# Interpreting the LSND anomaly: sterile neutrinos or CPT-violation or...?

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## Abstract

We first study how sterile neutrinos can fit the $5\sigma$ $\bar{\nu}_\mu \to \bar{\nu}_e$ LSND anomaly: 3+1 solutions give a poor fit, but better than than 2+2 solutions (the best fit regions are somewhat different, so that MiniBooNE could discriminate). If instead MiniBooNE will find no $\nu_\mu \to \nu_e$ signal, we will have a hint for CPT violation. Already now, unlike sterile neutrinos, CPT-violating neutrino masses can nicely accomodate all safe and unsafe data. We study how much CPT must be conserved according to atmospheric and K2K data and list which CPT-violating signals could be discovered by forthcoming solar and long-baseline experiments.

Oscillations between the three Standard Model neutrinos are described by two independent squared neutrino mass differences, allowing to explain only two of the three neutrino anomalies (atmospheric [1], solar [2] and LSND [3]) as oscillations. A joint fit is not possible even if one trusts only the safest data from atmospheric, solar and reactor [4] neutrino experiments: the the up/down atmospheric asymmetries and a $\sim 1/2$ disappearance of solar $\nu_e$. Most global fits of neutrino data drop the LSND anomaly because the other ones are considered as more solid. In quantitative terms, we have a $7\sigma$ solar anomaly (although it can be reduced to $4\sigma$ by dropping solar model predictions), a $14\sigma$ atmospheric anomaly and a $5\sigma$ LSND anomaly*. The 'number of standard deviations' is here naïvely computed as $(\Delta\chi^2)^{1/2} = (\chi^2_{\rm SM} - \chi^2_{\rm best})^{1/2}$, where $\chi^2_{\rm best}$ is the $\chi^2$ value corresponding to the best-fit oscillation, and $\chi^2_{\rm SM}$ corresponds to massless SM neutrinos.

In section 1 we discuss how and how well oscillations with extra sterile neutrinos can fit the LSND anomaly [5]. In particular we study which one of the two different kind of four-neutrino spectra (1+3 or 2+2) is favoured by the present data, and by an eventual future confirmation of the LSND data. An extra sterile neutrino can improve the situation, but some contradiction between different sets of data remains, expecially in the 2+2 scheme.

This situation suggests to look for alternative interpretations of the LSND anomaly. One possibility is that either the atmospheric or the solar anomaly is not due to oscillations. We will not consider this possibility, although various mechanisms (even unplausible ones) can fit the data as well as oscillations [6, 7].

In view of this situation, it is interesting that all data can be consistently fitted by the CPT-violating neutrino spectrum illustrated in fig. 1. This solution was proposed in [8] when the initial $2.6\sigma$ LSND hint for $\nu_\mu \to \nu_e$ [9] decreased down to $0.6\sigma$, leaving an anomaly only in $\bar{\nu}_\mu \to \bar{\nu}_e$ [3]. Unlike sterile neutrinos, this solution also satisfies (unsafe?) bounds from nucleosynthesis and SN1987A. Despite the lack of theoretical grounds, this bold speculation is interesting because can be tested soon. If CPT violation were the right answer, MiniBooNE [10] (the experiment designed to test LSND, looking for $\nu_\mu \to \nu_e$) will not see the LSND oscillations; a $\bar{\nu}_\mu \to \bar{\nu}_e$ experiment is also needed to directly test this possibility. If CPT is badly violated as in fig. 1, one generically expects detectable CPT-violating signals in atmospheric and solar oscillations. In any case it remains interesting to constrain CPT-violation in neutrino masses. In section 2 we compute the present bounds and list the possible CPT-violating signals and surprises that could appear in forthcoming solar and long-baseline experiments.

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* The $\bar{\nu}_\mu \to \bar{\nu}_e$ LSND anomaly is presented as an evidence for a $\mu \to e$ oscillation probability of $(0.264 \pm 0.081)\%$ [3], that differs from zero only by slightly more than $3\sigma$. However, from a table of the likelihood $\mathcal{L}$, obtained from the LSND collaboration and computed on an event-by-event basis, we read

$$\chi^2_{\rm SM} - \chi^2_{\rm best} = -2\ln\frac{\mathcal{L}_{\rm best}}{\mathcal{L}_{\rm SM}} = 29 \qquad \text{rather than } \sim 10.$$

Apparently, some mark of oscillations that cannot be summarized by the number of $\bar{\nu}_e$ events is hidden in the full LSND data, maybe in the energy distribution. However a large range of different $\Delta m^2$ values fits almost equally well the LSND data.

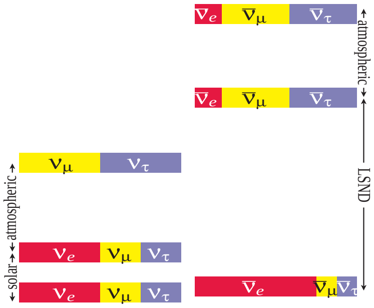


Figure 1: *The CPT-violating spectrum proposed in [8].*

# 1 Sterile neutrinos

### 3+1 neutrinos

In the jargon 3+1 indicates that the additional sterile neutrino is separated by the large LSND mass gap from the 3 active neutrinos, separated among them only by the small solar and atmospheric mass differences. A theoretical remark is in order. If the $4 \times 4$ neutrino mass matrix $m_{ii'}$ ($i = \{\ell, s\}$ and $\ell = \{e, \mu, \tau\}$) has the naïve form

$$m_{\ell s} = m_{s\ell} = \theta_{\ell s} m_{ss}, \quad m_{ss} = m_{\rm LSND}, \quad m_{\ell\ell'} \ll m_{ss}$$

the sterile neutrino gives a contribution to the solar mass splitting of order[†]

$$\delta\Delta m^2_{\rm sun} \sim \Delta m^2_{\rm LSND} \sin^2 2\theta_{\rm LSND} \approx 10^{-(3\div 1)}\,{\rm eV}^2$$

that is too large in most of the region allowed by solar and LSND data. One needs either a cancellation or a mass matrix of the special form $m_{ii'} \simeq \theta_{is}\theta_{i's} m_{\rm LSND}$ (in the case of active neutrinos only, such a rank one matrix can be naturally obtained from a see-saw with a single heavy state).

Even ignoring this potential theoretical problem, 3+1 oscillations present a phenomenological problem, because predict that $\nu_\mu \to \nu_e$ oscillations at the LSND frequency proceed trough $\nu_\mu \to \nu_s \to \nu_e$ and $\nu_{e,\mu} \to \nu_s$ are strongly constrained by disappearance experiments. More precisely, keeping only oscillations at the dominant LSND frequency

$$S \equiv \sin^2(\Delta m^2_{\rm LSND} L/4E_\nu)$$

one has

$$\begin{aligned} P(\nu_e \to \nu_e) &= 1 - S\sin^2 2\theta_{es} \\ P(\nu_\mu \to \nu_\mu) &= 1 - S\sin^2 2\theta_{\mu s} \\ P(\nu_e \to \nu_\mu) &= S \sin^2 2\theta_{\rm LSND} \end{aligned}$$

with $\theta_{\rm LSND} \approx \theta_{es}\theta_{\mu s}$, or more precisely [11]

$$\sin^2 2\theta_{\rm LSND} = \frac{1}{4}\sin^2 2\theta_{es}\sin^2 2\theta_{\mu s}. \qquad (1)$$

The $\theta_{es}$ mixing angle is constrained by Bugey, CHOOZ [4], SuperKamiokande (SK) atmospheric data [1] and the $\theta_{\mu s}$ mixing angle by CDHS [12] and SK. Furthermore $\nu_\mu \to \nu_e$ oscillations are also directly constrained by Karmen [13]. Fig. 2 illustrates how accurately we reproduce such bounds[‡].

The crucial question is if these bounds are too strong for allowing the oscillations suggested by LSND. At first sight the answer is that they are [11], but this negative conclusion was questioned in [15] and the first accurate statistical analysis of this issue was performed in [16] with Bayesian techniques. Our result, shown in fig. 3 basically agrees with [16]. Working in gaussian approximation[§] we find that all 97% CL LSND confidence region has been excluded at, at least, 97% CL level. Therefore 3+1 solutions have some goodness-of-fit problem. One needs to invoke a statistical fluctuation with around % probability to explain why only LSND sees the sterile oscillations.

Even if this conclusion is self-evident, we justify the statistical strategy we have adopted. As discussed in [17], due to the large number of d.o.f. (about 200) a naïve Pearson global $\chi^2$ test is unable to notice this problem and would erroneously suggest that 3+1 oscillations give a good fit. While it is difficult to develop a general and efficient goodness-of-fit test, in this particular case the fit is bad for one specific reason: different sets of data are mutually exclusive (up to a 97% CL) within our theoretical assumptions. In such a situation the goodness-of-fit problem is efficiently recognized by fitting separately the two incompatible data. This is what is done in fig. 3.

Ignoring the poor quality of the fit, the best combined fit region for the LSND parameters is shown in fig. 4a. It agrees reasonably well with the corresponding fig. in [18], taking into account that we show values of

$$\chi^2(\theta_{\rm LSND}, \Delta m^2_{\rm LSND}) = \min_p \chi^2(p, \theta_{\rm LSND}, \Delta m^2_{\rm LSND})$$

(where $p$ are all other parameters in which we are not interested), so that we convert values of $\chi^2 - \chi^2_{\rm best}$ into

[†] More precisely, assuming $\theta_{\ell s} \ll 1$, maximal atmospheric mixing and $\theta_{13} = 0$, and taking into account the larger atmospheric mass splitting, one has $\delta\Delta m^2_{\rm sun} = (\theta^2_{es} + \theta^2_{\perp s})^2 \Delta m^2_{\rm LSND}$ where $\theta_{\perp s} \approx (\theta_{\mu s} - \theta_{\tau s})/\sqrt{2}$.

[‡] We used the SK atmospheric results [1] after 79 kton·year (55 data), K2K [14] (at the moment K2K finds 44 events, versus an expected no-oscillation signal of 64±6 events), the latest solar results from Homestake, Gallex, SAGE, GNO, SK, SNO (41 data), the final Bugey (60 data), CHOOZ (14 data), CDHS (15 data) and LSND results, and the Karmen data after 7160 C accumulated protons. *We plan to soon update our results, when the final Karmen data will be released.* We use the likelihoods computed by the Karmen and LSND collaborations on an event-by-event basis. We have not included data from Macro [1] (that confirms the atmospheric anomaly) and from earlier atmospheric experiments because are less statistically significant than SK. Most of the present work consisted in fitting carefully all these data. The data are combined by multiplying all likelihoods $\mathcal{L}$ (i.e. by summing all $\chi^2 = -2\ln\mathcal{L}$). We deprecate CDHS experimentalists that, instead of giving the $\mu$ energy, preferred to write the $\mu$ range in their detector.

[§] So that $\Delta\chi^2 = 7$ corresponds to 97% CL level for the two parameters $\theta_{\rm LSND}$ and $\Delta m^2_{\rm LSND}$. The Gaussian approximation is not fully satisfied (e.g. our best fit regions are not ellipses). A Bayesian analysis can shift 97% to $\sim 95$ or $\sim 98$, with 'reasonable' choiches of the prior probability distribution. (the arbitrariety remains until there are 'large' allowed regions). As discussed in [17], a similar shift is typically obtained in a frequentist analysis, that cannot however be performed in a reasonable computing time. Therefore we stick to the Gaussian approximation.

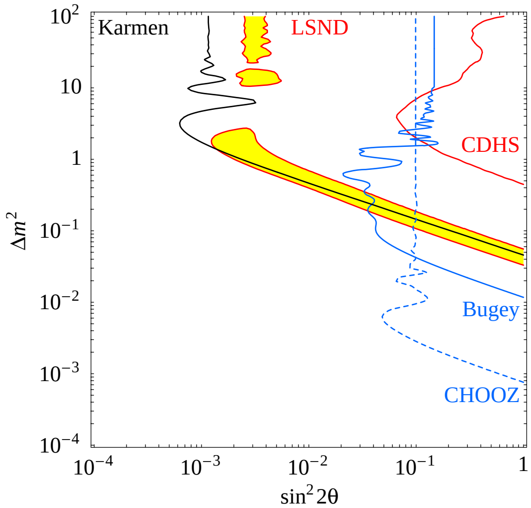


Figure 2: 90% *CL regions from Karmen, CDHS, Bugey, Chooz and LSND (shaded). The mixing angle $\theta$ on the horizontal axis is different for the different experiments.*

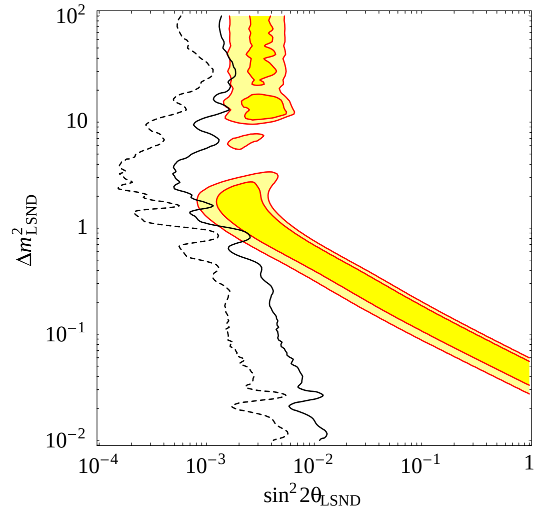


Figure 3: *The LSND region at* 90% *and* 99% *CL, compared with the* 90% *(dashed line) and* 99% *CL (continuous line) combined exclusion bounds from data in fig.* 2 *and SK.*

confidence levels using the gaussian values appropriate for 2 d.o.f. (the 2 LSND parameters), while a statistically less efficient procedure with more d.o.f. is employed in [18].

### 2+2 neutrinos

In the jargon 2+2 indicates 2 couples of neutrinos (one generates the solar anomaly, and the other one the atmospheric anomaly), separated by the large LSND mass gap.

2+2 oscillations do not have the same problem of 3+1 oscillations, but of course this does not guarantee that 2+2 oscillations can give a good fit. In fact they present a bigger problem, such that at the end the best-fit global $\chi^2$ is even worse than in the 3+1 case [19, 18]. The problem of 2+2 oscillations is that sterile oscillations are predicted to be the source of either solar or atmospheric oscillations (or both), in contradiction with experimental data. Let us summarize the present experimental status of this issue.

- **Solar data** give a $\gtrsim 3.3\sigma$ indication for pure active solar oscillations versus pure sterile oscillations. In fact, our global fit of solar data gives[¶]

$$\chi^2_{\rm sun}(\text{best sterile}) - \chi^2_{\rm sun}(\text{best active}) = 10 \div 14.$$

In particular, SNO/SK find a $3.3\sigma$ direct indication for $\nu_{\mu,\tau}$ appearance. Unless it is a statistical fluctuation, with more statistics this indication could reach the $6\sigma$ level.

- **Atmospheric data** data give a $6.3\sigma$ indication for pure active atmospheric oscillations versus pure sterile oscillations. In fact, a global fit of atmospheric data gives [1]

$$\chi^2_{\rm atm}(\text{best sterile}) - \chi^2_{\rm atm}(\text{best active}) = 40.$$

This strong evidence is obtained combining independent sets of data. SK claims [1] that pure sterile is disfavoured by the up/down ratio in a NC-enriched sample (3.4 standard deviations) and by matter effects in partially contained events ($\approx 2.9\sigma$) and upward through-going muons ($\approx 2.9\sigma$): in total $5.4\sigma$. Matter effects in MACRO [1] give another $3.1\sigma$ signal. Furthermore SK finds a direct $2\sigma$ hint for $\tau$ appearance.

A large amount of these atmospheric data is not included in theoretical reanalyses (because not yet accessible outside the SK collaboration in a form that allows to recompute them) that therefore obtain a much smaller $\Delta\chi^2 \approx 15$ [18, 24] in place of 40. This poor result means that at the moment it is not possible to perform a sensible analysis of mixed sterile and active atmospheric oscillations[‖]. Therefore we only consider the two extreme cases: all the sterile in atmospheric oscillations and all the sterile in solar oscillations. The large difference in the value of the best

[¶] Arbitrary choices become more relevant when fitting disfavoured data (for example: the error is evaluated at the experimental point or at the theoretical point?). Global solar fits performed by different authors disagree on how much a sterile fit is disfavoured. Using the same data, the value of $\Delta\chi^2_{\rm sun}$ ranges from 6.8 [20] to 10.5 [21] to 11.1 [22] up to 18.5 [23]. In our fit all sterile solutions (usually named LMA, LOW, SMA, energy-independent, JustSo2) have $\Delta\chi^2 \approx 14$, but solutions with a fine-tuned $\Delta m^2_{\rm sun}$ around $5\,10^{-10}\,{\rm eV}^2$ have $\Delta\chi^2 \approx 10$. This happens because the survival probability of $^7$Be neutrinos fastly oscillates in space (giving a clean seasonal signal at the forthcoming Borexino and maybe KamLAND experiments) and, in view of the small excentricity of the earth orbit, can be somewhat adjusted to a convenient value by tuning $\Delta m^2_{\rm sun}$.

[‖] However, we expect that the naïve interpolation $\Delta\chi^2 \approx 40\eta_{\rm s}$ approximates reasonably well our present bounds on the sterile fraction $0 \le \eta_{\rm s} \le 1$ involved in atmospheric oscillations.

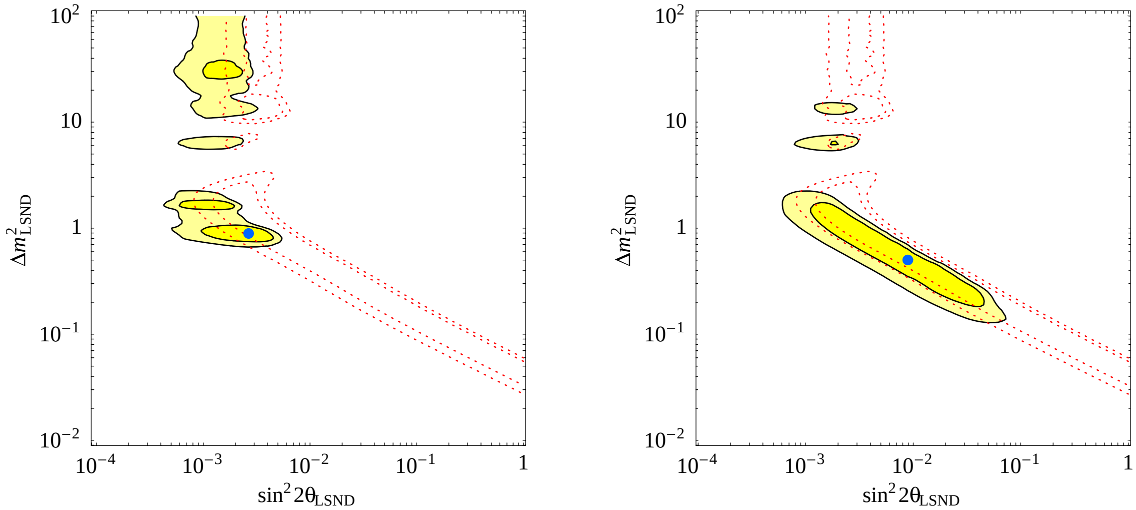


Figure 4: *Best-fit regions of the LSND and all other data assuming 3+1 (left) and 2+2 (right) oscillations at 90% and 99% CL (2 d.o.f.). The dotted lines show the regions suggested by only the LSND data. The dots show the best fit points (3+1 gives a better fit than 2+2, see table 1).*

$\chi^2$ suggests that the absolute 2+2 best fit is (very close to) $\nu_e \to \nu_{\rm s}$ solar and $\nu_\mu \to \nu_\tau$ atmospheric oscillations.

Our result is shown in table 1. We see that 2+2 oscillations give a worse best-fit than 3+1 oscillations. Even if the quality of the fit is poor, the best-fit region for the LSND parameters assuming 2+2 oscillations can be reliably computed**, since it is unaffected by the problematic solar and atmospheric data. This region extends to values of the LSND parameters not accessible within 3+1 oscillations, see fig. 4. Therefore the value of $P(\nu_\mu \to \nu_e)$ that will be measured at MiniBooNE could discriminate between the two spectra: roughly, 2+2 (3+1) oscillations prefer a value of $P(\nu_\mu \to \nu_e)$ around (somewhat smaller than) the one suggested by LSND.

### Many sterile neutrinos

Each of the 2+2 and 3+1 cases can be realized with a variety of spectra. Since at the moment (and in the near future) no experiment can resolve the difference we do not need to consider all possibilities.

As shown in the last paper in [15], many sterile neutrinos cannot give a much better 3+1 fit than a single sterile neutrino. Of particular interest are minimal models where right-handed neutrinos live in a single extra dimension of radius $R$ [25], that could be identified with the LSND scale. In such $3+\infty$ models the problematic prediction (1) of 3+1 oscillations becomes slightly more problematic [7]. In fact, for small mixing angles and in the limit of averaged sterile oscillations, we now have $\theta_{\rm LSND} \approx \sqrt{7/10}\,\theta_{es}\theta_{\mu s}$ in place of $\theta_{\rm LSND} \approx \theta_{es}\theta_{\mu s}$. Furthermore the effective active/sterile mixing angles are now predicted to be

$$\theta^2_{\ell s} = \frac{\pi^2}{3}|V_{\ell 3}|^2 \Delta m^2_{\rm atm} R^2$$

(for a hierarchical spectrum of active neutrinos, the other cases are more problematic). The CHOOZ bound on $V_{e3}$ (that will soon be tested and eventually strengthened by long-baseline experiments) now gives another constraint on $\theta_{es}$, making this minimal model more problematic than 3+1 oscillations. One can consider a large variety of non-minimal extra dimensional models.

In the case of 2+2 oscillations, many sterile neutrinos can instead be less disfavoured that a single sterile neutrino. As discussed above, pure atmospheric sterile oscillations are mostly, but not only, disfavoured by matter effects (in the earth), that suppress $\nu_\mu \to \nu_s$ at large energy: SK data are better fitted by $\nu_\mu \to \nu_\tau$ oscillations, unsuppressed by matter effects. Even in the solar case, matter effects (in the sun) contribute to determine how much SMA sterile oscillations are disfavoured [26]. In presence of a tower of many sterile neutrinos, matter effects do not suppress sterile oscillations at large energy or density, until there is a sufficiently heavy sterile resonance to cross. However, sterile oscillations must be strongly matter suppressed within a supernova. As discussed in [7] supernovæ strongly constrain sterile towers that continue up to masses of $10^{4\div 5}$ eV. This is e.g. the case of an extra-dimensional Kaluza-Klein tower that continues up to the TeV scale [25]. In conclusion, $(2+\text{many})$ oscillations can be less disfavoured than $2+2$ oscillations. However, even forgetting the lack of theoretical motivation, it does not seem possible to achieve a really satisfactory fit.

**We prefer not to present the best fit regions of a global analysis where 3+1 and 2+2 oscillations are treated as different parameter regions of a four neutrino framework, because none of them gives a good fit. By studying separately the two cases, we factor out the statistical fluctuations unrelated to the determination of parameters.

## 2 CPT violation

### Theory

The only safe result is that CPT is conserved in Lorentz-invariant local quantum field theories (QFT). Therefore CPT-violating effects can be obtained by speculating on abandoning locality or Lorentz invariance:

1. In local QFT, CPT violation can be induced if the Lorentz symmetry is broken, e.g. spontaneously by vacuum expectation values of fields with spin 1 or higher, or cosmologically by interactions with some 'æther', or by a non-trivial extra-dimensional background or...

This first possibility seem not promising for LSND: like anomalous matter effects and unlike oscillations, new effects are not enhanced at low neutrino energy. Therefore old experiments [27] done at energies $2 \div 3$ orders of magnitude higher than LSND, disfavour the best fit Karmen/LSND region. We do not perform a dedicated analysis, and focus on the second possibility, that could explain the LSND anomaly [8]:

2. Strings, branes, quantum foams, wormholes, non commutative geometry (and other non local things like that) suggest CPT-violating effects, maybe suppressed by only one power of the quantum gravity scale $M$ (this case gives rise to interesting signals even for $M \sim 10^{19}\,\mathrm{GeV}$ [28]).

If an effect at that level were an unavoidable phenomenon, quantum gravity at the TeV scale would be excluded by bounds on the $K_0\bar{K}_0$ mass difference:

$$m_{K_0} - m_{\bar{K}_0} < 0.4\ 10^{-9}\,\mathrm{eV}.$$

The mass difference between neutrinos and anti-neutrinos that could explain LSND is larger by many orders of magnitude.

The generic Hamiltonian that describes non relativistic systems (e.g. Kaons) violates CPT, if the constraints from the underlying local relativistic QFT are not imposed. In the case of relativistic systems (e.g. neutrinos) one can mimic the standard Hamiltonian demanded by local relativistic QFT (particles together with anti-particles) but without imposing all the constraints demanded by QFT (particles degenerate with anti-particles), so that the generic Hamiltonian that describes free propagation of Dirac neutrinos has different mass terms for $\nu$ and $\bar{\nu}$. The social duty of studying how CPT-violating neutrino masses can arise in popular fundamental models has been exploited in [29], obtaining the imprimatur from string brane-world orbifolds. Non commutative geometry was invoked in [8].

We do not consider other possible CPT violations in neutrino interactions, because experiments with (mainly) $\nu_\mu$, $\bar{\nu}_\mu$ beams and precision electroweak data [30, 31] find that neutrino NC couplings cannot differ from the SM prediction by more than few %. A global fit of electroweak precision data [31] shows that the CC couplings of $e$ and $\mu$ neutrinos agree with the SM with few per-mille accuracy

### Fit of SK and K2K data

In absence of oscillations, the number of $\nu_\mu$-induced events at SK would be roughly double than the number of $\bar{\nu}_\mu$-induced events (the ratio is higher at sub-GeV energies. This is mainly due to the different $\nu_\mu$ and $\bar{\nu}_\mu$ cross-sections on matter, that we compute by summing the elastic and deep-inelastic cross sections [32]). We assume that SK has an equal efficiency for $\nu$ and $\bar{\nu}$-induced events.

We use a (hopefully) self-explanatory notation for the $\nu$ and $\bar{\nu}$ parameters. An over-bar marks anti-neutrino parameters. For example, $\bar{\theta}_{\rm atm}$ and $\Delta\bar{m}^2_{\rm atm}$ parameterize the atmospheric $\bar{\nu}_\mu \to \bar{\nu}_\tau$ oscillations.

To begin, we assume that $\theta_{\rm sun}$, $\theta_{\rm CHOOZ}$, $\bar{\theta}_{\rm CHOOZ}$, $\bar{\theta}_{\rm LSND}$ have negligible effect on atmospheric oscillations, that are therefore described by $\Delta m^2_{\rm atm}$, $\Delta\bar{m}^2_{\rm atm}$, $\theta_{\rm atm}$ and $\bar{\theta}_{\rm atm}$.

A simple approximation captures the main properties of the fit. The up/down asymmetry in the number of multi-GeV muon events is [1]

$$A \equiv \frac{N_\downarrow - N_\uparrow}{N_\downarrow + N_\uparrow} = 0.327 \pm 0.045$$

Assuming maximal mixings, in the CPT-conserving case one has

$$\Delta\bar{m}^2_{\rm atm} = \Delta m^2_{\rm atm} \approx 3\ 10^{-3}\,\mathrm{eV}^2 : \qquad A \approx 1/3$$

The asymmetry is smaller in CPT-violating cases, e.g.

$$\begin{aligned}
\Delta\bar{m}^2_{\rm atm} &\gg \Delta m^2_{\rm atm} \approx 3\ 10^{-3}\,\mathrm{eV}^2 : && A \approx 1/4\\
\Delta\bar{m}^2_{\rm atm} &\ll \Delta m^2_{\rm atm} \approx 3\ 10^{-3}\,\mathrm{eV}^2 : && A \approx 1/5\\
\Delta m^2_{\rm atm} &\gg \Delta\bar{m}^2_{\rm atm} \approx 3\ 10^{-3}\,\mathrm{eV}^2 : && A \approx 1/7\\
\Delta m^2_{\rm atm} &\ll \Delta\bar{m}^2_{\rm atm} \approx 3\ 10^{-3}\,\mathrm{eV}^2 : && A \approx 1/11
\end{aligned}$$

and even smaller if mixings are non maximal. These considerations allow to understand the main features of our numerical result. In fig. 5 we show the $\chi^2$ minimized with respect to the mixing angles. We see that, while $\Delta m^2_{\rm atm}$ is almost as strongly constrained as in a CPT-conserving fit, $\Delta\bar{m}^2_{\rm atm}$ about one order of magnitude larger or smaller that $\Delta m^2_{\rm atm}$.[††] The global $\chi^2$ for SK data is here obtained by summing the $\chi^2$ corresponding to the individual zenith-angle distributions of sub-GeV and multi-GeV (10 $e$-like bins and 10 $\mu$-like bins each), stopping $\mu$ (5 bins) and upward-through-going $\mu$ (10 bins) events. The overall

[††] Unlike in [33], where an analogous fit of some SK data has been performed, our fit does not give any strong evidence for CPT-violation. We also disagree with another CPT-violating fit presented in [34]: from the point of view of forthcoming long-baseline experiments the difference is qualitatively significant even in the CPT-conserving limit.

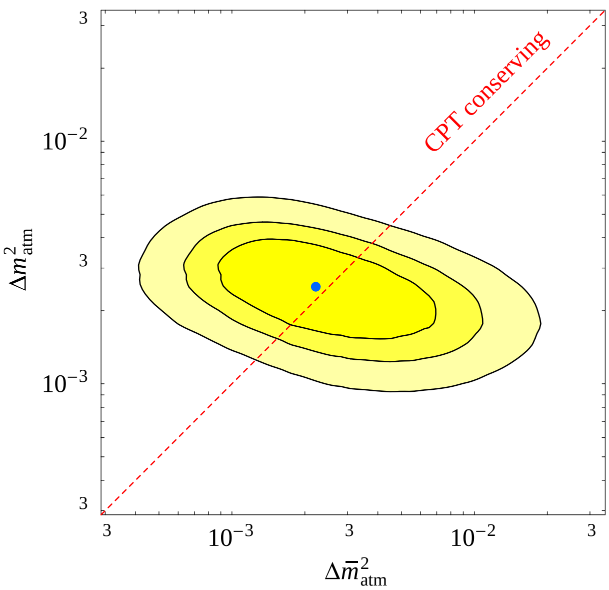


Figure 5: *Fit of SK and K2K data for the neutrino and anti-neutrino atmospheric mass splitting at 68, 90, 99% CL (2 d.o.f.).*

normalization in each kind of events has been considered as a free parameter.

Alternatively, one can try to take into account the theoretical predictions for the overall fluxes as in [35] employing a $55 \times 55$ correlation matrix. This second approach gives a slightly different bound on CPT-violation: larger values of $\Delta\bar{m}^2_{\rm atm}$ would not be significantly disfavoured up to the right border of fig. 5. In the case of K2K data (sensitive to neutrinos) we just fitted the total number of events ignoring the information about e.g. their energy, finding a result in agreement with [36].

Since the best fit is obtained for almost CPT-conserving oscillations, the fit for the mixing angles is quite simple, and we do not need to show a dedicated figure. In the CPT-conserving case $\sin^2 2\theta_{\rm atm}$ has to be close to one. We find that in the CPT-violating case the same bound applies replacing

$$\sin^2 2\theta_{\rm atm} \to \frac{2}{3}\sin^2 2\theta_{\rm atm} + \frac{1}{3}\sin^2 2\bar{\theta}_{\rm atm}$$

so that both $\theta_{\rm atm}$ and (to a lesser extent) $\bar{\theta}_{\rm atm}$ have to be close to maximal.

We now discuss the effects of the other mixing angles that we have so far neglected. Some of them are allowed to be large, but cannot significantly affect our CPT-violating atmospheric fit shown in fig. 5.

In anti-neutrinos, the LSND, Bugey and CHOOZ experiments require a small value of $\bar{\theta}_{\rm LSND}$ (the mixing angle that induces $\bar{\nu}_e \to \bar{\nu}_\mu$ oscillations at the LSND frequency). The CHOOZ experiment requires a small $\bar{\theta}_{\rm CHOOZ}$ (that induces $\bar{\nu}_e \to \bar{\nu}_\mu$ at the atmospheric frequency $\Delta\bar{m}^2_{\rm atm}$) *unless* $\Delta\bar{m}^2_{\rm atm}$ is below $\Delta m^2_{\rm atm}$ and below the CHOOZ sensitivity. This $\bar{\nu}$ angle is only weakly bounded by atmospheric data.

In neutrinos, solar experiments require $\theta_{\rm sun} \sim 1$ as in the CPT-conserving case. Unlike in the standard case CHOOZ does not force $\Delta m^2_{\rm sun}$ to be smaller than about $0.7\ 10^{-3}\,{\rm eV}^2$. Atmospheric data allow a $\Delta m^2_{\rm sun}$ just below the atmospheric mass scale [37]. This possibility gives energy-independent oscillations of solar neutrinos. Few years ago this solution was strongly disfavoured by solar data, but not by safe ones [37]. With the latest solar data, this solution is now within best-fit regions, although the absolute best solar fit is obtained for a smaller $\Delta m^2_{\rm sun} \approx 5\ 10^{-5}\,{\rm eV}^2$ in the LMA range [38, 21, 23, 17, 20, 22]. The angle $\theta_{\rm CHOOZ}$ (that induces $\nu_\mu \to \nu_e$ oscillations at the atmospheric frequency; we improperly adopt the name used in CPT-conserving analyses) is not bounded by CHOOZ (i.e. by disappearance of $\bar{\nu}_e$), but only by global fits of solar and atmospheric data, that weakly prefer a small value of $\theta_{\rm CHOOZ}$ but allow for a large $\theta_{\rm CHOOZ}$ [39].

At the light of these results, we can now list the CPT violating signals that could appear in forthcoming experiments

- MiniBooNE will not see the LSND oscillations, if will only search them as $\nu_\mu \to \nu_e$ rather than as $\bar{\nu}_\mu \to \bar{\nu}_e$.

While this signal is mandatory if the CPT-violating interpretation of the LSND anomaly [8] is correct, the following signals can but need not to appear, depending on the values of the unknown parameters:

- We would have a signal for CPT violation if KamLAND will find no solar oscillations in its reactor data, and Borexino will indirectly favour LMA by finding a $\sim 1/2$ suppression and no matter or seasonal effects (however, as summarized in [19], Borexino cannot discriminate between LMA and oscillations with $\Delta m^2_{\rm sun} \approx 10^{-8}\,{\rm eV}^2$, not significantly disfavoured by present data).

- If $\bar{\theta}_{\rm CHOOZ}$ were large, KamLAND would discover its effects and misinterpret them as LMA oscillations. In particular this implies that, if KamLAND will confirm LMA, the CPT-violating spectrum of [8] would not be immediately excluded. In general, few possibilities (not listed) could happen, depending on future Borexino (or KamLAND) solar results.

- According to our fit in fig. 5, long-baseline experiments that plan to employ a $\nu_\mu$ beam (like K2K, Minos and CNGS) have almost the same capabilities of confirming atmospheric oscillations as in the CPT-conserving case. Using a a $\bar{\nu}_\mu$ beam they can also test if $\Delta\bar{m}^2_{\rm atm}$ is higher than $\Delta m^2_{\rm atm}$ (if $\Delta\bar{m}^2_{\rm atm}$ is as large as possible, a 5% $\bar{\nu}_\mu$ contamination in the $\nu_\mu$ beam could also give detectable $\tau$-appearance effects).

| model and number of free parameters | | $\Delta\chi^2_{\rm tot}$ | $\Delta\chi^2_{\rm sun}$ | $\Delta\chi^2_{\rm atm}$ | $\Delta\chi^2_{\rm LSND}$ | $\Delta\chi^2_{\rm bounds}$ |
|---|---|---|---|---|---|---|
| normal 3 neutrinos | 5 | 25 | 0 | 0 | 28.8 | 0 |
| $3+1: \ \Delta m^2_{\rm sterile} = \Delta m^2_{\rm LSND}$ | 9 | 8 | 0 | 0 | 3.4 | 8.3 |
| $2+2: \ \Delta m^2_{\rm sterile} = \Delta m^2_{\rm sun}$ | 9 | $10 \div 14$ | $10 \div 14$ | 0 | 0.4 | 3.1 |
| $2+2: \ \Delta m^2_{\rm sterile} = \Delta m^2_{\rm atm}$ | 9 | 40 | 0 | 40 | 0.4 | 3.1 |
| 3 neutrinos and ~~CPT~~ | 10 | 0 | 0 | 0 | 0.4 | 3.1 |

Table 1: *Few interesting models, the number of their relevant parameters and their minimal global $\Delta\chi^2$ (so that the best-fit corresponds to 0). The last 4 columns show the minimal $\Delta\chi^2$ restricted to solar data, atmospheric data, LSND data, and to experiments compatible with no oscillations (mainly CHOOZ, Bugey and CDHS).*

- These long-baseline experiments can test if $\theta_{\rm CHOOZ}$ is larger than what allowed in the CPT-conserving case by looking at $\nu_\mu \to \nu_e$.
- In the same way, they could discover solar $\nu_\mu \to \nu_e$, if the solar frequency $\Delta m^2_{\rm sun}$ were large enough.

In longer terms, an atmospheric experiment that separately measures $\Delta m^2_{\rm atm}$ and $\Delta \bar{m}^2_{\rm atm}$ (and sees the first oscillation dip) seems feasible [40], although KEK, CERN and FermiLab preferred to pursue 3 long-baseline experiments.

With a hierarchical $\bar\nu$ spectrum (rather than with the inverted spectrum motivated in [8]) planned $\beta$-decay experiments like KATRIN [41] can test the upper part of the $\Delta m^2$ range suggested by LSND [29]. Planned neutrino-less double $\beta$-decay experiments [42] have brighter perspectives of improvement than $\beta$-decay experiments, but CPT-violating neutrino masses seem to require Dirac (rather than Majorana) neutrinos, if the Lorentz symmetry is unbroken.

In the far future, with a neutrino factory it should be possible to test CPT conservation in atmospheric oscillations at the % level [43].

# 3 Conclusions

A possible global explanation of the three neutrino anomalies (atmospheric, solar and LSND) is that an extra sterile neutrino generates one of them. The relatively better global fit is obtained with a 3+1 spectrum (sterile LSND oscillations) rather than with a 2+2 spectrum (sterile solar or atmospheric oscillations). However the fit is not good: within the 3+1 scheme the LSND anomaly conflicts with $\nu_e$ or $\nu_\mu$ disappearance experiments. One needs to invoke a statistical fluctuation with around % probability to understand why Bugey, CHOOZ, CDHS or SK have not seen sterile effects.

Within the 2+2 scheme, solar data contradict atmospheric data, because sterile neutrinos are predicted to generate solar or atmospheric oscillations (or a part of both), but solar and atmospheric data prefer active oscillations. Since atmospheric bounds are stronger, the best 2+2 fit is obtained for solar sterile oscillations and should be definitively tested by SNO with more statistics.

Ignoring that a satisfactory sterile fit is never obtained, by combining the data from all relevant experiments we find that at the moment the best fit LSND regions are somewhat different in the 3+1 and 2+2 cases, as shown in fig. 4. In particular, this means that MiniBooNE could discriminate the two cases.

Our main results are summarized in table 1. Despite some differences, we agree with a recent similar analysis [18] on the general conclusions.

Many sterile neutrinos (motivated e.g. in extra dimensional models) can somewhat improve the fit in both cases, but it does not seem possible to obtain a good sterile solution.

In view of these unsatisfactory sterile fits, and of the latest LSND results [3]

$$\begin{aligned} P(\nu_\mu \to \nu_e) &= (1.0 \pm 1.6)\, 10^{-3} \\ P(\bar\nu_\mu \to \bar\nu_e) &= (2.6 \pm 0.8)\, 10^{-3} \end{aligned}$$

one might want to speculate on CPT-violation. A satisfactory global fit of all neutrino data (see table 1) can be obtained with the CPT-violating neutrino masses proposed in [8]. Theory gives no useful restriction, and in particular does not tell if CPT should be violated also in atmospheric oscillations, although it looks plausible. Fig. 2 shows how present SK and K2K data restrict the atmospheric oscillation parameters $\Delta m^2_{\rm atm}$ and $\Delta \bar{m}^2_{\rm atm}$. They can differ by about one order of magnitude. In section 2 we studied which CPT-violating oscillations are compatible with present data, and listed the unusual signals that could be seen at forthcoming solar (KamLAND, Borexino) and long-baseline experiments (K2K, MINOS, CNGS) — and of course at MiniBooNE.

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