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Some aspects of collisional sources for electroweak baryogenesis

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We consider the dynamics of fermions with a spatially varying mass which couple to bosons through a Yukawa interaction term and perform a consistent weak coupling truncation of the relevant kinetic equations. We then use a gradient expansion and derive the CP-violating source in the collision term for fermions which appears at first order in gradients. The collisional sources together with the semiclassical force constitute the CP-violating sources relevant for baryogenesis at the electroweak scale. We discuss also the absence of sources at first order in gradients in the scalar equation, and the limitations of the relaxation time approximation.

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1 Introduction

The main unsolved problem of electroweak baryogenesis is a systematic computation of the relevant sources in transport equations. We shall now present a method for controlled derivation of leading CP-violating sources appearing as a consequence of collisions of chiral fermions with scalar particles in presence of a scalar field condensate. We assume the following picture of baryogenesis at a first order electroweak phase transition: when the Universe supercools, the bubbles of the Higgs phase nucleate and grow into the sea of the hot phase. For species that couple to the Higgs condensate in a CP-violating manner that CP-violating currents are created at the phase boundary (bubble wall). These currents then bias baryon number violating interactions mostly in the hot (symmetric) phase, where the B-violating processes are unsuppressed. The baryons then diffuse to the Higgs phase, where the B-violating interactions are suppressed, resulting in baryogenesis.

Kimmo Kainulainen [1] has explained how to systematically derive the CP-violating source in the flow term of the kinetic equation for fermions. For details see Paper I [4]. The source is universal in that its form is independent on interactions. It can be represented as the semiclassical force originally introduced for baryogenesis in two-Higgs doublet models in [2], and subsequently adapted to the Minimal Supersymmetric Standard Model (MSSM) in [3]. This problem involves computation of CP-violating sources from charginos, which couple to the Higgs condensate in a manner that involves fermionic mixing. Here we show how to compute the CP-violating source in the collision term that arises at first order in gradients. We work in the simple model of chiral fermions coupled to a complex scalar field *via* the Yukawa interaction with the Lagrangian of the form [4, 9]

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi - \bar{\psi}_L m \psi_R - \bar{\psi}_R m^* \psi_L + \mathcal{L}_{\rm yu}, \tag{1}$$

where $\mathcal{L}_{\rm yu}$ denotes the Yukawa interaction term

$$\mathcal{L}_{\rm yu} = -y\phi\bar{\psi}_L\psi_R - y\phi^*\bar{\psi}_R\psi_L, \tag{2}$$

and m is a complex, spatially varying mass term

$$m(u) \equiv y'\Phi_0 = m_R(u) + i m_I(u) = |m(u)|{\rm e}^{i\theta(u)}. \tag{3}$$

Such a mass term arises naturally from an interaction with a scalar field condensate $\Phi_0 = \langle\hat{\Phi}\rangle$. This situation is realised for example by the Higgs field condensate of a first order electroweak phase transition in supersymmetric models. When ϕ in (2) is the Higgs field the coupling constants y and y' are identical; our considerations are however not limited to this case.

The dynamics of quantum fields can be studied by considering the equations of motion arising from the two-particle irreducible effective action (2PI) [5] in the Schwinger-Keldysh closed-time-path formalism [6, 7]. This formalism is for example appropriate for studying thermalization in quantum field theory [8].

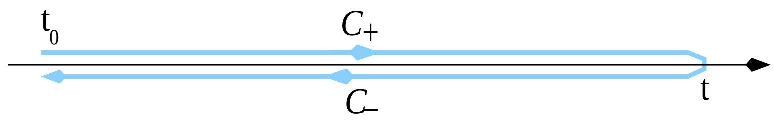

Figure 1: The Schwinger *closed-time-path* (CTP) used in the derivation of the 2PI effective action (6).

We are interested in the dynamics of the fermionic and bosonic two-point functions

$$S_{\alpha\beta}(u,v) = -i\left\langle T_{\mathcal{C}}[\psi_\alpha(u)\bar{\psi}_\beta(v)]\right\rangle \tag{4}$$
$$\Delta(u,v) = -i\left\langle T_{\mathcal{C}}[\phi(u)\phi^\dagger(v)]\right\rangle \tag{5}$$

where the time ordering $T_{\mathcal{C}}$ is along the Schwinger contour shown in figure 1, which is suitable for the dynamics of out-of-equilibrium quantum fields.

2 Effective action and self-energies

The 2PI effective action can be in general written in the form [5, 7]

$$\Gamma_{\text{eff}} = \Gamma_0 + \Gamma_1 + \Gamma_{\text{2PI}}, \tag{6}$$

where the tree-level and one-loop actions read

$$\Gamma_0 = i\int_{\mathcal{C}} d^4u\, d^4v[\Delta_0^{-1}(u,v)\Delta(v,u) - S_0^{-1}(u,v)S(v,u)] \tag{7}$$
$$\Gamma_1 = i\int_{\mathcal{C}} d^4u[\ln\Delta^{-1}(u,u) - \ln S^{-1}(u,u)], \tag{8}$$

and Δ_0^{-1} and S_0^{-1} are the tree-level *inverse* propagators:

$$S_0^{-1}(u,v) = (i\partial\!\!\!/_u - m(u)P_R - m^*(u)P_L)\delta_{\mathcal{C}}(u-v) \tag{9}$$
$$\Delta_0^{-1}(u,v) = (-\partial_u^2 - m_\phi^2(u))\delta_{\mathcal{C}}(u-v). \tag{10}$$

Here $P_{L,R} = (1\mp\gamma^5)/2$, and $\delta_{\mathcal{C}}$ is the δ-function along the contour $\mathcal{C}$. Γ_{2PI} in (6) contains higher order quantum corrections, which can be for example studied in the loop expansion. At two loops Γ_{2PI} reads

$$\Gamma_{\text{2PI}} \to \Gamma_2[S,\Delta] = -y^2\int_{\mathcal{C}} d^4u\, d^4v\ \text{Tr}[P_R S(u,v) P_L S(v,u)]\,\Delta(u,v), \tag{11}$$

which can be easily computed from the two-loop diagram in figure 2. The one-loop self-energies are then obtained from (11) by taking the functional derivatives

$$\Sigma'(u,v) = -i\frac{\delta\Gamma_{\text{2PI}}}{\delta S(v,u)} = iy^2\,[P_L S(u,v)P_R\Delta(v,u) + P_R S(u,v)P_L\Delta(u,v)] \tag{12}$$
$$\Pi'(u,v) = i\frac{\delta\Gamma_{\text{2PI}}}{\delta\Delta(v,u)} = -iy^2\,\text{Tr}\,[P_L S(v,u)P_R S(u,v)]. \tag{13}$$

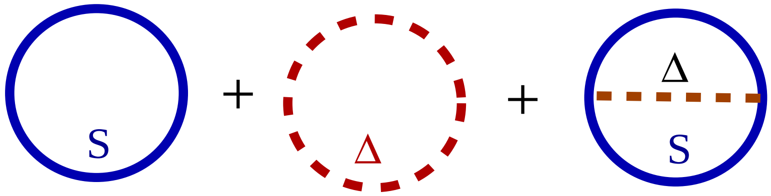

Figure 2: The diagrams contributing to the 2PI effective action (7-11) up to two loops for the Lagrangian (1) with the fermion-scalar Yukawa coupling term (2). The full (dressed) fermionic and bosonic propagators are denoted by S (*solid blue lines*) and Δ (*dashed red lines*), respectively.

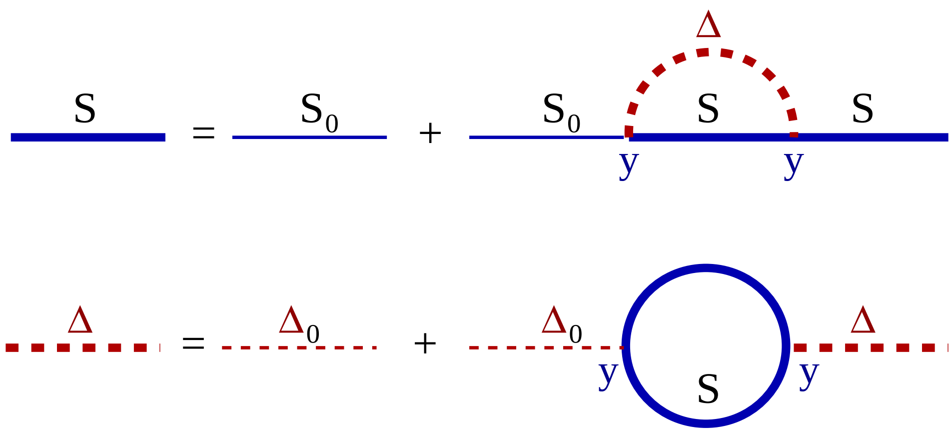

Figure 3: The Dyson-Schwinger equations at one loop obtained by varying the 2PI effective action (7-11) with respect to the fermionic and bosonic propagators. The corresponding tree-level Lagrangian is given by (1-2). The full (dressed) fermionic and bosonic propagators are denoted by S (*solid blue lines*) and Δ (*dashed red lines*), respectively.

The equation of motion for S and Δ are obtained by varying the effective action with respect to S and Δ. The resulting equations are simply

$$S_0^{-1} \otimes S = \delta_{\mathcal{C}} + \Sigma' \otimes S \tag{14}$$

$$\Delta_0^{-1} \otimes \Delta = \delta_{\mathcal{C}} + \Pi' \otimes \Delta, \tag{15}$$

where $\otimes$ denotes a convolution with respect to contour integration. These are the Dyson-Schwinger integro-differential equations diagrammatically shown in figure 3. The simple look of these equations is deceptive, since they involve integration over the closed-time-path in figure 1. To proceed we use the Keldysh reformulation of the problem, according to which the contour $\mathcal{C}$ is split into two parts: (t_0, t) in the positive time direction and (t, t_0) in the negative time direction; finally we set $t_0 \to -\infty$. This corresponds to the replacements

$$\int_{\mathcal{C}} d^4u \longrightarrow \sum_{a=\pm 1} a \int_{t_0 \to -\infty}^{t} d^4u$$

$$\begin{aligned} S(u,v) &\longrightarrow S^{ab}(u,v) \\ \Delta(u,v) &\longrightarrow \Delta^{ab}(u,v) \\ \delta_{\mathcal{C}}(u-v) &\longrightarrow a\delta_{ab}\delta(u-v) \end{aligned} \tag{16}$$

in the effective action (6-11). This procedure naturally leads to the Keldysh 2×2 formulation for the two-point functions (4-5) in which the off-diagonal elements of the fermionic Wigner functions correspond to

$$\begin{aligned} S^{<}(u,v) &\equiv S^{+-}(u,v) = i\langle\bar{\psi}(v)\psi(u)\rangle \\ S^{>}(u,v) &\equiv S^{-+}(u,v) = -i\langle\psi(u)\bar{\psi}(v)\rangle, \end{aligned} \tag{17}$$

and similarly the bosonic ones (5) are

$$\begin{aligned} \Delta^{<}(u,v) &\equiv \Delta^{+-}(u,v) = -i\langle\phi^{\dagger}(v)\phi(u)\rangle \\ \Delta^{>}(u,v) &\equiv \Delta^{-+}(u,v) = -i\langle\phi(u)\phi^{\dagger}(v)\rangle. \end{aligned} \tag{18}$$

In the Keldysh representation it is convenient to redefine the self-energies (12-13) as

$$\Sigma^{ac}(u,v) \equiv a\Sigma'^{ac}(u,v)c \tag{19}$$

$$\Pi^{ac}(u,v) \equiv a\Pi'^{ac}(u,v)c. \tag{20}$$

3 Kinetic equations

When written explicitly the kinetic equations for fermions (14) and bosons (15) become

$$(\frac{i}{2}\not{\partial} + \not{k} - \hat{m}P_R - \hat{m}^*P_L)S^{<} - e^{-i\diamond}\{\Sigma_R, S^{<}\} - e^{-i\diamond}\{\Sigma^{<}, S_R\} = \mathcal{C}_{\psi} \tag{21}$$

$$(-\frac{1}{4}\partial^2 + k^2 + ik\cdot\partial - \hat{m}^2)\Delta^{<} - e^{-i\diamond}\{\Pi_R, \Delta^{<}\} - e^{-i\diamond}\{\Pi^{<}, \Delta_R\} = \mathcal{C}_{\phi}, \tag{22}$$

where

$$S^{<}(x,k) = \int d^4r\, e^{ik\cdot r} S^{<}(x+r/2, x-r/2) \tag{23}$$

defines the Wigner functions for fermions, and there is a similar expression for bosons. Π_R, Σ_R, S_R and Δ_R denote the hermitean parts of the self-energies and two-point functions, respectively. We postpone a discussion of the physical relevance of the contributions involving the hermitean parts to a later publication. Eq. (23) defines a Wigner transform, which is useful to separate the dependence on the slowly varying average (*macroscopic*) coordinate $x = (u+v)/2$ from that on the *microscopic* coordinate $r = u - v$.

The bilinear operator $\diamond$ in Eqs. (21-22) is defined as

$$\diamond\{A,B\} \equiv \frac{1}{2}\Big(\partial_x A\cdot\partial_k B - \partial_k A\cdot\partial_x B\Big), \tag{24}$$

the mass terms read

$$\hat{m} = m(x)e^{-\frac{i}{2}\overleftarrow{\partial}\cdot\,\partial_k} \tag{25}$$

$$\hat{m}^2_\phi = m^2_\phi(x)e^{-\frac{i}{2}\overleftarrow{\partial}\cdot\,\partial_k}, \tag{26}$$

and the collision terms are of the form

$$\mathcal{C}_\psi = -\frac{1}{2}e^{-i\diamond}\Big(\{\Sigma^>, S^<\} - \{\Sigma^<, S^>\}\Big), \tag{27}$$

$$\mathcal{C}_\phi = -\frac{1}{2}e^{-i\diamond}\Big(\{\Pi^>, \Delta^<\} - \{\Pi^<, \Delta^>\}\Big). \tag{28}$$

The one-loop expressions for the self-energies can be inferred from Eqs. (19-20) and (12-13):

$$\begin{aligned}\Sigma^{<,>}(k,x) &\equiv \Sigma^{+-,-+}(k,x)\\ &= iy^2\int\frac{d^4k'd^4k''}{(2\pi)^8}\,[(2\pi)^4\delta(k-k'+k'')P_LS^{<,>}(k',x)P_R\Delta^{>,<}(k'',x)\\ &\qquad +(2\pi)^4\delta(k-k'-k'')P_RS^{<,>}(k',x)P_L\Delta^{<,>}(k'',x)]\end{aligned} \tag{29}$$

$$\begin{aligned}\Pi^{<,>}(k,x) &\equiv \Pi^{+-,-+}(k,x)\\ &= -iy^2\!\int\frac{d^4k'd^4k''}{(2\pi)^8}(2\pi)^4\delta(k+k'-k'')\mathrm{Tr}\,[P_RS^{>,<}(k',x)P_LS^{<,>}(k'',x)].\end{aligned} \tag{30}$$

4 Wigner functions

We are interested in modeling the dynamics of fermions and bosons in the presence of a Higgs field condensate of growing bubbles at a first order electroweak phase transition. When the bubble walls are thick we can expand in gradients of the condensate. This expansion is accurate for quasiparticles whose de Broglie wavelength ℓ_{dB} is small when compared to the scale of variation of the background, which is specified by the phase boundary thickness L_w. Since equilibrium considerations yield $L_w \sim 5-15$, and the de Broglie wavelength is typically given by the inverse temperature, $\ell_{dB} \sim 1/T$, we have $\ell_{dB}\partial_x \sim \ell_{dB}/L_w \ll 1$, so that the expansion in gradients is justified.

The problem can be further simplified by noting that typically large bubbles are almost planar and slow, such that it suffices to keep the leading order terms in the bubble wall velocity, and expand to leading nontrivial order in gradients. So when written in the rest frame of the bubble wall (wall frame), Eqs. (25-26) simplify to $\hat{m}(z) = m + \frac{i}{2}m'\partial_{k_z} + ..$ and $\hat{m}^2_\phi(z) = m^2_\phi + \frac{i}{2}m^{2\prime}_\phi\partial_{k_z} + \ldots$

In Paper I we have shown that the fermionic Wigner function (17) acquires a nontrivial contribution at first order in gradients. In the presence of a scalar condensate, when one neglects particle interactions, a wall moving in z-direction conserves spin in z-direction.

This implies that one can write (*cf.* Paper II) the Wigner function in the following spin-diagonal form

$$S^{<,>} = \sum_{s=\pm} S_s^{<,>}, \tag{31}$$

where

$$S_s^{<,>} = iP_s\Big[s\gamma^3\gamma^5 g_0^{s<,>} - \gamma^3 s g_3^{s<,>} + g_1^{s<,>} - i\gamma^5 g_2^{s<,>}\Big]. \tag{32}$$

Here P_s denotes the spin projector

$$P_s \;=\; \frac{1}{2}(1 + sS_z), \tag{33}$$

where

$$S_z \;=\; \frac{1}{\tilde{k}_0}(k_0\gamma^0 - \vec{k}_\parallel \cdot \vec{\gamma}_\parallel)\,\gamma^3\gamma^5 \tag{34}$$

is the spin operator in z-direction, and $\tilde{k}_0 = \mathrm{sign}[k_0]\sqrt{k_0^2 - \vec{k}_\parallel^2}$.

The quantity $g_0^{s<,>}$ in Eq. (32) is a measure for the phase space density of particles with spin s. When written in the wall frame and to first order in gradients, the solution for $g_0^{s<,>}$ has the form

$$\begin{aligned} g_0^{s<} \;&=\; 2\pi\delta\Big(k^2 - |m|^2 + \frac{1}{\tilde{k}_0}s|m|^2\theta'\Big)|\tilde{k_0}|n(k,x) \\ g_0^{s>} \;&=\; -2\pi\delta\Big(k^2 - |m|^2 + \frac{1}{\tilde{k}_0}s|m|^2\theta'\Big)|\tilde{k_0}|(1 - n(k,x)). \end{aligned} \tag{35}$$

In thermal equilibrium we have

$$n(k,x) \to n_0 = \frac{1}{e^{\hat{k}_0/T} + 1}, \qquad \hat{k}_0 = \gamma_w(k_0 + v_w k_z), \tag{36}$$

where $\vec{v}_w = v_w\hat{z}$ is the wall velocity and $\gamma_w = 1/\sqrt{1 - v_w^2/c^2}$. The quantities $g_i^{s<,>}$ ($i = 1,2,3$) in (32) can be related to $g_0^{s<,>}$ by (see Paper I & II)

$$\begin{aligned} g_1^{s<,>} \;&=\; \frac{1}{\tilde{k}_0}\Big[m_R g_0^{s<,>} - \frac{s}{2\tilde{k}_0}\partial_z(m_I g_0^{s<,>}) - \frac{s m_I'}{2\tilde{k}_0}\partial_{k_z}(k_z g_0^{s<,>})\Big] \\ g_2^{s<,>} \;&=\; \frac{1}{\tilde{k}_0}\Big[m_I g_0^{s<,>} + \frac{s}{2\tilde{k}_0}\partial_z(m_R g_0^{s<,>}) + \frac{s m_R'}{2\tilde{k}_0}\partial_{k_z}(k_z g_0^{s<,>})\Big] \\ g_3^{s<,>} \;&=\; \frac{1}{\tilde{k}_0}\Big[s k_z g_0^{s<,>} + \frac{|m|^2\theta'}{2\tilde{k}_0}\partial_{k_z} g_0^{s<,>}\Big]. \end{aligned} \tag{37}$$

It is now easy to check that, to leading order in gradients, the Kubo-Martin-Schwinger (KMS) condition

$$S_0^{>} = -e^{\hat{k}_0/T} S_0^{<} \tag{38}$$

holds when $n = n_0$. The first order solution (32) with $n = n_0$ does *not* obey the KMS condition however, except in the limit of a static wall $v_w \to 0$.

The solution for the scalar field Wigner function is much simpler. One can show [4] that, to the order $\hbar$ in gradient expansion, there is no source for bosons, so that the bosonic Wigner function in the wall frame can be approximated by

$$\begin{aligned}\Delta^< &= \frac{\pi}{\omega_\phi}\Big[\delta(k_0-\omega_\phi)f_\phi + \delta(k_0+\omega_\phi)(1+\bar f_\phi)\Big]\\ \Delta^> &= \frac{\pi}{\omega_\phi}\Big[\delta(k_0-\omega_\phi)(1+f_\phi) + \delta(k_0+\omega_\phi)\bar f_\phi\Big]\end{aligned} \tag{39}$$

where $f_\phi(k_z)$ and $\bar f_\phi(-k_z)$ denote the occupation number of particles and antiparticles, respectively, such that in thermal equilibrium we get the standard Bose distribution function

$$f_{\phi 0} = \frac{1}{e^{\gamma_w(\omega_\phi + v_w k_z)/T} - 1}, \qquad \bar f_{\phi 0} = \frac{1}{e^{\gamma_w(\omega_\phi - v_w k_z)/T} - 1} \tag{40}$$

with $\omega_\phi = \sqrt{\vec k^2 + m_\phi^2}$. Note that in thermal equilibrium the KMS condition, $\Delta_0^> = e^{k_0/T}\Delta_0^<$, is satisfied.

5 Collisional sources

5.1 Collisional source in the scalar equation

Making use of equations (22), (28) and (30-37), one can show [9] that to first order in gradients the scalar field collision integral $\mathcal{C}_\phi$ is proportional to delta functions:

$$\mathcal{C}_\phi \propto \delta(k_0 - \omega_\phi) + \delta(k_0 + \omega_\phi). \tag{41}$$

Because of the positive relative sign between f_ϕ and $\bar f_\phi$ in the scalar Wigner function (39), the source arising from $\mathcal{C}_\phi$ gives rise to a CP-even and spin independent contribution to the scalar kinetic equation (22), which is thus of no relevance for baryogenesis. As a consequence, there is no CP-violating source for *stops* [11] in the collision term at first order in gradients. Our analysis is however based on a one-loop approximation for the self-energies. To make the study more complete, it would be desirable to perform a two-loop calculation of the source.

5.2 Collisional source in the fermionic equation

Before proceeding to calculate the source in the fermionic kinetic equation, we first prove that *there has to be* a nonvanishing source in the fermionic collision term.

Since the equilibrium solution (32-36) for the Wigner function $S^{<,>}$ does not satisfy the KMS condition, the collision term does not vanish in the equilibrium defined by

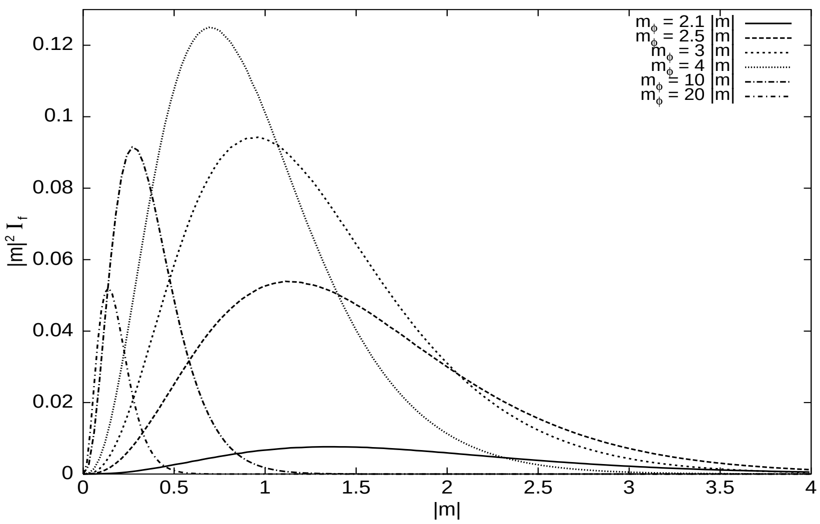

Figure 4: Shown as a function of $|m|/T$ is the expression $|m|^2\mathcal{I}_f(|m|, m_\phi)$, which appears in the collisional source (46) arising in the fermionic kinetic equation at one loop. For simplicity we have set $T = 1$.

Ansatz (36). We shall now show that it is not possible to avoid this outcome by choosing n differently.

First recall that for a static wall with $v_w = 0$ the equilibrium solution (32-36) *does* satisfy the KMS condition. Now try to modify the equilibrium solution for $S^{<,>}$ as follows

$$S^< \;=\; 2i\mathcal{A}(n_0 + \delta n), \qquad S^> = -2i\mathcal{A}(1 - n_0 - \delta n), \tag{42}$$

where δn is some scalar function of first order in gradients. The spectral function

$$\mathcal{A} = \mathcal{A}_0 + \mathcal{A}_1 \tag{43}$$

contains the spinor structure at leading and first order in derivatives, respectively, and the δ-function, $\delta(k^2 - |m|^2 + s|m|^2\theta'/\tilde{k}_0)$, as specified by equations (37). $\mathcal{A}_1$ contains derivatives which spoil the KMS condition when acting on $n_0(\hat{k}_0)$. We now seek δn such that $S^> = -e^{\beta\hat{k}_0}S^<$ be satisfied. Working to first order in gradients one arrives at the following condition

$$\mathcal{A}_0\delta n \;=\; n_0\mathcal{A}_1 1 - \mathcal{A}_1 n_0. \tag{44}$$

where $\mathcal{A}_1$ acts as an operator on 1 and n_0, respectively. Since $\mathcal{A}_0$ and $\mathcal{A}_1$ have different spinor structure, there is no scalar function δn that solves this equation. Hence there are indeed collisional sources that cannot be removed by a change of thermal equilibrium that is local in momentum.

In order to obtain the relevant collisional source in the kinetic equation for fermions, we need to multiply Eq. (27) by $-P_s$, take the real part of the spinorial trace and truncate at first order in gradients.

To get the collisional source in the vector current continuity equation one should integrate the collision term over the momenta. It is not hard to see that the integrand is symmetric under the change of variables $k_z \to -k_z$ and $k'_z \to -k'_z$, which then implies that

$$\int_{\pm} \frac{d^4k}{(2\pi)^4} \mathcal{C}_{\psi(0)} = 0, \tag{45}$$

where $\mathcal{C}_{\psi(0)} = -\mathrm{Tr}[P_s \mathcal{C}_\psi]$ and $\int_\pm$ denotes integration over the positive and negative frequencies, respectively. This means that there is no collisional source in the continuity equation for the vector current.

The source in the first velocity moment equation, the Euler fluid equation, is however nonvanishing. One can show that, to leading order in v_w, the source can be written in the following form

$$2\int_{\pm} \frac{d^4k}{(2\pi)^4} \frac{k_z}{\omega_0} \mathcal{C}_{\psi(0)} \;=\; \pm v_w y^2 \frac{s|m|^2\theta'}{64\pi^3 T} \mathcal{I}_f(|m|, m_\phi) \tag{46}$$

where the function $\mathcal{I}_f(|m|, m_\phi)$ contains a complicated integral expression. To get a quantitative estimate of the source, we perform numerical integration of $\mathcal{I}_f$ and plot the result in figure 4. Note that for $|m| \gg T$ ($T = 1$ in the figure) the source is Boltzmann suppressed, while for $|m| \ll T$ it behaves as $\propto (|m|/T)^{3/2}$. The source peak shifts towards the infrared as $m_\phi/|m|$ increases, and the magnitude drops drastically when one approaches the mass threshold $m_\phi = 2|m|$, as requested for scalar particle decay and inverse decay processes.

5.3 Collisional sources in the relaxation time approximation

The methods used in literature for computation of sources from the collision term in scalar and fermionic equations [10, 11] can be in many cases rephrased as the relaxation time approximation:

$$\mathcal{C}_{\psi si(0)} \approx -\Gamma_{si} \left(f_{si} - f_{si0}\right), \tag{47}$$

where f_{si} denotes the true particle density of spin s and flavour i, f_{si0} is the equilibrium particle density (for a moving wall), and Γ_{si} is the relevant relaxation rate, which is usually assumed to be given by the elastic scattering rate. In Paper II we show that Eq. (35) implies the following form for the CP-violating contribution to the particle density

$$n^0_{si} \equiv n^0_{si+} - n^0_{si-} = \frac{s|M_i|^2\Theta'_i}{16\pi^2} \, \mathcal{J}_0(|M_i|/T), \tag{48}$$

where $n^0_{si\pm} \equiv \int_\pm (d^4k/(2\pi)^4) g^s_{00}$ and M_i and Θ_i denote the masses and CP-violating phases of fermions. The detailed form of the function $\mathcal{J}_0$ is discussed in Paper II and it is not of importance for the purpose of this talk.

We note that the CP-violating source for charginos in the MSSM shows the parametric dependence $|M_i|^2\Theta_i' \propto (h_1h_2)'$ on the Higgs field *vev*'s h_1 and h_2. This is to be compared with Refs. [12, 13], where a source proportional to $h_1h_2' - h_2h_1'$ was found and claimed to be important for baryogenesis.

Apart from parametric dependences issues, the most worrisome feature about the relaxation time approximation is that it yielded a nonvanishing source (48) on the *vector* continuity equation, which we found to vanish based on equation (45). Moreover, the source (48) does not vanish in the static limit $v_w \to 0$, and hence is clearly unphysical with no apparent relevance for baryogenesis.

6 Conclusions

We have studied collisional sources appearing at first order in gradients of a spatially varying mass for the model of chiral fermions interacting with a scalar field *via* a standard Yukawa interaction. This model is relevant for baryogenesis from fermions interacting with the Higgs condensate on growing bubbles at a strongly first order electroweak phase transition. The self-energies have been approximated by one-loop expressions. We have argued that, at first order in gradients, there is no collisional source in the scalar kinetic equation, indicating that baryogenesis sourced by scalar particles is highly suppressed. We have then proven that there is a CP-violating source in the fermionic equation and performed a quantitative analysis of the source. This source, together with the one from the semiclassical force, comprise the relevant sources for baryogenesis at the electroweak scale. Finally we have shown that analysis of CP-violating sources in the relaxation time approximation yields incorrect results. In order to perform a full quantitative assessment of collisional sources, a two-loop analysis of self-energies is required, which is a work in progress.

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