, the cross section of a bound nucleon is equal to the cross section of a free nucleon multiplied by a factor of  $(1 - D/N)$ . For neutrons,

$$D = \begin{cases} Z & 2z \leq w - v, \\ \frac{1}{2}A \left[1 - \frac{3z}{4}(v^2 + w^2) + \frac{z^3}{3} + \frac{3}{32z}(w^2 - v^2)^2\right] & w - v \leq 2z \leq w + v, \\ 0 & 2z \geq w + v, \end{cases} \tag{126}$$

with  $z = \sqrt{(q^2 + m_\ell^2)^2 / (4M^2) - q^2 / (2k_f)}$ ,  $w = (2N/A)^{1/3}$ , and  $v = (2Z/A)^{1/3}$ . Here  $A$ ,  $Z$ ,  $N$  are, respectively, the nucleon, proton and neutron numbers and  $k_f$  is the Fermi momentum,  $k_f = 0.225(0.26) \text{ GeV}$  for oxygen (iron). For protons, the same formula applies with the exchange  $N \leftrightarrow Z$ . The effect of this factor is to decrease the cross section. The decrease is larger for smaller neutrino energy. For energies above 1 GeV the nuclear effects lead to an 8% decrease on the quasi-elastic cross section. As seen in Fig. 25, quasi-elastic interactions dominate for  $E_\nu \lesssim 1 \text{ GeV}$  and induce most of the observed contained events.

An important role in the interpretation of the zenith angle distribution of the contained events is played by the relative angle between the direction of the incoming neutrino (carrying the information on the neutrino path length) and the direction of the produced charged lepton (which is measured). In Fig. 26 we plot this angle as a function of the measured charged lepton energy for quasi-elastic interactions. For energies below 1 GeV, the opening angle is rather large: for sub-GeV events the correlation between the measured  $\ell$  direction and the distance traveled by the neutrinos is weak. For  $E \gtrsim$  a few GeV, the two directions are almost aligned (for example, it is in average  $1^\circ$  for upgoing muons) and the  $\ell$  direction gives a very good measurement of the neutrino path length.

Single pion production,  $\nu N \rightarrow \ell^- N' \pi$ , is dominated by the  $\Delta(1232)$  resonance (Fogli and Nardulli, 1979; Nakahata *et. al.*, 1986). It is most relevant at  $E_\nu \simeq 1 \text{ GeV}$  (see Fig. 25).

Deep inelastic processes,  $\nu N \rightarrow \ell^- X$  where  $X$  represents any hadronic system, dominate atmospheric neutrino interactions for  $E_\nu \gtrsim$  a few GeV. The parton model cross section is given by

$$\frac{d\sigma_{DIS}(\nu)}{dx dy} = \frac{G_F^2 s x}{4\pi} \left[ F_1 - F_3 + (F_1 + F_3)(1 - y)^2 \right], \tag{127}$$

where  $y = 1 - E_\ell/E_\nu$  and  $x = -q^2/(2ME_\nu y)$ . For  $\bar{\nu}$ , a similar formula applies, with  $F_3 \rightarrow -F_3$ .  $F_1$  and  $F_3$  are given in terms of the parton distributions. For isoscalar targets  $F_1 = 2\sum_i(q_i + \bar{q}_i)$  and  $F_3 = \sum_i(\bar{q}_i - q_i)$ . In order to avoid double counting of the single pion production, the deep inelastic contribution must be integrated in the region of hadronic masses  $W_X > W_c$  ( $W_c = 1.4$  GeV), which implies  $2ME_\nu y(1-x) \geq W_c^2 - M^2$ .

### C. Two-Neutrino Oscillation Analysis

The simplest and most direct interpretation of the atmospheric neutrino anomaly is that of muon neutrino oscillations (Learned, Pakvasa, and Weiler, 1988; Barger and Whisnant, 1988; Hidaka, Honda and Midorikawa, 1988). The estimated value of the oscillation parameters can be easily derived in the following way:

- The angular distribution of contained events shows that, for  $E \sim 1$  GeV, the deficit comes mainly from  $L \sim 10^2 - 10^4$  km. The corresponding oscillation phase must be maximal,  $\frac{\Delta m^2(\text{eV}^2)L(\text{km})}{2E(\text{GeV})} \sim 1$ , which requires  $\Delta m^2 \sim 10^{-4} - 10^{-2}$  eV<sup>2</sup>.
- Assuming that all upgoing  $\nu_\mu$ 's which would lead to multi-GeV events oscillate into a different flavor while none of the downgoing ones do, the up-down asymmetry is given by  $|A_\mu| = \sin^2 2\theta/(4 - \sin^2 2\theta)$ . The present one sigma bound reads  $|A_\mu| > 0.27$  [see Eq. (118)], which requires that the mixing angle is close to maximal,  $\sin^2 2\theta > 0.85$ .

In order to go beyond these rough estimates, one must compare in a statistically meaningful way the experimental data with the detailed theoretical expectations. We now describe how to obtain the allowed region. We consider two neutrino cases, where  $\nu_\mu$  oscillates into either  $\nu_e$  or  $\nu_\tau$  or a sterile neutrino  $\nu_s$ .

#### 1. Predicted number of events

For a given neutrino oscillation channel, the expected number of  $\mu$ -like and  $e$ -like contained events,  $N_\alpha$  ( $\alpha = \mu, e$ ), can be computed as:

$$N_\mu = N_{\mu\mu} + N_{e\mu}, \quad N_e = N_{ee} + N_{\mu e}, \quad (128)$$

where

$$N_{\alpha\beta} = n_t T \int \frac{d^2\Phi_\alpha}{dE_\nu d(\cos\theta_\nu)} \kappa_\alpha(h, \cos\theta_\nu, E_\nu) P_{\alpha\beta} \frac{d\sigma}{dE_\beta} \varepsilon(E_\beta) dE_\nu dE_\beta d(\cos\theta_\nu) dh. \quad (129)$$

Here  $P_{\alpha\beta}$  is the conversion probability of  $\nu_\alpha \rightarrow \nu_\beta$  for given values of  $E_\nu$ ,  $\cos\theta_\nu$  and  $h$ , *i.e.*,  $P_{\alpha\beta} \equiv P(\nu_\alpha \rightarrow \nu_\beta; E_\nu, \cos\theta_\nu, h)$ . In the SM, the only non-zero elements are the diagonal ones, *i.e.*  $P_{\alpha\alpha} = 1$  for all  $\alpha$ . In Eq. (129),  $n_t$  denotes the number of target particles,  $T$  is the experiment running time,  $E_\nu$  is the neutrino energy,  $\Phi_\alpha$  is the flux of atmospheric  $\nu_\alpha$ 's,  $E_\beta$  is the final charged lepton energy,  $\varepsilon(E_\beta)$  is the detection efficiency for such a charged lepton,  $\sigma$  is the neutrino-nucleon interaction cross section, and  $\theta_\nu$  is the angle between the vertical direction and the incoming neutrinos ( $\cos\theta_\nu=1$  corresponds to the down-coming neutrinos).

In Eq. (129),  $h$  is the slant distance from the production point to the sea level for  $\alpha$ -type neutrinos with energy  $E_\nu$  and zenith angle  $\theta_\nu$ , and  $\kappa_\alpha$  is the slant distance distribution, normalized to one. (We use in our calculations  $\kappa_\alpha$  from Gaisser and Stanev, 1998).

To obtain the expectation for the angular distribution of contained events one must integrate the corresponding bins for  $\cos\theta_\beta$ , where  $\theta_\beta$  is the angle of the detected lepton, taking into account the opening angle between the neutrino and the charged lepton directions as determined by the kinematics of the neutrino interaction.

As discussed above, the neutrino fluxes, in particular in the sub-GeV range, depend on the solar activity. In order to take this fact into account, one uses in Eq. (129) a linear combination of atmospheric neutrino fluxes  $\Phi_\alpha^{\max}$  and  $\Phi_\alpha^{\min}$  which correspond to the most (solar maximum) and least (solar minimum) active phases of the Sun, respectively, with different weights which depend on the specific running period.

Experimental results on upgoing muons are presented in the form of measured muon fluxes. To obtain the effective muon fluxes for both stopping and through-going muons, one must convolute the survival probabilities for  $\nu_\mu$ 's with the corresponding muon fluxes produced by the neutrino interactions with the Earth. One must further take into account the muon energy loss during propagation both in the rock and in the detector, and also the effective detector area for both types of events, stopping and through-going. Schematically,

$$\Phi_\mu(\theta)_{S,T} = \frac{1}{A(L_{\min}, \theta)} \int_{E_{\mu,\min}}^{\infty} \frac{d\Phi_\mu(E_\mu, \cos\theta)}{dE_\mu d\cos\theta} A_{S,T}(E_\mu, \theta) dE_\mu, \quad (130)$$

where

$$\begin{aligned} \frac{d\Phi_\mu}{dE_\mu d\cos\theta} &= N_A \int_{E_\mu}^{\infty} dE_{\mu 0} \int_{E_{\mu 0}}^{\infty} dE_\nu \int_0^{\infty} dX \int_0^{\infty} dh \kappa_{\nu_\mu}(h, \cos\theta, E_\nu) \\ &\quad \frac{d\Phi_{\nu_\mu}(E_\nu, \theta)}{dE_\nu d\cos\theta} P_{\mu\mu} \frac{d\sigma(E_\nu, E_{\mu 0})}{dE_{\mu 0}} F_{\text{rock}}(E_{\mu 0}, E_\mu, X). \end{aligned} \quad (131)$$

Here  $N_A$  is the Avogadro number,  $E_{\mu 0}$  is the energy of the muon produced in the neutrino interaction,  $E_\mu$  is the muon energy when entering the detector after traveling a distance  $X$  in the rock, and  $\cos\theta$  labels both the neutrino and the muon directions which, at the relevant energies, are collinear to a very good approximation. We denote by  $F_{\text{rock}}(E_{\mu 0}, E_\mu, X)$  the function which characterizes the energy spectrum of the muons arriving the detector. The Standard practice is to use an analytical approximation obtained by neglecting the fluctuations during muon propagation in the Earth. In this case the three quantities  $E_{\mu 0}$ ,  $E_\mu$ , and  $X$  are not independent:

$$\int_0^{\infty} F_{\text{rock}}(E_{\mu 0}, E_\mu, X) dX = \frac{1}{\langle d\mathcal{E}_\mu(E_\mu)/dX \rangle}, \quad (132)$$

where  $\langle d\mathcal{E}_\mu(E_\mu)/dX \rangle$  is the average muon energy loss due to ionization, bremsstrahlung,  $e^+e^-$  pair production and nuclear interactions in the rock (Lohmann, Kopp and Voss, 1985).

For SK, the pathlength traveled by the muon inside the detector is given by the muon range function in water:

$$L(E_\mu) = \int_0^{E_\mu} \frac{1}{\langle d\mathcal{E}_\mu(E'_\mu)/dX \rangle} dE'_\mu. \quad (133)$$

In Eq. (130),  $A(L_{\min}, \theta) = A_S(E_\mu, \theta) + A_T(E_\mu, \theta)$  is the projected detector area for internal path-lengths longer than  $L_{\min}$  ( $= 7$  m in SK). Here  $A_S$  and  $A_T$  are the effective areas for stopping and through-going muon trajectories. These effective areas can be computed using a simple geometrical picture given, for instance, in Lipari and Lusignoli (1998). For a given angle, the threshold energy cut for SK muons is obtained by equating Eq. (133) to  $L_{\min}$ , *i.e.*,  $L(E_\mu^{\text{th}}) = L_{\min}$ .

In contrast to SK, MACRO present their results as muon fluxes for  $E_\mu > 1$  GeV, after correcting for detector acceptances. Therefore in this case we compute the expected fluxes as in Eqs. (130) and (131) but without the inclusion of the effective areas.

## 2. Conversion probabilities

We consider a two-flavor scenario,  $\nu_\mu \rightarrow \nu_X$  ( $X = e, \tau$  or  $s$ ). The oscillation probabilities are obtained by solving the evolution equation of the  $\nu_\mu - \nu_X$  system in the matter background of the Earth (see section III.C):

$$i \frac{d}{dt} \begin{pmatrix} \nu_\mu \\ \nu_X \end{pmatrix} = \begin{pmatrix} H_\mu & H_{\mu X} \\ H_{\mu X} & H_X \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \nu_X \end{pmatrix}, \quad (134)$$

where

$$\begin{aligned} H_\mu &= V_\mu - \frac{\Delta m^2}{4E_\nu} \cos 2\theta, \\ H_X &= V_X + \frac{\Delta m^2}{4E_\nu} \cos 2\theta, \\ H_{\mu X} &= \frac{\Delta m^2}{4E_\nu} \sin 2\theta. \end{aligned} \quad (135)$$

The various neutrino potentials in matter are given by

$$\begin{aligned} V_e &= \frac{\sqrt{2}G_F\rho}{M} \left( Y_e - \frac{1}{2}Y_n \right), \\ V_\mu = V_\tau &= \frac{\sqrt{2}G_F\rho}{M} \left( -\frac{1}{2}Y_n \right), \\ V_s &= 0. \end{aligned} \quad (136)$$

Here  $\rho$  is the matter density in the Earth (Dziewonski and Anderson, 1981),  $M$  is the nucleon mass, and  $Y_e(Y_n)$  is the electron (neutron) fraction. For anti-neutrinos, the signs of the potentials are reversed.

For  $X = \tau$ , we have  $V_\mu = V_\tau$  and consequently these potentials can be removed from the evolution equation. The solution of Eq. (134) is then straightforward and the probability takes the well-known vacuum form [Eq. (35)], which is equal for neutrinos and anti-neutrinos. For  $X = e$  or  $s$ , the effect of the matter potentials requires a numerical solution of the evolution equations in order to obtain  $P_{\alpha\beta}$  which, furthermore, is different for neutrinos and anti-neutrinos. As first approximation, one can use a constant Earth matter density. Then (see section III) the solutions take the same form as the vacuum probability [Eq. (35)], but with the mixing angle and the oscillation length replaced by their effective values in matter



[Eqs. (61) and (64) with  $A = 2E(V_\mu - V_X)$ ]. In Fig. 27 (from Lipari and Lusignoli, 1989) we show the survival probability of  $\nu_\mu$  for the different oscillation channels for  $\Delta m^2 = 5 \times 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta = 1$  and various values of  $E_\nu$ . As seen in the figure, matter effects damp the oscillation amplitude. For the chosen mass difference, they are important for neutrino energies of few 10 GeV and therefore are relevant mainly for upgoing muons. We return to this point below when describing the results of the analysis.

### 3. Statistical analysis

In order to define in a statistically meaningful way the regions of neutrino flavor parameters that are allowed by a given set of atmospheric neutrino observables, one can construct, for example, a  $\chi^2$  function:

$$\chi^2 \equiv \sum_{I,J} (N_I^{\text{data}} - N_I^{\text{th}}) \cdot (\sigma_{\text{data}}^2 + \sigma_{\text{th}}^2)_{IJ}^{-1} \cdot (N_J^{\text{data}} - N_J^{\text{th}}), \quad (137)$$

where  $I$  and  $J$  stand for any combination of experimental data sets and event-types considered, *i.e.*,  $I = (A, \alpha)$  and  $J = (B, \beta)$ . The latin indices  $A, B$  stand for the different experiments or different data samples in a given experiment. The greek indices denote electron-type or muon-type events, *i.e.*,  $\alpha, \beta = e, \mu$ . In Eq. (137),  $N_I^{\text{th}}$  stands for the predicted number of events (or for the predicted value of the flux, in the case of upgoing muons) calculated as discussed above, whereas  $N_I^{\text{data}}$  is the corresponding experimental measurement. In Eq. (137),  $\sigma_{\text{data}}^2$  and  $\sigma_{\text{th}}^2$  are the error matrices containing the experimental and theoretical errors, respectively. They can be written as

$$\sigma_{IJ}^2 \equiv \sigma_\alpha(A) \rho_{\alpha\beta}(A, B) \sigma_\beta(B), \quad (138)$$

where  $\rho_{\alpha\beta}(A, B)$  is the correlation matrix containing all the correlations between the  $\alpha$ -like events in the  $A$ -type experiment and  $\beta$ -like events in  $B$ -type experiment, whereas  $\sigma_\alpha(A)$  [ $\sigma_\beta(B)$ ] are the errors for the number of  $\alpha$ -like events in the  $A$  [ $\beta$ -like events in the  $B$ ] experiment. The dimension of the error matrix varies depending on the combination of experiments included in the analysis. Details on the statistical analysis of the atmospheric neutrino data and results from the analyses with various data samples can be found in Fogli, Lisi and Montanino (1994,1995); Fogli and Lisi (1995b); Gonzalez-Garcia *et al.* (1998,1999); Gonzalez-Garcia, Fornengo and Valle (2000); Foot, Volkas and Yasuda (1998); Yasuda (1998); Akhmedov *et al.* (1999).

Using the definitions given above, one calculates  $\chi^2$  of Eq. (137) as a function of the neutrino parameters. By minimizing  $\chi^2$  with respect to  $\sin^2 2\theta$  and  $\Delta m^2$ , one determines the best fit results, while the allowed regions are determined by the conditions:  $\chi^2 \equiv \chi_{\text{min}}^2 + 4.61$  (6.1) [9.21] for a confidence level (CL) of 90 (95) [99] %.

### 4. $\nu_\mu \rightarrow \nu_e$

At present  $\nu_\mu \rightarrow \nu_e$  is excluded with high CL as the explanation to the atmospheric neutrino anomaly for two different reasons:

(i) SK high precision data show that the  $\nu_e$  contained events are very well described by the SM prediction both in normalization and in their zenith angular dependence [see Fig. 21]. The  $\nu_\mu$  distribution, however, shows an angle-dependent deficit.  $\nu_\mu \rightarrow \nu_e$  oscillations can explain the angular dependence of the  $\nu_\mu$  flux only at the price of introducing angular dependence of the  $\nu_e$  flux, in contrast to the data. Furthermore, even the best fit point for  $\nu_\mu \rightarrow \nu_e$  oscillations does not generate the observed up-down asymmetry in the Multi-GeV muon sample. This is illustrated in Fig. 28, where we show the predicted angular distribution of contained events at SK for the best fit points of the different oscillation channels. For  $\nu_\mu \rightarrow \nu_e$  oscillations, the asymmetry in the Multi-GeV muon distribution is much smaller than in the  $\nu_\mu \rightarrow \nu_\tau$  or  $\nu_\mu \rightarrow \nu_s$  channels.

(ii) Explaining the atmospheric data with  $\nu_\mu \rightarrow \nu_e$  transition has direct implications for the  $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$  transition. In particular, there should be a  $\bar{\nu}_e$  deficit in the CHOOZ reactor experiment. Thus the neutrino parameters not only give a poor fit to the atmospheric data but are actually excluded by the negative results from the CHOOZ reactor experiment (see Fig. 28).

## 5. $\nu_\mu \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_s$

In Fig. 29 we show the values of the oscillation parameters ( $\Delta m^2, \sin^2 \theta$ ) which describe various sets of data for these two oscillation channels. The upper panels (labeled as FINKS) and the central panels refer to contained events. The upper panels take into account only the total rates, and the central ones only the angular distribution (from both Kamiokande and SK). The lower panels correspond to the angular distribution for upgoing muons from SK. The three shaded regions are allowed at the 90, 95 and 99% CL. In Fig. 30 we plot the allowed regions from the global analysis, including all the atmospheric neutrino data. Also shown are the expected sensitivity from  $\nu_\mu$  disappearance at the long baseline (LBL) experiments K2K and MINOS discussed in VI.D. (These figures are an update of the results presented in Gonzalez-Garcia *et al*, 2000.) We emphasize the following points:

(i) The allowed regions from the various data samples overlap. The oscillation hypothesis can then consistently explain the atmospheric neutrino data.

(ii) The information from the total event rates alone is consistent with arbitrarily high  $\Delta m^2$  values. The reason is that no information on the minimum oscillation length can be inferred from the data [see section III.A].

(iii) The allowed regions for  $\nu_\mu \rightarrow \nu_\tau$  transition are symmetric with respect to maximal mixing. This must be the case because the corresponding probabilities take the vacuum expression and therefore depend on  $\sin^2 2\theta$ .

(iv) The allowed regions for  $\nu_\mu \rightarrow \nu_s$  transition are asymmetric due to Earth matter effects. These effects are more pronounced when the condition for maximal matter effect,  $\Delta m^2 \cos 2\theta \sim 2EV_s$ , is fulfilled. Since in the chosen convention the potential difference  $A = 2E(V_\mu - V_s) < 0$ , the matter effects enhance neutrino oscillations for  $\cos 2\theta < 0$  ( $\sin^2 \theta > 0.5$ ). The opposite situation holds for antineutrinos, but neutrino fluxes are larger and dominate in the resulting effect. As a result the allowed regions are *wider* in the  $\sin^2 \theta > 0.5$  side of the plot.

(v) The best fit to the full data for  $\nu_\mu \rightarrow \nu_\tau$  corresponds to  $\Delta m^2 = 2.6 \times 10^{-3} \text{ eV}^2$  and  $\sin^2 2\theta = 0.97$ . The best fit for  $\nu_\mu \rightarrow \nu_s$  lies at  $\Delta m^2 = 3 \times 10^{-3} \text{ eV}^2$  and  $\sin^2 \theta = 0.61$ .

In order to discriminate between the  $\nu_\mu \rightarrow \nu_\tau$  and  $\nu_\mu \rightarrow \nu_s$  options, one can use the difference in the survival probabilities due to the presence of matter effects for oscillations into sterile neutrinos (Lipari and Lusignoli, 1998). As discussed above, the effect is important mainly for the higher energy neutrinos which lead to through-going muons events. In Fig. 31 we plot the expected distributions for the best fit points for the two channels. As seen in the figure, the distribution is steeper for  $\nu_\mu \rightarrow \nu_\tau$  while for  $\nu_\mu \rightarrow \nu_s$  a flattening is observed for neutrinos coming close to the vertical due to the damping of the oscillation amplitude [see Fig. 27]. The data favours the steeper distributions and this translates into a better global fit for oscillations into  $\nu_\tau$ . For the global analysis, an update of the results of Gonzalez-Garcia, *et al.* (2000) shows that for  $\nu_\tau$  oscillations  $\chi^2_{min} = 56/63$  d.o.f while for  $\nu_s$  oscillations  $\chi^2_{min} = 72/63$  d.o.f. As we will see in Sec. VII, this difference has an important role in 4- $\nu$  mixing schemes.

SK has also used other methods to distinguish between the  $\nu_\tau$  and  $\nu_s$  hypotheses for explaining atmospheric  $\nu_\mu$  disappearance. One is to examine events likely to have been caused by NC interactions. While  $\nu_\tau$ 's readily undergo such interactions,  $\nu_s$ 's do not, resulting in a relative suppression of the NC signal (Vissani and Smirnov, 1998; Hall and Murayama, 1998). Another method attempts to observe appearance of the newly created  $\nu_\tau$ , even if only on a statistical basis, by selecting enriched samples. All methods strongly favour  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations over  $\nu_\mu \leftrightarrow \nu_s$  (Fukuda *et al.*, 2000; Toshito *et al.*, 2001).

## VI. LABORATORY EXPERIMENTS

Laboratory experiments to search for neutrino oscillations are performed with neutrino beams produced at either accelerators or nuclear reactors. In *disappearance* experiments, one looks for the attenuation of a neutrino beam primarily composed of a single flavor due to the mixing with other flavors. In *appearance* experiments, one searches for interactions by neutrinos of a flavor not present in the original neutrino beam.

Most of the past and present laboratory experiments did not have an oscillation signal. In such a case, as discussed in Sec. III.A, the experiment sets a limit on the corresponding oscillation probability. Appearance experiments set limits  $\langle P_{\alpha\beta} \rangle < P_L$  for given flavors  $\alpha \neq \beta$ . Disappearance experiments set limits  $\langle P_{\alpha\alpha} \rangle > 1 - P_L$  for a given flavor  $\alpha$  which, in the two neutrino case, can be translated into  $\langle P_{\alpha\beta} \rangle < P_L$  for  $\beta \neq \alpha$ . The results are usually interpreted in a two neutrino framework as exclusion regions in the  $(\Delta m^2, \sin^2 2\theta)$  plane. One can take the upper bound on the mixing angle in the asymptotic large  $\Delta m^2$  range and translate it back into the value of  $P_L$ :  $\sin^2 2\theta_{lim} = 2P_L$  [see discussion in Sec. III.A]. The probability  $P_L$  is the relevant quantity when interpreting the results in the more-than-two neutrino framework.

### A. Short Baseline Experiments at Accelerators

Conventional neutrino beams from accelerators are mostly produced by  $\pi$  decays, with the pions produced by the scattering of the accelerated protons on a fixed target:



$$\begin{aligned}
\mu^+ &\rightarrow e^+ \nu_e \bar{\nu}_\mu \\
\pi^- &\rightarrow \mu^- \bar{\nu}_\mu \\
\mu^- &\rightarrow e^- \bar{\nu}_e \nu_\mu.
\end{aligned}$$

Thus the beam can contain both  $\mu$ - and  $e$ -neutrinos and antineutrinos. The final composition and energy spectrum of the neutrino beam is determined by selecting the sign of the decaying  $\pi$  and by stopping the produced  $\mu$  in the beam line.

Most oscillation experiments performed so far with neutrino beams from accelerators have characteristic distances of the order of hundreds of meters. We call them *short baseline (SBL) experiments*. With the exception of the LSND experiment, which we discuss below, all searches have been negative. In table VI we show the limits on the various transition probabilities from the negative results of the most restricting SBL experiments. In Fig. 32 (from Astier *et al.*, 2001) we show the excluded regions corresponding to (the absence of)  $\nu_\mu \rightarrow \nu_\tau$  and  $\nu_e \rightarrow \nu_\tau$  oscillations. Due to the short path length, these experiments are not sensitive to the low values of  $\Delta m^2$  invoked to explain either the solar or the atmospheric neutrino data but they are relevant for 4- $\nu$  mixing schemes as we will see in Sec. VII.B.

## B. LSND and KARMEN

The only positive signature of oscillations at a laboratory experiment comes from the Liquid Scintillator Neutrino Detector (LSND) running at Los Alamos Meson Physics Facility (Athanasopoulos *et al.*, 1995, 1996, 1998). The primary neutrino flux comes from  $\pi^+$ 's produced in a 30-cm-long water target when hit by protons from the LAMPF linac with 800 MeV kinetic energy. The detector is a tank filled with 167 metric tons of dilute liquid scintillator, located about 30 m from the neutrino source.

Most of the produced  $\pi^+$ 's come to rest and decay through the sequence  $\pi^+ \rightarrow \mu^+ \nu_\mu$ , followed by  $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ . The  $\bar{\nu}_\mu$ 's so produced have a maximum energy of 52.8 MeV. This is called the *decay at rest* (DAR) flux and is used to study  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  oscillations. The energy dependence of the  $\bar{\nu}_\mu$  flux from decay at rest is very well known, and the absolute value is known to 7%. The open space around the target is short compared to the pion decay length. Thus only 3% of the  $\pi^+$ 's *decay in flight* (DIF). The DIF  $\nu_\mu$  flux is used to study  $\nu_\mu \rightarrow \nu_e$  oscillations.

In 1995, the LSND experiment published data showing candidate events that are consistent with  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  oscillations. Further supporting evidence was provided by the signal in the  $\nu_\mu \rightarrow \nu_e$  channel. We summarize here their main results in both the DAR and DIF channels.

For DAR related measurements,  $\bar{\nu}_e$ 's are detected in the quasi elastic process  $\bar{\nu}_e p \rightarrow e^+ n$ , in correlation with a monochromatic photon of 2.2 MeV arising from the neutron capture reaction  $np \rightarrow d\gamma$ . The main background is due to the  $\bar{\nu}_e$  component in the beam that is produced in the decay chain starting with  $\pi^-$ 's. This background is suppressed by three factors. First, the  $\pi^+$  production rate is about eight times the  $\pi^-$  production rate in the beam stop. Second, 95% of the  $\pi^-$ 's come to rest and are absorbed before decay in the beam stop. Third, 88% of the  $\mu^-$ 's from  $\pi^-$ 's DIF are captured from atomic orbit, a process which does not give a  $\bar{\nu}_e$ . Thus, the relative yield, compared to the positive channel, is estimated to be  $\sim (1/8) \times 0.05 \times 0.12 = 7.5 \times 10^{-4}$ .

In Athanassopoulos *et al.* (1995) LSND report a total of 22 events with  $e^+$  energy between 36 and 60 MeV when  $4.6 \pm 0.6$  background events are expected. They fit the full  $e^+$  event sample in the energy range  $20 < E_e < 60$  MeV and the result yields  $64.3_{-16.7}^{+18.5}$  beam-related events. Subtracting the estimated neutrino background with a correlated gamma of  $12.5 \pm 2.9$  events results in an excess of  $51.2_{-16.9}^{+18.7} \pm 8.0$  events. The interpretation of this anomaly in terms of  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  oscillations requires  $P_{e\mu} = (3.1 \pm 1.2 \pm 0.5) \times 10^{-3}$ .

For DIF related measurements, the  $\nu_e$ 's are observed via the detection of electrons produced in the process  $\nu_e C \rightarrow e^- X$  with energy  $60 < E_e < 200$  MeV. Using two independent analyses,  $27.7 \pm 6.4$  events are observed. The neutrino induced backgrounds are dominated by  $\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e$  and  $\pi^+ \rightarrow e^+ \nu_e$  decays in flight in the beam stop area and are estimated to be  $9.6 \pm 1.9$  events. The excess above the expected background from conventional processes is then  $18.1 \pm 6.6 \pm 4.0$  events. The excess events are consistent with  $\nu_\mu \rightarrow \nu_e$  oscillations with  $P_{\mu e} = (2.6 \pm 1.0 \pm 0.5) \times 10^{-3}$ .

The LSND results have been recently updated to include the runs till 1998 (Aguilar *et al.*, 2001) and the total fitted excess is of  $87.9 \pm 22.4 \pm 6$  events, corresponding to an oscillation probability of  $(2.64 \pm 0.67 \pm 0.45) \times 10^{-3}$ . In the two-family formalism these results lead to the oscillation parameters shown in Fig. 33. The shaded regions are the 90 % and 99 % likelihood regions from LSND. The best fit point corresponds to  $\Delta m^2 = 1.2 \text{ eV}^2$  and  $\sin^2 2\theta = 0.003$ .

The region of parameter space which is favoured by the LSND observations has been partly tested by other experiments like the old BNL E776 experiment (Borodovsky *et al.*, 1992) and more recently by the KARMEN experiment (Gemmeke *et al.*, 1990). The KARMEN experiment is performed at the neutron spallation facility ISIS of the Rutherford Appleton Laboratory. Neutrinos are produced by stopping the 800 MeV protons in a massive beam stop target, thereby producing pions. The  $\pi^-$ 's are absorbed by the target nuclei whereas  $\pi^+$ 's decay at rest producing muon neutrinos via  $\pi^+ \rightarrow \mu^+ \nu_\mu$ . The low momentum  $\mu^+$ 's are also stopped within the massive target and decay at rest,  $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ . The  $\pi^+ - \mu^+$  decay chain at rest gives a neutrino source with identical intensities for  $\nu_\mu$ ,  $\nu_e$  and  $\bar{\nu}_\mu$  emitted isotropically. There is a minor fraction of  $\pi^-$  decaying in flight (DIF) with the following  $\mu^-$  DAR in the target station which leads to a very small contamination of  $\bar{\nu}_e/\nu_e < 6.2 \cdot 10^{-4}$ . The energy spectra of the  $\nu$ 's are well defined due to the DAR of both the  $\pi^+$  and  $\mu^+$ .

The neutrinos are detected in a rectangular tank filled with 56 t of a liquid scintillator. The signature for the detection of  $\bar{\nu}_e$  is a spatially correlated delayed coincidence of positrons from  $\bar{p}(\bar{\nu}_e, e^+)n$  with energies up to  $E_{e^+} = E_{\bar{\nu}_e} - Q = 51.0 \text{ MeV}$  and  $\gamma$  emission of either of the two neutron capture processes:  $p(n, \gamma)d$  with one  $\gamma$  of  $E(\gamma) = 2.2 \text{ MeV}$  or  $\text{Gd}(n, \gamma)$  with  $3\gamma$ -quanta on average and  $\sum E(\gamma) = 8 \text{ MeV}$ .

The raw data presented in the EPS HEP 2001 conference (Wolf *et al.*, 2001) correspond to  $\sim 9400$  C protons on target. Analyzing the data results in 11 sequential events which satisfy all cuts. This number is in good agreement with the total background expectation of  $12.3 \pm 0.6$ . Applying a Bayesian renormalization procedure, an upper limit of  $N(\text{osc}) < 6.3$  at 90%CL can be extracted. However using the spectral information and a maximum likelihood analysis, KARMEN find a best fit value  $N(\text{osc}) = 0$  within the physically allowed range of parameters, which can be translated into an upper limit of 3.8 and 3.1 oscillation events for  $\Delta m^2 < 1 \text{ eV}^2$  and  $\Delta m^2 > 20 \text{ eV}^2$ , respectively. These numbers are based on

a complete frequentist approach as suggested by G. Feldman and R. Cousins. The corresponding exclusion curve in the two-neutrino parameter space is given in Fig. 33 together with the favoured region for the LSND experiment (from Wolf *et al.*, 2001). At high  $\Delta m^2$ , KARMEN results exclude the region favored by LSND. At low  $\Delta m^2$ , KARMEN leaves some allowed space, but the reactor experiments at Bugey and CHOOZ add stringent limits for the larger mixing angles. This figure represents the final status of the LSND oscillation signal.

The MiniBooNE experiment (Bazarko *et al.*, 2000) searches for  $\nu_\mu \rightarrow \nu_e$  oscillations and is specially designed to make a conclusive statement about the LSND's neutrino oscillation evidence. They use a  $\nu_\mu$  beam of energy 0.5 – 1.0 GeV initiated by a primary beam of 8 GeV protons from the Fermilab Booster, which contains only a small intrinsic  $\nu_e$  component (less than 0.3%). They search for an excess of electron neutrino events in a detector located approximately 500 m from the neutrino source. The MiniBooNE neutrino detector consists of 800 tons of pure mineral oil contained in a 12.2 m diameter spherical tank. A structure in the tank supports phototubes, which detect neutrino interactions in the oil by the Cerenkov and scintillation light that they produce.

The  $L/E$  ratio is similar to that of LSND, giving MiniBooNE sensitivity to the same mode of oscillations. However, neutrino energies are more than an order of magnitude higher than at LSND, so that the search at MiniBooNE employs different experimental techniques. In Fig. 33 we show the 90% CL limits that MiniBooNE can achieve. Should a signal be found then the next step would be the BooNe experiment.

### C. Disappearance Experiments at Reactors

Neutrino oscillations are also searched for using neutrino beams from nuclear reactors. Nuclear reactors produce  $\bar{\nu}_e$  beams with  $E_\nu \sim \text{MeV}$ . Due to the low energy,  $e$ 's are the only charged leptons which can be produced in the neutrino CC interaction. If the  $\bar{\nu}_e$  oscillated to another flavor, its CC interaction could not be observed. Therefore oscillation experiments performed at reactors are disappearance experiments. They have the advantage that smaller values of  $\Delta m^2$  can be accessed due to the lower neutrino beam energy. In table VII we show the limits on  $P_{ee}$  from the negative results of the reactor experiments Gosgen (Zacek, G., 1986), Krasnoyarsk (Vidyakin *et al.*, 1994), Bugey (Achkar *et al.*, 1995), and CHOOZ (Apollonio *et al.*, 1999). Gosgen, Krasnoyarsk and Bugey have relatively short baselines while CHOOZ is the first long baseline (LBL) reactor experiment.

In Fig. 34 we show the corresponding excluded regions in the parameter space for two neutrino oscillations. Bugey sets the strongest constraint on the allowed mixing in the  $\Delta m^2$  range that is interesting for the LSND signal. Due to its longer baseline, CHOOZ is sensitive to the lowest values of  $\Delta m^2$ , low enough to be in the range of interest for atmospheric neutrinos oscillations and the upper sector of the LMA solution for solar neutrinos.

The CHOOZ experiment searches for disappearance of  $\bar{\nu}_e$ 's produced in a power station with two pressurized-water nuclear reactors with a total thermal power of 8.5 GW. At the detector, located at  $L \simeq 1$  km from the reactors, the  $\bar{\nu}_e$  reaction signature is the delayed coincidence between the prompt  $e^+$  signal and the signal due to the neutron capture in the Gd-loaded scintillator. The ratio between the measured and expected fluxes averaged over the neutrino energy spectrum is given by

$$R = 1.01 \pm 2.8 \%(\text{stat}) \pm 2.7 \%(\text{syst}). \quad (140)$$

Thus no evidence was found for a deficit in the flux. The negative result is translated into exclusion regions in the  $(\Delta m^2, \sin^2 2\theta)$  plane shown in Fig. 34. The resulting 90% CL limits include  $\Delta m^2 < 7 \times 10^{-4} \text{ eV}^2$  for maximal mixing, and  $\sin^2 2\theta < 0.10$  for large  $\Delta m^2$ . The CHOOZ results are significant in excluding part of the region that corresponds to the LMA solution of the solar neutrino problem (see Sec. IV). Furthermore, the CHOOZ bound rules out with high significance the possibility that  $\nu_\mu \rightarrow \nu_e$  oscillations explain the atmospheric neutrino deficit. The constraint on the mixing angle is also relevant to the interpretation of the atmospheric neutrino anomaly in the framework of three-neutrino mixing. We return to this issue in Sec. V.

Smaller values of  $\Delta m^2$  can be accessed at future reactor experiments using longer baseline. Pursuing this idea, the KamLAND experiment (Piepke *et al.*, 2001), a 1000 ton liquid scintillation detector, is currently in operation in the Kamioka mine in Japan. This underground site is conveniently located at a distance of 150-210 km from several Japanese nuclear power stations. The measurement of the flux and energy spectrum of the  $\bar{\nu}_e$ 's emitted by these reactors will provide a test to the LMA solution of the solar neutrino anomaly. In Fig. 34 we plot the expected 90% sensitivity for the KamLAND experiment after 3 years of data taking (from Piepke *et al.*, 2001). The experiment will, for the first time, provide a completely solar model independent test of this particle physics solution of the solar neutrino problem. After a few years of data taking, it should be capable of either excluding the entire LMA region or, not only establishing  $\nu_e \leftrightarrow \nu_{\text{other}}$  oscillations, but also measuring the oscillation parameters with unprecedented precision. Data taking is expected to commence in 2002.

#### D. Long Baseline Experiments at Accelerators

Smaller values of  $\Delta m^2$  can also be accessed using accelerator beams at long baseline (LBL) experiments. In these experiments the intense neutrino beam from an accelerator is aimed at a detector located underground at a distance of several hundred kilometers. The main goal of these experiments is to test the presently allowed solution for the atmospheric neutrino problem by searching for either  $\nu_\mu$  disappearance or  $\nu_\tau$  appearance. At present there are three such projects approved: K2K (Nishikawa *et al.*, 1997; Ahn *et al.*, 2001; Nishikawa, *et al.*, 2001), which runs with a baseline of about 235 km from KEK to SK, MINOS (Ables *et al.*, 1995; Wojcicki, 2001) under construction with a baseline of 730 km from Fermilab to the Soudan mine where the detector will be placed, and OPERA (Shibuya *et al.*, 1997; Cocco *et al.*, 2000), under construction with a baseline of 730 km from CERN to Gran Sasso. With their expected sensitivities, these experiments can cover either some fraction or all of the parameter region suggested by the atmospheric neutrino anomaly discussed in Sec. V.

In the K2K experiment, a wide-band, almost pure  $\nu_\mu$  beam from  $\pi^+$  decays is generated in the KEK 12-GeV/c Proton Synchrotron and a neutrino beam-line. The detector is in SK at a distance of 250 km. Various beam monitors along the beam line and two different types of front detectors (FDs) have also been constructed at the KEK site. The FDs are a 1kt water Cerenkov detector, which is a miniature of the SK detector, and a so-called fine-grained detector which is composed of a scintillating fiber tracker trigger counters, lead

glass counters and a muon range detector. The characteristics of the neutrino beam in the KEK site – direction, intensity, stability, energy spectrum and  $\nu_e - \nu_\mu$  composition – are examined using FDs and beam monitors. They are then extrapolated to the SK site and used to obtain the expected number of events and the energy spectrum.

The K2K experiment had a successful start in early 1999, and data were recorded during several periods in 2000/1. The accumulated beam intensity during the 2000 runs was  $22.9 \times 10^{18}$  protons on target (p.o.t.) (Ahn *et al.*, 2001), increased to  $\sim 38 \times 10^{18}$  p.o.t. with the 2001 run till the summer (Nishikawa, *et al.*, 2001) which was about 40% of the goal of the experiment,  $10^{20}$  p.o.t.. The no-oscillation prediction for this sample, based on the data from the FDs, is  $63.9_{-6.6}^{+6.1}$  events while a total of 44 events have been observed in 22.5kt of the fiducial volume. The statistical probability that the observation would be equal to or smaller than 44 is  $\sim 8\%$ . In the presence of oscillations the expected number of events would be 41.5 for  $\Delta m^2 = 3 \times 10^{-3} \text{ eV}^2$  and maximal mixing. Although the central value of the number of observed events is consistent with the data from atmospheric neutrinos, the discrepancy with the no-oscillation prediction is still within the statistical error. K2K has also studied the energy distribution of these events compared to the expectation based on the pion monitor data and a Monte-Carlo simulation, but the statistics is too poor to test the neutrino energy spectrum.

MINOS is designed to detect neutrinos delivered by the Main Injector accelerator at Fermilab (NuMI) with average energies of  $\sim 5\text{-}15$  GeV depending on the beam configuration. Two detectors, functionally identical, will be placed in the NuMI neutrino beam: one at Fermilab and the second one in Soudan iron mine, 732 km away. The MINOS detectors are iron/scintillator sampling calorimeters with a toroidal magnetic field in the iron. Observed interactions of  $\nu_\mu$  can be divided into two classes: CC-like, with an identified  $\mu$  track, and NC-like, muonless. The ratio of the observed numbers of the CC and NC-like events in the two detectors provides a sensitive test for oscillations. With its expected sensitivity MINOS will be able to precisely measure (roughly at the level of 10%) the oscillation parameters in the  $\nu_\mu \rightarrow \nu_\tau$  channel. The primary measurement for this is the comparison of the rate and spectrum of the CC events in the Far Detector with those in the Near Detector. Comparing the NC/CC ratios in the two detectors, the experiment can also be sensitive to the presence of  $\nu_\mu \rightarrow \nu_s$  oscillations. MINOS is scheduled to start data taking at the end of 2004.

Both K2K and MINOS have also the capability for detecting the appearance of  $\nu_e$ 's due to  $\nu_\mu \rightarrow \nu_e$  oscillations. This signal however suffers from large backgrounds due to both the  $\nu_e$ 's in the beam and the NC events with a topology similar to the  $\nu_e$  interaction. These backgrounds can be partially suppressed using the information from the near detectors. In particular, MINOS may be able to improve the CHOOZ bounds.

OPERA is designed to search for  $\nu_\mu \rightarrow \nu_\tau$  oscillations in the Gran Sasso Laboratory. It will study the interaction of 20 GeV neutrinos produced at CERN. The goal is to observe the appearance of  $\nu_\tau$ 's in a pure  $\nu_\mu$  beam. The detector is based on a massive lead/nuclear emulsion target. Nuclear emulsions are exploited for the direct observation of the decay of the  $\nu_\tau$  in a very low background environment.



## E. Direct Determination of Neutrino Masses

Oscillation experiments have provided us with important information on the differences between the neutrino masses-squared,  $\Delta m_{ij}^2$ , and on the leptonic mixing angles,  $U_{ij}$ . But they are insensitive to the absolute mass scale for the neutrinos,  $m_i$ .

Of course, the results of an oscillation experiment do provide a lower bound on the heavier mass in  $\Delta m_{ij}^2$ ,  $|m_i| \geq \sqrt{\Delta m_{ij}^2}$  for  $\Delta m_{ij}^2 > 0$ . But there is no upper bound on this mass. In particular, the corresponding neutrinos could be approximately degenerate at a mass scale that is much higher than  $\sqrt{\Delta m_{ij}^2}$ . Moreover, there is neither upper nor lower bound on the lighter mass  $m_j$ .

Information on the neutrino masses, rather than mass differences, can be extracted from kinematic studies of reactions in which a neutrino or an anti-neutrino is involved. In the absence of mixing the present limits are (Groom *et al.*, 2000)

$$m_{\nu_\tau} < 18.2 \text{ MeV (95\% CL) from } \tau^- \rightarrow n\pi + \nu_\tau \quad (141)$$

$$m_{\nu_\mu} < 190 \text{ keV (90\% CL) from } \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \quad (142)$$

$$m_{\nu_e} < 2.2 \text{ eV (95\% CL) from } {}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e, \quad (143)$$

where for the bound on  $m_{\nu_e}$  we take the latest limit from the Mainz experiment (Bonn *et al.*, 2001). A similar bound is obtained by Troitsk experiment (Lobashev *et al.*, 2001). A new experimental project, KATRIN, is under consideration with an estimated sensitivity limit:  $m_{\nu_e} \sim 0.3 \text{ eV}$ .

In the presence of mixing these limits have to be modified and in general they involve more than a single flavor parameter. The limit that is most relevant to our purposes is the most sensitive one from tritium beta decay. In presence of mixing, the electron neutrino is a combination of mass eigenstates and the tritium beta decay spectrum is modified as (Shrock, 1980):

$$\frac{dN}{dE} = R(E) \sum_i |U_{ei}|^2 [(E_0 - E)^2 - m_i^2]^{1/2} \Theta(E_0 - E - m_i), \quad (144)$$

where  $E$  is the energy of electron,  $E_0$  is the total decay energy and  $R(E)$  is  $m_\nu$ -independent. The step function,  $\Theta(E_0 - E - m_i)$ , reflects the fact that a given neutrino can only be produced if the available energy is larger than its mass. According to Eq. (144), there are two important effects, sensitive to the neutrino masses and mixings, on the electron energy spectrum: (i) Kinks at the electron energies  $E_e^{(i)} = E \sim E_0 - m_i$  with sizes that are determined by  $|U_{ei}|^2$ ; (ii) A shift of the end point to  $E_{\text{ep}} = E_0 - m_1$ , where  $m_1$  is the lightest neutrino mass. The situation simplifies considerably if we are interested in constraining the possibility of quasi-degenerate neutrinos with mass  $\sim m_\nu$ . In this case the distortion of the spectrum can be described by a single parameter (Vissani, 2001a; Farzan, Peres and Smirnov, 2001),  $m_\beta = \frac{\sum_i m_i |U_{ei}|^2}{\sum_i |U_{ei}|^2} \sim m_\nu$ . So the limit in Eq. (143) applies to the unique neutrino mass scale.

Direct information on neutrino masses can also be obtained from neutrinoless double beta decay ( $2\beta 0\nu$ ) searches:

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^-. \quad (145)$$

The rate of this process is proportional to the *effective Majorana mass* of  $\nu_e$ ,

$$m_{ee} = \left| \sum_i m_i U_{ei}^2 \right| \quad (146)$$

which, in addition to five parameters that affect the tritium beta decay spectrum, depends also on the three leptonic CP violating phases. Notice that in order to induce the  $2\beta 0\nu$  decay,  $\nu_e$  must be a Majorana particle.

The present strongest bound from  $2\beta 0\nu$ -decay is obtained by Heidelberg-Moscow group (Klapdor-Kleingrothaus *et al.*, 2001):

$$m_{ee} < 0.34 \text{ (0.26) eV,} \quad 90 \% \text{ (68\% C.L..)} \quad (147)$$

Taking into account systematic errors related to nuclear matrix elements, the bound may be weaker by a factor of about 3. A sensitivity of  $m_{ee} \sim 0.1$  eV is expected to be reached by the currently running NEMO3 experiment (Marquet *et al.*, 2000), while a series of new experiments (CUORE, EXO, GENIUS) is planned with sensitivity of up to  $m_{ee} \sim 0.01$  eV.

The knowledge of  $m_{ee}$  will provide information on the mass and mixing parameters that is independent of the  $\Delta m_{ij}^2$ 's. However, to infer the values of neutrino masses, additional assumptions are required. In particular, the mixing elements are complex and may lead to strong cancellation,  $m_{ee} \ll m_1$ . Yet, the combination of results from  $2\beta 0\nu$  decays and Tritium beta decay can test and, in some cases, determine the mass parameters of given schemes of neutrino masses (Vissani, 1999; Farzan, Peres and Smirnov, 2001; Bilenky, Pascoli and Petcov, 2001a, 2001b; Klapdor-Kleingrothaus, Pas and Smirnov, 2001).

## VII. THREE- AND FOUR-NEUTRINO MIXING

In the previous sections we discussed the three pieces of evidence for neutrino masses and mixing (solar neutrinos, atmospheric neutrinos and the LSND results) as usually formulated in the framework of two-neutrino oscillations. The results are summarized in Fig. 35 where we show the ranges of masses and mixing implied by these signals at 90 and 99% CL for 2 d.o.f., as well as relevant constraints from negative searches in laboratory experiments.

The three pieces of evidence correspond to three values of mass-squared differences of different orders of magnitude. Consequently, there is no consistent explanation to all three signals based on oscillations among the three known neutrinos. The argument for this statement is very simple. With three neutrinos, there are only two independent mass-squared differences, since the following relation must hold:

$$\Delta m_{21}^2 + \Delta m_{32}^2 + \Delta m_{13}^2 = 0. \quad (148)$$

This relation cannot be satisfied by three  $\Delta m_{ij}^2$  that are of different orders of magnitude. One may wonder if this naive extrapolation from the two-neutrino oscillation picture holds once the full mixing structure of the three-neutrino oscillations is taken into account or, on the contrary, some special configuration of the three-neutrino parameters could fit the three pieces of evidence. The combined fit to the data performed by Fogli, Lisi, Montanino and Scioscia (1999a) shows that this is not the case.

Whereas in the case of the solar and atmospheric neutrino indications, several experiments agree on the existence of the effect, the third indication is presently found only by the LSND experiment. Therefore, in many studies the LSND result is left out and the analysis of the solar and atmospheric data is performed in the framework of mixing between the three known neutrinos. In Sec. VII.A we discuss the derived masses and mixing in these scenarios. On the other hand, if all three indications in favour of neutrino oscillations are confirmed, one needs a minimum of four neutrinos. The present phenomenological status of the possibility of mixing between four neutrinos is discussed in Sec. VII.B.

### A. Three-Neutrino Mixing

The combined description of both solar and atmospheric anomalies requires that all three known neutrinos take part in the oscillations. The mixing parameters are encoded in the  $3 \times 3$  lepton mixing matrix (Maki, Nakagawa and Sakata, 1962; Kobayashi and Maskawa, 1973). The two Majorana phases do not affect neutrino oscillations (Bilenky, Hosek and Petcov, 1980; Langacker *et al.*, 1987). The Dirac phase (that is the analog of the KM phase of the quark sector) does affect neutrino oscillations in general, but for the purposes of this section we can set it to zero.

In this case the mixing matrix can be conveniently parametrized in the standard form (Groom *et al.*, 2000):

$$U = \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12} & -s_{23}c_{12} - s_{13}s_{12}c_{23} & c_{23}c_{13} \end{pmatrix}, \quad (149)$$

where  $c_{ij} \equiv \cos \theta_{ij}$  and  $s_{ij} \equiv \sin \theta_{ij}$ .

As we have seen in the previous sections, in most of the parameter space of solutions for solar and atmospheric oscillations, the required mass differences satisfy

$$\Delta m_{\odot}^2 \ll \Delta m_{\text{atm}}^2. \quad (150)$$

In this approximation the angles  $\theta_{ij}$  can be taken without loss of generality to lie in the first quadrant,  $\theta_{ij} \in [0, \pi/2]$ . There are two possible mass orderings which we chose as

$$\Delta m_{21}^2 = \Delta m_{\odot}^2 \ll \Delta m_{32}^2 \simeq \Delta m_{31}^2 = \Delta m_{\text{atm}}^2 > 0; \quad (151)$$

$$\Delta m_{21}^2 = \Delta m_{\odot}^2 \ll -\Delta m_{31}^2 \simeq -\Delta m_{32}^2 = |\Delta m_{\text{atm}}^2| > 0. \quad (152)$$

We refer to the first option, Eq. (151), as the *direct* scheme, and to the second one, Eq. (152), as the *inverted* scheme. The direct scheme is naturally related to hierarchical masses,  $m_1 \ll m_2 \ll m_3$ , for which  $m_2 \simeq \sqrt{\Delta m_{21}^2}$  and  $m_3 \simeq \sqrt{\Delta m_{32}^2}$ , or to quasi-degenerate masses,  $m_1 \simeq m_2 \simeq m_3 \gg \Delta m_{21}^2, \Delta m_{32}^2$ . On the other hand, the inverted scheme implies that  $m_3 < m_1 \simeq m_2$ .

One may wonder how good an approximation it is to set the CP violating phases to zero. It turns out to be an excellent approximation for the analysis of solar, atmospheric and laboratory data if Eq. (150) holds. In this case, as discussed below, no simultaneous effect of the two mass differences is observable in any  $\nu$ -appearance transition.

## 1. Probabilities

The determination of the oscillation probabilities for both solar and atmospheric neutrinos requires that one solves the evolution equation of the neutrino system in the matter background of the Sun or the Earth. In a three-flavor framework, this equation reads:

$$i\frac{d\vec{\nu}}{dt} = H\vec{\nu}, \quad H = U \cdot H_0^d \cdot U^\dagger + V, \quad (153)$$

where  $U$  is the lepton mixing matrix,  $\vec{\nu} \equiv (\nu_e, \nu_\mu, \nu_\tau)^T$ ,  $H_0^d$  is the vacuum hamiltonian,

$$H_0^d = \frac{1}{2E_\nu} \text{diag} \left( -\Delta m_{21}^2, 0, \Delta m_{32}^2 \right), \quad (154)$$

and  $V$  is the effective potential that describes charged-current forward interactions in matter:

$$V = \text{diag} \left( \pm\sqrt{2}G_F N_e, 0, 0 \right) \equiv \text{diag} (V_e, 0, 0). \quad (155)$$

In Eq. (155), the sign  $+$  ( $-$ ) refers to neutrinos (antineutrinos), and  $N_e$  is electron number density in the Sun or the Earth.

In what follows we focus on the direct scheme of Eq. (151), for which the five relevant parameters are related to experiments in the following way:

$$\Delta m_{\odot}^2 = \Delta m_{21}^2, \quad \Delta m_{\text{atm}}^2 = \Delta m_{32}^2, \quad (156)$$

$$\theta_{\odot} = \theta_{12}, \quad \theta_{\text{atm}} = \theta_{23}, \quad \theta_{\text{reactor}} = \theta_{13}. \quad (157)$$

For transitions in vacuum, the results apply also to the inverted scheme of Eq. (152). In the presence of matter effects, the direct and inverted schemes are no longer equivalent, although the difference is hardly recognizable in the current solar and atmospheric neutrino phenomenology as long as Eq. (150) holds (Fogli, Lisi, Montanino and Scioscia, 1997). Under this approximation, the results obtained for the direct scheme can be applied to the inverted scheme by replacing  $\Delta m_{32}^2 \rightarrow -\Delta m_{32}^2$ .

In general the transition probabilities present an oscillatory behaviour with two oscillation lengths. However, the hierarchy in the splittings, Eq. (150), leads to important simplifications.

Let us first consider the analysis of solar neutrinos. A first simplification occurs because  $L_{32}^{\text{osc}} = 4\pi E/\Delta m_{32}^2$  is much shorter than the distance between the Sun and the Earth. Consequently, the oscillations related to  $L_{32}^{\text{osc}}$  are averaged in the evolution from the Sun to the Earth. A second simplification occurs because, for the evolution in both the Sun and the Earth,  $\Delta m_{32}^2 \gg 2\sqrt{2}G_F N_e E_\nu \sin^2 2\theta_{13}$ . Consequently, matter effects on the evolution of  $\nu_3$  can be neglected. The result of these two approximations is that the three-flavor evolution equations decouple into an effective two-flavor problem for the basis (Kuo and Pantaleone, 1986; Shi and Schramm, 1992)

$$\nu_{e'} = c_{12}\nu_1 + s_{12}\nu_2, \quad \nu_{\mu'} = -s_{12}\nu_1 + c_{12}\nu_2, \quad (158)$$

with the substitution of  $N_e$  by the effective density

$$N_e \Rightarrow N_e \cos^2 \theta_{13}. \quad (159)$$

Thus the survival probability takes the following form:

$$P_{ee,\odot}^{3\nu} = \sin^4 \theta_{13} + \cos^4 \theta_{13} P_{e'e',\odot}^{2\nu}, \quad (160)$$

where  $P_{e'e',\odot}^{2\nu}$  is the two-flavor survival probability in the  $(\Delta m_{21}^2, \theta_{12})$  parameter space but with the modified matter density of Eq. (159). We conclude that the analysis of the solar data constrains three of the five independent oscillation parameters:  $\Delta m_{21}^2, \theta_{12}$  and  $\theta_{13}$ .

Eq. (160) reveals what is the dominant effect of a non-vanishing  $\theta_{13}$  in the solar neutrino survival probability: the energy dependent part of the probability,  $P_{e'e',\odot}^{2\nu}$ , gets damped by the factor  $\cos^4 \theta_{13}$ , while an energy independent term,  $\sin^4 \theta_{13}$ , is added. Thus increasing  $\theta_{13}$  makes the solar neutrino survival probability more and more energy independent.

Let us now consider the analysis of atmospheric neutrinos. Here  $L_{21}^{\text{osc}} = 4\pi E/\Delta m_{21}^2$  is much larger than the relevant distance scales. Consequently, the corresponding oscillating phase is negligible. In this approximation one can rotate away the corresponding angle  $\theta_{12}$ . Thus the resulting survival probabilities do not depend on  $\Delta m_{21}^2$  and  $\theta_{12}$ . For instance for constant Earth matter density, the various  $P_{\alpha\beta}$  can be written as follows:

$$\begin{aligned} P_{ee} &= 1 - 4s_{13,m}^2 c_{13,m}^2 S_{31}, \\ P_{\mu\mu} &= 1 - 4s_{13,m}^2 c_{13,m}^2 s_{23}^4 S_{31} - 4s_{13,m}^2 s_{23}^2 c_{23}^2 S_{21} - 4c_{13,m}^2 s_{23}^2 c_{23}^2 S_{32}, \\ P_{\tau\tau} &= 1 - 4s_{13,m}^2 c_{13,m}^2 c_{23}^4 S_{31} - 4s_{13,m}^2 s_{23}^2 c_{23}^2 S_{21} - 4c_{13,m}^2 s_{23}^2 c_{23}^2 S_{32}, \\ P_{e\mu} &= 4s_{13,m}^2 c_{13,m}^2 s_{23}^2 S_{31}, \\ P_{e\tau} &= 4s_{13,m}^2 c_{13,m}^2 c_{23}^2 S_{31}, \\ P_{\mu\tau} &= -4s_{13,m}^2 c_{13,m}^2 s_{23}^2 c_{23}^2 S_{31} + 4s_{13,m}^2 s_{23}^2 c_{23}^2 S_{21} + 4c_{13,m}^2 s_{23}^2 c_{23}^2 S_{32}. \end{aligned} \quad (161)$$

Here  $\theta_{13,m}$  is the effective mixing angle in matter:

$$\sin 2\theta_{13,m} = \frac{\sin 2\theta_{13}}{\sqrt{(\cos 2\theta_{13} - 2E_\nu V_e/\Delta m_{32}^2)^2 + (\sin 2\theta_{13})^2}} \quad (162)$$

and  $S_{ij}$  are the oscillating factors in matter:

$$S_{ij} = \sin^2 \left( \frac{\Delta \mu_{ij}^2}{4E_\nu} L \right). \quad (163)$$

In Eq. (163),  $\Delta \mu_{ij}^2$  are the effective mass-squared differences in matter:

$$\begin{aligned} \Delta \mu_{21}^2 &= \frac{\Delta m_{32}^2}{2} \left( \frac{\sin 2\theta_{13}}{\sin 2\theta_{13,m}} - 1 \right) - E_\nu V_e, \\ \Delta \mu_{32}^2 &= \frac{\Delta m_{32}^2}{2} \left( \frac{\sin 2\theta_{13}}{\sin 2\theta_{13,m}} + 1 \right) + E_\nu V_e, \\ \Delta \mu_{31}^2 &= \Delta m_{32}^2 \frac{\sin 2\theta_{13}}{\sin 2\theta_{13,m}}. \end{aligned} \quad (164)$$

and  $L$  is the pathlength of the neutrino within the Earth, which depends on its direction. We conclude that the analysis of the atmospheric data constrains three of the five independent oscillation parameters:  $\Delta m_{32}^2, \theta_{23}$  and  $\theta_{13}$ .

So we find that in the approximation of Eq. (150) the mixing angle  $\theta_{13}$  is the only parameter common to both solar and atmospheric neutrino oscillations and which may potentially allow for some mutual influence. The main effect of the three-neutrino mixing is that now atmospheric neutrinos can oscillate simultaneously in both the  $\nu_\mu \rightarrow \nu_\tau$  and  $\nu_\mu \rightarrow \nu_e$  (and, similarly,  $\nu_e \rightarrow \nu_\tau$  and  $\nu_e \rightarrow \nu_\mu$ ) channels. The oscillation amplitudes for channels involving  $\nu_e$  are controlled by the size of  $\sin^2 \theta_{13} = |U_{e3}|^2$ . We learn that in the approximation of Eq. (150) solar and atmospheric neutrino oscillations decouple in the limit  $\theta_{13} = 0$ . This angle is constrained by the CHOOZ reactor experiment. To analyze the CHOOZ constraints we need to evaluate the survival probability for  $\bar{\nu}_e$  of average energy  $E \sim$  few MeV at a distance of  $L \sim 1$  Km. For these values of energy and distance, one can safely neglect Earth matter effects. The survival probability takes the analytical form:

$$\begin{aligned}
P_{ee}^{\text{CHOOZ}} &= 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \left( \frac{\Delta m_{21}^2 L}{4E_\nu} \right) \\
&\quad - \sin^2 2\theta_{13} \left[ \cos^2 \theta_{12} \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E_\nu} \right) + \sin^2 \theta_{12} \sin^2 \left( \frac{\Delta m_{32}^2 L}{4E_\nu} \right) \right] \\
&\simeq 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{32}^2 L}{4E_\nu} \right),
\end{aligned} \tag{165}$$

where the second equality holds under the approximation  $\Delta m_{21}^2 \ll E_\nu/L$  which can only be safely made for  $\Delta m_{21}^2 \lesssim 3 \times 10^{-4} \text{eV}^2$ . Thus in general the analysis of the CHOOZ reactor data involves four oscillation parameters:  $\Delta m_{32}^2$ ,  $\theta_{13}$ ,  $\Delta m_{21}^2$ , and  $\theta_{12}$  (Gonzalez-Garcia, Maltoni, Peña-Garay and Valle, 2001; Bilenky, Nicolo and Petcov, 2001)

## 2. Allowed masses and mixing

There are several three-neutrino oscillation analyses in the literature which include either solar (Fogli, Lisi and Montanino, 1996; Fogli, Lisi, Montanino and Palazzo, 2000a, 2000b; Gago, Nunokawa and Zukanovich, 2001) or atmospheric (Fogli, Lisi, Montanino and Scioscia, 1997; Fogli, Lisi, Marrone and Scioscia, 1999b; Fogli, Lisi, Marrone, 2001b; de Rujula, Gavela and Hernandez, 2001; Teshima and Sakai 1999) neutrino data. Combined studies have also been performed (Barger, Whistnant and Phillips, 1980; Fogli, Lisi and Montanino, 1994, 1995; Barbieri *et al.*, 1998; Barger and Whistnant, 1999). We follow and update here the results from the latest analysis of Gonzalez-Garcia, Maltoni, Peña-Garay and Valle (2001).

As discussed above, in the approximation of Eq. (150), solar and atmospheric neutrino oscillations decouple in the limit  $|U_{e3}| = \sin \theta_{13} = 0$ . In this limit the allowed values of the parameters can be obtained directly from the results of the analyses in terms of two-neutrino oscillations presented in Sec. IV and Sec. V. Deviations from the two-neutrino scenario are then determined by the size of  $\theta_{13}$ . Thus the first question to answer is how the presence of this additional angle affects the analysis of the solar and atmospheric neutrino data.

In Fig. 36 we show the allowed regions for the oscillation parameters  $\Delta m_{21}^2$  and  $\tan^2 \theta_{12}$  from the global analysis of the solar neutrino data in the framework of three-neutrino oscillations for different values of  $\sin^2 \theta_{13}$  (updated from Gonzalez-Garcia, Maltoni, Peña-Garay and Valle, 2001). The allowed regions for a given CL are defined as the set of points satisfying the condition  $\chi^2(\Delta m_{12}^2, \tan^2 \theta_{12}, \tan^2 \theta_{13}) - \chi_{\text{min}}^2 \leq \Delta\chi^2(\text{CL}, 3 \text{ d.o.f})$  where, for instance,

$\Delta\chi^2(\text{CL}, 3 \text{ d.o.f})=6.25, 7.83, \text{ and } 11.36$  for  $\text{CL}=90, 95, \text{ and } 99\%$ , respectively. The global minimum used in the construction of the regions lies in the LMA region and corresponds to  $\tan^2\theta_{13} = 0$ , that is, to the *decoupled* scenario.

As seen in the figure, the modifications to the decoupled case are significant only if  $\theta_{13}$  is large. As  $\sin^2\theta_{13}$  increases, all the allowed regions disappear, leading to an upper bound on  $\sin^2\theta_{13}$  that is independent of the values taken by the other parameters in the three-neutrino mixing matrix. For instance, no region of parameter space is allowed (at 99% C.L. for 3 d.o.f) for  $\sin^2\theta_{13} = |U_{e3}|^2 > 0.80$ . This fact is also illustrated in Fig. 37 where we plot the shift in  $\chi^2$  as a function of  $\sin^2\theta_{13}$  when the mass and mixing parameters  $\Delta m_{12}^2$  and  $\tan^2\theta_{12}$  are chosen to minimize  $\chi^2$ . In the figure we show the curves for the analysis of the total rates and for the global analysis. From Fig. 37 we also learn that the upper bound on  $\sin^2\theta_{13}$  obtained when including only the total event rates in the analysis is actually stronger than after adding the data on the day-night spectrum. This fact can be seen as a consequence of a *tension* between the data on the total event rates, which favour smaller  $\theta_{13}$  values, and the day-night spectra which allow larger values. It can also be understood as the tension between the energy dependent and the constant pieces of the electron survival probability in Eq. (160).

In Fig. 38 we show the allowed regions for the oscillation parameters  $\Delta m_{32}^2$  and  $\tan^2\theta_{23}$  from the analysis of the atmospheric neutrino data in the framework of three-neutrino oscillations for different values of  $\sin^2\theta_{13}$  (updated from Gonzalez-Garcia, Maltoni, Peña-Garay and Valle, 2001). In the upper-left panel  $\tan^2\theta_{13} = 0$  which corresponds to pure  $\nu_\mu \rightarrow \nu_\tau$  oscillations. Note the exact symmetry of the contour regions under the transformation  $\theta_{23} \rightarrow \pi/4 - \theta_{23}$ . This symmetry follows from the fact that in the pure  $\nu_\mu \rightarrow \nu_\tau$  channel matter effects cancel out and the oscillation probability depends on  $\theta_{23}$  only through the double-valued function  $\sin^2 2\theta_{23}$  (see Sec. V.C). For  $\theta_{13} \neq 0$  this symmetry breaks due to the three-neutrino mixing structure even if matter effects are neglected. The analysis of the full atmospheric neutrino data in the framework of three-neutrino oscillations clearly favours the  $\nu_\mu \rightarrow \nu_\tau$  oscillation hypothesis. As a matter of fact, the best fit corresponds to a very small value,  $\theta_{13} = 6^\circ$ , but it is statistically indistinguishable from the decoupled scenario,  $\theta_{13} = 0^\circ$ . No region of parameter space is allowed (at 99% C.L. for 3 d.o.f) for  $\sin^2\theta_{13} = |U_{e3}|^2 > 0.40$ . The physics reason for this limit is clear from the discussion of the  $\nu_\mu \rightarrow \nu_e$  oscillation channel in Sec. V.C: large values of  $\theta_{13}$  imply a too large contribution of the  $\nu_\mu \rightarrow \nu_e$  channel and would spoil the otherwise successful description of the angular distribution of contained events. This situation is illustrated also in Fig. 37 where we plot the shift of  $\chi^2$  for the analysis of atmospheric data in the framework of oscillations between three neutrinos as a function of  $\sin^2\theta_{13}$  when the mass and mixing parameters  $\Delta m_{32}^2$  and  $\tan^2\theta_{23}$  are chosen to minimize  $\chi^2$ .

For any value of the mixing parameters, the mass-squared difference relevant for the atmospheric analysis is restricted to lie in the interval  $1.25 \times 10^{-3} < \Delta m_{32}^2/\text{eV}^2 < 8 \times 10^{-3}$  at 99 % CL. Thus it is within the range of sensitivity of the CHOOZ experiment. Indeed, as illustrated in Fig. 37, the limit on  $\sin^2\theta_{13}$  becomes stronger when the CHOOZ data are combined with the atmospheric and solar neutrino results.

One can finally perform a global analysis in the five dimensional parameter space combining the full set of solar, atmospheric and reactor data. Such analysis leads to the following allowed  $3\sigma$  ranges for individual parameters (that is, when the other four parameters have

been chosen to minimize the global  $\chi^2$ ):

$$\begin{aligned}
1.9 \times 10^{-5} < \Delta m_{21}^2 / \text{eV}^2 < 2.7 \times 10^{-4} & \quad \text{LMA} , \\
0.22 < \tan^2 \theta_{12} < 0.71 & \quad \text{LMA} , \\
4.3 \times 10^{-8} < \Delta m_{21}^2 / \text{eV}^2 < 1.8 \times 10^{-7} & \quad \text{LOW} , \\
0.47 < \tan^2 \theta_{12} < 1.1 & \quad \text{LOW} , \\
1.4 \times 10^{-3} < \Delta m_{32}^2 / \text{eV}^2 < 6.0 \times 10^{-3}, & \\
0.4 < \tan^2 \theta_{23} < 3.0, & \\
\sin^2 \theta_{13} < 0.05. & \quad (166)
\end{aligned}$$

These results can be translated into our present knowledge of the moduli of the mixing matrix  $U$ :

$$|U| = \begin{pmatrix} 0.67 - 0.91 & 0.41 - 0.73 & < 0.22 \\ 0.21 - 0.70 & 0.20 - 0.77 & 0.51 - 0.87 \\ 0.05 - 0.63 & 0.36 - 0.83 & 0.49 - 0.85 \end{pmatrix}. \quad (167)$$

In conclusion, we learn that at present the large mixing-type solutions provide the best fit. As concerns  $|U_{e3}|$ , both solar and atmospheric data favour small values and this trend is strengthened by the reactor data.

## B. Four-Neutrino Mixing

If all three indications of neutrino oscillations are confirmed, then a minimum of four neutrinos will be required to accommodate the data. As discussed in Sec. II, the measurement of the decay width of the  $Z^0$  boson into neutrinos makes the existence of three, and only three, light (that is,  $m_\nu \lesssim m_Z/2$ ) active neutrinos an experimental fact (Eq. 6). Therefore, a fourth neutrino must not couple to the standard electroweak current, that is, it must be sterile.

One of the most important issues in the context of four-neutrino scenarios is the four-neutrino mass spectrum. There are six possible four-neutrino schemes, shown in Fig. 39, that can accommodate the results from solar and atmospheric neutrino experiments as well as the LSND result. They can be divided in two classes: (3+1) and (2+2) (Barger *et al.*, 2001). In the (3+1) schemes, there is a group of three close-by neutrino masses that is separated from the fourth one by a gap of the order of 1 eV<sup>2</sup>, which is responsible for the SBL oscillations observed in the LSND experiment. In (2+2) schemes, there are two pairs of close masses separated by the LSND gap. The main difference between these two classes is the following: if a (2+2)-spectrum is realized in nature, the transition into the sterile neutrino is a solution of either the solar or the atmospheric neutrino problem, or the sterile neutrino takes part in both, whereas with a (3+1)-spectrum the sterile neutrino could be only slightly mixed with the active ones and mainly provide a description of the LSND result.

As concerns the mixing parameters, we emphasize that the mixing matrix that describes CC interactions in these schemes is a  $3 \times 4$  matrix. The reason is that there are three charged lepton mass eigenstates ( $e, \mu, \tau$ ) and four neutrino mass eigenstates ( $\nu_1, \nu_2, \nu_3, \nu_4$ ).



As discussed in Sec. II.D, if we choose an interaction basis where the charged leptons are the mass eigenstates, then the CC mixing matrix  $U$  is a sub-matrix of the  $4 \times 4$  unitary matrix  $V^\nu$  that rotates the neutrinos from the interaction basis to the mass basis, where the line corresponding to  $\nu_s$  is removed.

### 1. Status of (3+1) schemes

It has been argued in the literature that the (3+1)-spectra are strongly disfavoured by the data from SBL laboratory experiments (Okada and Yasuda, 1997; Bilenky, Giunti, and Grimus, 1998; Barger, Pakvasa, Weiler and Whisnant, 1998a; Bilenky, Giunti, Grimus and Schwetz, 1999). At what level they can actually be ruled out has been a matter of debate and the answer has also changed with the changes in the analysis of the LSND and KARMEN collaborations (Barger *et al.*, 2000; Giunti and Laveder, 2001; Peres and Smirnov, 2001).

The arguments which disfavour the (3+1)-mass spectra are based on exclusion curves from SBL experiments, and on some simplified treatment of the solar and atmospheric neutrino data. We summarize here these arguments and the present phenomenological status as given in Grimus and Schwetz (2001) and Maltoni, Schwetz, and Valle (2001a,2001b).

The probability  $P_{\mu e}$  that is relevant for LSND (and also for KARMEN and NOMAD) is given by

$$P_{\mu e} = P_{\bar{\mu} \bar{e}} = 4|U_{e4}U_{\mu4}|^2 \sin^2 \frac{\Delta m^2 L}{4E}, \quad (168)$$

where  $L$  is the distance between source and detector. In this expression solar and atmospheric splittings have been neglected as they are too small to give any observable effect at the  $L/E$  that is relevant for LSND. In this approximation, for schemes (3+1)A,  $\Delta m^2 \equiv \Delta m_{41}^2 = \Delta m_{42}^2 = \Delta m_{43}^2$ .

The LSND experiment gives then an allowed region in the  $\Delta m^2 - |U_{e4}U_{\mu4}|^2$  plane which can be directly obtained from the two-neutrino oscillation region shown in Fig. 33 with the identification  $\sin^2 2\theta \rightarrow 4|U_{e4}U_{\mu4}|^2$ . In the same way, the KARMEN experiment gives an excluded region in the same plane which can be directly obtained from Fig. 33.

Further constraints on  $|U_{e4}U_{\mu4}|^2$  can be obtained by combining the bounds on  $|U_{e4}|$  and  $|U_{\mu4}|$  from reactor and accelerator experiments in combination with the information from solar and atmospheric neutrinos. The strongest constraints in the relevant  $\Delta m^2$  region are given by the Bugey (Achkar *et al.*, 1995) and CDHS (Dydak *et al.*, 1984) experiments. Using their limits on the survival probabilities one finds

$$4|U_{e4}|^2(1 - |U_{e4}|^2) < D_e^{\text{Bugey}}(\Delta m^2),$$

$$D_e^{\text{Bugey}} < 0.001 - 0.1 \quad (\text{for } 0.1 \lesssim \Delta m^2/\text{eV}^2 \lesssim 10), \quad (169)$$

$$4|U_{\mu4}|^2(1 - |U_{\mu4}|^2) < D_\mu^{\text{CDHS}}(\Delta m^2),$$

$$D_\mu^{\text{CDHS}} < 0.05 - 0.1 \quad (\text{for } \Delta m^2 \gtrsim 0.5 \text{ eV}^2), \quad (170)$$

while for lower  $\Delta m^2$  the CDHS bound weakens considerably.

In principle, the inequalities in Eq. (169) and Eq. (170) can be satisfied with either small mixing parameters  $|U_{\alpha4}|^2 \lesssim D_\alpha/4$ , or close to maximal mixing,  $|U_{\alpha4}|^2 \gtrsim 1 - D_\alpha/4$ . This is the generalization to these schemes of the symmetry  $\theta \leftrightarrow \frac{\pi}{2} - \theta$  of two-neutrino vacuum

oscillations (see Sec. III.A). Solar and atmospheric data are invoked in order to resolve this ambiguity. Since in any of the allowed regions for solar oscillations  $P_{ee}^{\odot} \lesssim 0.5$ , only the small values of  $|U_{e4}|$  are possible. For atmospheric neutrinos one can use the fact that oscillations with the large  $\Delta m^2$  would wash-out the up-down asymmetry and this wash-out grows with the projection of the  $\nu_{\mu}$  over states separated by the large  $\Delta m^2$ , which is controlled by the mixing parameter  $|U_{\mu 4}|$ . Consequently only the small values of  $|U_{\mu 4}|$  are allowed. Thus naively one obtains the bound

$$4|U_{e4}U_{\mu 4}|^2 < 0.25D_e^{\text{Bugey}}(\Delta m^2)D_{\mu}^{\text{CDHS}}(\Delta m^2), \quad (171)$$

which further strengthens the exclusion curve corresponding to the KARMEN bound. A detailed and statistically meaningful evaluation of the final combined limit, including also the results from other experiments like NOMAD and CHOOZ, leads to the results presented in Fig. 40 (Grimus and Schwetz, 2001; Maltoni, Schwetz and Valle, 2001a). The figure shows that there is no overlap of the region allowed by the combined bound at 95% CL with the region allowed by LSND at 99% CL. For the combined bound at 99% CL there are marginal overlaps with the 99% CL LSND allowed region at  $\Delta m^2 \sim 0.9$  and  $2 \text{ eV}^2$ , and a very marginal overlap region still exists around  $6 \text{ eV}^2$ .

## 2. (2+2) schemes: active-sterile admixtures

The main feature of (2+2)-spectra is that either solar or atmospheric oscillations must involve the sterile neutrino. Such oscillations are, however, disfavoured for both the solar (see Sec. IV) and atmospheric (see Sec. V) neutrinos. One expects then that the (2+2) schemes are disfavoured. However, as first discussed by Dooling, Giunti, Kang and Kim (2000), within (2+2) schemes, oscillations into pure active or pure sterile states are only limiting cases of the most general possibility of oscillations into an admixture of active and sterile neutrinos. One can wonder then whether some admixture of active-sterile oscillations gives an acceptable description of both solar (Giunti, Gonzalez-Garcia and Peña-Garay, 2000) and atmospheric (Yasuda, 2000; Fogli, Lisi and Marrone, 2001a, 2001b) data. To address this issue we follow here the results from the combined analysis performed by Gonzalez-Garcia, Maltoni and Peña-Garay (2001).

For the phenomenology of neutrino oscillations, the (2+2)A and (2+2)B schemes are equivalent up to the relabeling of the mass eigenstates (or, equivalently, of the mixing angles). Thus in what follows we consider the scheme B, where the mass spectrum presents the following hierarchy:

$$\Delta m_{\odot}^2 = \Delta m_{21}^2 \ll \Delta m_{\text{atm}}^2 = \Delta m_{43}^2 \ll \Delta m_{\text{LSND}}^2 = \Delta m_{41}^2 \simeq \Delta m_{42}^2 \simeq \Delta m_{31}^2 \simeq \Delta m_{32}^2. \quad (172)$$

Neglecting possible CP phases, and choosing a convention that is convenient for the study of solar and atmospheric neutrinos, the matrix  $V_{\alpha i}^{\nu}$  ( $\alpha = e, s, \mu, \tau$ ) can be written as follows:

$$V^{\nu} = V_{24} V_{23} V_{14} V_{13} V_{34} V_{12}, \quad (173)$$

where  $V_{ij}$  represents a rotation of angle  $\theta_{ij}$  in the  $ij$  plane. Since the parametrizations of the leptonic mixing matrix  $U$  and of the corresponding lines in  $V^{\nu}$  are the same, and in

particular involve six mixing angles, we will concentrate below on the allowed values of the  $4 \times 4$  neutrino mixing matrix  $V^\nu$ .

This general form can be further simplified by taking into account the negative results from the reactor experiments (in particular the Bugey experiment) which in the range of  $\Delta m_{41}^2$  relevant to the LSND experiment imply that

$$|V_{e3}^\nu|^2 + |V_{e4}^\nu|^2 = c_{14}^2 s_{13}^2 + s_{14}^2 \lesssim 10^{-2}. \quad (174)$$

For our purposes, the two angles  $\theta_{13}$  and  $\theta_{14}$  can then be safely neglected and the  $U$  matrix takes the effective form:

$$V^\nu = \begin{pmatrix} c_{12} & s_{12} & 0 & 0 \\ -s_{12}c_{23}c_{24} & c_{12}c_{23}c_{24} & s_{23}c_{24}c_{34} - s_{24}s_{34} & s_{23}c_{24}s_{34} + s_{24}c_{34} \\ s_{12}s_{23} & -c_{12}s_{23} & c_{23}c_{34} & c_{23}s_{34} \\ s_{12}c_{23}s_{24} & -c_{12}c_{23}s_{24} & -s_{23}s_{24}c_{34} - c_{24}s_{34} & -s_{23}s_{24}s_{34} + c_{24}c_{34} \end{pmatrix}. \quad (175)$$

The full parameter space relevant to solar and atmospheric neutrino oscillation can be covered with three of the angles  $\theta_{ij}$  in the first quadrant,  $\theta_{ij} \in [0, \pi/2]$  while one (which we choose to be  $\theta_{34}$ ) is allow to vary in the range  $-\frac{\pi}{2} \leq \theta_{34} \leq \frac{\pi}{2}$ .

In this scheme solar neutrino oscillations are generated by the mass-squared difference between  $\nu_2$  and  $\nu_1$  while atmospheric neutrino oscillations are generated by the mass-squared difference between  $\nu_3$  and  $\nu_4$ . It is clear from Eq. (175) that the survival of solar  $\nu_e$ 's depends mainly on  $\theta_{12}$  while atmospheric  $\nu_e$ 's are not affected by the four-neutrino oscillations in the approximation  $\theta_{13} = \theta_{14} = 0$  and neglecting the effect of  $\Delta m_{21}^2$  in the range of atmospheric neutrino energies. The survival probability of atmospheric  $\nu_\mu$ 's depends mainly on the  $\theta_{34}$ .

Thus solar neutrino oscillations occur with a mixing angle  $\theta_{12}$  between the states

$$\nu_e \rightarrow \nu_\alpha \quad \text{with} \quad \nu_\alpha = c_{23}c_{24} \nu_s + \sqrt{1 - c_{23}^2 c_{24}^2} \nu_a, \quad (176)$$

where  $\nu_a$  is a linear combination of  $\nu_\mu$  and  $\nu_\tau$ ,

$$\nu_a = \frac{1}{\sqrt{1 - c_{23}^2 c_{24}^2}} (s_{23}\nu_\mu + c_{23}s_{24}\nu_\tau). \quad (177)$$

We remind the reader that  $\nu_\mu$  and  $\nu_\tau$  cannot be distinguished in solar neutrino experiments, because their matter potential and their interaction in the detectors are equal, due to only NC weak interactions. Thus solar neutrino oscillations cannot depend on the mixing angle  $\theta_{34}$  and depend on  $\theta_{23}$  and  $\theta_{24}$  through the combination  $c_{23}^2 c_{24}^2$ .

Atmospheric neutrino oscillations, *i.e.* oscillations with the mass difference  $\Delta m_{34}^2$  and mixing angle  $\theta_{34}$ , occur between the states

$$\nu_\beta \rightarrow \nu_\gamma \quad \text{with} \quad \nu_\beta = s_{23}c_{24}\nu_s + c_{23}\nu_\mu - s_{23}s_{24}\nu_\tau \quad \text{and} \quad \nu_\gamma = s_{24}\nu_s + c_{24}\nu_\tau. \quad (178)$$

We learn that the mixing angles  $\theta_{23}$  and  $\theta_{24}$  determine two projections. First, the projection of the sterile neutrino onto the states in which the solar  $\nu_e$  oscillates is given by

$$c_{23}^2 c_{24}^2 = 1 - |V_{a1}^\nu|^2 - |V_{a2}^\nu|^2 = |V_{s1}^\nu|^2 + |V_{s2}^\nu|^2. \quad (179)$$

Second, the projection of the  $\nu_\mu$  over the solar neutrino oscillating states is given by

$$s_{23}^2 = |V_{\mu 1}^\nu|^2 + |V_{\mu 2}^\nu|^2 = 1 - |V_{\mu 3}^\nu|^2 - |V_{\mu 4}^\nu|^2 \quad (180)$$

One expects  $s_{23}$  to be small in order to explain the atmospheric neutrino deficit. We will see that this is indeed the case. Furthermore, the negative results from the CDHS and CCFR searches for  $\nu_\mu$ -disappearance also constrain such a projection to be smaller than 0.2 at 90% CL for  $\Delta m_{\text{LSND}}^2 \gtrsim 0.4 \text{ eV}^2$ .

We distinguish the following limiting cases:

(i) If  $c_{23} = 1$  then  $V_{\mu 1}^\nu = V_{\mu 2}^\nu = 0$ . The atmospheric  $\nu_\mu = \nu_\beta$  state oscillates into a state  $\nu_\gamma = c_{24}\nu_\tau + s_{24}\nu_s$ . We will denote this case as *restricted* ( $\text{ATM}_R$ ). In particular:

- If  $c_{23} = c_{24} = 1$ ,  $V_{a 1}^\nu = V_{a 2}^\nu = 0$  ( $V_{s 3}^\nu = V_{s 4}^\nu = 0$ ) and we have the limit of pure two-generation solar  $\nu_e \rightarrow \nu_s$  transitions and atmospheric  $\nu_\mu \rightarrow \nu_\tau$  transitions.
- If  $c_{24} = 0$  then  $V_{s 1}^\nu = V_{s 2}^\nu = 0$  and  $V_{\tau 3}^\nu = V_{\tau 4}^\nu = 0$ , corresponding to the limit of pure two-generation solar  $\nu_e \rightarrow \nu_\tau$  transitions and atmospheric  $\nu_\mu \rightarrow \nu_s$  transitions.

(ii) If  $c_{23} = 0$  then  $V_{s 1}^\nu = V_{s 2}^\nu = 0$  corresponding to the limit of pure two-generation solar  $\nu_e \rightarrow \nu_a$  with  $a = \mu$  and there are no atmospheric neutrino oscillations as the projection of  $\nu_\mu$  over the relevant states cancels out ( $V_{\mu 3}^\nu = V_{\mu 4}^\nu = 0$ ).

Notice that in the restricted case  $\theta_{23} = 0$ , there is an additional symmetry in the relevant probabilities so that the full parameter space can be spanned by  $0 \leq \theta_{34} \leq \frac{\pi}{2}$ . The reason for this is that we now have effectively two-neutrino oscillations for both the solar ( $\nu_e \rightarrow \nu_\alpha$ ) and atmospheric ( $\nu_\mu \rightarrow \nu_\gamma$ ) cases.

To summarize, solar neutrino oscillations depend on the new mixing angles only through the product  $c_{23}c_{24}$  and therefore the analysis of the solar neutrino data in four-neutrino mixing schemes is equivalent to the two-neutrino analysis but taking into account that the parameter space is now three-dimensional ( $\Delta m_{21}^2, \tan^2 \theta_{12}, c_{23}^2 c_{24}^2$ ). Atmospheric neutrino oscillations are affected independently by the angles  $\theta_{23}$  and  $\theta_{24}$ , and the analysis of the atmospheric neutrino data in the four-neutrino mixing schemes is equivalent to the two-neutrino analysis, but taking into account that the parameter space is now four-dimensional ( $\Delta m_{43}^2, \theta_{34}, c_{23}^2, c_{24}^2$ ).

As an illustration we show in Fig. 41 the allowed regions for the oscillation parameters  $\Delta m_{21}^2$  and  $\tan^2 \theta_{12}$  from the global analysis of the solar neutrino data for different values of  $c_{23}^2 c_{24}^2$  (from Gonzalez-Garcia, Maltoni and Peña-Garay 2001). The global minimum corresponds to pure  $\nu_e$ -active oscillations in the LMA region. As seen in the figure, with increasing sterile component the allowed regions become smaller till they totally disappear. This behaviour is also illustrated in Fig. 42.b where we show the shift in  $\chi^2$  as a function of the active-sterile admixture  $|V_{s 1}^\nu|^2 + |V_{s 2}^\nu|^2 = c_{23}^2 c_{24}^2$ . From the figures we conclude that the solar neutrino data favour pure  $\nu_e \rightarrow \nu_a$  oscillations but sizeable active-sterile admixtures are still allowed. For instance at 99% CL, both the LMA and LOW-QVO regions are allowed for maximal active-sterile mixing,  $c_{23}^2 c_{24}^2 = 0.5$ .

Similar analysis can be performed for the atmospheric neutrino data (Gonzalez-Garcia, Maltoni and Peña-Garay, 2001) to obtain the allowed regions for the oscillation parameters  $\Delta m_{43}^2$  and  $\tan^2 \theta_{34}$  from the global analysis for different values of  $\theta_{23}$  and  $\theta_{24}$  (or, equivalently, of the projections  $|V_{\mu 1}^\nu|^2 + |V_{\mu 2}^\nu|^2$  and  $|V_{s 1}^\nu|^2 + |V_{s 2}^\nu|^2$ ). The global minimum

corresponds to almost pure atmospheric  $\nu_\mu - \nu_\tau$  oscillations and the allowed regions become considerably smaller for increasing values of the mixing angle  $\theta_{23}$ , which determines the size of the projection of  $\nu_\mu$  over the neutrino states oscillating with  $\Delta m_{43}^2$ , and for increasing values of the mixing angle  $\theta_{24}$ , which determines the active-sterile admixture in which the *almost*- $\nu_\mu$  oscillates. Fig. 42.a shows the allowed region in the parameter space ( $s_{23}^2 = |V_{\mu 1}^\nu|^2 + |V_{\mu 2}^\nu|^2$ ,  $c_{23}^2 c_{24}^2 = |V_{s 1}^\nu|^2 + |V_{s 2}^\nu|^2$ ) from this analysis. Therefore the atmospheric neutrino data give an upper bound on both mixings which further implies a lower bound on the combination  $c_{23}^2 c_{24}^2 = |V_{s 1}^\nu|^2 + |V_{s 2}^\nu|^2$ . The same combination is limited from above by the solar neutrino data. Fig. 42.b shows the shift in  $\chi^2$  for the analysis of the atmospheric data as a function of the active-sterile admixture  $|V_{s 1}^\nu|^2 + |V_{s 2}^\nu|^2 = c_{23}^2 c_{24}^2$  in the general case (in which the analysis is optimized with respect to the parameter  $s_{23}^2 = |V_{\mu 1}^\nu|^2 + |V_{\mu 2}^\nu|^2$ ) as well as the restricted case (in which  $s_{23}^2 = 0$ ). The 90% (99%) CL (for 1 d.o.f) lower bounds on  $c_{23}^2 c_{24}^2$  from the analysis of the atmospheric neutrino data are:

$$|V_{s 1}^\nu|^2 + |V_{s 2}^\nu|^2 > \begin{cases} 0.64 \text{ (0.52)} & \text{general,} \\ 0.83 \text{ (0.74)} & \text{restricted.} \end{cases} \quad (181)$$

In summary, the analysis of the solar data favours the scenario in which the solar oscillations in the 1 – 2 plane are  $\nu_e - \nu_\alpha$  oscillations, and gives an upper bound on the projection of  $\nu_s$  on this plane. On the other hand, the analysis of the atmospheric data prefers the oscillations of the 3 – 4 states to occur between a close-to-pure  $\nu_\mu$  and an active ( $\nu_\tau$ ) neutrino and gives an upper bound on the projection of the  $\nu_s$  over the 3 – 4 states or, equivalently, a lower bound on its projection over the 1 – 2 states. From Fig. 42 we see that the exclusion curves from the solar and atmospheric analyses are only marginally compatible at  $\sim 99\%$  CL.

The question is then what is the best scenario for the active-sterile admixture once these two bounds are put together. The answer is that the combined analysis still favours close-to-pure active and sterile oscillations (which gives a bad fit to either the solar or the atmospheric data) and disfavors oscillations into a near-maximal active-sterile admixture which could, in principle, lead to a compromise between the fits to the two data samples (Gonzalez-Garcia, Maltoni and Peña-Garay, 2001). This can be interpreted as a hint of incompatibility between the present solar, atmospheric and LSND results even in the context of 2+2 four-neutrino mixing although, at present, these schemes cannot be ruled out from the pure statistical point of view.

Thus the conclusion concerning the four-neutrino scenarios is that both 3+1 and 2+2 schemes seem to be equally unsatisfactory as explanations of the three experimental pieces of evidence but both are still marginally acceptable (see also Maltoni, Schwetz and Valle, 2001b).

## VIII. IMPLICATIONS OF THE NEUTRINO MASS SCALE AND FLAVOR STRUCTURE

### A. New Physics

The simplest and most straightforward lesson of the evidence for neutrino masses is also the most striking one: there is NP beyond the SM. This is the first experimental result that is inconsistent with the SM.

Most likely, the NP is related to the existence of singlet fermions at some high energy scale that induce, at low energies, the effective terms (13) through the see-saw mechanism. The existence of heavy singlet fermions is predicted by many extensions of the SM, most noticeably GUTs (beyond  $SU(5)$ ) and left-right symmetric theories.

There are of course other possibilities. One could induce neutrino masses without introducing any new fermions beyond those of the SM. This requires the existence of a scalar  $\Delta_L(1, 3)_{+1}$ , that is a triplet of  $SU(2)_L$ . The smallness of neutrino masses is then related to the smallness of the VEV  $\langle \Delta_L^0 \rangle$  (required also by the success of the  $\rho = 1$  relation) and does not have a generic natural explanation.

In left-right symmetric models, however, where the breaking of  $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$  is induced by the VEV of an  $SU(2)_R$  triplet,  $\Delta_R$ , there must exist also an  $SU(2)_L$  triplet scalar. Furthermore, the Higgs potential leads to an order of magnitude relation between the various VEVs,  $\langle \Delta_L^0 \rangle \langle \Delta_R^0 \rangle \sim v^2$  (where  $v$  is the electroweak breaking scale), and the smallness of  $\langle \Delta_L^0 \rangle$  is related to the high scale of  $SU(2)_R$  breaking. This situation can be thought of as a see-saw of VEVs. In this model there are, however, also singlet fermions. The light neutrino masses arise from both the see-saw mechanism and the triplet VEV.

Neutrino masses could also be of the Dirac type. Here, again, singlet fermions have to be introduced, but lepton number conservation needs to be imposed by hand. This possibility is disfavored by theorists since it is likely that global symmetries are violated by gravitational effects. Furthermore, the lightness of neutrinos (compared to charged fermions) is again unexplained.

Another possibility is that neutrino masses are generated by mixing with singlet fermions but the mass scale of these fermions is not high. Here again the lightness of neutrino masses remains a puzzle. The best known example of such a scenario is the framework of supersymmetry without R-parity.

Let us emphasize that the see-saw mechanism or, more generally, the extension of the SM with non-renormalizable terms, is the simplest explanation of neutrino masses. Models where neutrino masses are produced by NP at low energy imply a much more dramatic modification of the SM. Furthermore, the existence of see-saw masses is an unavoidable prediction of various extensions of the SM. In contrast, many (but not all) of the low energy mechanisms are introduced for the specific purpose of generating neutrino masses.

In this and in the next section, where we discuss the implications of the experimental data for theories beyond the SM, we choose to focus on models that explain the atmospheric and solar neutrino data through mixing among three active neutrinos. In other words, we assume three-neutrino mixing with the oscillation parameters derived in Sec. VII.A. We do not review models that try to incorporate the LSND data by adding light sterile neutrinos and we only comment on this possibility in the context of theories with new extra dimensions.

## B. The Scale of New Physics

Given the relation (14),  $m_\nu \sim v^2/\Lambda_{\text{NP}}$ , it is straightforward to use measured neutrino masses to estimate the scale of NP that is relevant to their generation. In particular, if there is no quasi-degeneracy in the neutrino masses, the heaviest of the active neutrino masses can be estimated,

$$m_h = m_3 \sim \sqrt{\Delta m_{\text{atm}}^2} \approx 0.05 \text{ eV}. \quad (182)$$

(In the case of inverted hierarchy the implied scale is  $m_h = m_2 \sim \sqrt{|\Delta m_{\text{atm}}^2|} \approx 0.05 \text{ eV}$ ). It follows that the scale in the non-renormalizable term (13) is given by

$$\Lambda_{\text{NP}} \sim v^2/m_h \approx 10^{15} \text{ GeV}. \quad (183)$$

We should clarify two points regarding Eq. (183):

1. There could be some level of degeneracy between the neutrino masses that are relevant to the atmospheric neutrino oscillations. In such a case (see Sec. VI.E), Eq. (182) is modified into a lower bound and, consequently, Eq. (183) becomes an upper bound on the scale of NP.

2. It could be that the  $Z_{ij}$  couplings of Eq. (13) are much smaller than one. In such a case, again, Eq. (183) becomes an upper bound on the scale of NP. On the other hand, in models of approximate flavor symmetries, there are relations between the structures of the charged lepton and neutrino mass matrices that give quite generically  $Z_{33} \gtrsim m_\tau^2/v^2 \sim 10^{-4}$ . We conclude that the likely range of  $\Lambda_{\text{NP}}$  that is implied by the atmospheric neutrino results is given by

$$10^{11} \text{ GeV} \lesssim \Lambda_{\text{NP}} \lesssim 10^{15} \text{ GeV}. \quad (184)$$

The estimates (183) and (184) are very exciting. First, the upper bound on the scale of NP is well below the Planck scale. This means that there is a new scale in Nature which is intermediate between the two known scales, the Planck scale  $m_{\text{Pl}} \sim 10^{19} \text{ GeV}$  and the electroweak breaking scale,  $v \sim 10^2 \text{ GeV}$ .

It is amusing to note in this regard that the solar neutrino problem does not necessarily imply such a new scale. If its solution is related to vacuum oscillations with  $\Delta m_{21}^2 \sim 10^{-10} \text{ eV}^2$ , it can be explained by  $\Lambda_{\text{NP}} \sim m_{\text{Pl}}$ . However (see Sec. IV.D.4), the favoured explanation for the solar neutrino deficit is the LMA solution which again points towards NP scale in the range of Eq. (184).

Second, the scale  $\Lambda_{\text{NP}} \sim 10^{15} \text{ GeV}$  is intriguingly close to the scale of gauge coupling unification. We will say more about this fact when we discuss GUTs.

### C. Implications for Flavor Physics

#### 1. The flavor parameters

*Flavor physics* refers to interactions that are not universal in generation space. In the Standard Model interaction basis, this is the physics of the Yukawa couplings. In the mass basis, this term refers to fermion masses and mixing parameters. There are two main reasons for the interest in flavor physics:

1. The SM has thirteen flavor parameters. These are the nine charged fermion masses or, equivalently, Yukawa couplings,  $Y_i = \frac{m_i}{v/\sqrt{2}}$ :

$$\begin{aligned} Y_t &\sim 1, & Y_c &\sim 10^{-2}, & Y_u &\sim 10^{-5}, \\ Y_b &\sim 10^{-2}, & Y_s &\sim 10^{-3}, & Y_d &\sim 10^{-4}, \\ Y_\tau &\sim 10^{-2}, & Y_\mu &\sim 10^{-3}, & Y_e &\sim 10^{-6}, \end{aligned} \quad (185)$$

and the four CKM parameters:

$$|V_{us}| \sim 0.2, \quad |V_{cb}| \sim 0.04, \quad |V_{ub}| \sim 0.004, \quad \sin \delta_{\text{KM}} \sim 1. \quad (186)$$

One can easily see that the flavor parameters are hierarchical (that is, they have very different magnitudes from each other) and all but two (the top-Yukawa and the CP violating phase) are small. The unexplained smallness and hierarchy pose *the flavor puzzle* of the SM. Its solution may direct us to physics beyond the SM.

2. The smallness of flavor changing neutral current (FCNC) processes, such as  $\Delta m_K$  and  $\mu \rightarrow e\gamma$ , is explained within the SM by the GIM mechanism. In many extensions of the SM there is, generically, no such mechanism. In some cases, experimental bounds on flavor changing processes are violated. The solution of such flavor problems leads to refinement of the models. In other cases, the model predictions are within present bounds but well above the SM range. Then we can hope to probe this NP through future measurements of flavor changing processes.

Many mechanisms have been proposed in response to either or both of these flavor aspects. For example, approximate horizontal symmetries, broken by a small parameter, can lead to selection rules that both explain the hierarchy of the Yukawa couplings and suppress flavor changing couplings in sectors of new physics.

In the extension of the SM with three active neutrinos that have Majorana-type masses, there are nine new flavor parameters in addition to those of the SM with massless neutrinos. These are three neutrino masses, three lepton mixing angles and three phases in the mixing matrix. The counting of parameters is explained in Sec. II.D. Of these, four are determined from existing measurements of solar and atmospheric neutrino fluxes: two mass-squared differences and two mixing angles. This adds significantly to the input data on flavor physics and provides an opportunity to test and refine flavor models.

If neutrino masses arise from effective terms of the form (13), then the overall scale of neutrino masses is related to the scale  $\Lambda_{\text{NP}}$  and, in most cases, does not teach us about flavor physics. The more significant information for flavor models can then be written in terms of three dimensionless parameters whose values can be read from the results of the global analysis in Eq. (166); First, the mixing angles that are relevant to atmospheric neutrinos:

$$|U_{\mu 3} U_{\tau 3}^*| \sim 0.4 - 0.5; \quad (187)$$

Second, the mixing angles that are relevant to solar neutrinos:

$$|U_{e1} U_{e2}^*| \sim 0.35 - 0.5; \quad (188)$$

Third, the ratio between the respective mass-squared differences:

$$\frac{\Delta m_{21}^2}{|\Delta m_{32}^2|} \sim \begin{cases} 10^{-1} - 10^{-3} & \text{LMA,} \\ 10^{-4} - 10^{-6} & \text{LOW.} \end{cases} \quad (189)$$

In addition, the upper bound on the third mixing angle from the CHOOZ experiments often plays a significant role in flavor model building:

$$|U_{e3}| \lesssim 0.22. \quad (190)$$



## 2. Special features of the neutrino flavor parameters

There are several features in the numerical values of these parameters that have drawn much attention and drove numerous investigations:

(i) Large mixing and strong hierarchy: The mixing angle that is relevant to the atmospheric neutrino problem,  $U_{\mu 3}$ , is large, of order one. On the other hand, the ratio of mass-squared differences  $\Delta m_{21}^2/|\Delta m_{32}^2|$  is small. If there is no degeneracy in the neutrino sector then the small ratio of mass-squared differences implies a small ratio between the masses themselves,  $m_2/m_3 \ll 1$ . (In the case of inverted hierarchy, the implied hierarchy is  $m_3/m_2 \ll 1$ .) It is difficult to explain in a natural way a situation where, in the 2 – 3 generation sector, there is large mixing but the corresponding masses are hierarchical. Below we discuss in detail the difficulties and various possible solutions.

(ii) If the LMA solution to the solar neutrino problem holds, then the data can be interpreted in a very different way. In this case, the two measured mixing angles are of order one. Moreover,  $\Delta m_{21}^2/|\Delta m_{32}^2| \sim 10^{-2}$  means that the ratio between the masses themselves (which, for fermions, are after all the fundamental parameters) is not very small,  $m_2/m_3 \sim 10^{-1}$ . Such a value could easily be an accidentally small number, without any parametric suppression. If this is the correct way of reading the data, the measured neutrino parameters may actually reflect the absence of any hierarchical structure in the neutrino mass matrices (Hall, Murayama and Weiner (2000), Haba and Murayama (2001), Berger and Siyeon (2001), Hirsch and King (2001)). Obviously, this interpretation is plausible only if the LMA solution to the solar neutrino problem holds, and will be excluded if the SMA (small mixing angle) or LOW (small mass ratio) solutions hold. Another important test of this idea will be provided by a measurement of  $|U_{e3}|$ . If indeed the entries in  $M_\nu$  have random values of the same order, all *three* mixing angles are expected to be of order one. If experiments measure  $|U_{e3}| \sim 10^{-1}$ , that is close to the present CHOOZ bound, it can be again argued that its smallness is accidental. The stronger the upper bound on this angle will become, the more difficult it will be to maintain this view.

(iii) A special case of large mixing is that of *maximal* mixing. In a two generation case, with a single mixing angle, maximal mixing is defined as  $\sin^2 2\theta = 1$ . In the three generation case, what we mean by maximal mixing is that the disappearance probability is equivalent to that for maximal two neutrino mixing at the relevant mass scale (Barger *et al.*, 1998b). Maximal atmospheric neutrino mixing corresponds to  $4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) = 1$ , which leads to

$$|U_{\mu 3}|^2 = 1/2. \quad (191)$$

Maximal solar neutrino mixing corresponds to  $4|U_{e1}U_{e2}|^2 = 1$ , which leads to

$$|U_{e1}|^2 = |U_{e2}|^2 = 1/2, \quad |U_{e3}|^2 = 0. \quad (192)$$

As seen in Sec VII, present data are consistent with atmospheric neutrino mixing being near-maximal and solar neutrino mixing being large but not maximal. The possibility that both mixings are near-maximal is, however, not entirely excluded. This scenario, where both atmospheric and solar neutrino mixing are near-maximal, means that the structure of the leptonic mixing matrix is given to a good approximation by

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (193)$$

The case of equation (193) is commonly called *bi-maximal* mixing. We would like to make the following comments, regarding maximal or bi-maximal mixing:

1. Theoretically, it is not difficult to construct models that explain near-maximal solar neutrino mixing in a natural way. We will encounter some examples below. Experimentally, it may be difficult to make a convincing case for near-maximal (rather than just order one) solar neutrino mixing (Gonzalez-Garcia, Peña-Garay, Nir and Smirnov, 2001).

2. It is highly non-trivial to construct models that explain near-maximal atmospheric neutrino mixing in a natural way. The reason is that near-maximal mixing is often related to quasi-degeneracy. So when solar neutrino data are also taken into account, approximate degeneracy among all three neutrinos is required and models of three quasi-degenerate neutrinos are, in general, not easy to construct in a natural way.

3. The case of bi-maximal mixing is also very challenging for theory. Many of the attempts in the literature involve fine-tuning. (Alternatively, the term *bi-maximal mixing* is sometimes used to denote the case of two large, rather than maximal, mixing angles.)

### 3. Large mixing and strong hierarchy

In this subsection, we focus our attention on the *large mixing – strong hierarchy* problem. We explain the challenge to theory from this feature and present various solutions that have been proposed to achieve this challenge.

A large mixing angle by itself should not be a surprise. After all, the naive guess would be that all dimensionless couplings [such as the  $Z_{ij}$  couplings of Eq. (13)] are naturally of order one and consequently all mixing angles are of order one. However, the quark mixing angles are small, and this situation has led to a prejudice that so would the lepton mixing angles be. Second, and more important, flavor models have a built-in mechanism to naturally induce smallness and hierarchy in the quark parameters (and perhaps also in charged lepton masses). For example, a mechanism that has been intensively studied in the literature is that of selection rules due to an approximate symmetry. Within such frameworks, numbers of order one are as difficult (or as easy) to account for as small numbers: one can assign charges in such a way that small flavor parameters correspond to terms in the Lagrangian that carry charge different from zero, while the order one parameters correspond to terms that carry no charge.

The combination of a large mixing angle and strong hierarchy is however a true puzzle. To understand the difficulty in this combination, let us assume that both the mixing and the hierarchy can be understood from the relevant  $2 \times 2$  block in the mass matrix for light (Majorana) neutrinos. The generic form of this block is

$$M_\nu = \frac{v^2}{\Lambda_{\text{NP}}} \begin{pmatrix} A & B \\ B & C \end{pmatrix}. \quad (194)$$

This matrix would lead to a large mixing angle if

$$|B| \gtrsim |C - A|, \quad (195)$$

and to strong mass hierarchy if (for simplicity we assume real entries):

$$|AC - B^2| \ll \max(A^2, B^2, C^2). \quad (196)$$

If we examine the two neutrino sector alone, these conditions mean fine-tuning. Hence the challenge to find models where the hierarchy is naturally induced.

To make the problem even sharper, let us explicitly discuss models with a horizontal  $U(1)_H$  symmetry (Froggatt and Nielsen, 1979). A horizontal symmetry is one where different generations may carry different charges, in contrast to the SM gauge group. We assume that the symmetry is broken by a small parameter  $\lambda$ . To be concrete, we take  $\lambda = 0.2$ , close to the value of the Cabibbo angle. One can derive selection rules by attributing charge to the breaking parameter. Our normalization is such that this charge is  $H(\lambda) = -1$ . While coefficients of order one are not determined in this framework, one can derive the parametric suppression of all couplings and consequently have an order of magnitude estimate of the physical parameters. Take, for example, the case that the  $H$ -charges of the left-handed lepton doublets  $L_i$  are positive. The parametric suppression of an entry in the neutrino mass matrix is determined by the charge that it carries,  $(M_\nu)_{ij} \sim \lambda^{H(L_i)+H(L_j)+2H(\phi)}$ . The parametric suppression of the mixing angles and mass ratios can then be estimated (see *e.g.* Leurer, Nir and Seiberg, 1993)

$$|U_{ij}| \sim \lambda^{H(L_i)-H(L_j)}, \quad m(\nu_i)/m(\nu_j) \sim \lambda^{2[H(L_i)-H(L_j)]} \quad (i \leq j). \quad (197)$$

If the various generations of left-handed fields (quarks or leptons) carry different  $H$ -charges then, in general, the (quark or lepton) mixing angles are suppressed. For example,  $\sin\theta_C \sim \lambda$  is naturally induced if the charges of the first two quark-doublet generations are chosen appropriately:  $H(Q_1) - H(Q_2) = 1$ . A mixing angle of order one can be naturally induced if the charges of the corresponding lepton-doublet fields are equal to each other. Equation (197) shows however that in this class of models, independent of the charge assignments (as long as they are all positive), we have (Grossman and Nir, 1995)

$$m(\nu_i)/m(\nu_j) \sim |U_{ij}|^2. \quad (198)$$

Hence, for order one mixing, there is no parametric suppression of the corresponding neutrino mass ratio, and no hierarchy induced.

There is another possibility to induce large lepton mixing which is unique to the case of Majorana neutrinos. Here one assigns, for example, opposite charges,  $H(L_i) = -H(L_j)$ , to the relevant lepton-doublet fields. The selection rules lead to the following structure of the mass matrix:

$$M_\nu \sim \frac{v^2}{\Lambda_{\text{NP}}} \begin{pmatrix} A\lambda^{2|H(L_i)|} & B \\ B & C\lambda^{2|H(L_i)|} \end{pmatrix}, \quad A, B, C = \mathcal{O}(1). \quad (199)$$

(In a supersymmetric framework, the combination of supersymmetry and the horizontal symmetry (Leurer *et al.*, 1993, 1994) leads to a situation where either  $A$  or  $C$  vanish.) This mass matrix has a *pseudo-Dirac* structure and it leads to near-maximal mixing and near-degeneracy of masses (see, for example, Nir (2000), Joshipura and Rindani (2000b)):

$$\sin^2 \theta = 1/2 - \mathcal{O}(\lambda^{2|H(L_i)|}), \quad \frac{\Delta m^2}{m^2} = \mathcal{O}(\lambda^{2|H(L_i)|}). \quad (200)$$

Of course, a pseudo-Dirac structure is inconsistent with mass hierarchy.

Here are a few mechanisms that have been suggested in the literature to induce strong hierarchy simultaneously with large mixing angle:

(i) Accidental hierarchy: the mass matrix (194) has  $A, B, C = \mathcal{O}(1)$  and (196) is accidentally fulfilled. In the context of Abelian horizontal symmetries, this means that the mass ratio is numerically but not parametrically suppressed (Binetruy *et al.*, 1996; Irges *et al.*, 1998; Elwood *et al.*, 1998; Vissani, 1998, 2001b; Ellis *et al.*, 1999; Sato and Yanagida, 2000). We would like to emphasize that this is not an unlikely resolution if the LMA solution of the solar neutrino problem is correct. While the oscillation experiments give the ratio of masses-squared, the relevant quantity for theories is the ratio of masses. For the LMA solution,  $m_2/m_3 \sim 0.1$  which is not a particularly small number and could easily arise from a combination of order one terms.

(ii) Several sources for neutrino masses: the leading contribution to the neutrino mass matrix, which is responsible to the order one mixing, is rank one. The lighter generation masses come from a different, sub-dominant source. This is, for example, the generic situation in supersymmetric models without R-parity, discussed in Sec. IX.C. There tree level mixing with neutralinos can lead to large mixing but gives a mass to only one neutrino generation, while the lighter masses arise at the loop level. Another realization of this principle is the scenario of a see-saw mechanism with a single right-handed neutrino (Davidson and King, 1998; King, 1998, 1999, 2000; de Gouvea and Valle, 2001). For another example, within Supersymmetric theories, see Borzumati and Nomura, 2001.

(iii) Large mixing from the charged lepton sector: it is possible that the neutrino mass matrix is hierarchical and nearly diagonal in the 2–3 sector, and the large mixing is coming from the diagonalization of the charged lepton mass matrix. A variety of models have been constructed that give this structure of mass matrices, based on discrete horizontal symmetries (Grossman *et al.*, 1998; Tanimoto, 1999b), holomorphic zeros (Grossman *et al.*, 1998) and  $U(1)_H$  with two breaking parameters (Nir and Shadmi, 1999). For example, Grossman *et al.* (1998) work in the supersymmetric framework with a horizontal  $U(1) \times U(1)$  symmetry. By an appropriate choice of horizontal charges, they obtain the following structure of lepton mass matrices (arbitrary coefficients of order one are omitted in the various entries):

$$M_\nu \sim \frac{\langle \phi_u \rangle^2}{\Lambda_{\text{NP}}} \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 0 & 0 \\ \lambda & 0 & 1 \end{pmatrix}, \quad M_{\ell^\pm} \sim \langle \phi_d \rangle \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^7 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^5 & \lambda^3 \end{pmatrix}, \quad (201)$$

where the zero entries are a consequence of holomorphy. These matrices lead to

$$\Delta m_{21}^2/|\Delta m_{32}^2| \sim \lambda^3, \quad |U_{\mu 3}| \sim 1, \quad (202)$$

where the dominant contribution to the mixing angle comes from diagonalization of  $M_{\ell^\pm}$ . The solar neutrino parameters of this model correspond to the LMA solution.

(iv) Large mixing from the see-saw mechanism: it is possible that sterile neutrinos play a significant role in the flavor parameters of the light, dominantly active neutrinos. In other

words, an effective mass matrix of the form (194) with the condition (196) can be a result of selection rules applied in the full high energy theory to a mass matrix that includes both active and sterile neutrinos and integrating out the heavy, dominantly sterile ones (Barbieri *et al.*, 1998; Altarelli and Feruglio, 1998; Eyal and Nir, 1999). It is interesting to note that in a large class of such models, the induced hierarchy is too strong for the LMA and SMA solutions, unless at least three sterile neutrinos play a role in determining the low energy parameters (Nir and Shadmi, 1999). An interesting example for a model of this type is presented by Altarelli and Feruglio (1998). They consider a horizontal  $U(1)$  symmetry broken by two parameters of opposite charges. By an appropriate choice of horizontal charges for the lepton fields, they obtain the following neutrino Dirac mass matrix  $M_D$  and Majorana mass matrix for the sterile neutrinos  $M_N$ :

$$M_D \sim \langle \phi_u \rangle \begin{pmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda' & 1 \\ \lambda & \lambda' & 1 \end{pmatrix}, \quad M_N \sim \Lambda_{\text{NP}} \begin{pmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{pmatrix}. \quad (203)$$

Integrating out of the three heavy neutrinos, the following Majorana mass matrix for the light neutrinos is obtained:

$$M_\nu \sim \frac{\langle \phi_u \rangle^2}{\Lambda_{\text{NP}}} \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & A & B \\ \lambda^2 & B & C \end{pmatrix}, \quad A, B, C = \mathcal{O}(1), \quad (204)$$

$$|AC - B^2| = \mathcal{O}(\lambda\lambda'). \quad (205)$$

If one were to apply the selection rules directly on  $M_\nu$ , the generic structure of Eq. (204) would be reproduced, but not the relation between the order one coefficients in Eq. (205).

(v) A three generation mechanism: approximate  $L_e - L_\mu - L_\tau$ . One of the more interesting frameworks that produces all the observed features of neutrino flavor parameters is intrinsically a three generation framework. One applies an approximate  $L_e - L_\mu - L_\tau$  symmetry to the mass matrices of light, active neutrinos and of charged leptons (Barbieri *et al.*, 1998, 1999; Frampton and Glashow, 1999; Joshipura and Rindani, 2000a; Mohapatra *et al.*, 2000a; Cheung and Kong, 2000; Shafi and Tavartkiladze, 2000d; Nir and Shadmi, 2000; Nir, 2000). For the most general case, the symmetry is broken by small parameters,  $\epsilon_+$  and  $\epsilon_-$ , of charges +2 and -2, respectively. The lepton mass matrices have the following form:

$$M_\nu \sim \frac{\langle \phi_u \rangle^2}{\Lambda_{\text{NP}}} \begin{pmatrix} \epsilon_- & 1 & 1 \\ 1 & \epsilon_+ & \epsilon_+ \\ 1 & \epsilon_+ & \epsilon_+ \end{pmatrix}, \quad M_\ell \sim \langle \phi_d \rangle \begin{pmatrix} \lambda_e & \lambda_\mu \epsilon_- & \lambda_\tau \epsilon_- \\ \lambda_e \epsilon_+ & \lambda_\mu & \lambda_\tau \\ \lambda_e \epsilon_+ & \lambda_\mu & \lambda_\tau \end{pmatrix}, \quad (206)$$

where the  $\lambda_i$ 's allow for generic approximate symmetry that acts on the  $SU(2)$ -singlet charged leptons. The resulting neutrino masses are as follows:

$$m_{1,2} = m (1 \pm \mathcal{O}[\max(\epsilon_+, \epsilon_-)]), \quad m_3 = m\mathcal{O}(\epsilon_+). \quad (207)$$

Note that the quasi-degenerate pair of mass eigenstates involved in the solar neutrino anomaly is heavier than the third, separated state. For the mixing angles, one finds

$$|U_{\mu 3}| = \mathcal{O}(1), \quad U_{e3} = \mathcal{O}[\max(\epsilon_+, \epsilon_-)], \quad |U_{e2}|^2 = 1/2 - \mathcal{O}[\max(\epsilon_+, \epsilon_-)]. \quad (208)$$

The overall picture is that, somewhat surprisingly, the lepton parameters (187), (188) and (189) are not easy to account for with Abelian flavor symmetries. The simplest and most predictive models have difficulties in accommodating the large 2 – 3 mixing together with the strong 2 – 3 hierarchy. One can find more complicated models that naturally induce these parameters, but often at the cost of losing predictive power. In particular, it may be the case that, specifically for neutrinos, one cannot ignore the existence of heavy degrees of freedom (sterile neutrinos), well beyond the reach of direct experimental production, that affect the flavor parameters of the low energy effective theory. If true, this situation would mean that measuring the low-energy neutrino parameters cannot by itself make a convincing case for the idea of Abelian flavor symmetries. (An exception are models of approximate  $L_e - L_\mu - L_\tau$  symmetry.)

Similar difficulties are encountered in the framework of non-Abelian symmetries. Again, the simplest models do not work and have to be extended in rather complicated ways (Barbieri *et al.*, 1999a,b). In some cases, the non-Abelian symmetries can give testable (almost) exact relations between masses and mixing angles. For example, the model of Barbieri *et al.* (1999c) predicts  $\sin \theta_{12} = \sqrt{m_e/m_\mu} \cos \theta_{23}$  and  $\sin \theta_{13} = \sqrt{m_e/m_\mu} \sin \theta_{23}$ . If it turns out that all three light neutrinos are quasi-degenerate, non-Abelian symmetries will become an unavoidable ingredient in the flavor model building, but the task of constructing realistic models will be very challenging (see, for example, Carone and Sher, 1998; Fukugita, Tanimoto and Yanagida, 1998, 1999; Fritzsch and Xing, 1996,2000; Barbieri *et al.*, 1999d; Wetterich, 1999; Tanimoto, 1999a,2000; Tanimoto, Watari and Yanagida, 1999; Wu, 1999a,b,c; Perez, 2000; Ma and Rajasekaran, 2001). Radiative corrections are an important issue when examining the naturalness of various models that account for quasi-degeneracy among the neutrinos (see, for example, Casas *et al.*, 1999, 2000).

## IX. IMPLICATIONS FOR MODELS OF NEW PHYSICS

In this section we review the implications of neutrino physics for various extensions of the SM. We do not attempt to describe all relevant models of NP, but take three representative frameworks. We choose to focus on well motivated models that were not constructed for the special purpose of explaining the neutrino parameters. Thus, the neutrino parameters either test these frameworks or provide further guidance in distinguishing among different options for model building within each framework. The three frameworks are the following:

(i) GUTs: here the overall scale of the neutrino masses and some features of the mixing provide interesting tests.

(ii) Large extra dimensions: in the absence of a high energy scale, the see-saw mechanism is not operative in this framework. The lightness of the neutrinos is a challenge.

(iii) Supersymmetry without  $R$  parity: here there are many new sources of neutrino masses. Neutrino masses are both a non-trivial challenge and an excellent guide for model building.

### A. Grand Unified Theories

There are two significant facts about the gauge symmetries of the Standard Model and the structure of its fermionic representations that motivate the idea of supersymmetric grand

unification. First, GUTs provide a unification of the SM multiplets (for a recent review, see Wilczek, 2001). Second, in the framework of the supersymmetric Standard Model, the three gauge couplings unify (Dimopoulos, Raby and Wilczek, 1981; Ibañez and Ross, 1981). The unification scale is given by (see, for example, Langacker and Polonsky, 1995)

$$\Lambda_{\text{GU}} \sim 3 \times 10^{16} \text{ GeV}. \quad (209)$$

Further support for grand unification comes from the flavor sector: the masses of the bottom quark and the tau lepton are consistent with equal masses at  $\Lambda_{\text{GU}}$ , as predicted by SU(5) and its extensions.

The evidence for neutrino masses from atmospheric neutrinos, the implied scale of  $\Delta m_{\text{atm}}^2$  and the required mixing of order one should be considered as three further triumphs of the grand unification idea.

First, as mentioned above, SO(10) theories and their extensions require that there exist singlet fermions. Neutrino masses are then a *prediction* of these theories.

Second, SO(10) theories naturally give the singlet fermions heavy masses at the SO(10) breaking scale and, furthermore, relate the neutrino Dirac mass matrix  $M_\nu^D$  to the up quark mass matrix  $M_u$ . Specifically, the naive SO(10) relation reads  $M_\nu^D = M_u$ . It follows that the mass of the heaviest among the three light neutrinos can be estimated:

$$m_3 \sim m_t^2 / \Lambda_{\text{SO}(10)} \sim 10^{-3} \text{ eV}. \quad (210)$$

It only takes that the lightest of the three singlet fermion masses is a factor of fifty below the unification scale to induce neutrino masses at the scale that is appropriate for the atmospheric neutrino anomaly. In other words, the intriguing proximity of  $\Lambda_{\text{NP}}$  of Eq. (183) to  $\Lambda_{\text{GU}}$  of Eq. (209) finds a natural explanation in the framework of SO(10). In this sense the see-saw mechanism of Gell-Mann, Ramond and Slansky (1979), and of Yanagida (1979), predicted neutrino masses at a scale that is relevant to atmospheric neutrinos.

The third triumph,  $|U_{\mu 3}| = \mathcal{O}(1)$ , is more subtle but still quite impressive (Harvey, Reiss and Ramond, 1982; Babu and Barr, 1996; Sato and Yanagida, 1998). Unlike the previous points that were related to SO(10) GUT, here the consistency relates to SU(5) GUT. SU(5) theories relate the charged lepton mass matrix  $M_\ell$  to the down quark mass matrix  $M_d$ . Specifically, the naive SU(5) relation reads  $M_\ell = M_d^T$ . It follows that

$$|U_{\mu 3} V_{cb}| \sim m_s / m_b. \quad (211)$$

Given the experimental values  $|V_{cb}| \sim 0.04$  and  $m_s / m_b \sim 0.03$  we conclude that the naive SU(5) relations predict  $|U_{\mu 3}| \sim 1$ . Of course,  $|U_{\mu 3}|$  is also affected by the diagonalization of the light neutrino mass matrix, but there is no reason to assume that this contribution cancels against the charged lepton sector in such a way that  $|U_{\mu 3}| \ll 1$ .

Many of the other naive SO(10) relations fail, as do many of the naive SU(5) relations. Specifically, SO(10) predicts vanishing CKM mixing and mass ratios such as  $m_c / m_t = m_s / m_b$ . SU(5) predicts  $m_s = m_\mu$  and  $m_d = m_e$ . It is possible however that these bad predictions are corrected by subleading effects while all the successful predictions (particularly  $m_b = m_\tau$ ,  $m_\mu / m_\tau \sim m_s / m_b$  and  $|U_{\mu 3} V_{cb}| \sim m_s / m_b$ ) are retained since they depend on the leading contributions. This is demonstrated in a number of specific GUT models (Albright, Babu and Barr, 1998; Albright and Barr, 1998, 1999a, 1999b, 2000a,

2000b, 2001; Altarelli and Feruglio, 1998, 1999a, 1999b; Altarelli, Feruglio and Masina, 2000a, 2000b; Berezhiani and Rossi, 1999, 2001; Hagiwara and Okamura, 1999; Babu, Pati and Wilczek, 2000; Shafi and Tavartkiladze, 1999, 2000a, 2000b, 2000c; Maekawa, 2001).

The flavor structure of the first two neutrino generations depends on both the Majorana mass matrix for the singlet fermions and the subdominant effects that correct the flavor parameters of the first two generation quarks and charged leptons. This part of GUTs is therefore much more model dependent. Explicit GUT models were constructed that accommodate the various solutions of the solar neutrino problem. We will not describe them in any detail here.

## B. Extra Dimensions

New ideas concerning the possibility of large additional dimensions and the world on a brane can lead to sources of neutrino masses that are very different from the ones that we discussed so far. For example, the small mass could be related to the large volume factor of the extra dimensions which suppresses the coupling to bulk fermions (Arkani-Hamed *et al.*, 2002; Dienes *et al.*, 1999; Mohapatra *et al.*, 1999; Mohapatra and Perez-Lorenzana, 2000; Lukas *et al.*, 2000, 2001; Dienes and Sarcevic, 2001; Mohapatra *et al.*, 2000b), to the breaking of lepton number on a distant brane (Arkani-Hamed *et al.*, 2002), or to the warp factor in the Randall-Sundrum framework (Grossman and Neubert, 2000). In this section, we briefly describe these three mechanisms.

The existence of large extra dimensions does not only provide new ways of generating small neutrino masses but can also lead to interesting phenomenological consequences. In particular, the phenomenology of matter oscillations in the Sun can be affected (Dvali and Smirnov, 1999; Barbieri *et al.*, 2000; Lukas *et al.*, 2000; Ma *et al.*, 2000; Caldwell *et al.*, 2001a, 2001b). Other phenomenological implications can be used to constrain the parameters of the extra dimensions (Faraggi and Pospelov, 1999; Ioannisian and Pilaftsis, 2000; Das and Kong, 1999).

### 1. Coupling to bulk fermions

It is possible that we live on a brane with three spatial dimensions that is embedded in a spacetime with  $n$  additional large spatial dimensions (Arkani-Hamed *et al.*, 1998). This idea has the potential of providing an understanding of the hierarchy between the gravitational mass scale,  $M_{\text{Pl}}$ , and the electroweak scale,  $m_Z$ . The hope is to solve the hierarchy problem by avoiding a fundamental high energy (that is Planck or GUT) scale altogether. The observed Planck scale,  $M_{\text{Pl}} = (G_N)^{-1/2} \sim 10^{19}$  GeV, is related to the fundamental Planck scale (most likely the string scale) of the  $4 + n$  dimensional theory,  $M_*$ , by Gauss law:

$$M_{\text{Pl}}^2 = M_*^{n+2} V_n, \quad (212)$$

where  $V_n$  is the volume of the  $n$ -dimensional extra space. This picture has dramatic phenomenological consequences for particle physics and cosmology. Such a situation also poses an obvious problem for neutrino physics. If there is no scale of physics as high as (184), the see-saw mechanism for suppressing the light neutrino masses cannot be implemented.



Conversely, if there are singlet neutrinos that are confined to the three dimensional brane where the active neutrinos live, one expects that their mass is at the string scale,  $M_*$ , which in these models is much smaller than the four dimensional  $M_{\text{Pl}}$ , perhaps as small as a few TeV, and the resulting light neutrino masses are well above the scales of atmospheric or solar neutrinos.

The first implication within this framework of the evidence for neutrino masses is that there better be no singlet fermions confined to the brane. Alternatively, the model must include some special ingredients to avoid ordinary see-saw masses.

On the other hand, it is typical in this framework that there are singlet fermions in the bulk. This would be the case, for example, with modulinos, the fermionic partners of the moduli fields that are generic to string theories. The crucial point is that the Yukawa interaction between bulk singlet fermions and brane active neutrinos is highly suppressed by a large volume factor of the  $n$  extra dimensions,  $1/\sqrt{V_n}$ . This suppression factor reflects the small overlap between the wave functions of the sterile neutrino in the bulk and the active one on the brane. By construction, this factor provides a suppression of the neutrino Dirac mass by the ratio  $v/M_{\text{Pl}}$ . Consequently, large extra dimensions provide a natural source of very small Dirac mass terms for the active neutrinos.

The consequences for neutrino masses and mixing depend on the details of the physics of the bulk neutrinos. The possible scenarios are clearly described by Lukas *et al.* (2000). There an explicit derivation of the effective four dimensional action from a five dimensional one is given. The final result for the Dirac mass is as follows:

$$m_i^{\text{Dir}} = \frac{vY_i}{\sqrt{V_n M_*^n}} = Y_i v \frac{M_*}{M_{\text{Pl}}}. \quad (213)$$

Here  $Y_i$  is the Yukawa coupling between the lepton doublet  $L_i$  and a bulk singlet  $M_*$  is the string scale and  $V_n$  is the volume of the  $n$ -dimensional extra space. A usual Dirac mass is of order  $Yv$ , but in Eq. (213) we see explicitly the  $(M_*/M_{\text{Pl}})$  suppression factor in the effective four-dimensional Yukawa couplings, leading to highly suppressed Dirac mass terms.

Being a bulk state, the singlet fermion has a whole tower of Kaluza-Klein (KK) associated states which are all coupled to the left-handed brane neutrinos. In the simplest scenario and for one extra dimension of radius  $R$ , the masses of all KK states are determined by the scale  $1/R$ :

$$m_n^2 = \frac{n^2}{R^2}. \quad (214)$$

In more general scenarios there can be other bulk mass terms and the masses of the KK states receive additional contributions. Then the lightest KK mass can be taken as an independent parameter while the mass splitting between the states is still determined by the scale  $1/R$ .

Let us denote by  $M_{\text{min}}$  the lightest mass in the Kaluza-Klein spectrum and by  $\Delta m$  the mass scale that is relevant to atmospheric or to solar neutrino oscillations. Then one can distinguish three cases:

1.  $1/R \gg \Delta m$  and  $M_{\text{min}} \gg \Delta m$ . The Kaluza-Klein states play no direct role in these oscillations. Their main effect is to give see-saw masses to the active brane neutrinos.

2.  $1/R \gg \Delta m$  and  $M_{\min} \lesssim \Delta m$ . The situation is equivalent to conventional models containing a small number of sterile states. In particular, for  $M_{\min} = 0$  it is a possible framework for light Dirac neutrinos. For small but non-vanishing  $M_{\min}$  it may lead to the four-neutrino mixing scenarios discussed in Sec. VII.B. However, as argued there, these scenarios are not particularly favoured.

3.  $1/R \lesssim \Delta m$  and  $M_{\min} \lesssim \Delta m$ . A large number of bulk modes can take direct part in the oscillation phenomena. This situation modifies in a very interesting way the solution to the solar neutrino problem that involves MSW resonance conversion into a sterile neutrino. Now the  $\nu_e$  can oscillate into a whole set of Kaluza-Klein states. However, as discussed in Sec. IV, solar oscillations into sterile states are now strongly disfavoured by the comparison of the SNO and SK data.

In summary, large extra dimensions with sterile bulk fermions provide a natural explanation for light neutrino masses. In the simplest realization, in which there are no bulk mass terms and lepton number conservation is imposed in the bulk-brane couplings, light Dirac masses are generated. In the general case, the light neutrinos can be either Dirac or Majorana particles, depending on the details of the physics of the bulk neutrinos.

## 2. Lepton number breaking on a distant brane

In its simplest realization, the mechanism described above provides a natural way of generating light Dirac masses for the neutrinos. In the framework of extra dimensions one could also generate small Majorana masses by an alternative mechanism in which lepton number is broken on a distant brane (Arkani-Hamed *et al.*, 2002). Imagine that lepton number is spontaneously broken at the scale  $M_*$  on a different brane located a distance  $r$  from our brane. Further assume that the information about this breaking is communicated to our brane by a bulk field with a mass  $m$ . For the sake of concreteness we take the case that  $mr \gg 1$  and there are two extra dimensions. The resulting Majorana mass for the active neutrinos can be estimated as follows:

$$m_\nu \sim \frac{v^2}{M_*} e^{-mr}. \quad (215)$$

We learn that the naive see-saw mass, of order  $v^2/M_*$ , is suppressed by a small exponential factor. The consequences for the neutrino spectrum depend on various model dependent features: the number of large extra dimensions, the string scale, the distance between our brane and the brane where lepton number is broken, and the mass of the mediating bulk field.

A variant of the above two mechanisms can be implemented if the SM fields are confined to a thick wall (Arkani-Hamed and Schmaltz, 2000). Dirac masses are suppressed if left-handed and right-handed neutrinos are located at different points in the wall and consequently there is an exponentially small overlap of their wave functions. Majorana masses are suppressed if lepton number is spontaneously broken by a VEV that is localized within the wall but at some distance from the lepton doublet fields.

### 3. The warp factor in the Randall-Sundrum [RS] scenario

A different set-up for the extra dimensions (leading to a different explanation of the  $v/M_{\text{Pl}}$  hierarchy) was proposed by Randall and Sundrum (1999) [RS]. They considered one extra dimension parameterized by a coordinate  $y = r_c\phi$ , where  $-\pi \leq \phi \leq +\pi$ .  $r_c$  is the radius of the compact dimension, and the points  $(x, \phi)$  and  $(x, -\phi)$  are identified. A *visible* brane is located at  $\phi = \pi$  and a *hidden* one at  $\phi = 0$ . This set-up leads to the following non-factorizable metric:

$$ds^2 = e^{-2kr_c|\phi|} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2. \quad (216)$$

The parameter  $k$  is the bulk curvature. All dimensionful parameters in the effective theory on the visible brane are suppressed by the warp factor,  $\epsilon \equiv e^{-kr_c\pi}$ . With  $kr_c \approx 12$ , that is  $\epsilon \sim 10^{-16}$ , this mechanism produces physical masses of order  $v$  from fundamental masses of order  $M_{\text{Pl}}$ .

The two mechanisms described above to generate small neutrino masses do not work in this scenario because the extra dimensions are small and there is no volume suppression factor available. Grossman and Neubert (1999) proposed a different mechanism to generate small masses in the RS framework. (For a different mechanism, see Huber and Shafi, 2001.) With appropriate choice of orbifold boundary conditions, it is possible to locate the zero mode of a right-handed bulk neutrino on the hidden brane. If the fundamental mass scale  $m$  of the bulk fermions is larger than half the curvature  $k$  of the compact dimension, the wave function of the right-handed zero mode on the visible brane is power-suppressed in the ratio  $v/M_{\text{Pl}}$ . Coupling the bulk fermions to the Higgs and lepton doublet fields yields

$$m_\nu \sim v \left( \frac{v}{M_{\text{Pl}}} \right)^{\frac{m}{k} - \frac{1}{2}}. \quad (217)$$

Note that generically the relation between the neutrino mass and the weak scale is different from the see-saw mechanism (except for the special case  $m/k = 3/2$ ).

To summarize, in the presence of large extra dimensions, neutrino masses could be suppressed by the large volume factor if the left-handed neutrinos couple to the bulk fermions (being Dirac for massless bulk fermions), or by the distance to a brane where lepton number is broken (which generate Majorana masses). In the RS framework, the suppression can be induced by a power of the warp factor. The detailed structure of neutrino mass hierarchy and mixing is often related to the parameters that describe the extra dimensions.

### C. Supersymmetry without $R$ -parity

Supersymmetry is a symmetry that relates fermions and bosons. It is useful in solving the fine-tuning problem of the Standard Model (that is the stability of the small ratio between the electroweak breaking scale and the Planck scale against radiative corrections) and has many other virtues. Extending the Standard Model by imposing Supersymmetry requires that the particle content is also extended. In particular, one must add scalar superpartners to the three generations of quarks and leptons. (The supersymmetric multiplet that contains fermions and bosons with the same gauge quantum number is called a superfield.) These modifications make a significant difference as far as the accidental symmetries

of the Standard Model, Eq. (12), are concerned. One has the option of restoring baryon number conservation and total lepton number conservation by imposing a discrete symmetry,  $R_p = (-1)^{3B+L+2S}$ . Models where this symmetry is not imposed, and the accidental global symmetries of the Standard Model are all violated (or, at least, either baryon number or lepton number is not conserved) are therefore called models of *Supersymmetry without R-parity*.

Supersymmetry without  $R$ -parity provides a very rich phenomenology. In general, this framework has baryon number violation, lepton number violation, and many new sources of flavor changing and CP violating interactions. Since total lepton number and lepton flavor symmetries are violated, it is not surprising that this framework is relevant to the question of neutrino masses and mixing.

In some sense, the smallness of neutrino masses poses a problem for Supersymmetry without  $R_p$ . If dimensionful  $R_p$  violating (RPV) couplings were at the electroweak breaking scale, and if dimensionless  $R_p$  violating couplings were of order one, then one neutrino mass would be at the electroweak breaking scale, and the other two suppressed by a loop factor and bottom and tau Yukawa couplings. Thus the three masses would be well above the experimental bounds. This is part of the motivation for considering supersymmetry *with*  $R_p$ . (Proton decay and neutron-antineutron oscillations provide another source of motivation for  $R_p$ .) In other words, even if  $R_p$  is not an exact symmetry of Nature, it better be an approximate one. Various theories have been considered that lead to suppression of  $R_p$  violating couplings. The question of neutrino masses is usually considered in the framework of such special theories. (In particular, one usually assumes that baryon number is a good symmetry of the theory. While this assumption has no direct consequences for our discussion below, it implies that there is no need to consider constraints from proton decay on the lepton number violating couplings.)

One way to think about lepton number violation in the context of supersymmetry without  $R_p$  is to notice that in such a framework, there is no quantum number to distinguish between the down-Higgs and the lepton-doublet superfields. Consequently, there are four *flavors* of the  $(1, 2)_{-1/2}$  representation. It is convenient then to denote the four doublets by  $L_\alpha$ , where  $\alpha = 0, 1, 2, 3$ . There are bilinear and trilinear couplings that lead to lepton number violation. In the superpotential, we have the bilinear  $\mu$ -terms, and the trilinear  $\lambda$  and  $\lambda'$  terms:

$$W_{\mathcal{R}_p} = \mu_\alpha L_\alpha \phi_u + \lambda_{\alpha\beta k} L_\alpha L_\beta \bar{\ell}_k + \lambda'_{\alpha j k} L_\alpha Q_j \bar{d}_k. \quad (218)$$

Here  $\phi_u$  is the  $Y = +1/2$  Higgs superfield,  $\ell_k$  ( $k = 1, 2, 3$ ) are the  $SU(2)_L$  singlet charged lepton superfields, and  $Q_k$  ( $d_k$ ) are the  $SU(2)_L$  doublet (down singlet) quark superfields. Among the soft supersymmetry breaking terms in the scalar potential, we have the bilinear  $B$  terms and  $m^2$  terms and trilinear  $A$  terms:

$$V_{\mathcal{R}_p}^{\text{soft}} = B_\alpha L_\alpha \phi_u + m_{\alpha\beta}^2 L_\alpha L_\beta^\dagger + A_{\alpha\beta k} L_\alpha L_\beta \bar{\ell}_k + A'_{\alpha j k} L_\alpha Q_j \bar{d}_k + \text{h.c.}, \quad (219)$$

where we use the same notation for superfields in Eq. (218) and for their scalar components in Eq. (219).

There are many ways to define what one means by *the down Higgs field*, even when  $R_p$  is an approximate symmetry. These ways represent different choices of basis in the four-flavor space of  $L_\alpha$ . Let us denote the Higgs fields by  $L_0$  and the lepton fields by  $L_i$  ( $i = 1, 2, 3$ ). Then, for example, one can define  $L_0$  by choosing a basis where  $\mu_i = 0$ , or

by choosing a basis where  $\langle L_i \rangle = 0$  (vanishing sneutrino VEVs), or by choosing  $L_i$  to have the three lightest charged mass eigenstates. Physical results are of course independent of the choice of basis, but there is much confusion in the literature resulting from the use of different conventions. For a clarification of this subject and a very useful basis-independent formulation, see Davidson and Losada, 2000.

The most striking feature of neutrino masses in the context of RPV supersymmetry is that there are many different sources for them. Consequently, the hierarchy in neutrino masses can be the consequence of the interplay between these different sources. Relations between mixing and mass hierarchies that often apply when there is a single source of masses are easily modified in this framework. Below is a list of some of the contributions to neutrino masses that appear in a generic RPV supersymmetry. The estimates of the sizes of the various contributions is given in the convention that  $\langle L_i \rangle = 0$  and with the simplifying assumption that there is a single scale  $m_{\text{susy}}$  that characterizes all supersymmetry breaking and the  $\mu$  term.

1. Tree level contribution:

$$(M_\nu)_{ij} \sim \frac{\mu_i \mu_j}{m_{\text{susy}}}. \quad (220)$$

2. Sneutrino-neutralino loops:

$$(M_\nu)_{ij} \sim \frac{g^2}{64\pi^2 \cos^2 \beta} \frac{B_i B_j}{m_{\text{susy}}^3}. \quad (221)$$

3. Slepton-lepton loops:

$$(M_\nu)_{ij} \sim \frac{1}{8\pi^2} \lambda_{ink} \lambda_{jkn} \frac{m_{\ell_n} m_{\ell_k}}{m_{\text{susy}}}. \quad (222)$$

4. Squark-quark loops:

$$(M_\nu)_{ij} \sim \frac{3}{8\pi^2} \lambda'_{ink} \lambda'_{jkn} \frac{m_{d_n} m_{d_k}}{m_{\text{susy}}}. \quad (223)$$

There are many additional loop diagrams that involve various two insertions of RPV couplings. These could be two bilinear couplings, as in Eq. (221), two trilinear couplings, as in Eqs. (222) and (223), or one bilinear and one trilinear coupling. For a complete list, see Davidson and Losada, 2000.

A few comments are in order:

(i) The relative importance of the various contributions is model dependent.

(ii) The tree contribution vanishes if the following two conditions are fulfilled (Hall and Suzuki, 1984; Banks *et al.*, 1995):

$$B_\alpha \propto \mu_\alpha, \quad m_{\alpha\beta}^2 \mu_\beta = \tilde{m}^2 \mu_\alpha. \quad (224)$$

(iii) Even in case that (224) holds at a high energy scale, misalignment between the  $B$  and  $\mu$  terms will be induced by RGE, with the likely consequence that the tree contribution is dominant at low energy (Nardi, 1997).

(iv) The tree contribution gives mass to a single neutrino state. Thus at least two neutrino masses are induced by loop corrections.

(v) If there is a single small parameter that suppresses the leading RPV couplings, that is  $\mu_3/\mu_0 \sim B_3/B_0 \sim \lambda_{3jk}/\lambda_{0jk} \sim \lambda'_{3jk}/\lambda'_{0jk}$  (this is the typical case with an abelian flavor symmetry), then the sneutrino related loop of Eq. (221) dominates over the other loop contributions (Grossman and Haber, 1999).

There are many ways in which the lepton parameters deduced from atmospheric and solar neutrino measurements can be accommodated in the framework of RPV supersymmetry. The implications of the atmospheric neutrino results for the tree contributions were discussed by Bednyakov, Faessler and Kovalenko, 1998; Chun and Lee, 1999; Diaz *et al.*, 2000; Datta, Mukhopadhyaya and Vissani, 2000; Joshipura, Vaidya and Vempati, 2000; Suematsu, 2001. Implications, in addition, of solar neutrino results for the  $\lambda$  and  $\lambda'$  dependent loops were investigated by Chun *et al.*, 1999; Joshipura and Vempati, 1999a, 1999b; Mukhopadhyaya, Roy and Vissani, 1998; Choi *et al.*, 1999; Bhattacharyya, Klapdor-Kleingrothaus and Pas, 1999; Kong, 1999; Datta, Mukhopadhyaya and Roy, 2000; Takayama and Yamaguchi, 2000; Mira *et al.*, 2000. Contributions from misalignment of bilinear terms at tree and loop levels was analyzed by Grossman and Haber, 1999, 2001; Hambye, Ma and Sarkar, 2000; Romao *et al.*, 2000; Hirsch *et al.*, 2000. Contributions from trilinear terms only were considered in Rakshit, Bhattacharyya and Raychaudhuri, 1999; Drees *et al.*, 1998. Combinations of bilinear and trilinear couplings in the loop contributions were pointed out by Choi, Hwang and Chun, 1999; Kaplan and Nelson, 2000. Model independent descriptions of all contributions were given by Davidson and Losada, 2000; Abada and Losada, 2000a, 2000b; Choi, Chun and Hwang, 2000; Kong, 2000.

Since there are many independent RPV couplings, it is easy to make an ansatz that would be consistent with the data. A more interesting question is whether one can start from some theoretical guiding principle, such as an approximate flavor symmetry, and then the required mass hierarchy and mixing angles would arise naturally (for an early attempt, see, for example, Borzumati *et al.*, 1996). Some of the more interesting attempts in this direction are the following:

1. When  $\mu$  and  $B$  are misaligned, there are contributions related to bilinear couplings at both tree and loop level. The ratio between these contributions is roughly  $\frac{g^2}{64\pi^2} \sim 10^{-3}$ . This is a little (but certainly not far) below the hierarchy that is appropriate for  $(\Delta m_{21}^2/|\Delta m_{32}^2|)^{1/2}$  (see, for example, Grossman and Haber, 1999).

2. When  $\mu$  and  $B$  are aligned at a high scale, the ratio between the loop contribution from trilinear couplings and the tree contribution from RGE-induced misalignment is generally of the appropriate hierarchy (see, for example, Nardi, 1997).

3. If only trilinear couplings are significant, then the ratio between the leading contributions is likely to be of order  $m_\tau^2/3m_b^2$  or  $m_s/m_b$ , which is just the required hierarchy (see, for example, Drees *et al.*, 1998).

We conclude that future neutrino data will provide further guidance into model building in the framework of Supersymmetry without  $R$ -parity.

## X. CONCLUSIONS

Strong evidence for neutrino masses and mixing has been arising from various neutrino oscillation experiments in recent years. First, atmospheric neutrinos show deviation from the expected ratio between the  $\nu_\mu$ - and  $\nu_e$ -fluxes. Furthermore, the  $\nu_\mu$  flux has strong zenith angle dependence. The simplest interpretation of these results is that there are  $\nu_\mu - \nu_\tau$  oscillations with the following mass and mixing parameters:

$$1.9 \times 10^{-3} \text{ eV}^2 < \Delta m_{\text{atm}}^2 < 6 \times 10^{-3} \text{ eV}^2, \quad 0.4 < \tan^2 \theta_{\text{atm}} < 3.0. \quad (225)$$

The ranges quoted here correspond to the results of the global three-neutrino analysis presented in Sec. VII.A [Eq. (166)]. Second, the total rates of solar neutrino fluxes are smaller than the theoretical expectations. Furthermore, the suppression is different in the various experiments (which are sensitive to different energy ranges). The simplest interpretation of these results is that there are  $\nu_e - \nu_a$  oscillations (where  $\nu_a$  is some combination of  $\nu_\mu$  and  $\nu_\tau$ ) with one of the following sets of mass and mixing parameters:

$$\begin{aligned} 1.9 \times 10^{-5} \text{ eV}^2 < \Delta m_{\odot}^2 < 2.7 \times 10^{-4} \text{ eV}^2, & \quad 0.22 < \tan^2 \theta_{\odot} < 0.71 \quad (\text{LMA}), \\ 4.3 \times 10^{-8} \text{ eV}^2 < \Delta m_{\odot}^2 < 1.8 \times 10^{-7} \text{ eV}^2, & \quad 0.47 < \tan^2 \theta_{\odot} < 1.1 \quad (\text{LOW}), \end{aligned} \quad (226)$$

where again the ranges quoted correspond to the results of the global analysis given in Eq. (166). From the global analysis an upper bound on a third mixing angle arises driven mainly by the negative results of the reactor experiments in combination with the deduced  $\Delta m_{\text{atm}}^2$  from the atmospheric neutrino data:

$$\sin \theta_{\text{reactor}} \leq 0.22. \quad (227)$$

The smallness of this angle guarantees that the results of the three neutrino analysis combining the atmospheric and solar neutrino data are close to the two separate two neutrino analyses.

The evidence for neutrino masses implies that the SM cannot be a complete picture of Nature. In particular, if the SM is only a low energy effective theory, very light neutrino masses are expected. The scale at which the SM picture is not valid anymore is inversely proportional to the scale of neutrino masses. Specifically

$$m_\nu \gtrsim \sqrt{\Delta m_{\text{atm}}^2} \sim 0.05 \text{ eV} \implies \Lambda_{\text{NP}} \lesssim 10^{15} \text{ GeV}. \quad (228)$$

We learn that there is a scale of NP well below the Planck scale.

The scale of NP in Eq. (228) is intriguingly close to the scale of coupling unification. Indeed, since GUTs with an SO(10) (or larger) gauge group predict neutrino masses, plausibly at the 0.1 eV scale, the atmospheric neutrino data can be taken as another piece of support to the GUT idea. The large mixing angle in (225) also finds a natural and quite generic place in GUTs.

The measured values of the neutrino flavor parameters are useful in testing various ideas to explain the flavor puzzle. Quite a few of the simplest models that explain the smallness and hierarchy in the quark sector parameters fail to explain the neutrino parameters. The neutrino parameters have some features that are quite unique. In particular, the two mixing

angles  $\theta_{\text{atm}}$  and  $\theta_{\odot}$  are large. As concerns the mass hierarchy,  $\Delta m_{\odot}^2/\Delta m_{\text{atm}}^2$ , the situation is still ambiguous. If there is strong hierarchy, it is not easy to accommodate it together with large mixing angles. If the hierarchy is mild enough to be consistent with just accidental suppression, and if, in addition, the third mixing angle (bounded by reactor experiments) is not very small, it could well be that there is no hierarchy in the neutrino parameters at all. That would call for flavor frameworks that give a different structure for charged and neutral fermion parameters. Other possibilities, such as quasi-degeneracy in masses or bi-maximal mixing also call for special structure in the neutrino sector that is very different from the quark sector.

The mass scales involved in Eqs. (225) and (226) have implications to many other frameworks of NP. In particular, they can help discriminate between various options in the framework of models with extra dimensions, models of supersymmetry without  $R$  parity, left-right symmetric models, etc.

Another hint for neutrino masses comes from the LSND experiment. Here, however, the signal is presently observed by a single experiment, and further experimental testing is required. The simplest interpretation of the LSND data is that there are  $\nu_e \rightarrow \nu_{\mu}$  oscillations with  $\Delta m_{\text{LSND}}^2 = \mathcal{O}(1 \text{ eV}^2)$  and  $\sin^2 2\theta_{\text{LSND}} = \mathcal{O}(0.003)$ . The fact that  $\Delta m_{\text{LSND}}^2 \gg \Delta m_{\text{atm}}^2, \Delta m_{\odot}^2$  means that the three results cannot be explained with oscillations among the three active neutrinos alone.

If the LSND result is confirmed, then more dramatic modifications of the SM will be required. The simplest extension, that is, the addition of a light singlet neutrino, is not excluded but it does run into difficulties related to the fact that oscillations into purely sterile neutrinos fit neither the atmospheric nor the solar neutrino data.

The good news is that there has been a lot of progress in neutrino physics in recent years. Measurements of both atmospheric and solar neutrino fluxes make the case for neutrino masses and mixing more and more convincing. Many theoretical ideas are being excluded, while other have at last experimental guidance in constructing them. The other piece of good news is that there is a lot of additional experimental information concerning neutrino physics to come in the near future. We are guaranteed to learn much more.

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TABLES

TABLE I. Characteristic values of  $L$  and  $E$  for various neutrino sources and experiments.

Experiment	$L$ (m)	$E$ (MeV)	$\Delta m^2$ (eV <sup>2</sup> )
Solar	$10^{10}$	1	$10^{-10}$
Atmospheric	$10^4 - 10^7$	$10^2 - 10^5$	$10^{-1} - 10^{-4}$
Reactor	$10^2 - 10^3$	1	$10^{-2} - 10^{-3}$
Accelerator	$10^2$	$10^3 - 10^4$	$\gtrsim 0.1$
LBL Accelerator	$10^5 - 10^6$	$10^4$	$10^{-2} - 10^{-3}$

TABLE II. Effective potentials for  $\nu_e$  (upper sign) and  $\bar{\nu}_e$  (lower sign) in various media.

medium	$V_C$	$V_N$
$e^+$ and $e^-$	$\pm\sqrt{2}G_F(N_e - N_{\bar{e}})$	$\mp\frac{G_F}{\sqrt{2}}(N_e - N_{\bar{e}})(1 - 4\sin^2\theta_W)$
$p$ and $\bar{p}$	0	$\pm\frac{G_F}{\sqrt{2}}(N_p - N_{\bar{p}})(1 - 4\sin^2\theta_W)$
$n$ and $\bar{n}$	0	$\mp\frac{G_F}{\sqrt{2}}(N_n - N_{\bar{n}})$
Neutral ( $N_e = N_p$ )	$\pm\sqrt{2}G_F N_e$	$\mp\frac{G_F}{\sqrt{2}} N_n$

TABLE III. SSM predictions: solar neutrino fluxes and neutrino capture rates for the different experiments, with  $1\sigma$  uncertainties. The neutrino fluxes are the same as in the original BP00 model except for the  $^8\text{B}$  flux, which is increased because of the larger adopted value of  $S_{17}(0)$  (see Bahcall, Gonzalez-Garcia and Peña-Garay, 2001b).

Source	Flux ( $10^{10} \text{ cm}^{-2}\text{s}^{-1}$ )	Cl (SNU)	Ga (SNU)	SK ( $10^6 \text{ cm}^{-2}\text{s}^{-1}$ )	SNO(CC) ( $10^6 \text{ cm}^{-2}\text{s}^{-1}$ )
pp	$5.95 \left(1.00^{+0.01}_{-0.01}\right)$	0.0	69.7	0.0	0.0
pep	$1.40 \times 10^{-2} \left(1.00^{+0.015}_{-0.015}\right)$	0.22	2.8	0.0	0.0
hep	$9.3 \times 10^{-7}$	0.04	0.1	0.0093	0.0093
$^7\text{Be}$	$4.77 \times 10^{-1} \left(1.00^{+0.10}_{-0.10}\right)$	1.15	34.2	0.0	0.0
$^8\text{B}$	$5.93 \times 10^{-4} \left(1.00^{+0.14}_{-0.15}\right)$	6.76	14.2	5.93	5.93
$^{13}\text{N}$	$5.48 \times 10^{-2} \left(1.00^{+0.21}_{-0.17}\right)$	0.09	3.4	0.0	0.0
$^{15}\text{O}$	$4.80 \times 10^{-2} \left(1.00^{+0.25}_{-0.19}\right)$	0.33	5.5	0.0	0.0
$^{17}\text{F}$	$5.63 \times 10^{-4} \left(1.00^{+0.25}_{-0.25}\right)$	0.0	0.1	0.0	0.0
Total		$8.6^{+1.1}_{-1.2}$	$130^{+9}_{-7}$	$5.93^{+0.89}_{-0.83}$	$5.93^{+0.89}_{-0.83}$
Measured		$2.56 \pm 0.226$	$75.6 \pm 4.8$	$2.32 \pm 0.085$	$1.75 \pm 0.148$
$\frac{\text{Measured}}{\text{SSM}}$		$0.298 \pm 0.049$	$0.581 \pm 0.055$	$0.391 \pm 0.060$	$0.295 \pm 0.051$

TABLE IV. Best-fit parameters for the analysis of total rates only, corresponding to Fig. 16. The sterile SMA solution does not appear in Fig. 16 because  $\chi_{\min}^2$  is too large.

Solution	$\Delta m^2[\text{eV}^2]$	$\tan^2 \theta$	$\chi_{\min}^2$ (3 d.o.f)	g.o.f.
LMA	$1.9 \times 10^{-5}$	$3.0 \times 10^{-1}$	2.62	45%
SMA	$7.7 \times 10^{-6}$	$1.7 \times 10^{-3}$	4.40	22%
VAC	$7.7 \times 10^{-11}$	0.33(3.0)	5.83	12%
Sterile VAC	$9.2 \times 10^{-11}$	0.32(3.1)	10.1	1.7%
LOW	$1.0 \times 10^{-7}$	$6.1 \times 10^{-1}$	11.0	1.2%
Sterile SMA	$5.0 \times 10^{-6}$	$8.9 \times 10^{-4}$	22.8	0.4%

TABLE V. Best-fit global oscillation parameters with all solar neutrino data. This table corresponds to the global solution illustrated in Fig. 18. Sterile solutions do not appear in Fig. 18 because their  $\chi_{\min}^2$  is too large.

Solution	$\Delta m^2[\text{eV}^2]$	$\tan^2 \theta$	$\chi_{\min}^2$ (40 d.o.f)
LMA	$3.7 \times 10^{-5}$	$3.7 \times 10^{-1}$	34.5
LOW	$1.0 \times 10^{-7}$	$6.7 \times 10^{-1}$	40.8
VAC	$4.6 \times 10^{-10}$	2.5	42.3
Sterile VAC	$4.7 \times 10^{-10}$	3.0	49.1
SMA	$5.2 \times 10^{-6}$	$1.8 \times 10^{-3}$	49.9
Just $\text{So}^2$	$5.5 \times 10^{-12}$	0.61(1.6)	52.1
Sterile Just $\text{So}^2$	$5.5 \times 10^{-12}$	0.61(1.6)	52.1
Sterile SMA	$4.6 \times 10^{-6}$	$3.4 \times 10^{-4}$	52.3

TABLE VI. 90% CL limit on the neutrino oscillation probabilities from the negative searches at short baseline experiments.

Experiment	Beam	Channel	Limit (90%)	$\Delta m_{\min}^2$ (eV <sup>2</sup> )	Ref.
CDHSW	CERN	$\nu_\mu \rightarrow \nu_\mu$	$P_{\mu\mu} > 0.95$	0.25	Dydak <i>et al.</i> , 1984
E776	BNL	$\nu_\mu \rightarrow \nu_e$	$P_{e\mu} < 1.5 \times 10^{-3}$	0.075	Borodovsky <i>et al.</i> , 1992
E734	BNL	$\nu_\mu \rightarrow \nu_e$	$P_{e\mu} < 1.6 \times 10^{-3}$	0.4	Ahrens <i>et al.</i> , 1987
KARMEN2	Rutherford	$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$	$P_{e\mu} < 6.5 \times 10^{-4}$	0.05	Wolf <i>et. al.</i> , 2001
E531	FNAL	$\nu_\mu \rightarrow \nu_\tau$	$P_{\mu\tau} < 2.5 \times 10^{-3}$	0.9	Ushida <i>et al.</i> , 1986
CCFR	FNAL	$\nu_\mu \rightarrow \nu_\tau$	$P_{\mu\tau} < 4 \times 10^{-3}$	1.6	McFarland <i>et al.</i> , 1995
		$\nu_e \rightarrow \nu_\tau$	$P_{e\tau} < 0.10$	20	Naples <i>et al.</i> , 1999
		$\nu_\mu \rightarrow \nu_e$	$P_{\mu e} < 9. \times 10^{-4}$	1.6	Romosán <i>et al.</i> , 1997
Chorus	CERN	$\nu_\mu \rightarrow \nu_\tau$	$P_{\mu\tau} < 3.4 \times 10^{-4}$	0.6	Eskut <i>et al.</i> , 2001
		$\nu_e \rightarrow \nu_\tau$	$P_{e\tau} < 2.6 \times 10^{-2}$	7.5	Eskut <i>et al.</i> , 2001
Nomad	CERN	$\nu_\mu \rightarrow \nu_\tau$	$P_{\mu\tau} < 1.7 \times 10^{-4}$	0.7	Astier <i>et al.</i> , 2001
		$\nu_e \rightarrow \nu_\tau$	$P_{e\tau} < 7.5 \times 10^{-3}$	5.9	Astier <i>et al.</i> , 2001
		$\nu_\mu \rightarrow \nu_e$	$P_{\mu e} < 6 \times 10^{-4}$	0.4	Valuev <i>et al.</i> , 2001

TABLE VII. 90% CL lower bound on  $P_{ee}$  and  $\Delta m^2$  sensitivity from searches at reactor experiments.

Experiment	L	Limit (90%)	$\Delta m_{\min}^2$ (eV <sup>2</sup> )	Ref.
Bugey	15,40 m	$> 0.91$	$10^{-2}$	Achkar <i>et al.</i> , 1995
Krasnoyarsk	57, 230 m	$> 0.93$	$7 \times 10^{-3}$	Vidyakin <i>et al.</i> , 1994
Gosgen	38, 48, 65 m	$> 0.9$	0.02	Zacek, G., 1986
CHOOZ	1 km	$> 0.95$	$7 \times 10^{-4}$	Apollonio <i>et al.</i> , 1999
KamLAND	150–210 km	$> 0.85$	$5 \times 10^{-5}$	Piepke <i>et al.</i> , 2001

FIGURES

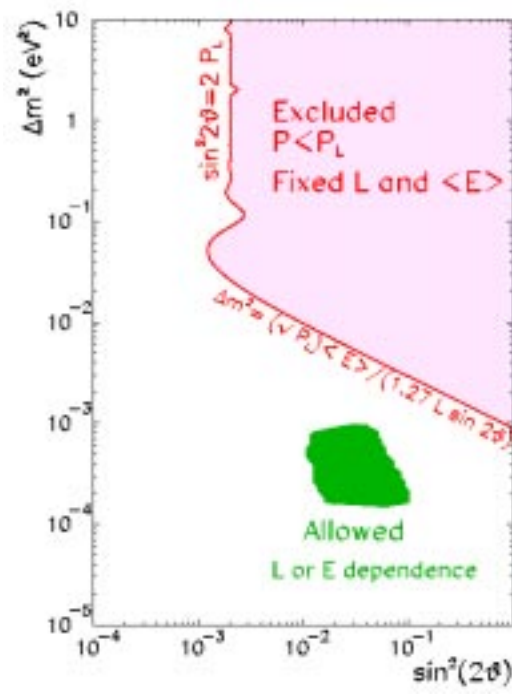


FIG. 1. The characteristic form of an excluded region from a negative search with fixed  $L/E$  and of an allowed region from a positive search with varying  $L/E$  in the two-neutrino oscillation parameter plane.

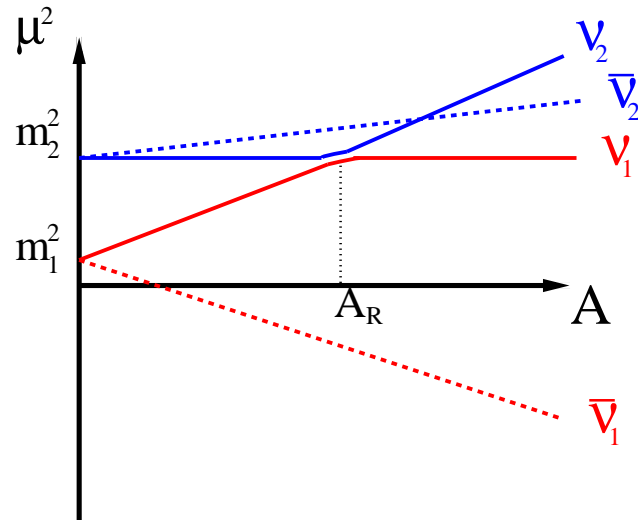


FIG. 2. Effective masses acquired in the medium by a system of two massive neutrinos as a function of the potential  $A$  [see Eq. (60)].

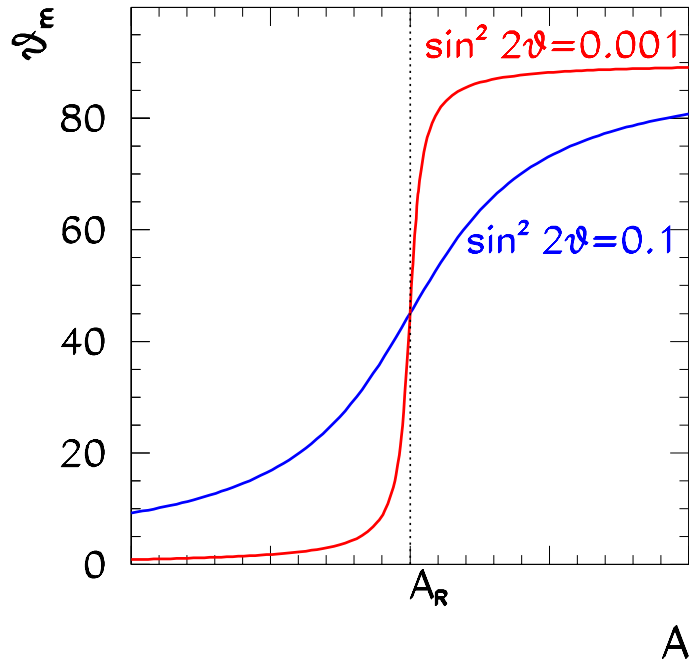


FIG. 3. The mixing angle in matter for a system of two massive neutrinos as a function of the potential  $A$  for two different values of the mixing angle in vacuum [see Eq. (61)].

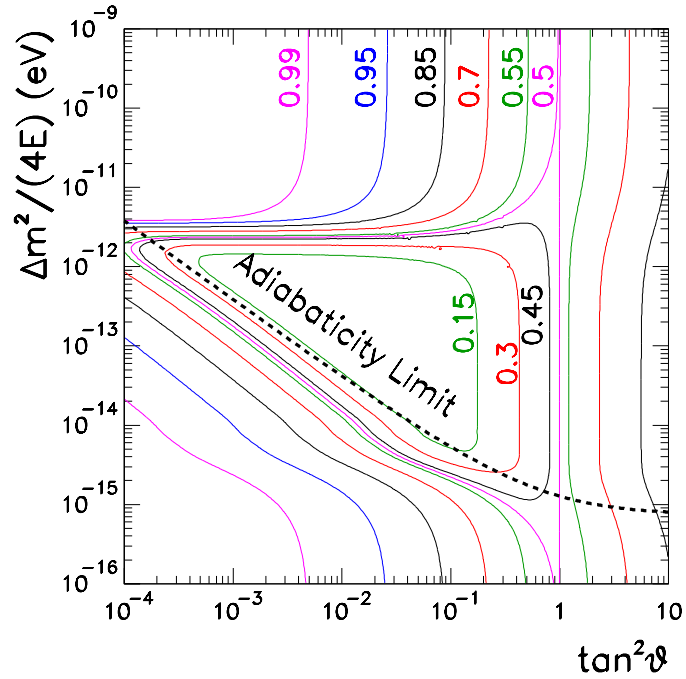


FIG. 4. Isocontours of the survival probability  $P_{ee}$  in the Sun. Also shown is the limit of applicability of the adiabatic approximation  $Q = 1$  (dashed line).

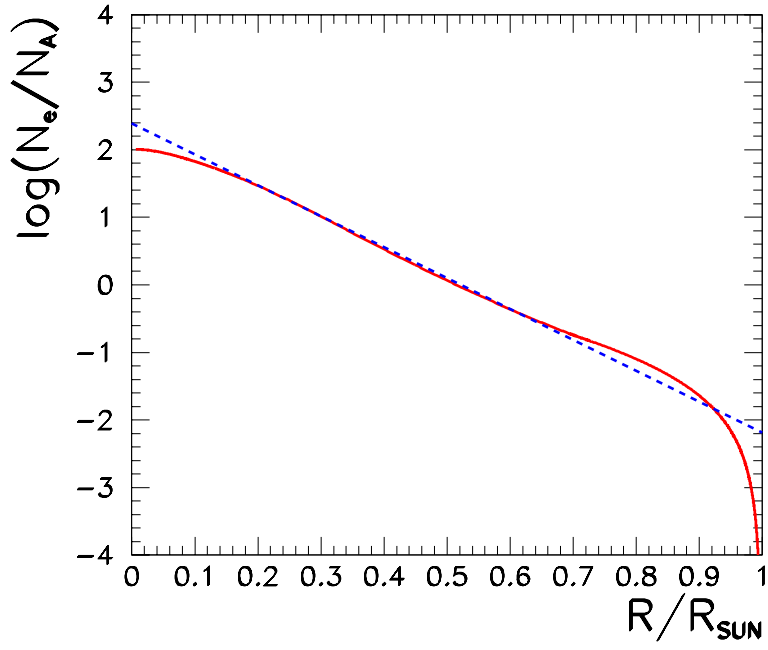


FIG. 5. The solar matter density profile for the BP00 model (full line), and the exponential approximation of Eq. (89) (dashed line).

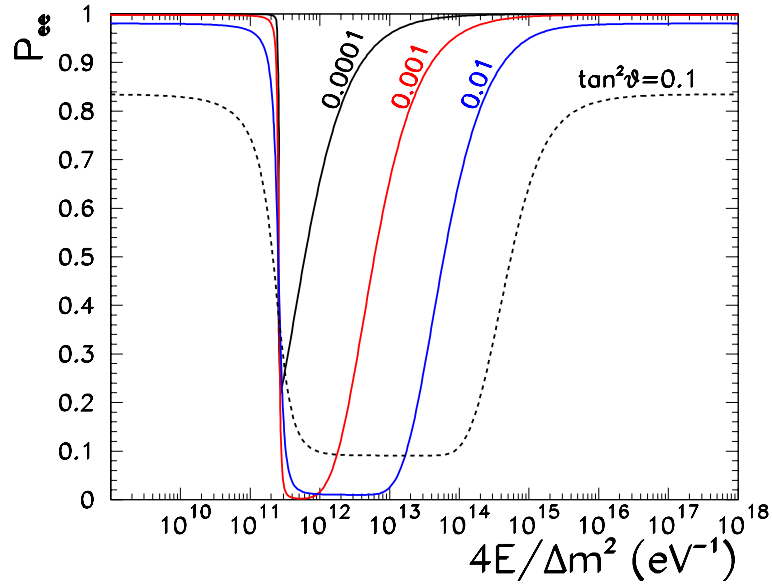


FIG. 6. The survival probability for a  $\nu_e$  state produced in the center of the sun as a function of  $E/\Delta m^2$  for various values of the mixing angle.

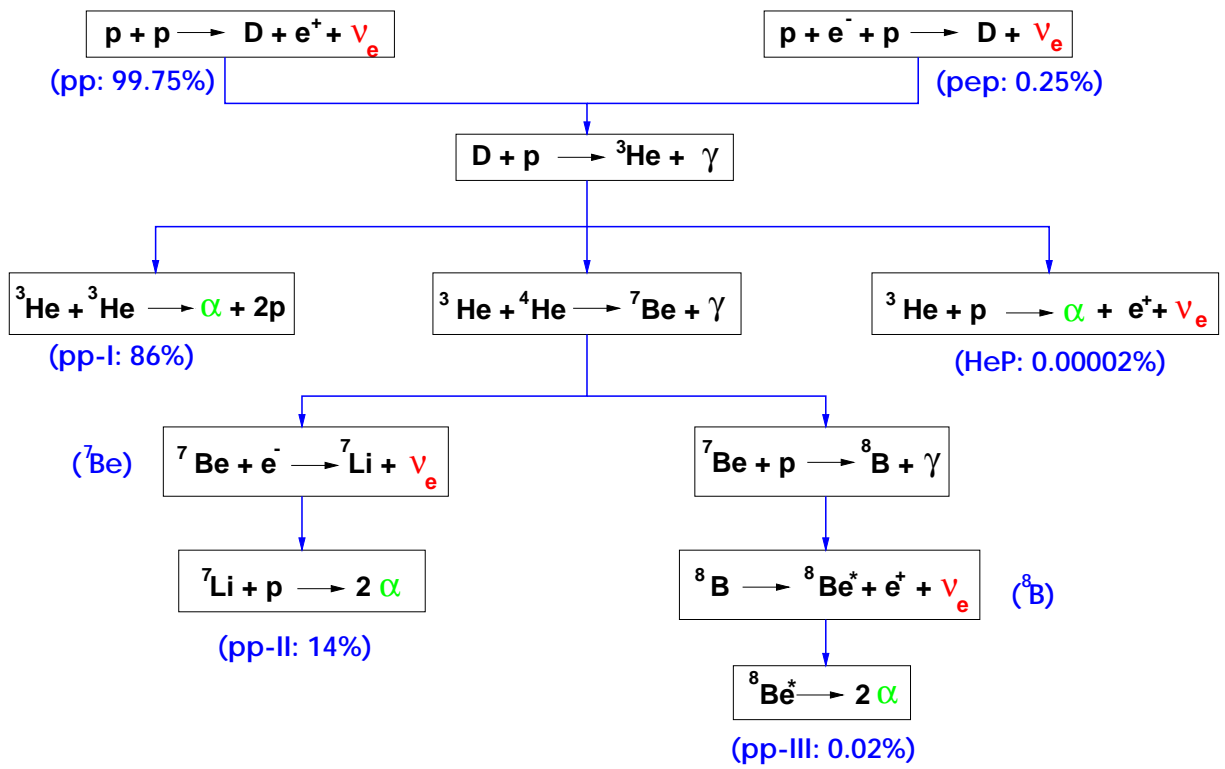


FIG. 7. The  $pp$  chain in the Sun.

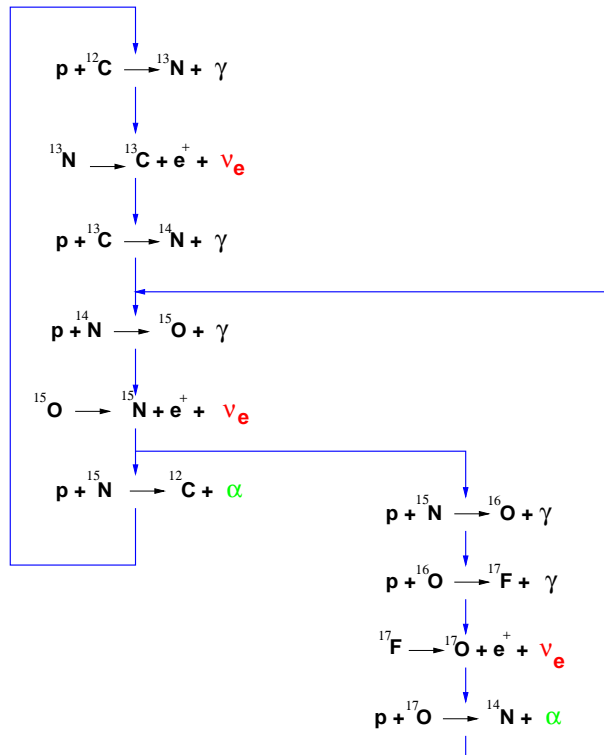


FIG. 8. The CNO cycle in the Sun.



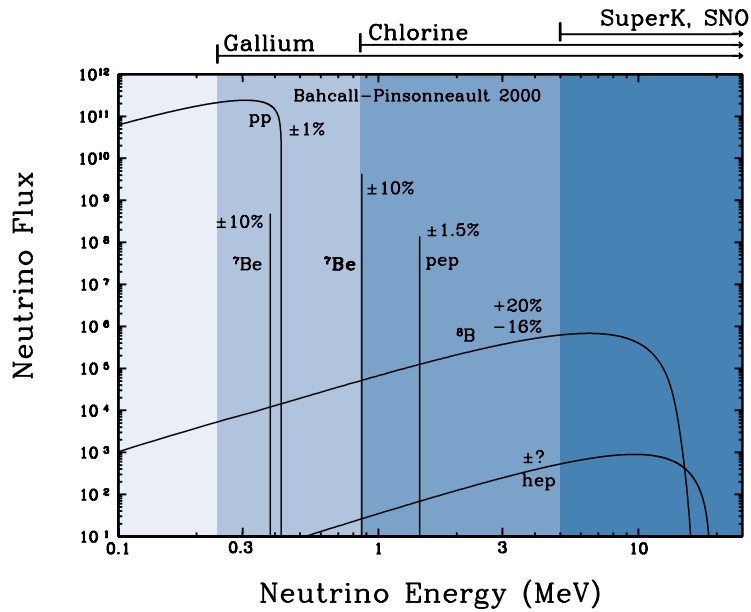


FIG. 9. Neutrino fluxes from the  $pp$  chain reactions as a function of the neutrino energy.

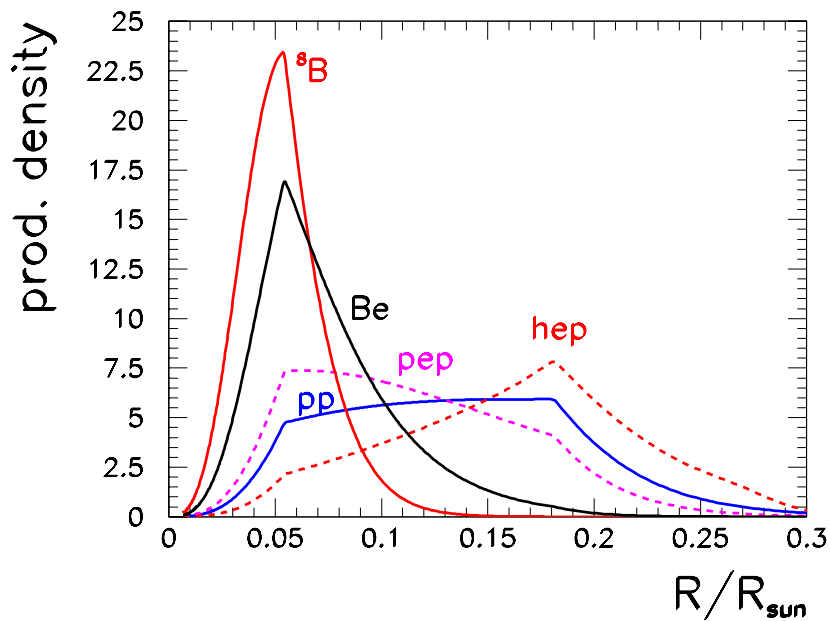


FIG. 10. Production point distribution for the  $pp$  chain neutrinos as a function of the distance from the solar center.

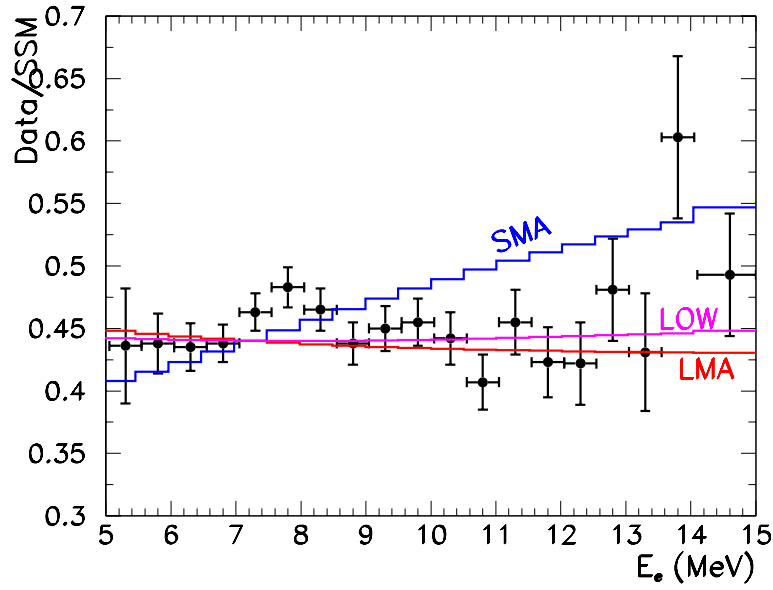


FIG. 11. The electron recoil energy spectrum measured in SK normalized to the SSM prediction, and the expectations for the best fit points for the LMA, SMA and LOW solutions in Tab. IV.

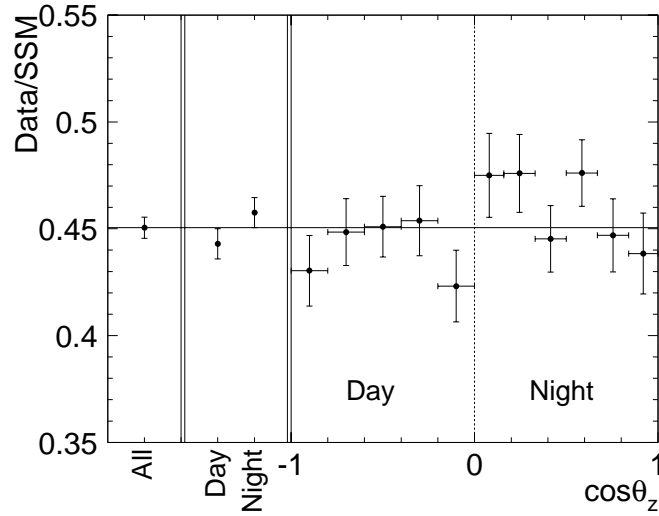


FIG. 12. The zenith angle dependence of the solar neutrino flux (statistical error only). The width of the night-time bins was chosen to separate solar neutrinos that pass through the Earth's dense core ( $\cos \theta_z \geq 0.84$ ) from those that pass through the mantle ( $0 < \cos \theta_z < 0.84$ ). The horizontal line shows the average flux.

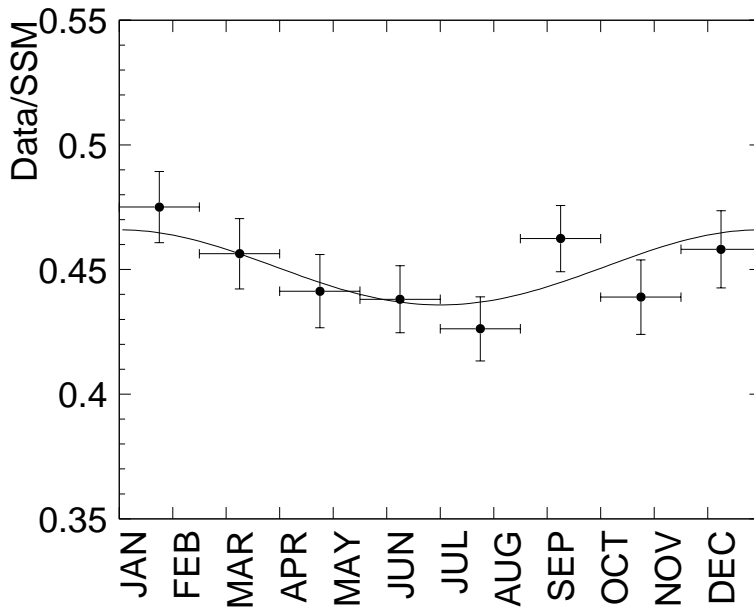


FIG. 13. Seasonal variation of the solar neutrino flux (statistical errors only). The curve shows the expected seasonal variation of the flux induced by the eccentricity of the Earth's orbit.

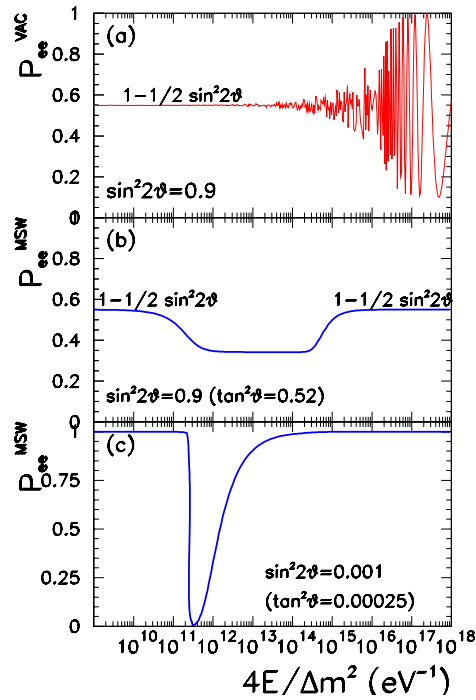


FIG. 14.  $P_{ee}$  as a function of  $4E/\Delta m^2$ . In panel (a) matter effects are ignored [see Eq. (106)]. Panels (b) (large mixing angle) and (c) (small mixing angle) take into account the MSW effect [see Eq. (108)].

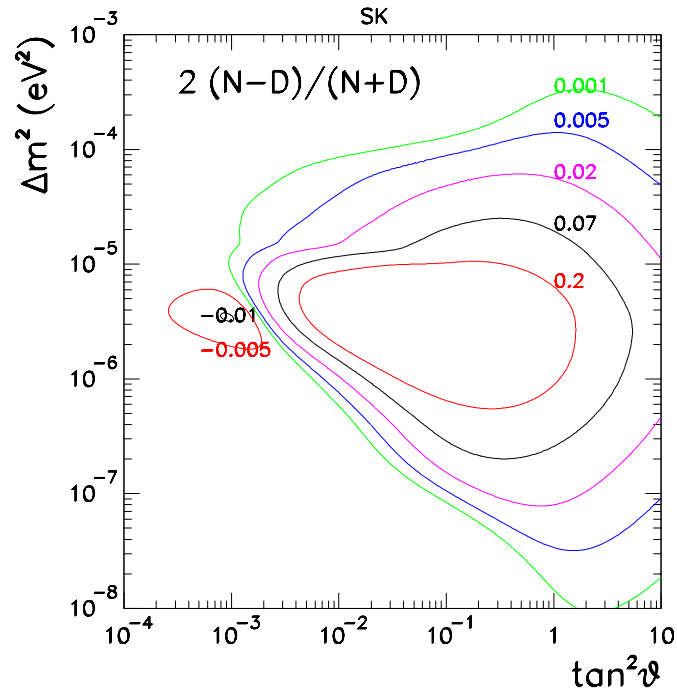


FIG. 15. Isocontours of the day-night asymmetry at SK.

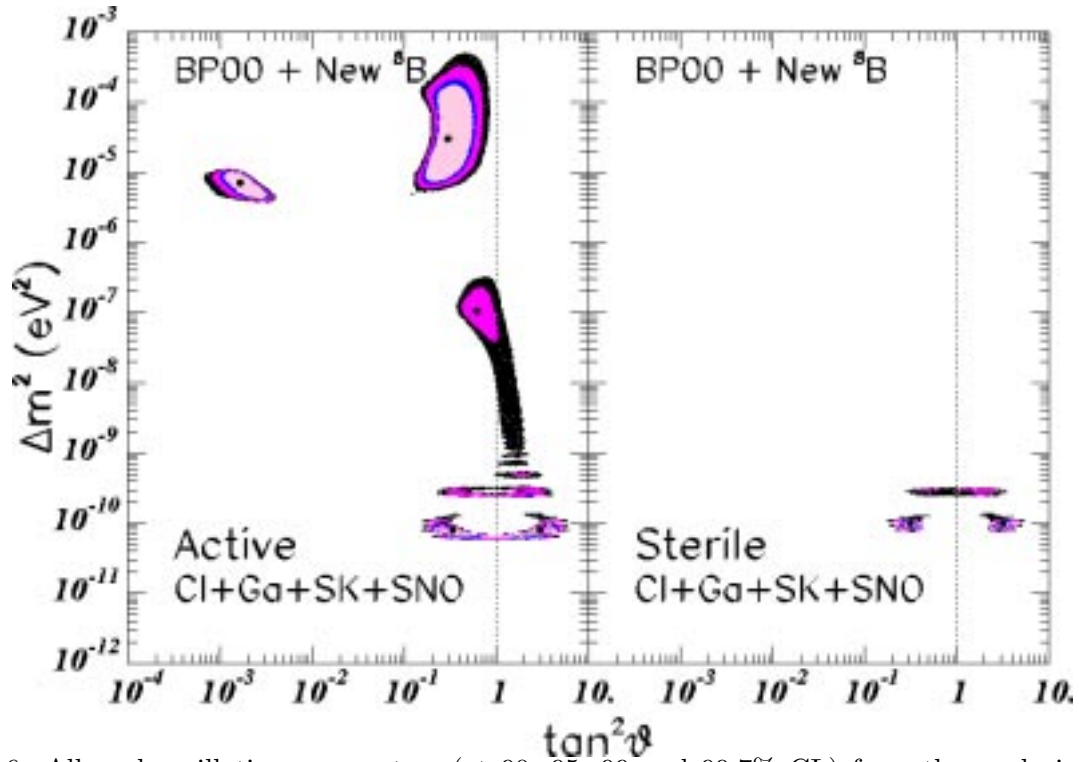


FIG. 16. Allowed oscillation parameters (at 90, 95, 99 and 99.7% CL) from the analysis of the total event rates of the Chlorine, Gallium, SK and SNO CC experiments. The best fit point is marked with a star.

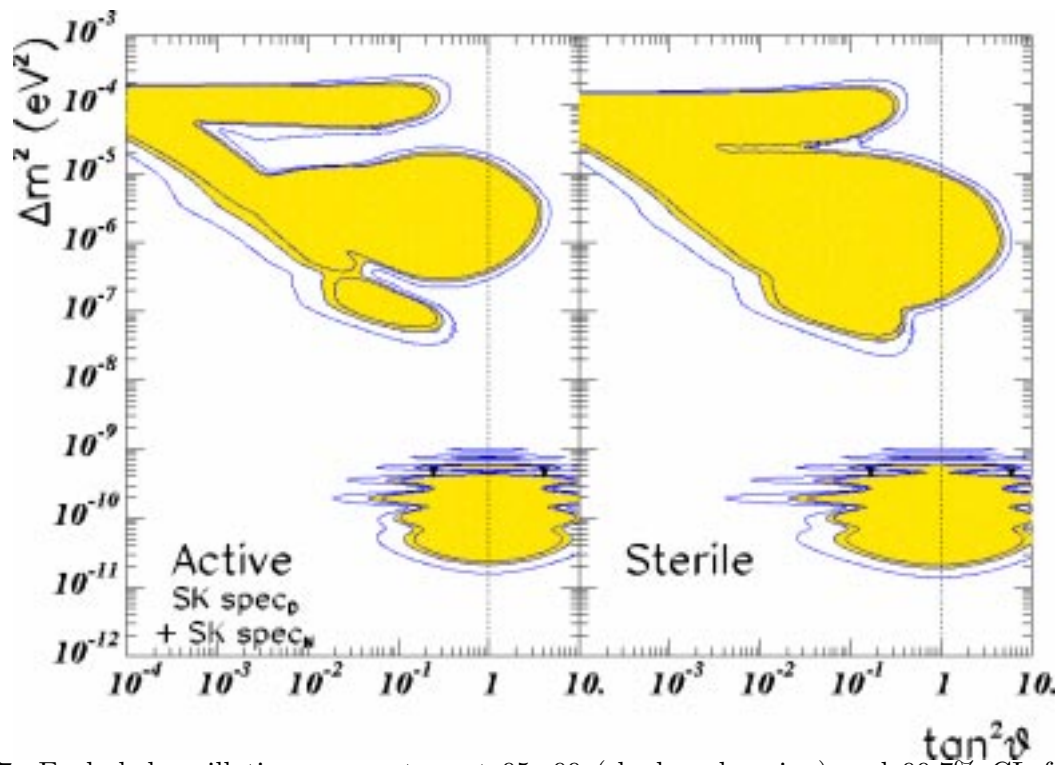


FIG. 17. Excluded oscillation parameters at 95, 99 (shaded region) and 99.7% CL from the analysis of the day-night spectrum data.

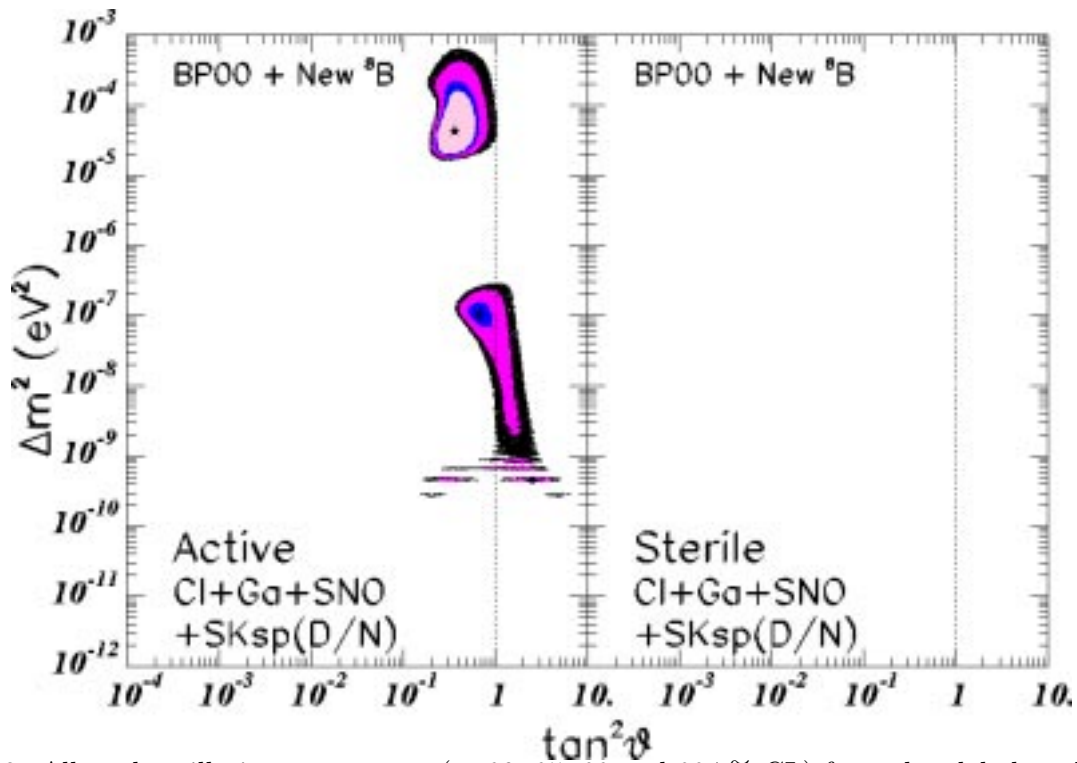


FIG. 18. Allowed oscillation parameters (at 90, 95, 99 and 99.7% CL) from the global analysis of the solar neutrino data. The best fit point (LMA active) is marked with a star.

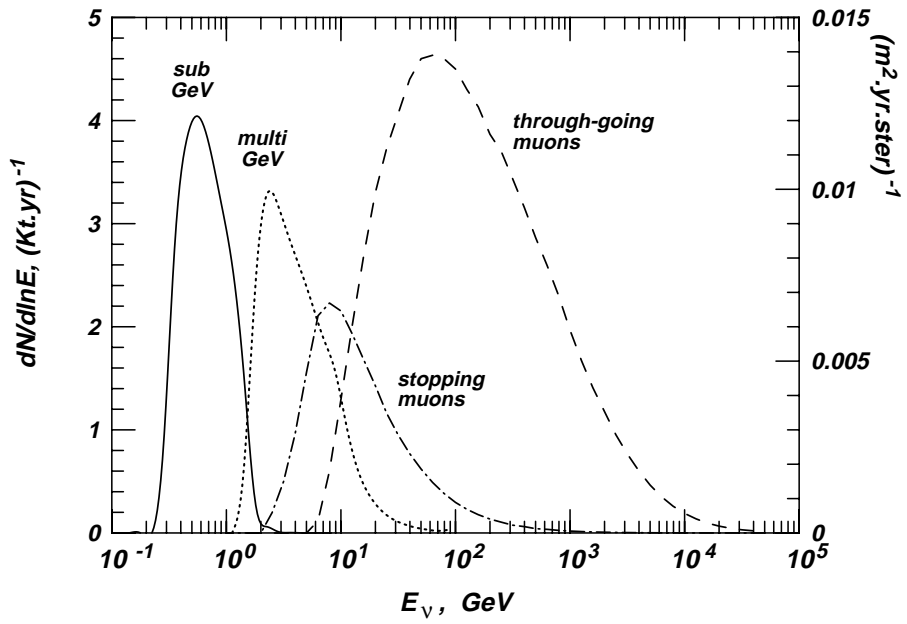


FIG. 19. Event rates as a function of neutrino energy for fully contained events, stopping muons, and through-going muons at SuperKamiokande.

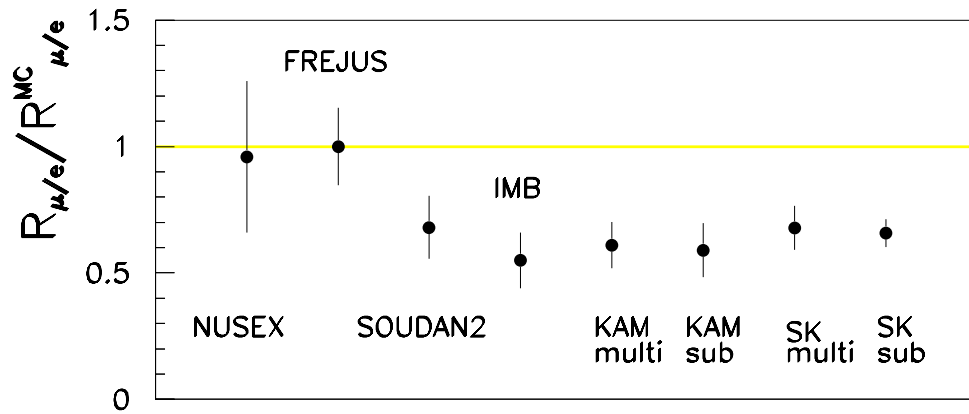


FIG. 20. The double ratio of  $\nu_\mu$  to  $\nu_e$  events, data divided by theoretical predictions, for various underground atmospheric neutrino detectors.

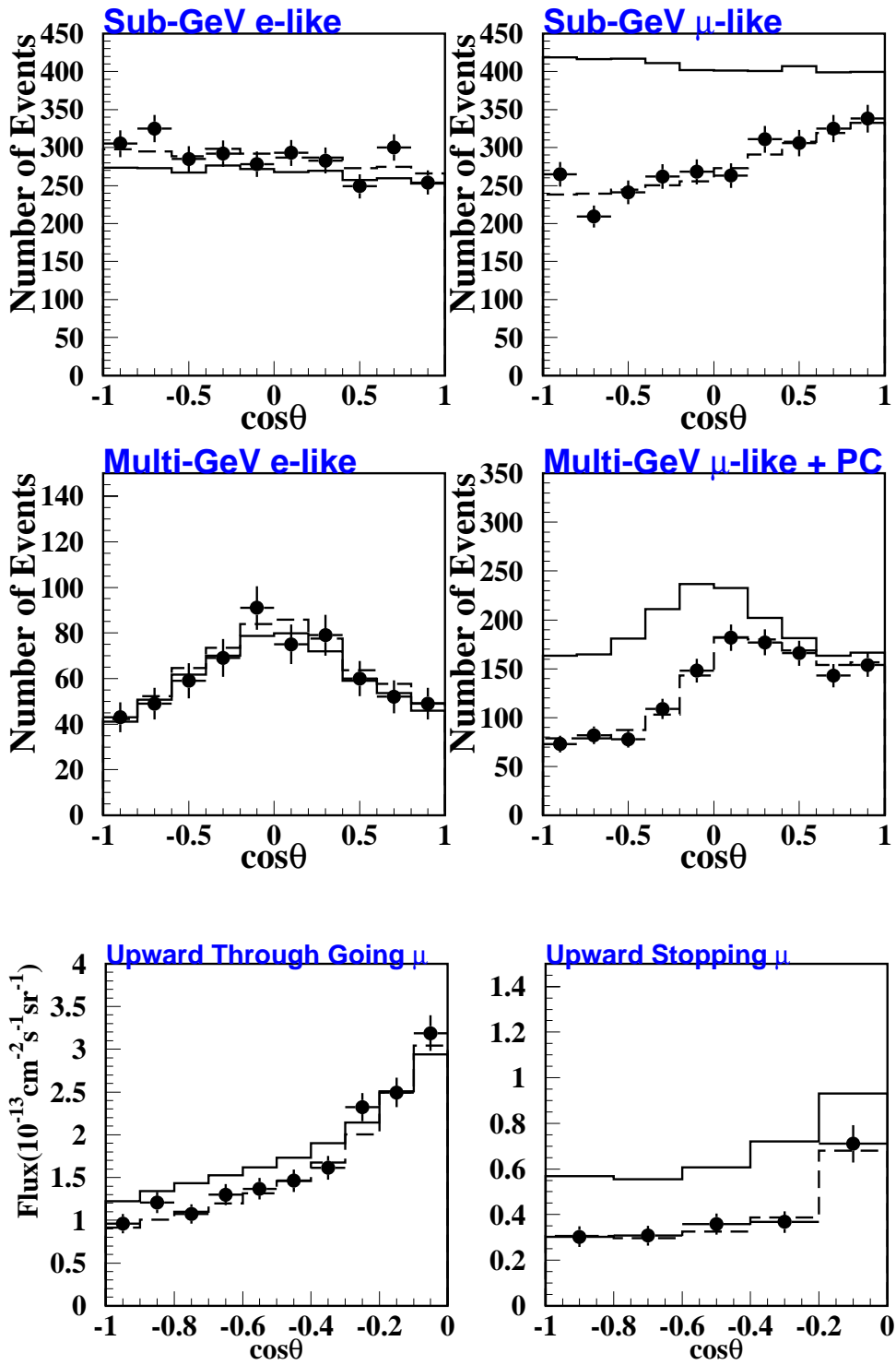


FIG. 21. Zenith angle distribution of SuperKamiokande 1289 days data samples. Dots, solid line and dashed line correspond to data, MC with no oscillation and MC with best fit oscillation parameters, respectively.

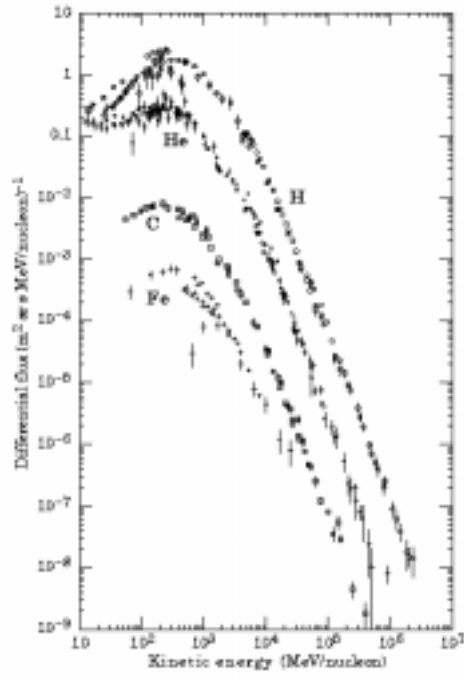


FIG. 22. The main components of the primary cosmic ray spectrum.

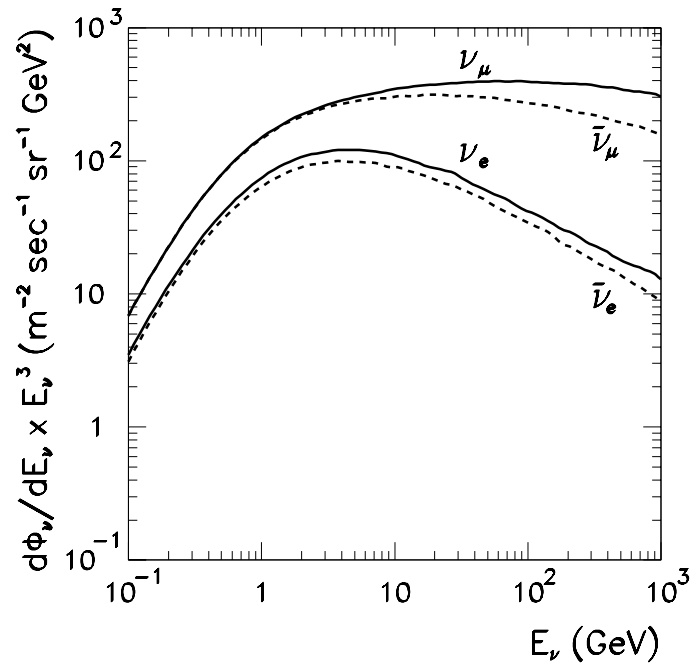


FIG. 23. Atmospheric neutrino fluxes as a function of the neutrino energy.



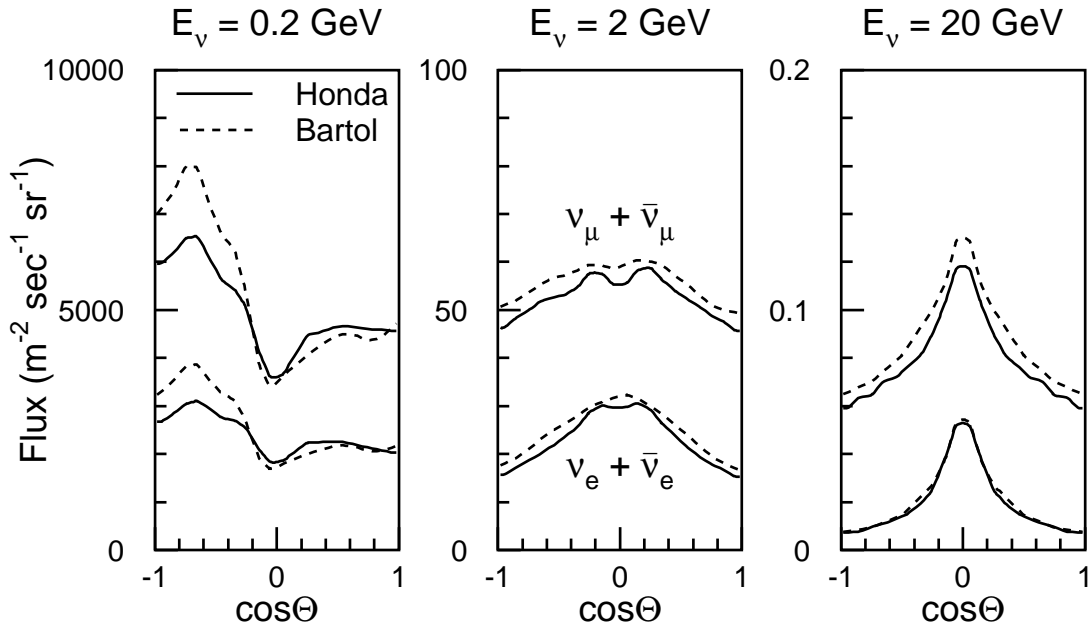


FIG. 24. Zenith angular dependence of the atmospheric neutrino fluxes.

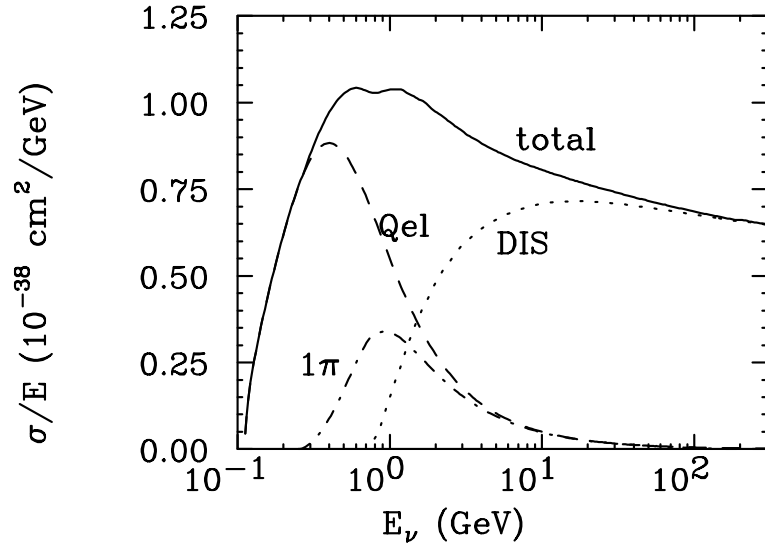


FIG. 25. The various contributions to the neutrino-nucleon cross section.

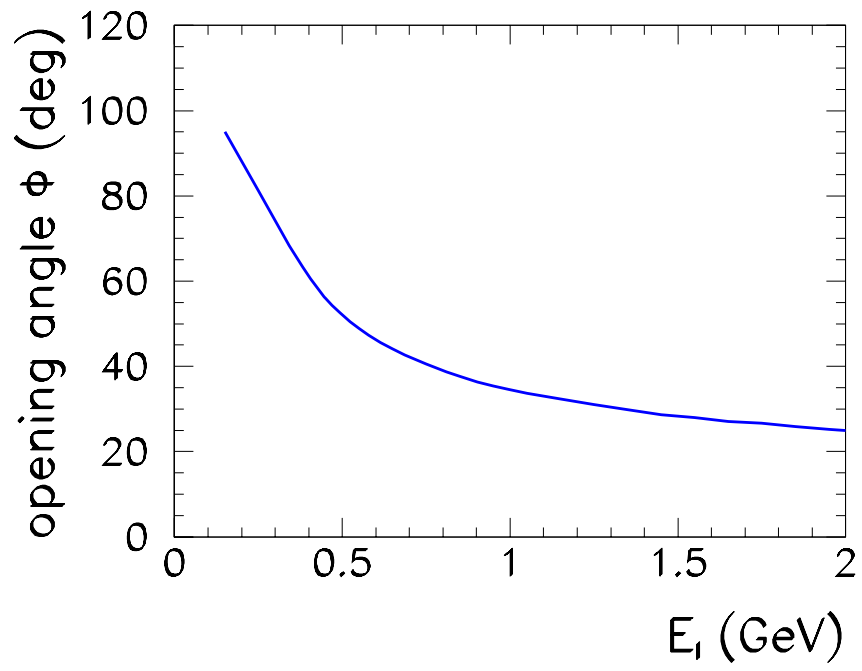


FIG. 26. The opening angle between the incoming neutrino and the produced charged lepton in QE interactions as a function of the charged lepton energy.

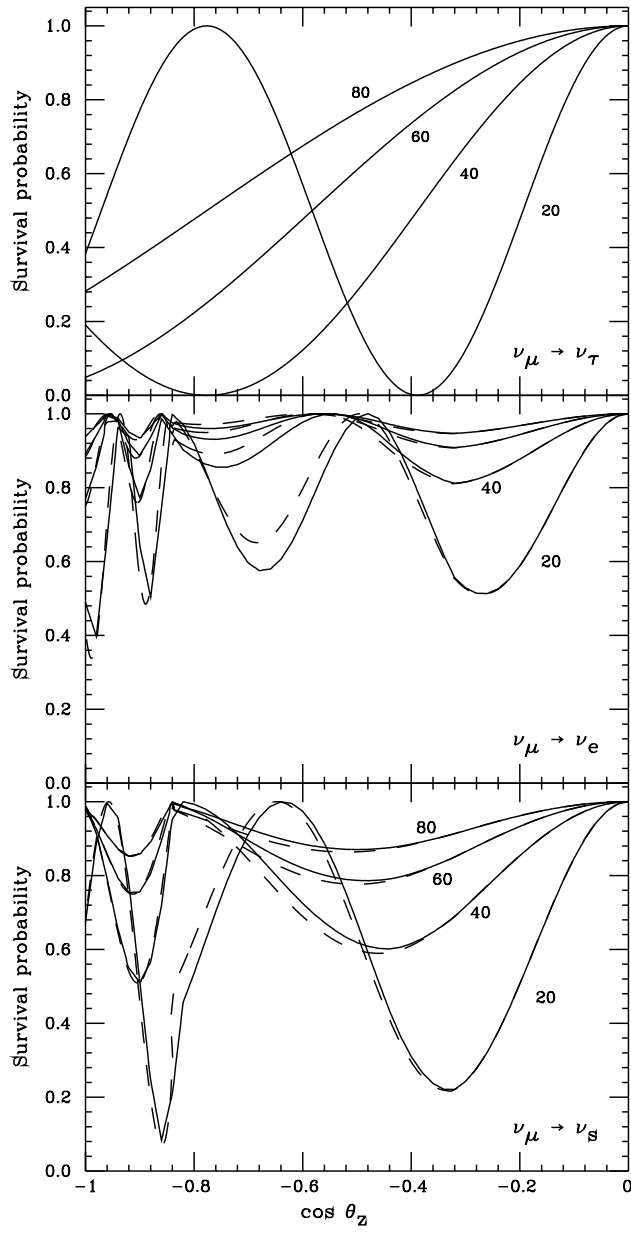


FIG. 27.  $P_{\mu\mu}$  as a function of the zenith angle for maximal mixing of  $\nu_\mu$  with  $\nu_\tau$  (upper panel),  $\nu_e$  (middle panel) and  $\nu_s$  (lower panel). For  $\Delta m^2 = 5 \times 10^{-3} \text{ eV}^2$  the curves correspond to  $E_\nu$  of 20, 40, 60 and 80 GeV. The dashed curves are calculated with the approximation of constant average densities in the mantle and in the core of the Earth.

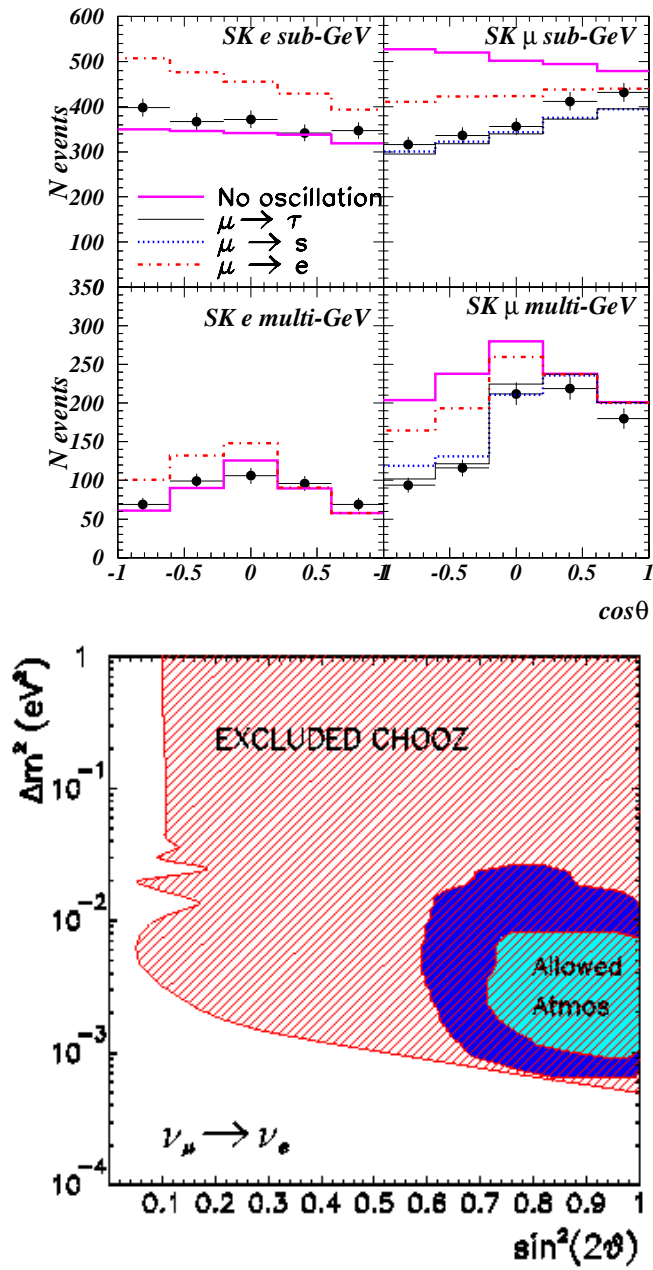


FIG. 28. The status of the  $\nu_\mu \rightarrow \nu_e$  oscillation solution to the atmospheric neutrino anomaly.

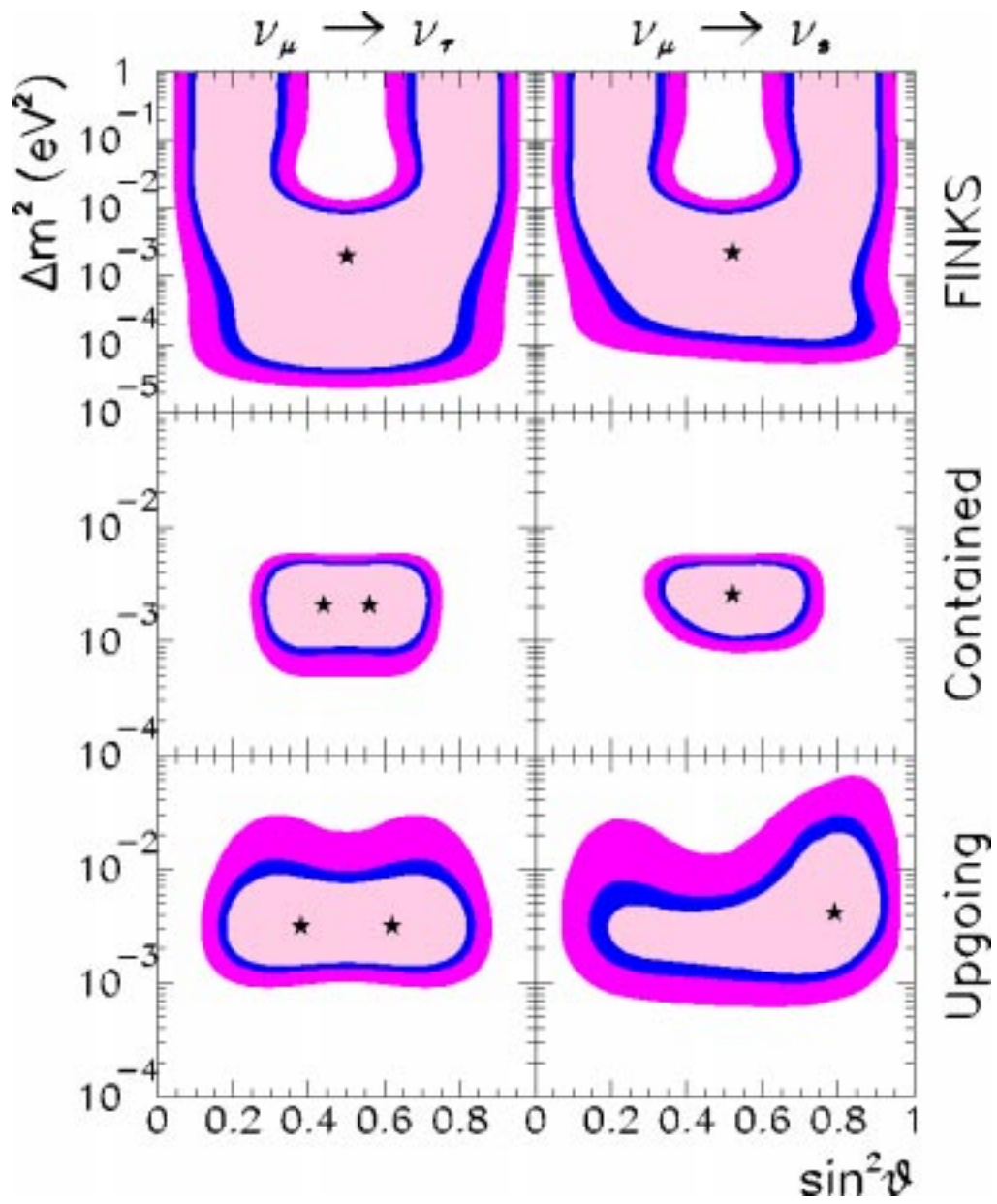


FIG. 29. Allowed regions (at 90, 95 and 99 % CL) from the analysis of various partial data samples of atmospheric neutrinos for the oscillation channels  $\nu_\mu \rightarrow \nu_\tau$  and  $\nu_\mu \rightarrow \nu_s$ . The best fit points are marked with a star (see text for details).

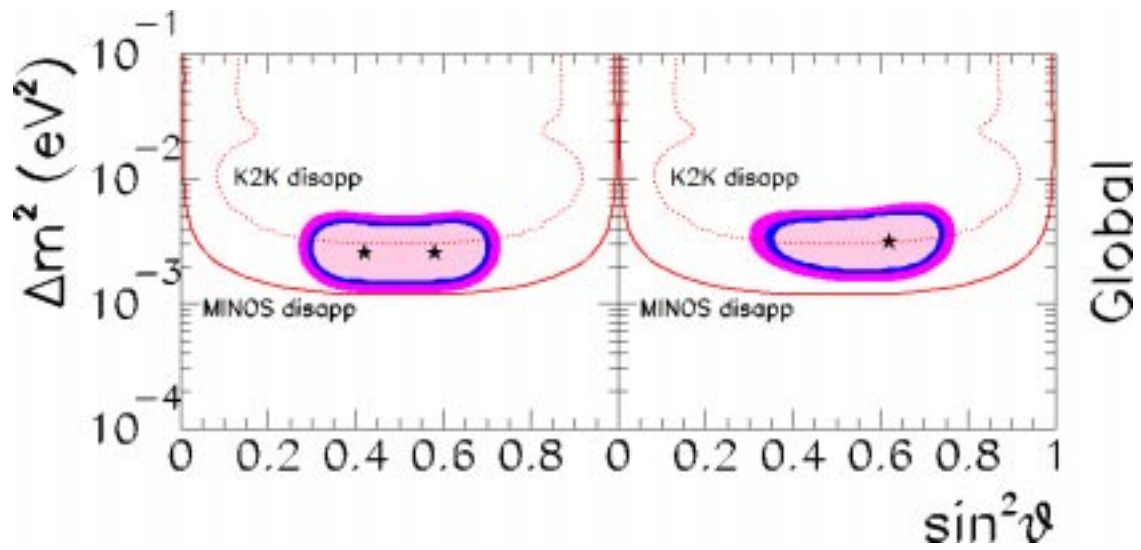


FIG. 30. Allowed regions (at 90, 95 and 99 % CL) from the analysis of the full data sample of atmospheric neutrinos for the oscillation channels  $\nu_\mu \rightarrow \nu_\tau$  and  $\nu_\mu \rightarrow \nu_s$ . The best fit points are marked with a star (see text for details). Also shown are the expected sensitivities from LBL experiments.

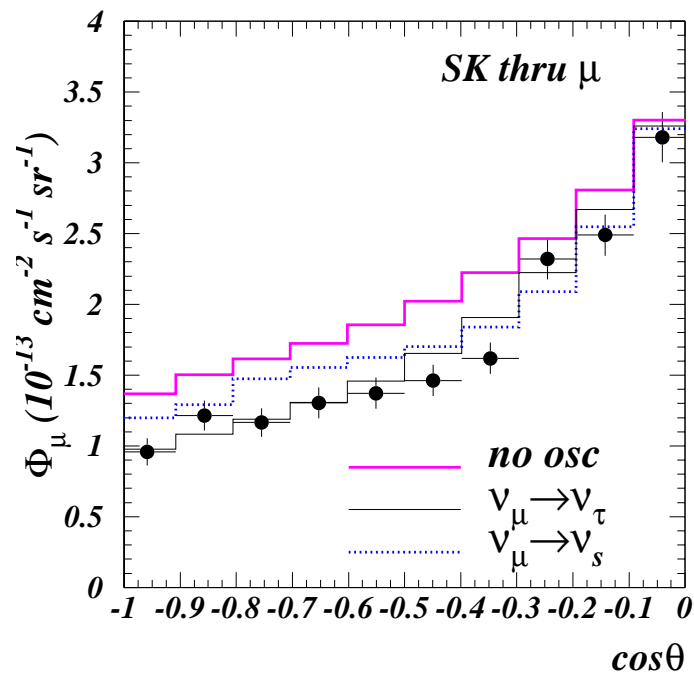


FIG. 31. Zenith angular dependence for through-going muon at SuperKamiokande compared with the predictions in the case of no-oscillation and for the best fit points of  $\nu_\mu \rightarrow \nu_\tau$  ( $\Delta m^2 = 2.6 \times 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta = 0.97$ ) and  $\nu_\mu \rightarrow \nu_s$  ( $\Delta m^2 = 3 \times 10^{-3} \text{ eV}^2$ ,  $\sin^2 \theta = 0.61$ ) oscillations.

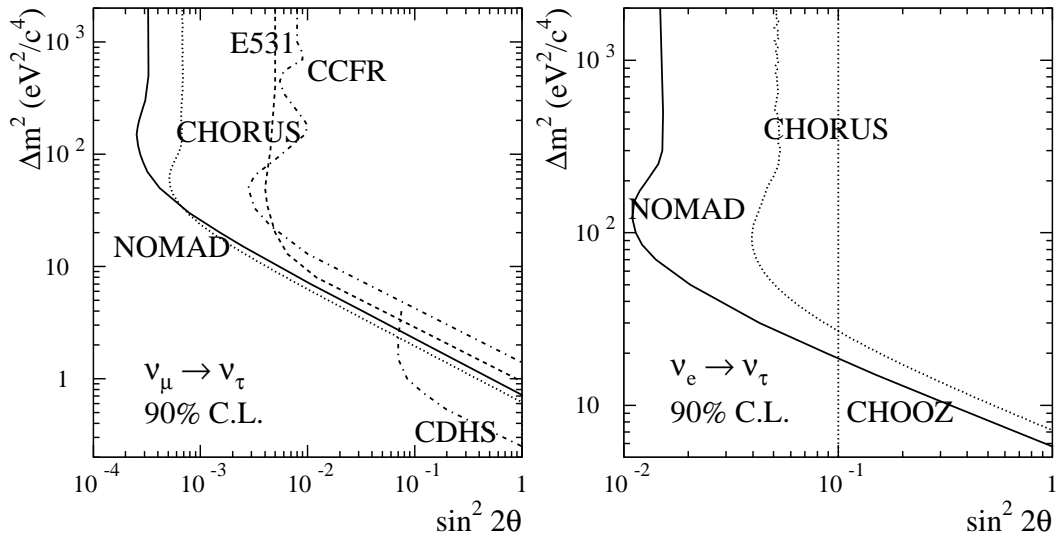


FIG. 32. Excluded regions at 90% in the  $\nu_\mu \rightarrow \nu_\tau$  and  $\nu_e \rightarrow \nu_\tau$  channels from SBL experiments.

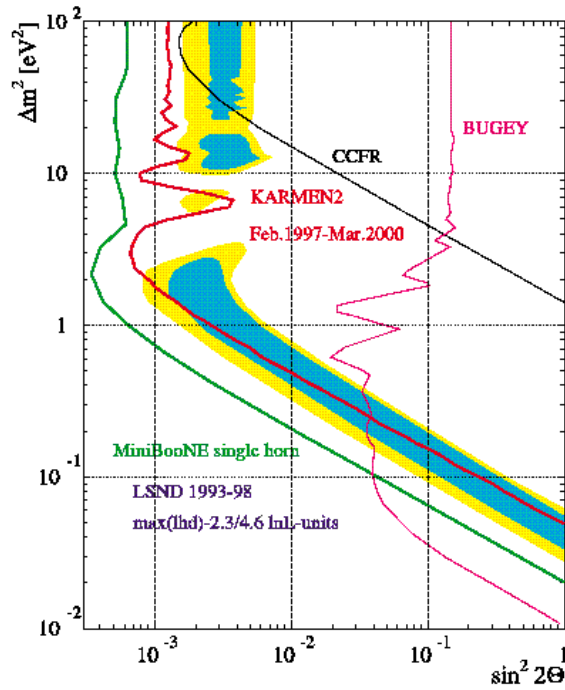


FIG. 33. Allowed regions (at 90 and 99 % CL) for  $\nu_e \rightarrow \nu_\mu$  oscillations from the LSND experiment compared with the exclusion regions (at 90% CL) from KARMEN2 and other experiments. The 90 % CL expected sensitivity curve for MinimBoNE is also shown.

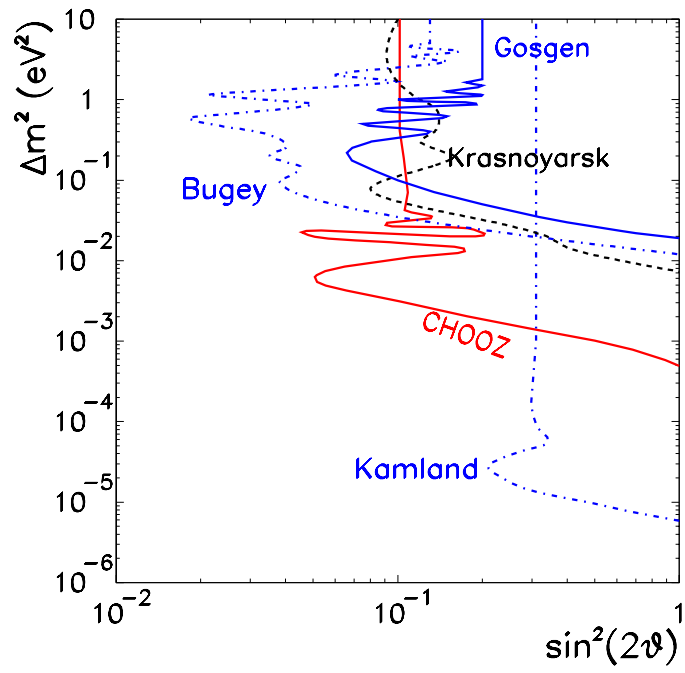


FIG. 34. Excluded regions at 90% for  $\nu_e$  oscillations from reactors experiments and the expected sensitivity from the KamLAND experiment.



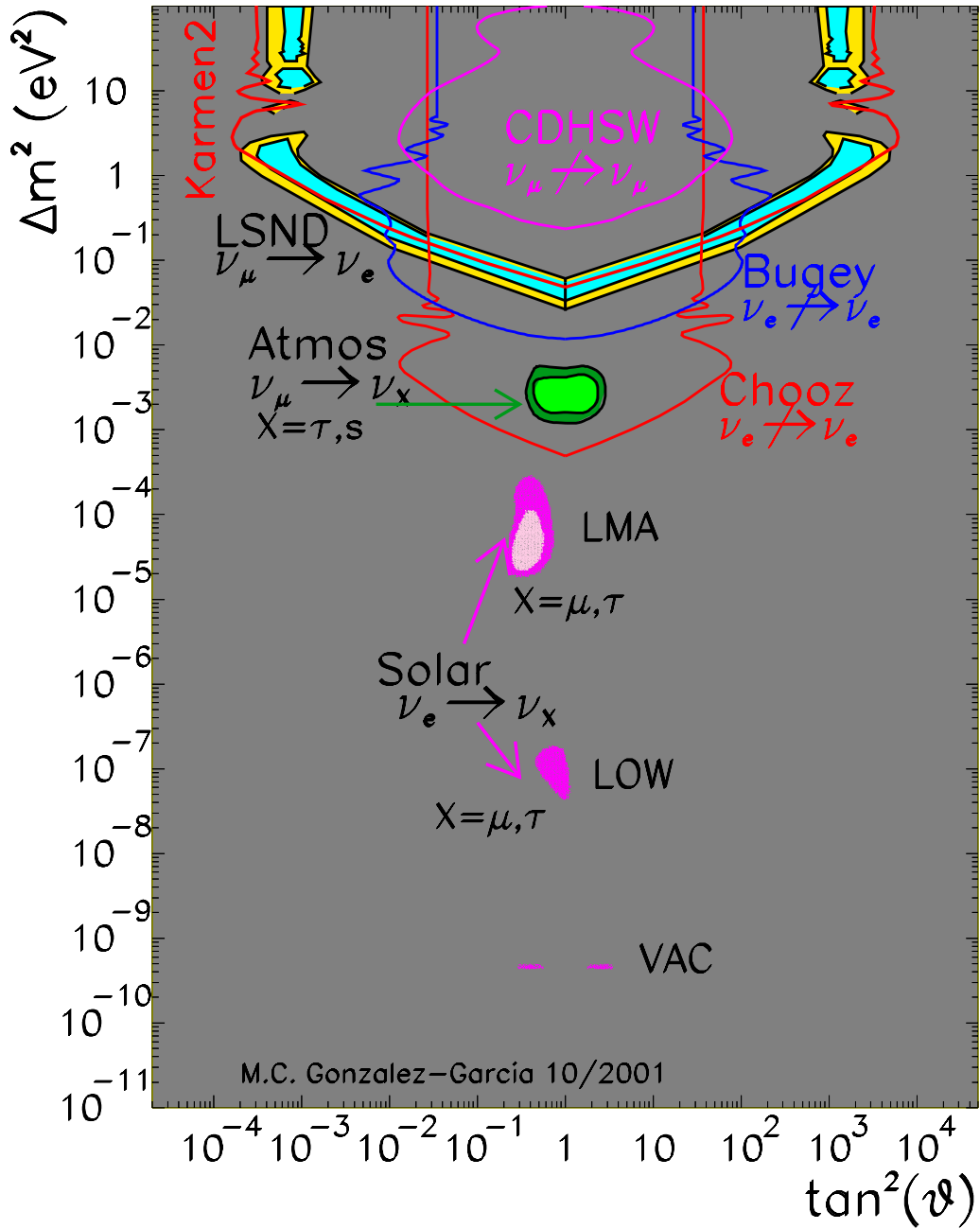


FIG. 35. Summary of the present pieces of evidence for neutrino masses and mixing as obtained from 2- $\nu$  oscillation analyses. The allowed regions correspond to 90% and 99% CL for 2 d.o.f. We also show relevant bounds from laboratory experiments.

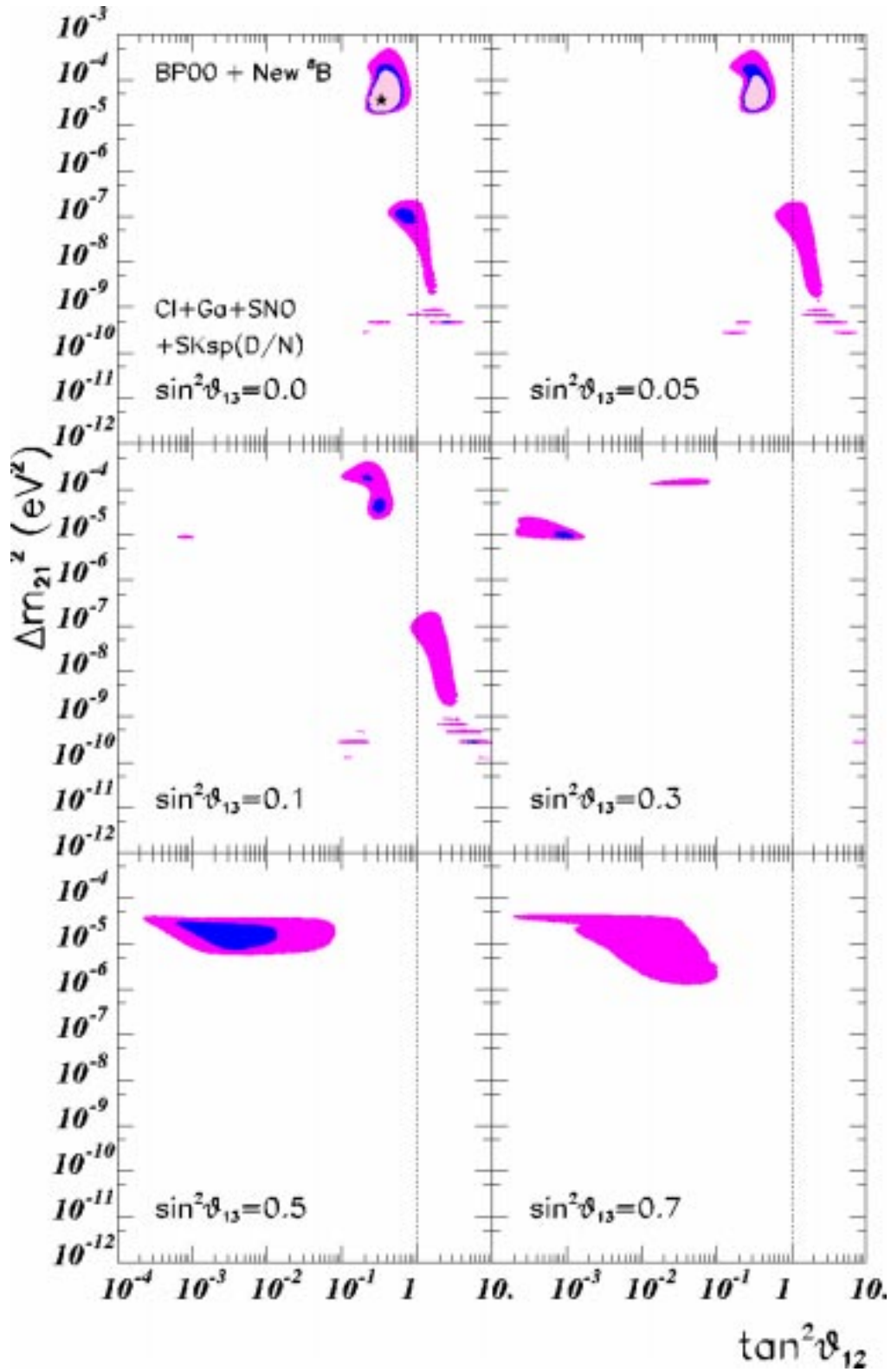


FIG. 36. Allowed regions (at 90, 95, and 99% CL) in the  $(\Delta m_{21}^2, \tan^2 \theta_{12})$  plane from the global analysis of the solar neutrino data in the framework of three-neutrino oscillations for various values of  $\sin^2 \theta_{13}$ . The global best fit point is marked by the star.

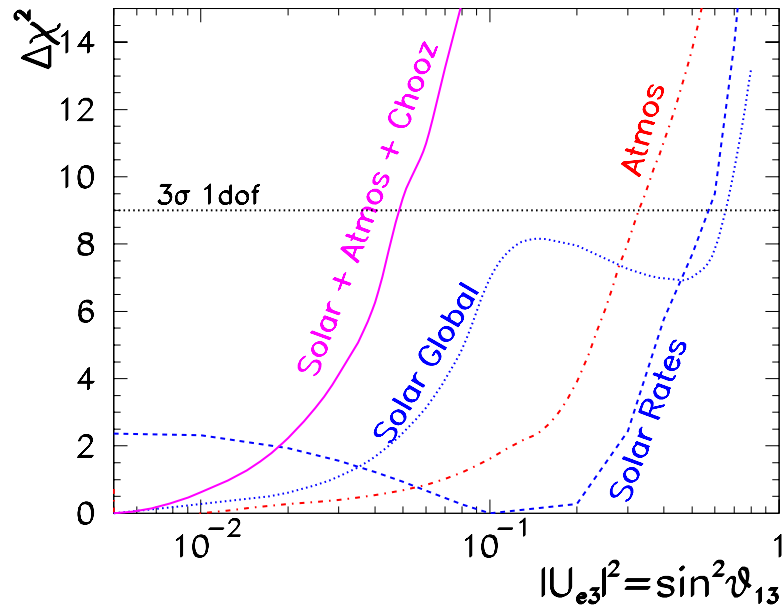


FIG. 37. Dependence of  $\Delta\chi^2$  on  $\sin^2\theta_{13}$  in the analysis of the atmospheric, solar and CHOOZ neutrino data. The dotted horizontal line corresponds to the  $3\sigma$  limit for a single parameter.

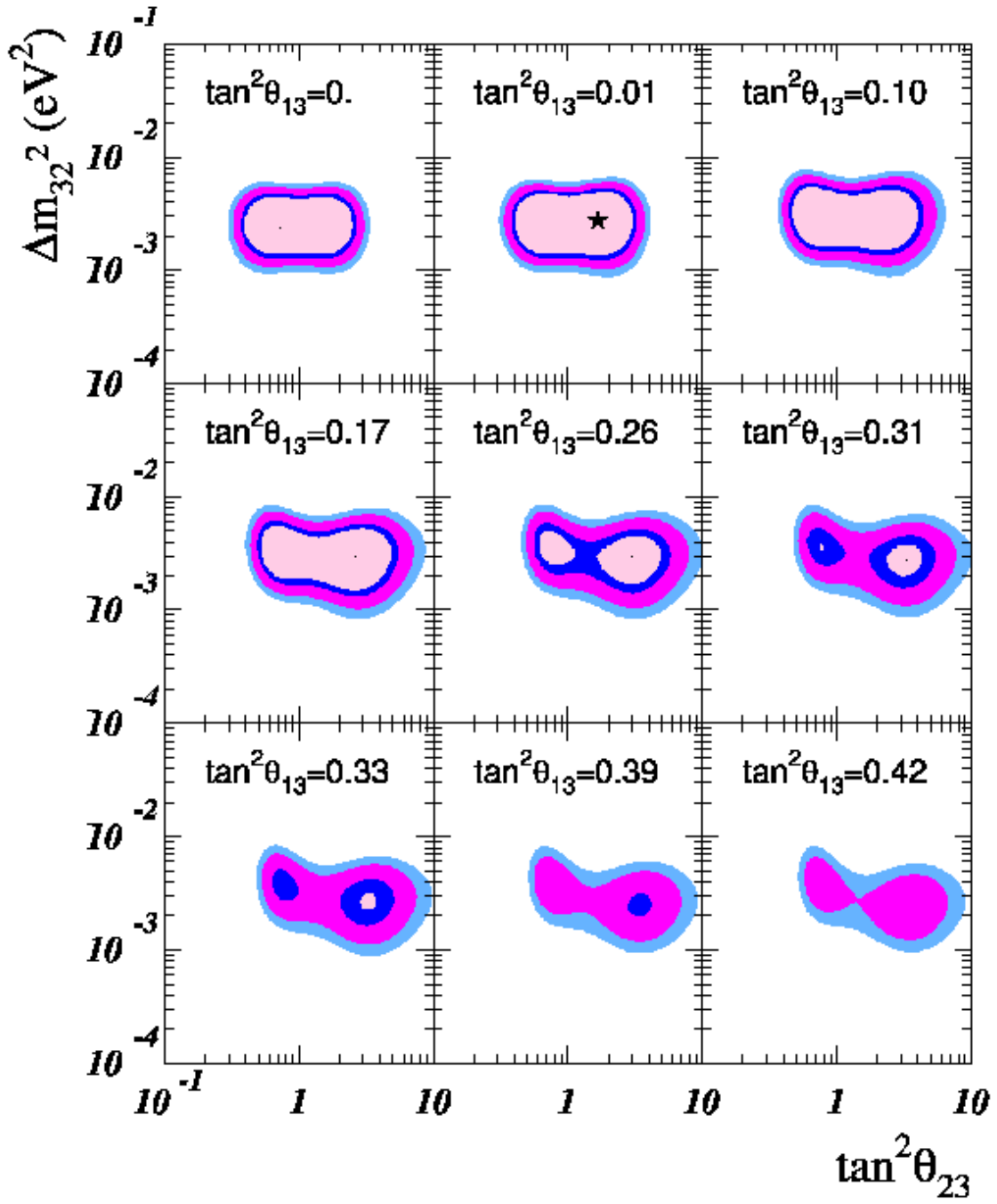


FIG. 38. Allowed regions (at 90, 95, 99, and 99.7% CL) in the  $(\Delta m_{32}^2, \tan^2 \theta_{23})$  plane from the global analysis of the atmospheric neutrino data in the framework of three-neutrino oscillations for various values of  $\tan^2 \theta_{13}$ . The global best fit point is marked by the star.

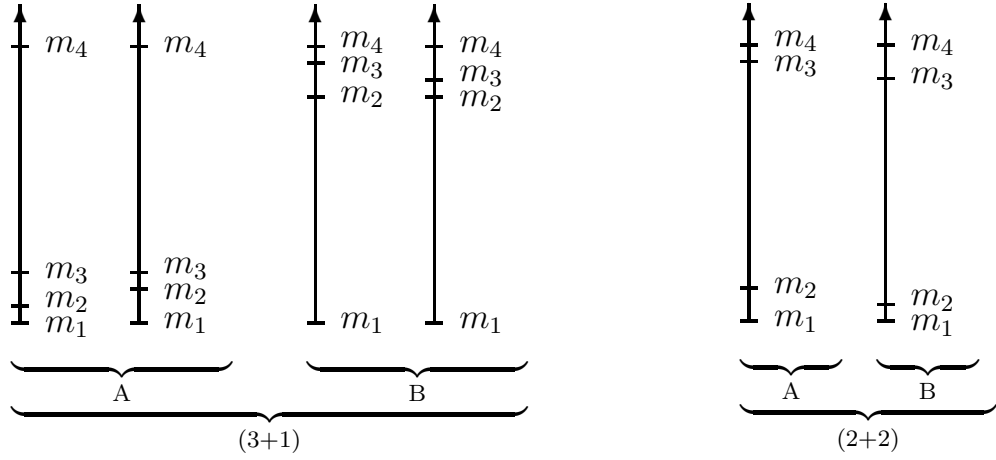


FIG. 39. The six types of 4-neutrino mass spectra. The different distances between the masses on the vertical axes represent the different scales of mass-squared differences required to explain solar, atmospheric and LSND data with neutrino oscillations.

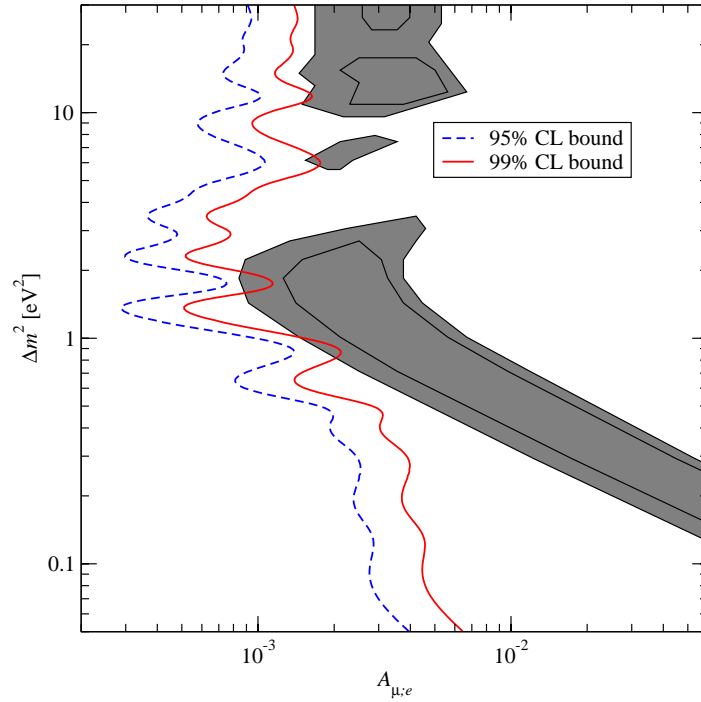


FIG. 40. Upper bounds (at 95 and 99% CL) on  $A_{\mu;e} \equiv 4|U_{e4}U_{\mu4}|^2$  in the context of (3+1)-schemes. The shaded regions are the regions allowed by LSND at 90 and 99% CL.

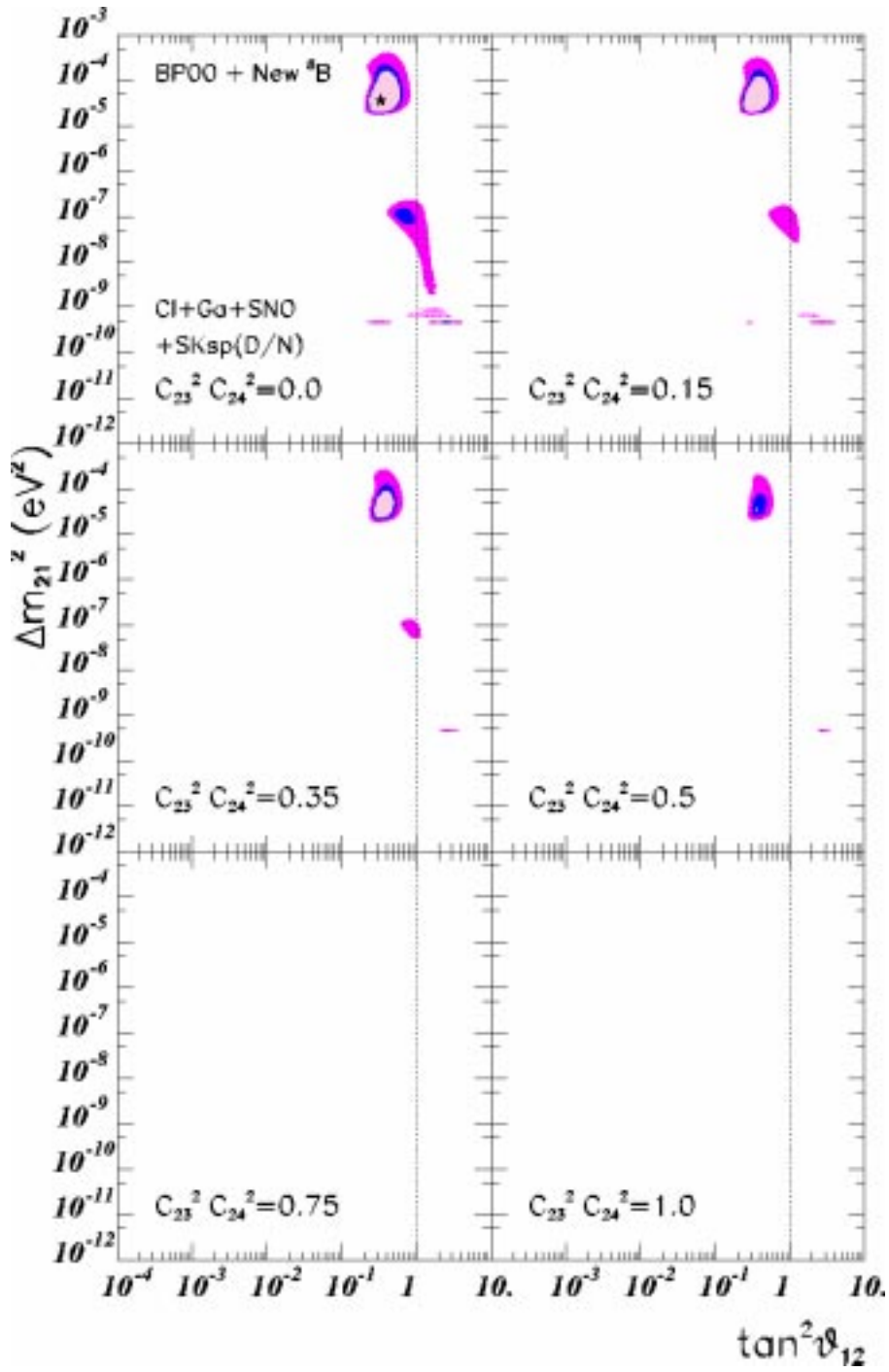


FIG. 41. Allowed regions in the  $(\Delta m_{21}^2, \tan^2 \theta_{12})$  plane in the (2+2)-schemes from the solar neutrino analysis. Each panel corresponds to a given value of the active-sterile admixture  $|V_{s1}^\nu|^2 + |V_{s2}^\nu|^2 = c_{23}^2 c_{24}^2$  and presents a section of the three-dimensional allowed regions at 90, 95 and 99% CL. The best-fit point in the three-parameter space is marked with a star.

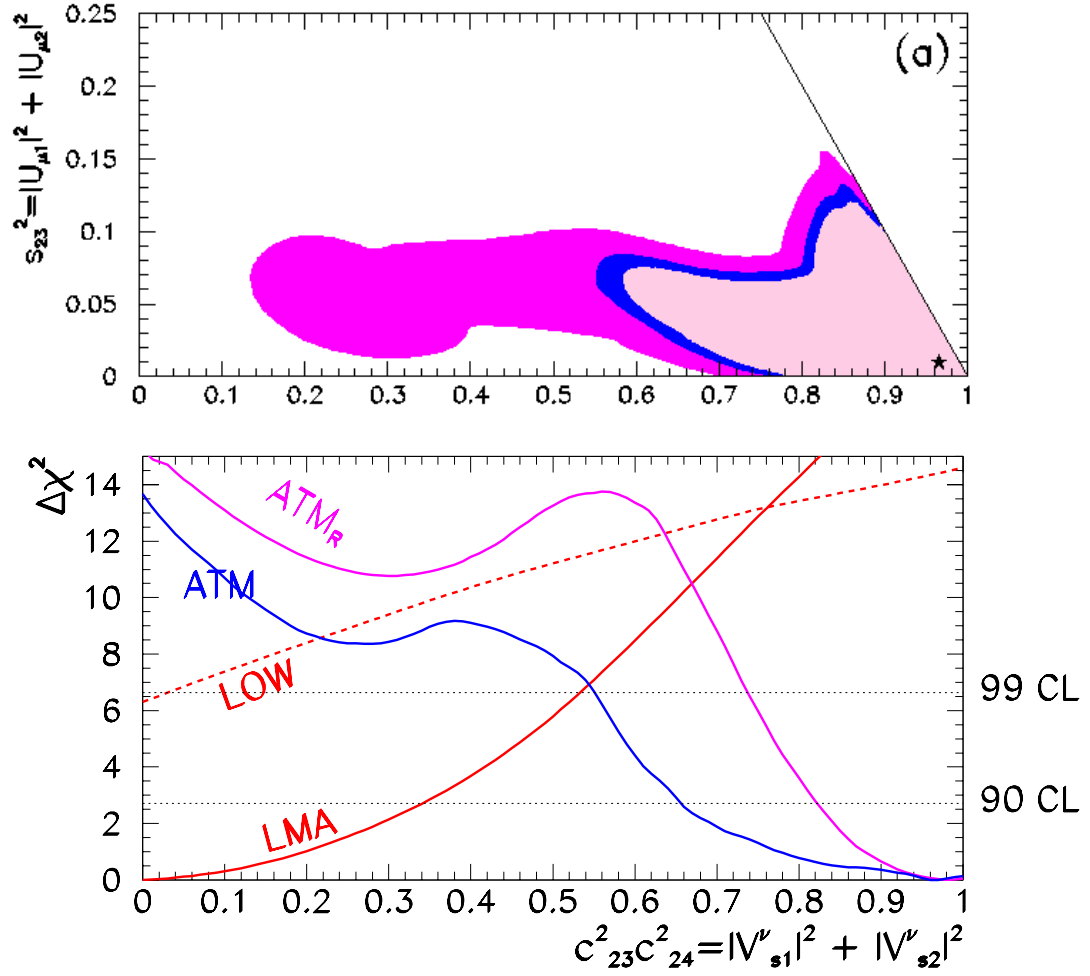


FIG. 42. (a) Allowed regions (at 90, 95 and 99% CL) in the  $(s_{23}^2, c_{23}^2 c_{24}^2)$  plane from the analysis of the atmospheric data in (2+2)-schemes. See text for details. (b)  $\Delta\chi^2$  as a function of the active-sterile admixture  $|V_{s1}^\nu|^2 + |V_{s2}^\nu|^2$  from the analysis of solar and atmospheric data in (2+2)-schemes.