Quantum Boltzmann equations for electroweak baryogenesis including gauge fields

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We review and extend to include the gauge fields our derivation of the semiclassical limit of the collisionless quantum transport equations for the fermions in presence of a CP-violating bubble wall at a first order electroweak phase transition. We show how the (gradient correction modified) Lorenz-force appears both in the Schwinger-Keldysh approach and in the semiclassical WKB-treatment. In the latter approach the inclusion of gauge fields removes the apparent phase reparametrization dependence of the intermediate calculations. We also discuss setting up the fluid equations for practical calculations in electroweak baryogenesis including the self-consistent (hyper)electric field and the anomaly.

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1 Introduction

Among the models put forward for creation of baryon asymmetry, those based on electroweak baryon number violation [1] during the electroweak phase transition (EWPT) stand out because the physics involved in the relevant processes is either being tested, or can soon be tested in accelerator experiments. A good example is the sphaleron wash-out constraint: to maintain an asymmetry created in EWPT, the baryon number violation must turn off in the broken phase. Whether this is the case, depends on the strength of the transition [2], which again depends on the model parameters, and can be studied using either effective potential or lattice methods. In particular it has been shown [3] that, given the current bound on the Higgs mass, the transition in the Minimal Standard Model (MSM) does not satisfy this constraint. So, to explain the origin of baryons one is led to consider extensions of the MSM, and perhaps the most natural is to introduce supersymmetry (SUSY). Among SUSY extensions of MSM, already the simplest Minimal Supersymmetric Standard Model (MSSM) [4] and its next to minimal extension (NMSSM) [5] contain additional scalars which can strengthen the phase transition as required for baryogenesis.

Because of the out-of-equilibrium requirement [6], baryons can only be created within the moving phase transition walls or in their immediate vicinity during EWPT. It is no surprise then that the ratio of the wall width ℓ_{wall} to the other characteristic scales, such as the scattering lengths $\ell_{\text{scatt}} \sim 1/\Gamma$ and de Broglie wave length $\ell_{\text{dB}} = 1/p \sim 1/T$ of the particles involved, strongly influences the mechanism of baryogenesis. The wall width also controls the CP-violation, which in general is introduced through complex phases in the mass matrices. When mass matrix elements depend on the higgs condensate, the phases of mass eigenstates can change over the wall profile influencing the propagation of particles and antiparticles in different ways. If the wall is wide in comparison with the typical de Broglie wave length $\ell_{\text{wall}} \ll \ell_{\text{dB}}$, these effects should be computable using semiclassical (SC) methods [7].

In supersymmetric models the bubble walls are found to be quite slow and thick [8, 9], so that the semiclassical approach [7] should be applicable. It has been used to compute the baryogenesis from chargino sector of MSSM [10, 11, 12] and in the NMSSM [13]. However, the semiclassical approach is based on the *assumption* that the plasma in a spatially varying background can be described by a collection of single particle WKB-excitations. Despite the obvious intuitive appeal, this assumption was lacking a fundamental proof until recently [14]. Proving the validity of the SC-approach was important, in particular because many other formulations of the transport equations in the limit of slowly varying walls had been but forward, which gave both quantitatively and qualitatively different results from the SC predictions [15]. Moreover, there was some initial confusion in identifying the correct physical excitations and their distribution functions even within the SC picture [7, 10], and a fully consistent SC-computation was introduced only in ref. [11].

Here we review our derivation of collisionless SC-transport equations for fermions with

spatially varying CP-violating mass terms [14] starting from Schwinger-Keldysh equations for the dynamical two point Green function (Wightman function) $G^<(u,v)$. Moreover, the analysis of [14] is extended to include the gauge fields, which give rise to the usual Lorenz-force on gauge-charged particles. We also generalize the WKB-analysis of ref. [11] to include the gauge fields, and show how the gauge degree of freedom is connected to the "reparametrization dependence" of canonical variables in the WKB-picture. As in [14] we concentrate on the 1+1-dimensional model; the full 3+1 dimensional case will be considered in ref. [16].

The most remarkable result of our analysis is that to first order in gradients (or equivalently in $\hbar$), the equations of motion admit a *spectral decomposition solution* for $G^<$ in terms of on-shell quasiparticle excitations; this qualitatively proves the validity of the above mentioned assumption of single particle picture underlying the SC-approach. Moreover, the on-shell momenta set by a dispersion relation derived from the equations of motion quantitatively agree with the SC results of [11] (and also with an earlier Schwinger-Dyson equations based calculation [17]). Finally, the on-shell distribution functions are defined for the quasiparticle excitations, and are shown to obey the usual kinetic (Vlasov) equations, with the SC group velocity $v_{s\pm} \equiv k_z/\omega_{s\pm}$ and the force $F_{s\pm} = \omega_{s\pm}dv_{s\pm}/dt$.

However, the equations of motion only admit a spectral decomposition solution to first order in $\hbar$. At higher orders more complicated structures arise and the SC-picture of a plasma consisting of single particle excitations breaks down. Quite fortuitiously for the EWBG-program this is all we need because, as we shall see, the dominant CP-violation terms appear precisely at order $\hbar$.

This contribution is organized as follows. In section 2 we derive the SC WKB-equations for a single Dirac fermion with a spatially varying complex mass term including the gauge field. In section 3 we derive the SC kinetic equation from Schwinger-Keldysh formalism, and in section 5 we study vector and axial vector divergences, and show how one sets up moment equations for practical baryogenesis computations. Finally, section 5 contains a discussion and summary.

2   WKB method

We will here derive the semiclassical dispersion relation for a fermionic field with a complex spatially varying mass term. More precisely, we take our system to be described by the effective lagrangian

$$\mathcal{L} = i\bar{\psi}(\slashed{\partial} + ie\slashed{A})\psi - \bar{\psi}_L m \psi_R - \bar{\psi}_R m^* \psi_L + \mathcal{L}_{\text{int}} \, , \tag{1}$$

where A_μ is the $U(1)$-gauge field, $\mathcal{L}_{\text{int}}$ contains (here unspecified) interactions and

$$m(z) = m_R(z) + im_I(z) = |m(z)|e^{i\theta(z)} \tag{2}$$

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is a mass term arising from an interaction with some CP-violating scalar field condensate with a planar symmetry. Planar symmetry effectively reduces the problem to 1+1 dimensions* (by use of boosts), and to keep the notation simple and consistent with the rest of the paper, we will assume this to be strictly the case here.

The basic assumption in the semiclassical treatment is that the plasma can be approximated by a collection of single particle excitations, whose dispersive properties can be studied by employing the WKB-methods. The lagrangian (1) implies the following Dirac equation for the spinor wave function $\psi(z, t)$:

$$\left(i\partial\!\!\!/ - e\,A\!\!\!/ - m_R - i\gamma^5 m_I\right)\psi = 0. \tag{3}$$

We are interested in stationary solutions of the form $\psi(t, z) = e^{-ik_0 t}\psi(k_0; z)$. Eq. (3) then becomes

$$\left(\gamma^0 \kappa_0 + i\gamma^3(\partial_z - ieA_z) - m_R - i\gamma^5 m_I\right)\psi(k_0; z) = 0, \tag{4}$$

where $\kappa_0 \equiv k_0 - eA_0$. Here spin is also a good quantum number and we can write the wave function as a direct product of chiral and spin degrees of freedom:

$$\psi \equiv \begin{pmatrix} L_s \\ R_s \end{pmatrix} \otimes \chi^s, \tag{5}$$

where χ^s are the eigenspinors of the spin operator σ^3. Eq. (4) then splits to two coupled equations or the left and right chiral wave functions:

$$\begin{align}
(\kappa_0 - esA_z - is\partial_z)L_s &= mR_s \\
(\kappa_0 + esA_z + is\partial_z)R_s &= m^*L_s, \tag{6}
\end{align}$$

One can use, say, the first of equations (6) to eliminate R_s from the second one, to obtain a second order equation for L_s. That equation can be analyzed by making the WKB-parametrization for L_s:

$$L_s \equiv u_s e^{i\int^z k_s dz'}. \tag{7}$$

In this way one finds the following two real valued equations

$$\kappa_0^2 - \kappa_3^2 - |m|^2 + (s\kappa_0 + \kappa_3)\theta' + \frac{u_s''}{u_s} - \frac{|m|'}{|m|}\frac{u_s'}{u_s} = 0 \tag{8}$$

$$\kappa_3' + (2\kappa_3 - \theta')\frac{u_s'}{u_s} - (s\kappa_0 + \kappa_3)\frac{|m|'}{|m|} = 0, \tag{9}$$

where $\kappa_3 \equiv k_s - eA_z$. These equations are exactly analogous to the ones obtained in [11], with the sole exception that the bare momentum of [11], has here been replaced by the one including the gauge field:

$$k_\mu \to \kappa_\mu \equiv k_\mu - eA_\mu. \tag{10}$$

*In partiuclar the gauge field components $A_{x,y}$ can be taken to vanish due to planar symmetry.

In absence of the gauge field, the WKB-equations of [11] were variant under phase redefinitions of the ψ-field, which led to the "reparametrization variance" of the canonical momentum. Equations (8) and (9) are clearly invariant under complete gauge transformations $\psi \to e^{i\alpha}\psi$ and $A_\mu \to A_\mu - (i/e)\partial_\mu\alpha$, and the reparametrization variance of [11] is here seen to correspond to the usual freedom of adding any four divergence of a scalar function $\partial_\mu\phi$ to the gauge potential A_μ.

While complex in appearance, equations (8-9) have the advantage of being readily solved iteratively in gradient expansion. This part of the analysis is analogous to one shown in [11], and hence we only quote the final results. To the first order in gradients the dispersion relation reads

$$\kappa_3 \equiv k_s - eA_\mu = p_0 + s_{\text{CP}}\frac{(sk_0 + p_0)\theta'}{2p_0}, \tag{11}$$

where $p_0 = \text{sign}(p_s)\sqrt{k_0^2 - |m|^2}$ and $s_{\text{CP}} = 1(-1)$ for particles (antiparticles). Equation (11) can be inverted to give the energy in terms of the momentum variable κ_3:

$$\kappa_0 \simeq \sqrt{(\kappa_3 - s_{\text{CP}}\frac{s\theta'}{2})^2 + |m|^2} - s_{\text{CP}}\frac{s\theta'}{2}. \tag{12}$$

Now it is easy to compute the physical momentum, which is defined as usual, in terms of the group velocity $k_z \equiv \kappa_0 v_g = \kappa_0(\partial_{\kappa_3}\kappa_0)_z$. After some algebra one finds

$$k_z = p_0 + s_{\text{CP}}\frac{s|m|^2\theta'}{2p_0\kappa_0}. \tag{13}$$

Finally, the physical force is defined as $F_z = \dot{k}_z$. The only difference between the computation here and in ref. [11] is that in the presence of an electric field the energy κ_0 is not conserved; indeed after some algebra one finds the expected result $\dot{\kappa}_0 = ev_gE_z$, where E_z is the electric field in the z-direction. Similarly, the classical force is found to be

$$F_z = \dot{\kappa}_0 v_g + \kappa_0\dot{v}_g = -\frac{|m|^{2'}}{2\kappa_0} + s_{\text{CP}}\frac{s(|m|^2\theta')'}{2\kappa_0^2} + eE_z\left(1 - s_{\text{CP}}\frac{s|m|^2\theta'}{2\kappa_0^3}\right). \tag{14}$$

We have here computed all expressions to the first order in gauge field and to the second order in gradients. If the electric field E_z is a self-consistent field created by the charge separation due to CP-violating force in (14), it is formally a second order term and the gradient correction multiplying eE_z should be dropped. According to the semiclassical assumption the Vlasov equation for the phase space density of collisionless WKB-excitations can now be written as

$$\partial_t f_s + v_g\partial_z f_s + F_z\partial_{k_z}f_s = 0. \tag{15}$$

The above computation is improvement upon ref. [11] in that we include also the self consistent electric field which is necessarily generated by the charge separation induced

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by the wall. It also better explains the phase reparametrization variance observed in connection with the canoncial momentum. The WKB-derivation however depends on the important assumption that the plasma in a spatially varying background can be described in a single particle WKB-picture. To determine whether this really is so, and to which accuracy, one has to go beyond the semiclassical approximation. This is what we do in the next section.

3 Schwinger-Keldysh method

The statistical properties of an out-of-equilibrium system can be described by the two point Wightman function [18]

$$G_{\alpha\beta}^{<}(u,v) = i\langle\bar{\psi}_{\alpha}(u)e^{ie\int_{v}^{u}dy\cdot A}\psi_{\beta}(v)\rangle, \tag{16}$$

where $\langle\cdot\rangle$ denotes the expectation value with respect to the initial state and the path for the gauge flux is defined as

$$\int_{v}^{u}dy\cdot A \equiv \int_{-1/2}^{1/2}ds\,r^{\mu}A_{\mu}(x+sr), \tag{17}$$

where $r \equiv u - v$ denotes the relative and $x = (u+v)/2$ the center-of-mass coordinate. The definition (16) is a gauge-invariant generalization [19] of the 2-point function used in [14]. $G^{<}$ corresponds to the off-diagonal part of the fermionic two-point function in the Schwinger-Keldysh formalism [20], which indeed is the method of choice to derive the equations of motion for $G^{<}$ including interactions. However, for noninteracting system studied here, all one needs is the familiar Dirac equation. Using Eq. (3) one finds:

$$\left(i\not{D}_{u} - m(u)^{*}P_{L} - m(u)P_{R} + e\gamma^{\mu}\frac{\partial}{\partial u_{\mu}}\int_{v}^{u}dx\cdot A\right)G^{<}(u,v) = 0. \tag{18}$$

In order to study equation (18) in gradient expansion we performa a Wigner transform of $G^{<}$ to the mixed representation:

$$G^{<}(x,k) \equiv \int d^{4}r\,e^{ik\cdot r}G^{<}(x+r/2, x-r/2), \tag{19}$$

where r and x are defined as in (17). In the mixed representation the equation of motion becomes

$$\left(\not{\Pi}(k,x) + \frac{i}{2}\not{D}(k,x) - m_{R}(x-\frac{i}{2}\partial_{k}) - im_{I}(x-\frac{i}{2}\partial_{k})\gamma^{5}\right)G^{<}(k,x) = 0. \tag{20}$$

where

$$m_{R,I}(x-\frac{i}{2}\partial_{k}) \equiv m_{R,I}(x)e^{-\frac{i}{2}\overleftarrow{\partial}_{x}\cdot\partial_{k}} \tag{21}$$

and the gauge generalized derivative and momentum terms are (in agreement with Zhuang and Heinz [19]):

$$D_\mu(x,k) \equiv \partial_\mu - e \int_{-1/2}^{+1/2} ds F_{\mu\nu}(x - is\partial_k)\partial_{k_\nu}$$

$$\Pi_\mu(x,k) \equiv k_\mu - ie \int_{-1/2}^{+1/2} dss F_{\mu\nu}(x - is\partial_k)\partial_{k_\nu}. \tag{22}$$

Above equations of motion are manifestly gauge invariant, and one can in fact always rewrite the $F_{\mu\nu}$-terms in any of the above formulas in terms of physical electric and magnetic fields. Using $F_{0i} = E_i$ and $F_{ij} = -\epsilon_{ijk}B_k$ one finds

$$\gamma^\mu F_{\mu\nu}\partial_{k_\nu} = -\gamma^0 \vec{E} \cdot \partial_{\vec{k}} + \vec{\gamma} \cdot \vec{E} \, \partial_{k_0} + \vec{\gamma} \cdot (\vec{B} \times \partial_{\vec{k}}). \tag{23}$$

The crucial advantage of the representation (19) is that it separates the internal fluctuation scales, described by momenta k, from the external ones which show up as a dependence of $G^<$ on the center-of-mass coordinate x, and thus gives us the chance of exploiting possible hierarchies between these scales. Indeed, when the external fields vary slowly, one can truncate the Taylor series operator expansions of the mass and gauge field operators appearing in (21-23) to some finite order in gradients (or equivalently in powers of $\hbar$). To the lowest order we get

$$D_\mu(x,k) \simeq \partial_\mu - e \, F_{\mu\nu}(x)\partial_{k_\nu}$$
$$\Pi_\mu(x,k) \simeq k_\mu. \tag{24}$$

In the planar symmetric case of an electroweak bubble wall, there can be no magnetic field in the plasma frame. Also $G^<$ can only depend on the spatial coordinate orthogonal to the wall, $z \equiv x^3$. Continuing to work in the strictly 1+1-dimensional case like in the previous section, we also drop the $k_\parallel$-term. With these simplifications the equation (20) reduces to

$$\left(\hat{k}_0 + \hat{k}_z\gamma^0\gamma^3 - \hat{m}_0\gamma^0 + i\hat{m}_5\gamma^0\gamma^5\right) i\gamma^0 G^< = 0, \tag{25}$$

where we used the following shorthand notations

$$\hat{k}_0 \equiv k_0 + \frac{i}{2}(\partial_t + eE_z\partial_{k_z})$$

$$\hat{k}_z \equiv k_z - \frac{i}{2}(\partial_z + eE_z\partial_{k_0}) \tag{26}$$

where E_z is the electric field in the z-direction, and

$$\hat{m}_{0(5)} \simeq m_{R(I)}(z) - \frac{i}{2}m'_{R(I)}\partial_{k_z}. \tag{27}$$

Apart from the electric field E_z appearing in the definitions (26), equation (25) is identical to the one we studied in [14]. Hence the further analysis proceeds in very a similar fashion,

we can skip many of the details here. Due to conservation of spin in z-direction $G^<$ can be decomposed as follows:

$$-i\gamma^0 G_s^< = \frac{1}{2}(1 + s\sigma^3) \otimes g_s^<, \tag{28}$$

where σ^3 is the usual Pauli matrix. Using this structure, and making the appropriate identifications $\gamma^0 \to \rho^1$, $-i\gamma^0\gamma^5 \to \rho^2$ and $-\gamma^5 \to \rho^3$, the original 4×4 problem reduces to a two-dimensional one:

$$\left(\hat{k}_0 - s\hat{k}_z\rho^3 - \rho^1\hat{m}_0 - \rho^2\hat{m}_5\right) g_s^< = 0, \tag{29}$$

where ρ^i are the Pauli matrices, which span the remaining chiral indices. The hermitean matrices $g_s^<$ can be expanded as

$$g_s^< \equiv \frac{1}{2}\left(g_0^s + g_i^s\rho^i\right), \tag{30}$$

where the signs and normalizations are chosen such that g_0^s measures the number density of particles with spin s in phase space.

Equation (29) still constitutes a set of four coupled complex differential equations for components of g_a^s ($a = 0, i$). They can be obtaned by multiplying (29) successively by 1 and ρ^i and taking the trace:

$$\hat{k}_0 g_0^s - s\hat{k}_z g_3^s - \hat{m}_0 g_1^s - \hat{m}_5 g_2^s = 0 \tag{31}$$
$$\hat{k}_0 g_3^s - s\hat{k}_z g_0^s - i\hat{m}_0 g_2^s + i\hat{m}_5 g_1^s = 0 \tag{32}$$
$$\hat{k}_0 g_1^s + is\hat{k}_z g_2^s - \hat{m}_0 g_0^s - i\hat{m}_5 g_3^s = 0 \tag{33}$$
$$\hat{k}_0 g_2^s - is\hat{k}_z g_1^s + i\hat{m}_0 g_3^s - \hat{m}_5 g_0^s = 0. \tag{34}$$

The real parts of these equations contain no time derivatives and hence are called *constraint equations* (CE). Their very important role is to constrain the available space of solutions for the four dynamical, *kinetic equations* (KE), which correspond to the imaginary parts of (31-34).

3.1   Constraint equations

We first use three out of the four constraint equations to iteratively solve g_i^s in terms of g_0^s and its derivatives. These expressions are then used to eliminate g_i^s from the last remaining CE. The most remarkable property of this equation is that all terms proportional to $\partial_{k_z} g_0^s$, also those involving the field E_z, cancel to first order in gradients (or $\hbar$). As a result g_0^s satisfies an *algebraic* equation

$$\left(k_0^2 - k_z^2 - |m|^2 + \frac{s}{k_0}|m|^2\theta'\right) g_0^s = 0, \tag{35}$$

which admits the spectral solution

$$g_0^s = \sum_{\pm} \frac{\pi}{2Z_{s\pm}} n_s\, \delta(k_0 \mp \omega_{s\pm})\,. \tag{36}$$

Here $n_s(k_0, k_z, z)$ are some nonsingular functions, the on-shell energies $\omega_{s\pm}$ are specified by the roots of equation

$$\Omega_s^2 \equiv k_0^2 - k_z^2 - |m|^2 + \frac{s}{k_0}|m|^2\theta' = 0, \tag{37}$$

and the normalization factor is defined as $Z_{s\pm} \equiv \frac{1}{2\omega_{s\pm}}|\partial_{k_0}\Omega_s^2|_{k_0=\pm\omega_{s\pm}}$.

The solution (36) is in many ways the most important result of our analysis, because it precisely proves the WKB-assumption that the plasma, to a certain accuracy, can be described by a collection of single particle on-shell excitations. Below we shall further show that also the WKB-dispersion relation and the semiclassical equations of motion of ref. [11] do arise in this picture. It is important to note however, that the confinement to sharp energy shells is only valid up to first order in gradients; beyond first order the constraint equation for g_0^s is no longer algebraic and hence does not admit a spectral solution, implying breakdown of the SC picture.

We end this subsection noting that, when treated to the first order in gradients, Eqn. (37) yields the solution

$$\omega_{s\pm} = \omega_0 \mp s\frac{|m|^2\theta'}{2\omega_0^2}\,, \qquad \omega_0 = \sqrt{k_z^2 + |m|^2}. \tag{38}$$

Inverting this to give k_z in terms of energy, one finds it to be identical with the WKB-relation (13) for the physical momentum.

3.2 Kinetic equations

We now turn our attention to the kinetic equations. One could choose any one out of the four equations to describe the dynamics along with the constraint equations. The quantity of most direct interest is g_0^s however, since it carries information on the particle density in phase space. From (31) one gets

$$(\partial_t + eE_z\partial_{k_z})g_0^s + s(\partial_z + eE_z\partial_{k_0})g_3^s - m_R'\partial_{k_z}g_1^s - m_I'\partial_{k_z}g_2^s = 0. \tag{39}$$

Using constraint equations to eliminate g_i^s as in the previous section one finds

$$\partial_t g_0^s + \frac{k_z}{k_0}\partial_z g_0^s + eE_z\partial_{k_0}\left(\frac{k_z}{k_0}g_0^s\right)$$
$$- \left(\frac{|m|^{2\prime}}{2k_0} - \frac{s}{2k_0^2}(|m|^2\theta')' - eE_z\left(1 - \frac{s|m|^2\theta'}{2k_0^3}\right)\right)\partial_{k_z}g_0^s = 0. \tag{40}$$

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We have so far used three out of four constraint equations. The fourth constraint (36) forces the solutions to the single particle on-shell, and is easily accounted for by inserting the spectral solution (36) into (40) and then separately integrating over the positive and negative frequencies k_0. The total divergence term with respect to k_0 drops and get the familiar form for the Vlasov equation

$$\partial_t f_{s\pm} + v_{s\pm}\partial_z f_{s\pm} + F_{s\pm}\partial_{k_z} f_{s\pm} = 0, \tag{41}$$

where $f_{s+} \equiv n_s(\omega_{s+}, k_z, z)$ and $f_{s-} \equiv 1 - n_s(-\omega_{s-}, -k_z, z)$ are the particle and antiparticle distribution functions with spin s, respectively, the quasiparticle group velocity is given by $v_{s\pm} \equiv k_z/\omega_{s\pm}$ and the spin-dependent and CP-violating *semiclassical force* reads

$$F_{s\pm} = -\frac{|m|^{2\prime}}{2\omega_{s\pm}} \pm \frac{s(|m|^2\theta')'}{2\omega_{s\pm}^2} + eE_z\left(1 \mp \frac{s|m|^2\theta'}{2\omega_{s\pm}^3}\right). \tag{42}$$

in complete analogy with the semiclassical result. The physcial significance of (42) is that due to derivative (quantum) corrections the spin degeneracy is lifted at first order in gradients and hence particles (antiparticles) of different spin experience different accelerations in a spatially varying background.

4   Continuity equations and sources for BG

We now investigate how the CP-violating sources appear in the equations for the divergences of the vector and axial vector currents. First note that any current operator can be computed as a trace in the Wigner representation, weighted by $G^<$:

$$\langle\bar\psi(x)X^\mu\psi(x)\rangle = \int\frac{d^2k}{(2\pi)^2}\mathrm{Tr}(-iG^<X^\mu). \tag{43}$$

For vector current $X^\mu = \gamma^\mu$ and for axial one $X^\mu = \gamma^\mu\gamma^5$. Using the above expressions and constraint equations from (31-34) one can verify that the divergence of the vector current is conserved and the axial one is broken by the complex mass as expected: [†]

$$\partial_\mu j^\mu = 0 \tag{44}$$

$$\partial_\mu j_5^\mu = 2iM_R\langle\bar\psi\gamma^5\psi\rangle - 2M_I\langle\bar\psi\psi\rangle, \tag{45}$$

[†]Note that the sign of the m_R-term in the axial current divergence was incorrect in our original work [14]. That error was compensated however, by a another trivial sign error in the definition of the pseudoscalar density in terms of g_2^s.

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where $\langle \bar{\psi}_s \psi_s \rangle = \int_k g_1^s$ denotes the scalar, and $\langle \bar{\psi}_s \gamma^5 \psi_s \rangle = -i \int_k g_2^s$ the pseudoscalar density, and $\int_k \equiv \int d^2k/(2\pi)^2$. If we define the fluid density[‡] $n_{s\pm}$ and fluid velocity moments as[§]

$$n_{s+} \equiv \int_+ \frac{d^2k}{(2\pi)^2} g_0^s$$

$$\langle v_{s+}^p \rangle \equiv \frac{1}{n_{s+}} \int_+ \frac{d^2k}{(2\pi)^2} \left(\frac{k_z}{k_0} \right)^p g_0^s, \tag{46}$$

then Eqs. (44-45) can be written as:

$$\partial_t n_{s\pm} + \partial_z (n_{s\pm} \langle v_{s\pm} \rangle) = 0, \tag{47}$$

$$\partial_t (n_{s\pm} \langle v_{s\pm} \rangle) + \partial_z \left(n_{s\pm} \langle v_{s\pm}^2 \rangle \right) = eE_z |m|^2 \mathcal{I}_{3s+} + S_{s\pm}, \tag{48}$$

where the quantity $S_{s\pm}$ is related to the baryogenesis source[¶] [11], and is given by the average over the semiclassical force (42) divided by k_0:

$$S_{s\pm} = -\frac{1}{2} |m|^{2\prime} \mathcal{I}_{2s\pm} + \frac{1}{2} s_{\text{CP}} s (|m|^2 \theta')' \mathcal{I}_{3s\pm}. \tag{49}$$

with

$$\mathcal{I}_{ps+} \equiv \int_+ \frac{d^2k}{(2\pi)^2} \frac{g_{0i}^s}{k_0^p}. \tag{50}$$

In anticipation of the fact that E_z is a self-consistent field, we dropped the gradient corrections to the Lorenz force above. The $|m|^2$-suppression in the E_z-term is particular to 1+1 dimensions, and to our use of velocity moments in fluid equations as in ref. [11]. In the actual baryogenesis computation one expects that the role of the electric field is expected to be taken by the hyperelectric field corresponding to the hypercharge $U(1)_B$ symmetry.

From expressions (47) and (48) it is obvious that the vector current divergence corresponds to the zeroth and the axial current divergence to the first velocity momentum of the kinetic equation (42). The former provides just the usual continuity equation, and the nontrivial CP-violating source and the electric field only appear in the latter. This connection with the axial current nicely explains why the semiclassical source appeares only at first order in moment expansion in earlier treatments [7, 11].

[‡]The fluid density $n_{s\pm}$ should not be confused with the phase space density n_s in (36).

[§]Here we defined integration over the positive frequencies. To obtain equations for antiparticles one integrate over negative frequencies taking into account the appropriate thermal equilibrium limit as in derivation of (42).

[¶]Note that the equations (47-48) are somewhat formal, and require more work before useful for practical computations. In particular the velocity-dependence of the actual source only comes about after one expands various quantities in the wall velocity around the equilibrium distribution. For details see [11] and [16].

Equations (47-48) do not close, because the field E_z is not yet determined and since the two equations contain three other kinetic unknowns $n_{s\pm}$, $\langle v_{s\pm} \rangle$ and $\langle v_{s\pm}^2 \rangle$. The former problem can be handled [22] by adding the Gauss law

$$\partial_z E_z = e \sum_s (n_{s+} - n_{s-}), \tag{51}$$

which completely defines the self-consistent electric field. The latter problem is a standard one for the moment expansions and requires a truncation approximation. For example one may set $\langle v_{s\pm}^2 \rangle \rightarrow \langle v_{s\pm} \rangle^2 + \sigma_{s\pm}^2$ and use the equilibrium distributions for $f_{s\pm}$'s to evaluating the variance $\sigma_{s\pm}^2 \equiv \langle v_{s\pm}^2 \rangle - \langle v_{s\pm} \rangle^2$ and the integrals $\mathcal{I}_{ps\pm}$ appearing in the source term (49).

The first source-term in (49) is primarily relevant for phase transition dynamics [8, 9], but it also contains a spin dependent and CP-violating part coming from the wave function renormalization [14]. That term, together with the second source provide the charge separation necessary for baryogenesis. To promote equations (47-48) and (51) into transport equations for baryogenesis calculations we still need to generalize them to include collisions. For a discussion of this problem see the talk of Tomislav Prokopec in these proceedings [23].

Let us finally discuss the role of the gauge anomaly in the fluid equations. In the 1+1-dimensional case the U(1)-theory is anomalous, and in general the anomaly [24]

$$(\partial_\mu j_5^\mu)_{\text{anomalous}} = -\frac{e}{2\pi} \epsilon_{\mu\nu} F^{\mu\nu} \tag{52}$$

should be added to the axial vector divergence (45). Because the axial current divergence, apart from the anomaly, has been here shown to coincide with the first velocity moment of the fluid equations [14], we can draw the following corollary: the first two moments of the fermionic transport equations, with the anomalous term added to the latter, together with the appropriate dynamical Langevin equations of motion for the gauge fields, provides a complete and self-consistent method for treating the CP-, P- and fermion number violating (baryon number violating) out-of-equilibrium interactions required for electroweak baryogenesis.

5   Discussion and summary

The question of a first principle derivation of CP-violating fluxes in transport equations has been the main theoretical challenge of recent work on electroweak baryogenesis. In this talk we reviewed our derivation [14] of the kinetic equations appropriate for EWBG in a systematic and controlled gradient expansion from Schwinger-Keldysh equations for two point Wightmann function, and extended it to include the gauge fields. We also generalized the derivation of WKB-dispersion relations [11] to include the gauge degree of freedom.

We have shown that to first order in $\hbar$ the collisionless kinetic equations for fermions allow a single particle description with a distribution function which obeys a familiar Vlasov equation where the group velocity and the semiclassical force terms contain all relevant quantum information, and in particular the CP-violating terms which source baryogenesis. These results agree with the SC equations derived in the WKB-picture in Ref. [11], originally developed for EWBG problem in [7]. A novel new feature discovered in our approach is that the semiclassical limit is *only* valid to first order in $\hbar$. Fortunately this is not a serious drawback to the EWBG-program, as it is just the first order term that carries the dominant source for baryogenesis.

For simplicity we have considered only the collisionless limit in 1+1 dimensions. One can show [16] that generalization to the 3+1 dimensional case does not affect results qualitatively, and some aspects of collision terms are considered by Tomislav Prokopec in these proceedings. For the lack of space within this presentation we also restricted us to the case of a single Dirac fermion. For the case of mixing fermionic fields see ref. [14].

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References

[1] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155** (1985) 36.

[2] M. E. Shaposhnikov, JETP Lett. **44** (1986) 465 [Pisma Zh. Eksp. Teor. Fiz. **44** (1986) 364]; M. E. Shaposhnikov, Nucl. Phys. B **287** (1987) 757.

[3] K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. B **493** (1997) 413 [hep-lat/9612006]; Phys. Rev. Lett. **77** (1996) 2887 [hep-ph/9605288]; Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020].

[4] M. Carena, M. Quiros and C. E. Wagner, Phys. Lett. B **380** (1996) 81 [hep-ph/9603420]; Nucl. Phys. B **524** (1998) 3 [hep-ph/9710401]; J.M. Cline and K. Kainulainen, Nucl. Phys. B **482** (1996) 73 [hep-ph/9605235]; Nucl. Phys. B **510** (1998) 88 [hep-ph/9705201]; M. Laine, Nucl. Phys. B **481** (1996) 43 [Erratum-ibid. B **548** (1996) 637] [hep-ph/9605283]; M. Losada, Phys. Rev. D **56** (1997) 2893 [hep-ph/9605266]; D. Bö deker, P. John, M. Laine and M. G. Schmidt, Nucl. Phys. B **497** (1997) 387 [hep-ph/9612364]; M. Laine and K. Rummukainen, Nucl. Phys. B **535** (1998) 423 [hep-lat/9804019]; Nucl. Phys. B **597** (2001) 23 [hep-lat/0009025].

[5] M. Pietroni, Nucl. Phys. B **402** (1993) 27 [hep-ph/9207227]; A. T. Davies, C. D. Frog-
gatt and R. G. Moorhouse, Phys. Lett. B **372** (1996) 88 [hep-ph/9603388]; S. J. Huber
and M. G. Schmidt, Eur. Phys. J. C **10** (1999) 473 [hep-ph/9809506].

[6] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. **5** (1967) 32 [JETP Lett. **5** (1967) 24].

[7] M. Joyce, T. Prokopec and N. Turok, Phys. Rev. Lett. **75** (1995) 1695 [Erratum-ibid.
75 (1995) 3375] [hep-ph/9408339]; M. Joyce, T. Prokopec and N. Turok, Phys. Rev.
D **53** (1996) 2958 [hep-ph/9410282].

[8] G. D. Moore and T. Prokopec, Phys. Rev. D **52** (1995) 7182 [hep-ph/9506475];
G. Moore and T. Prokopec, Phys. Rev. Lett. **75** (1995) 777 [hep-ph/9503296]. P. John
and M. G. Schmidt, Nucl. Phys. B **598** (2001) 291 [hep-ph/0002050].

[9] J. M. Moreno, M. Quiros and M. Seco, Nucl. Phys. B **526** (1998) 489 [hep-
ph/9801272]; J. M. Cline, G. D. Moore and G. Servant, Phys. Rev. D **60** (1999)
105035 [hep-ph/9902220]. G. D. Moore, JHEP **0003** (2000) 006 [hep-ph/0001274].
J. M. Cline and G. D. Moore, Phys. Rev. Lett. **81** (1998) 3315 [hep-ph/9806354].

[10] J. M. Cline, M. Joyce and K. Kainulainen, Phys. Lett. B **417** (1998) 79 [Erratum-ibid.
B **448** (1998) 321] [hep-ph/9708393].

[11] J. M. Cline and K. Kainulainen, Phys. Rev. Lett. **85** (2000) 5519 [hep-ph/0002272].
J. M. Cline, M. Joyce and K. Kainulainen, JHEP **0007** (2000) 018 [hep-ph/0006119].
K. Kainulainen, Proceedings of COSMO 99, Trieste, Italy, Oct 1999, p. 383, World
Scientific 2000 [hep-ph/0002273].

[12] S. J. Huber, P. John and M. G. Schmidt Euro. Phys. J. C. 20 (2001) 695 [hep-
ph/0101249].

[13] S. J. Huber and M. G. Schmidt Nucl.Phys.B606(2001)183 [hep-ph/0003122]; [hep-
ph/0011059]; S. J. Huber, P. John, M. Laine and M. G. Schmidt, Phys. Lett. B **475**
(2000) 104 [hep-ph/9912278].

[14] K. Kainulainen, T. Prokopec, M. G. Schmidt and S. Weinstock, JHEP **0106**, 031
(2001) [hep-ph/0105295].

[15] M. Carena, M. Quiros, A. Riotto, I. Vilja and C. E. Wagner, Nucl. Phys. B **503**
(1997) 387 [hep-ph/9702409]. M. Carena, J. M. Moreno, M. Quiros, M. Seco and
C. E. Wagner, Nucl. Phys. B **599** (2001) 158 [hep-ph/0011055].

[16] K. Kainulainen, T. Prokopec, M. G. Schmidt and S. Weinstock, in preparation.

[17] M. Joyce, K. Kainulainen and T. Prokopec, JHEP **0010** (2000) 029 [hep-ph/0002239].

[18] J. S. Schwinger, Phys. Rev. **128** (1962) 2425; J. S. Schwinger, Phys. Rev. Lett. **3** (1959) 296; D. Vasak, M. Gyulassy and H. T. Elze, Annals Phys. **173** (1987) 462.

[19] P. Zhuang and U. Heinz, Phys. Rev. D **57** (1998) 6525 [hep-ph/9801221].

[20] J. Schwinger, J. Math. Phys. **2** (1961) 407; L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47** (1964) 1515 [Sov. Phys. JETP **20** (1964) 1018].

[21] M. Joyce, K. Kainulainen and T. Prokopec, Phys. Lett. B **468** (1999) 128 [hep-ph/9906411].

[22] J. M. Cline and K. Kainulainen, Phys. Lett. B **356**, 19 (1995) [arXiv:hep-ph/9506285].

[23] K. Kainulainen, T. Prokopec, M. G. Schmidt and S. Weinstock, [arXiv:hep-ph/0201245].

[24] See, for example, M. E. Peskin and D. V. Schroeder, "An Introduction To Quantum Field Theory," *Reading, USA: Addison-Wesley (1995) 842 p.*