IPPP/01/64
DCPT/01/128
HIP-2001-67/TH
CERN-TH/2002-001

On the $B \to X_s\, l^+ l^-$ decays in general supersymmetric models

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Abstract

We analyze the inclusive semileptonic decays $B \to X_s\, l^+ l^-$ in the framework of the supersymmetric standard model with non-universal soft-breaking terms at GUT scale. We show that the general trend of universal and non-universal models is a decreasing of branching ratio (BR) and increasing of energy asymmetry (AS). However, only non–universal models can have chances to get very large enhancements in BR and AS, corresponding to large (negative) SUSY contributions to the $b \to s\gamma$ amplitude.

Flavor changing neutral current (FCNC) and CP violating phenomena can be considered as one of the best indirect probe for physics beyond the standard model (SM). Due to the absence of tree level FCNCs in the SM and the suppression of the Glashow-Iliopoulos-Maiani mechanism, they are particularly sensitive to any non standard physics contribution.

In the framework of low energy supersymmetric (SUSY) models, FCNC processes play an important role in severely constraining the soft breaking sector of supersymmetry [1]. As known, these constraints require an high degree of degeneracy in the squark mass matrices, suggesting that the mechanism which transmits the SUSY breaking to the observable sector should be flavour blind. For instance, minimal supergravity (mSUGRA) and gauge–mediation mechanisms successfully explain this degeneracy. In particular, in mSUGRA scenario all the tests on FCNCs can be satisfied due to the assumption of universality for the soft breaking terms at GUT scale. However, recently there has been a growing interest concerning supersymmetric models with non–universal SUSY soft–breaking terms [2]. This is motivated by the fact that superstring inspired models, where supergravity theories are derived, naturally favour non–universality in the soft-breaking terms [3]. This is mainly due to the fact that superstring theories live in extra–dimensions and after compactification, non–flat Kähler metrics and flavour–dependent SUSY soft–breaking terms can arise.

Particularly interesting among this class of models are the ones with non–universal trilinear soft–breaking terms in the scalar sector, the so–called A–terms. These models can have interesting phenomenological consequences. They could solve in principle the SUSY CP problem, satisfy all the FCNC constraints, and provide at the same time new significant contributions to the direct CP violating parameter ε'/ε as suggested by the recent experimental results on ε'/ε [4]. Moreover, it has been argued that this class of models should also pass the strong constraint on $B \to X_s \gamma$ decay [5] and gives rise to a large contribution to the CP asymmetry, of order $10\% - 15\%$ which can be accessible at B factories [6].

In this letter we analyze the impact of a large class of supersymmetric models with non–universal soft SUSY breaking terms (which is motivated by the string inspired scenarios) in the semileptonic (inclusive) $B \to X_s \, l^+ l^-$ decays (with $l = e, \mu$). As for the $B \to X_s \gamma$ decay, these processes are also very interesting for several reasons: first they are very sensitive to large $\tan\beta$ since they involve the magnetic dipole operator (Q_7) which allows the quark $b \to s\gamma$ transition. Second, they involve other operators as well, the semileptonic operators Q_9 and Q_{10}, and so can serve as complementary tests of the model. Third, they provide several measurable quantities, such as branching ratios and asymmetries. At present these decays are known in QCD at the next–to–leading (NLO) order logaritmhmic accuracy for the SM [7], and also $1/m_b$ nonperturbative contributions are small and well under control.

From the experimental side, the situation about these decay channels is quite exciting. The BELLE experiment has recently announced the first evidence for the exclusive process $B \to K^\star l^+ l^-$ [8], and upper bounds for the three body decays $B \to (K, K^\star) + (e^+ e^-, \mu + \mu -)$, reported by BABAR and BELLE, are very close the SM expectations [8, 9]. However, exclusive processes are affected by larger theoretical uncertainties than the inclusive ones due to model dependent calculations of hadronic matrix elements. For this reason we will restrict our analysis to the inclusive ones.

In the framework of supersymmetric models, there are several studies about $B \to X_s \, l^+ l^-$ decays in the literature [10–13]. However, a detailed analysis about SUSY models with non–universal soft breaking terms at GUT scale has not been considered. As suggested by a recent study [10], based on the low energy approach to supersymmetric models, the non–universality in the soft–breaking sector could generate significant departures from the SM in the semileptonic $B \to X_s \, l^+ l^-$ decays. In this analysis the mass insertion method has been used, where the pattern of flavour change is parametrized by the ratios

$$(\delta_{ij})^f_{AB} = \frac{(m^2_{ij})^f_{AB}}{M^2_{sq}} \qquad (1)$$

where $(m^2_{ij})^f_{AB}$ are the off–diagonal elements of the the $f = \tilde{u}, \tilde{d}$ scalar mass squared matrix which mixes flavour i, j for both left– and right–handed scalars ($A, B = \text{left}, \text{right}$), and M_{sq} is the average squark mass. The main conclusion of this work is that FCNCs constraints and vacuum stability bounds, which strongly constrain these δs, could not prevent large effects in $B \to X_s\, l^+ l^-$ decays. In particular, large SUSY contributions to the Wilson coefficients C_9 and C_{10} at EW scale, corresponding respectively to the semileptonic operators Q_9 and Q_{10}, are possible. Therefore, generic SUSY models (with non–universalities in the scalar sector and A-terms implemented at GUT scale) seem indeed an ideal scenario where these large effects could be found. However, it should be stressed that in the analysis of Ref.[10], the enhancement of C_9 and C_{10} is obtained by taking all the δs and other SUSY parameters at low energy as free parameters, in particular the gluino, the lightest stop mass and the bilinear Higgs couplings (the μ term). In the class of models analyzed here, we will see that these sizable effects to C_9 and C_{10} will not show up, leaving to potential large deviations only in the Wilson coefficient (C_7) of the magnetic–dipole operator Q_7. The main reason is due to the fact that the relevant (low energy) SUSY parameters for enhancing C_9 and C_{10} are strongly correlated, leaving the $B \to X_s\gamma$ and the experimental bounds on SUSY mass spectrum very effective in preventing such enhancements.

Furthermore, we will consider the effect of the SUSY models with non–abelian flavour symmetry on these semileptonic decays. The main effect of this symmetry is to prevent excessive FCNC effects in case that the mechanism of SUSY breaking should not be flavour blind. As a specific example, we will analyze here the model proposed in Ref.[14], in which the pattern of flavour violation is implemented by the breaking of an U(2) (horizontal) flavour symmetry. For the same reason given above, also these models have large potentialities to give sizable deviations in $B \to X_s\, l^+ l^-$ decays, since they contain a new flavour structure in addition to Yukawa matrices. However, we will see that the same conclusions about sizable contributions to C_9 and C_{10} will hold for these models as well.

Now we start with the SM results for the inclusive $B \to X_s\, l^+ l^-$ decays. Inclusive hadronic rates in B meson decays can be precisely calculated by using perturbative QCD and $1/m_b$ quark expansion. The effective Hamiltonian for the b quark semileptonic decay $b \to s\, l^+ l^-$ is given by

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V^{\star}_{ts} V_{tb} \sum_{i=1}^{10} C_i(\mu_b) Q_i(\mu_b) \ , \qquad (2)$$

where $Q_i(\mu)$ are the $\Delta B = 1$ transition operators evaluated at the renomalization scale $\mu \simeq \mathcal{O}(m_b)$. A complete list of operators involved in this decay are given in Refs.[7, 15].

The relevant operators that can be affected by the SUSY contributions are given by

$$\begin{aligned} Q_7 &= \frac{e}{16\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} b_R \; F_{\mu\nu} \;, && (3) \\ Q_8 &= \frac{g_s}{16\pi^2} m_b \bar{s}_L T^a \sigma^{\mu\nu} b_R \; G_{\mu\nu} \;, && (4) \\ Q_9 &= (\bar{s}_L \gamma_\mu b_L) \; \bar{l} \gamma^\mu l \;, && (5) \\ Q_{10} &= (\bar{s}_L \gamma_\mu b_L) \; \bar{l} \gamma^\mu \gamma_5 l \;. && (6) \end{aligned}$$

At one-loop, the SUSY contributions to these operators are given by Z and γ super-penguin and box diagrams, where inside the loop can run charged Higgs, charginos, gluinos, neutralinos, squarks, and sleptons [11, 12].

The general SUSY Hamiltonian also contains the operators $\tilde{Q}_i$ which have opposite chirality with respect to the Q_i ones. In the SM and minimal flavor SUSY models, these contributions are suppressed by $\mathcal{O}(m_s/m_b)$. However, in generic SUSY models, and in particular, in case of non–degenerate A–terms this argument is no longer true. For instance, the gluino contribution to these operators depend on $(\delta^d_{23})_{LR}$ and $(\delta^d_{23})_{RL}$. Here both of these mass insertions are linear combinations of the down type quark masses rather than m_b or m_s exclusively. Therefore, to be consistent, one has to include the contributions of these operators. Indeed, the effects of the operators $\tilde{Q}_{7,8}$ have been found to be very significant for the branching ratio of the $B \to X_s \gamma$ decay [5] and for the CP asymmetry of this decay as well [6]. The Wilson coefficients $C_i(\mu)$ can be decomposed as

$$C_i(\mu) = C_i^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_i^{(1)}(\mu) + \mathcal{O}(\alpha_s^2) \;, \tag{7}$$

where $C_i^{(0)}$ and $C_i^{(1)}$ refer to the LO and NLO results, respectively. For our purpose, the SUSY corrections from including the NLO and NNLO are unimportant. The new physics effects in $b \to s\, l^+ l^-$ can be paramterized by R_i and $\tilde{R}_i$, $i = 7, 8, 9, 10$ defined at the EW as

$$R_i = \frac{C_i^{(0)} - C_i^{(0)SM}}{C_i^{(0)SM}} \;, \qquad\qquad \tilde{R}_i = \frac{\tilde{C}_i^{(0)}}{C_i^{(0)SM}} \;. \tag{8}$$

Note that there is no SM contribution to $\tilde{C}_i$. In the minimal supersymmetric standard model (MSSM), the expressions for R_i and $\tilde{R}_i$ are given in Refs.[5, 11, 12]. However, we anticipate that in the class of models analyzed here, the SUSY contribution to R_9 and R_{10} is very small in comparison to R_7, the same conclusion hold for $\tilde{R}_9$ and $\tilde{R}_{10}$ as well. Therefore, in order to simplify our analysis, we will use the approximation in which the SUSY dependence in $b \to s\; l^+ l^-$ decay enters only through the ratios of Wilson coefficients R_7, R_9, R_{10}, and $\tilde{R}_7$. Note that the dependence on R_8 and $\tilde{R}_8$ is modest in $b \to s\; l^+ l^-$, due to the fact that the operator Q_8 mixes with Q_7 at the NLO. For this reason we will neglect their contribution. We have explicitly checked that this approximation does not significantly affect our results.

Using this parametrization, the non–resonant branching ratios (BR)* are expressed in terms of the new physics contribution as [12, 13].

$$\begin{aligned} \mathrm{BR}\;(\;B\to X_s\,e^+e^-) &= 7.29\times10^{-6}\Big(1+0.714\;R_{10}+0.357\;R_{10}^2+0.35\;R_7 \\ &+\;\;0.0947(R_7^2+\tilde{R}_7^2)+0.179R_9-0.0313\;R_7R_9+0.045\;R_9^2\Big), \end{aligned} \tag{9}$$

$$\begin{aligned} \mathrm{BR}\;(\;B\to X_s\,\mu^+\mu^-) &= 4.89\times10^{-6}\Big(1+1.07\;R_{10}+0.535\;R_{10}^2+0.0982\;R_7 \\ &+\;\;0.0491(R_7^2+\tilde{R}_7^2)+0.264R_9-0.0467\;R_7R_9+0.0671\;R_9^2\Big)\,. \end{aligned} \tag{10}$$

The SM values 7.29×10^{-6}, 4.89×10^{-6}, which correspond to $\mathrm{BR}(B\to X_s\,e^+e^-)$ and $\mathrm{BR}(B\to X_s\,\mu^+\mu^-)$ respectively, are recovered by setting $R_i=\tilde{R}_i=0$ in these formula. An important observation from Eqs.(9–10) is that the decay $b\to s\,l^+l^-$ is quite sensitive to R_{10} rather than the other variables. Therefore any enhancement for R_{10} could lead to significant changing in the prediction of BR of this decay without any consequences on $b\to s\gamma$ decay, which mainly depends on R_7. It is worth noticing that the different sensitivity in R_7 in Eqs.(9–10) is due to the fact that the coefficients proportional to R_7 come from integrating the $1/q^2$ pole (with q^2 the momentum square of the virtual photon) of the magnetic operator Q_7. Therefore, being the minimum value of q^2 proportional to the mass square of final leptons, the sensitivity to R_7 becomes larger in the electron channel.

We will also consider the lepton – anti–lepton energy asymmetry (AS) in the decay $b\to s\,l^+l^-$ which is defined as

$$\mathcal{A}=\frac{N(E_{l^-}>E_{l^+})-N(E_{l^+}>E_{l^-})}{N(E_{l^-}>E_{l^+})+N(E_{l^+}>E_{l^-})}, \tag{11}$$

where, for instance, $N(E_{l^-}>E_{l^+})$ is the number of the lepton pairs whose negative charged member is more energetic in the B meson rest frame than its positive partner. As for the BRs we will consider the AS in Eq.(11) integrated over non-resonant regions. With the above parametrization we find [12, 13]

$$\begin{aligned} \mathcal{A}_{ll} &= \frac{0.48\times10^{-6}}{\mathrm{R_{BR}(B\to X_s\,l^+l^-)}}\Big(1+0.911R_{10}-0.00882\;R_{10}^2-0.625\;R_7(R_{10}+1) \\ &+\;\;0.884\;R_9(R_{10}+1)\Big). \end{aligned} \tag{12}$$

where $R_{BR}=BR/BR^{SM}$.

Finally, regarding the $B\to X_s\,\gamma$ decay, we have used the following parametrization [5, 13]

$$\begin{aligned} \mathrm{BR}(B\to X_s\,\gamma) &= (3.29\pm0.33)\times10^{-4}\left(1+0.622R_7+0.090(R_7^2+\tilde{R}_7^2)\right. \\ &+\;\;\left.0.066R_8+0.019(R_7R_8+\tilde{R}_7\tilde{R}_8)+0.002(R_8^2+\tilde{R}_8^2)\right)\;, \end{aligned} \tag{13}$$

*In order to reduce the large non–perturbative contributions to the $B\to X_s\,l^+l^-$ decays, the resonant regions in the final invariant mass of the dilepton system l^+l^- should be avoided. This can be easily implemented by excluding some special areas from the integration regions in the dilepton invariant mass. The resulting BR where these regions have not been included, is usually called the non–resonant BR.

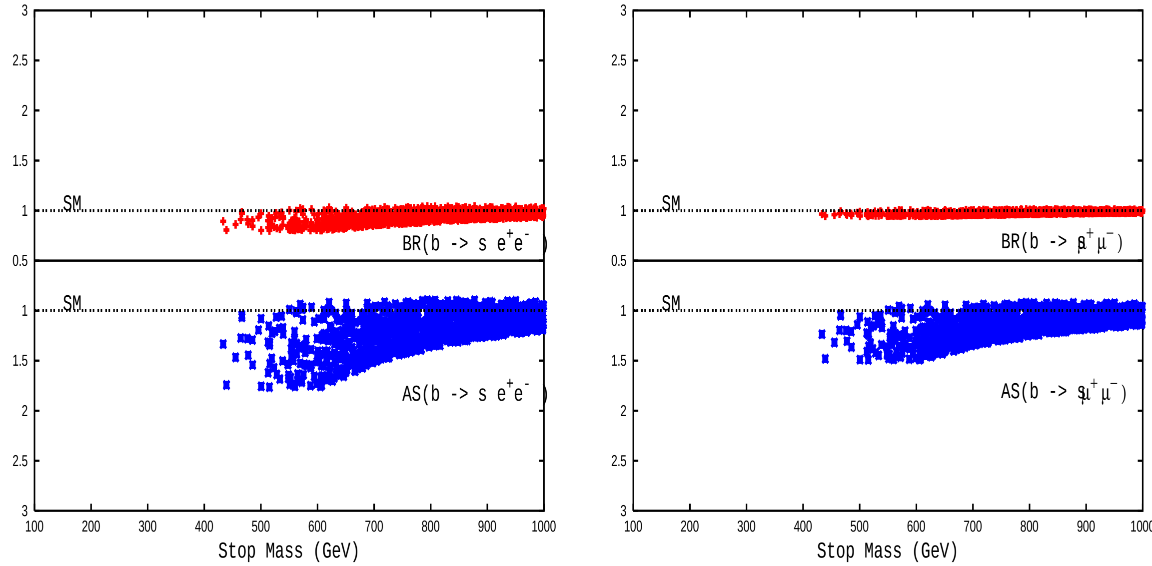

Figure 1: Branching ratio (BR) and energy asymmetry (AS) of $B \to X_s\, e^+e^-$ and $B \to X_s\, \mu^+\mu^-$ (normalized to the corresponding SM ones) versus the lightest stop mass in minimal SUGRA model.

where the overall factor corresponds to the SM value and its theoretical uncertainty.

We start our analysis by revisiting the predictions for the rate of these decays in the supersymmetric models with minimal flavor violation (such as the minimal SUGRA inspired model). In particular, we will show that the new bound on the Higgs mass [16] and the CLEO measurement for the BR of $B \to X_s\, \gamma$ decay [17]

$$2.0 \times 10^{-4} < BRB \to X_s\, \gamma) < 4.5 \times 10^{-4} \tag{14}$$

impose sever constraints on the parameter space of this class of models and it is no longer possible to have deviations on the non-resonant BR of $B \to X_s\, e^+e^-$ and $B \to X_s\, \mu^+\mu^-$ decays by more than 25% and 10% respectively, relative to their SM expectations.

The main reason for that is the following. As emphasized above, the main contributions to these processes are due to the operators Q_7, Q_9, and Q_{10} where their Wilson coefficients are proportional to the mass insertions $(\delta^{u,d}_{23})_{LR}$ and $(\delta^{u,d}_{23})_{LL}$. However in the minimal SUGRA scenario, and due to the universality assumption upon the soft SUSY breaking parameters at GUT scale, the flavor transitions are suppressed by the smallness of the CKM angles and/or the smallness of the Yukawa couplings. Moreover in this scenario, requiring the lightest Higgs mass to be $m_h > 110$ GeV implies that the universal gaugino masses $m_{1/2}$ has to be larger 250 GeV. This leads to a heavy stop mass and hence a further suppression for the SUSY contribution to $B \to X_s\, l^+l^-$ decays is found.

In our analysis we present our results for a specific choice of the $sign(\mu)$. This choice corresponds to the one which gives positive contributions to the $g-2$ of the muon, as it is favoured by the new experimental results on $g-2$ [18]. Incidentally, this specific choice of $sign(\mu)$ is the one for which the $B \to X_s\gamma$ constraints are less effective. In Fig. 1, we present the scatter plots of the BR and the AS for the decay $B \to X_s\, e^+e^-$ (which is the most sensitive semileptonic decay) and $B \to X_s\, \mu^+\mu^-$ as a function of the lightest stop mass. In obtaining these figures, we varied the universal soft scalar mass m_0 and gaugino mass $m_{1/2}$ from 50 GeV up to 1 TeV. The trilinear A-term is fixed to be $A_0 = m_0$ and $\tan\beta$ vary in the range $[3, 40]$. In our numerical analysis we assume the radiative electroweak symmetry breaking and impose the current experimental bounds on the SUSY spectra. We have also imposed the constraints which come from requiring vacuum stability (necessary to ensure that the potential is bounded from below) and from avoiding charge and color breaking minima deeper than the real one.[†] It turns out that the present experimental limit on the lightest Higgs mass sets the most important constraint in minimal SUGRA models. In particular, it excludes the parameter space that leads to stop masses lower than 400 GeV. It is clear that with such heavy stop masses the dominant contribution to $b \to s\, l^+l^-$, which comes from chargino exchanges, is quite suppressed.

As can be seen from Figs.1–3, the general trend of this class of models, for this particular choice of $sign(\mu)$, is in a decreasing of BR and increasing of AS with respect to the SM expectations, in both universal and non-universal models. The origin of this behaviour can be explained as follows. As discussed above, the variations of BR and AS are mainly due to R_7. For this choice of $sign(\mu)$ the $B \to X_s\gamma$ constraints are less restrictive and mostly allow for negative values of R_7. Negative values of R_7 (in the range of [-1,0]) will produce destructive and constructive interferences in BR and AS respectively, as can be understood from the parametrizations in Eqs.(9)–(12). However, we have also checked that for the other choice of $sign(\mu)$ the behaviour is opposite, giving an enhancement of BR and decreasing of AS, but with more moderate effects due to a stronger action of $B \to X_s\gamma$ constraints.

In the large $\tan\beta$ region, where chargino and Higgs contributions to R_7 are enhanced, (R_9 and R_{10} are moderately affected by $\tan\beta$), a sizeable changing in the BR and AS of $B \to X_s\, l^+l^-$ decays might arise. Nevertheless, R_7 gives also the major contribution to the BR of $B \to X_s\, \gamma$, and by imposing the CLEO limits we dismiss such large effects for $B \to X_s\, l^+l^-$ decay. Therefore, we can conclude that in SUSY models with universal soft breaking terms, it is not possible to get any significant enhancement for BR in $B \to X_s\, l^+l^-$ decays, while a decreasing up to 25% can be obtained in the electron channel. As explained above, the decreasing of BR is reflected in a large enhancement of AS, in particular up to 75% and 50% for the electron and muon channels respectively. However, we will see that in general SUSY models, mainly due to the non-universality in the scalar sector, the Higgs bounds can be relaxed and larger deviations on BR and AS for $B \to X_s\, l^+l^-$ decays can be achieved, deviations which correspond to the allowed region

[†]We stress that these last conditions may be automatically satisfied in minimal SUGRA, while in generic SUSY models, like those we will consider below, these conditions have to be explicitly checked.

of large negative values of R_7 (namely in the range of [-6,-4]).

Now we turn to the most general supersymmetric extension of the SM. In particular, we will consider SUSY models with non–degenerate A–terms and non–universal soft scalar and gaugino masses. Such models are naturally obtained from string inspired models [2] and some aspect of their phenomenological implications have been recently studied. Note that the squark mass matrices are often diagonal in string inspired models and this is what we will adopt here. Generic SUSY models might also have non–universality in the off-diagonal terms of squark mass matrices. Nevertheless, these off-diagonal terms are severely constrained by ΔM_K, ΔM_B, and ε_K. Models with flavor symmetries naturally avoid such constraints. We will consider later a model with $U(2)$ flavor symmetry as an example for this class models.

In order to parametrize the non-universality of a large class of string inspired models (with diagonal soft-breaking terms in the sfermion sector), we assume here the following soft scalar masses, gaugino masses M_a and trilinear couplings:

$$
\begin{aligned}
M_a &= \delta_a \, m_{1/2}, \quad a = 1,2,3, & (15)\\
m_Q^2 &= m_L^2 = m_0^2 \; diag\{1, 1, \delta_4\}, & (16)\\
m_U^2 &= m_0^2 \; diag\{1, 1, \delta_5\}, & (17)\\
m_D^2 &= m_E^2 = m_0^2 \; diag\{1, 1, \delta_6\}, & (18)\\
m_{H_1}^2 &= m_0^2 \; \delta_7, \; m_{H_2}^2 = m_0^2 \; \delta_8, & (19)\\
A^u &= A^d = A^l = m_0 \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}. & (20)
\end{aligned}
$$

where the parameters δ_i and a_{ij} can vary in the $[0,1]$ and $[-3,3]$ ranges respectively. It has recently been emphasized that these models could be free from the EDMs constraints and also have testable implications for the CP violation experiments [4]. In Ref.[5], the prediction for the BR of $B \to X_s \gamma$ decay has been considered in two representative examples for this class of models and it was found that $B \to X_s \gamma$ does not essentially constrain the non–universality of A–terms.

In our convention for the trilinear couplings, the A terms are defined such that $\hat{A}_{ij} = A_{ij} Y_{ij}$ (indices not summed) and Y_{ij} are the corresponding Yukawa couplings. We assume that the Yukawa matrices at EW scale are given by

$$
Y^d = \frac{1}{v_1} V_{CKM}^* \mathrm{diag}(m_d, m_s, m_b), \quad Y^u = \frac{1}{v_2} \mathrm{diag}(m_d, m_s, m_b) V_{CKM}^T \,, \qquad (21)
$$

For any value of the parameters m_0, δ_i, $m_{1/2}$, a_{ij} at GUT scale, and $\tan\beta$ (we determine the μ and B parameters from the electroweak breaking conditions) we compute the relevant SUSY spectrum and interaction vertices at low energy needed for the calculation of the $b \to s\, l^+ l^-$ decay amplitudes. In order to connect the high energy SUSY parameters,

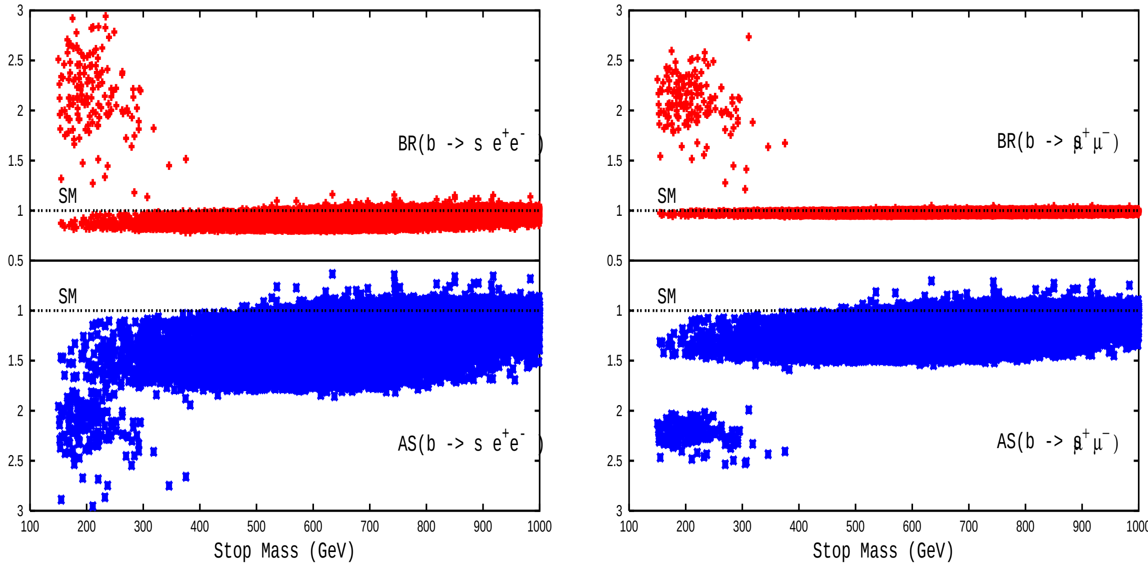

Figure 2: As in Fig. 1, but for SUSY model with non–universal soft breaking terms.

gauge and Yukawa couplings with the corresponding low energy ones, we have used the most general renormalization group equations in MSSM at 1-loop level. As stated above, we impose the current experimental bounds on SUSY spectrum, in particular lightest Higgs mass $m_h > 110$ GeV, and $B \to X_s\gamma$ constraints in Eq.(14).

In Fig. 2 we present scatter plots for the BR and AS for the $B \to X_s\, e^+e^-$ and $B \to X_s\, \mu^+\mu^-$ decays versus the lightest stop mass. As for the universal models, we varied the fundamental mass parameters m_0, $m_{1/2}$ from 50 GeV up 1 TeV, and $\tan\beta$ in the range $[3, 40]$. The parameters δ_i and a_{ij} have been also randomly selected in the ranges $[0, 1]$ and $[-3, 3]$ respectively. It is worth mentioning that the gluino contributions are negligible in the universal limit and the non–universality in the A–terms is essential for enhancing such contributions. Moreover, with non–universality in the gaugino masses we can have light chargino and stop masses close to their experimental limit and the Higgs mass bound satisfied. In this region of parameter space indeed the chargino contributions to R_7, R_9 and R_{10} are enhanced. However, it is noted that in all the parameter space, $R_{9,10}$ are much smaller than R_7 and still the main contributions to these processes are due to R_7 which also gives the main contribution to the $B \to X_s\,\gamma$ decay.

As can be seen from Fig. 2 there is a disconnected region of points, for stop masses lighter than 300 GeV, where very large enhancements in both BR and AS are reached. In particular, a factor 3 and 2.5 of enhancements in both BR and AS are obtained for electron and muon channels respectively. This region corresponds to the large (negative) SUSY contributions to R_7, roughly in the range of [-6,-4], obtained for $\tan\beta > 30$. Nevertheless, these huge enhancements belong to the less populated areas of scatter plots which means that a larger amount of fine tuning between the SUSY parameters is needed in this case.

The more populated areas in Fig.2 correspond to the other (disconnected) range of allowed values for R_7, namely $-1 < R_7 < 1$. In this region, the $B \to X_s\,\gamma$ constraints reduce the enhancements (with respect to the SM one) on the BR of $B \to X_s\, e^+e^-$ to be less than 20% and the decreasings up to 25%. More moderate effects are obtained for the muon channel, since it is less sensitive to R_7. In correspondence to these variations on the BR, larger effects are obtained for the AS. In particular up to 75% and 50% enhancements in the AS for electron and muon channel respectively, while a more moderate increasing (about 40% and 25% respectively) are expected.

Now we compare our results with the model independent analysis of Ref. [10], based on a low energy approach. Using the mass insertion approximations and general MSSM at low energy it was shown in Ref. [10] that the SUSY contributions to BR and AS of the semileptonic decays can get maximum enhancement (up to 4×10^{-5} for the $BR(B \to X_s\, e^+e^-)$ i.e 4 times the SM value). However, this needs the following values for the mass insertions (δ_{23}):

$$(\delta_{23}^{u,d})_{LL} \simeq -0.5, \quad (\delta_{23}^{u})_{LR} \simeq 0.9. \tag{22}$$

Such values can be obtained only in a very small region of the parameter space of the SUSY models with non–universal soft terms, specially after imposing the electroweak breaking conditions, the new bounds on the Higgs mass, and $B \to X_s\,\gamma$ contraints. However, we found that in general the typical values of these mass insertions are $|(\delta_{23}^{u,d})_{LL}| \simeq 10^{-2}$, $(\delta_{23}^{u})_{LR} \simeq 10^{-3}$. This, indeed, leads to a BR for the $B \to X_s\, e^+e^-$ decay of order 10^{-6} with at most 20% enhancement than the SM value.

Finally we proceed to consider SUSY models with non–abelian flavour symmetry. This class of models has a flavour structure in the soft scalar masses, and hence, the LL sector contains larger mixing than what is found in the previous models with diagonal squark masses. As mentioned, the ΔM_K and ε_K impose sever constraints on the squark mixing, namely $\sqrt{|Re(\delta_{12})^2_{LL}|} \lesssim 10^{-2}$ and $\sqrt{|Im(\delta_{12})^2_{LL}|} \lesssim 10^{-3}$ respectively [1].

Here as an illustrative example, we consider a model based on a $U(2)$ symmetry acting on the two light families [14] where the above mentioned constraints are satisfied. In this case, the Yukawa textures, at GUT scale, are given by [14]

$$Y_u = \frac{m_t}{v \sin\beta} \begin{pmatrix} 0 & c\varepsilon\varepsilon' & 0 \\ -c\varepsilon\varepsilon' & 0 & a\varepsilon \\ 1 & b\varepsilon & 1 \end{pmatrix}; \; Y_d = \frac{m_b}{v \cos\beta} \begin{pmatrix} 0 & \frac{\varepsilon'}{\sqrt{1+\rho^2 k^2}} & 0 \\ -\varepsilon' & 0 & a\varepsilon \\ 1 & \rho & 1 \end{pmatrix}; \tag{23}$$

and the squark mass matrices take the form

$$\begin{aligned} M_Q^2 &= m_{3/2} \begin{pmatrix} 1 & 0 & \alpha\varepsilon\varepsilon' \\ 0 & 1 & 0 \\ \alpha^*\varepsilon\varepsilon' & 0 & r_3 \end{pmatrix}; \; M_D^2 = m_{3/2} \begin{pmatrix} 1 & 0 & \alpha'\varepsilon\varepsilon' \\ 0 & 1+\lambda|\rho|^2 & \beta\rho^* \\ \alpha'^*\varepsilon\varepsilon' & \beta^*\rho & r_3' \end{pmatrix}; \\ M_U^2 &= m_{3/2} \begin{pmatrix} 1 & 0 & \alpha''\varepsilon\varepsilon' \\ 0 & 1 & 0 \\ \alpha''^*\varepsilon\varepsilon' & 0 & r_3'' \end{pmatrix}. \end{aligned} \tag{24}$$

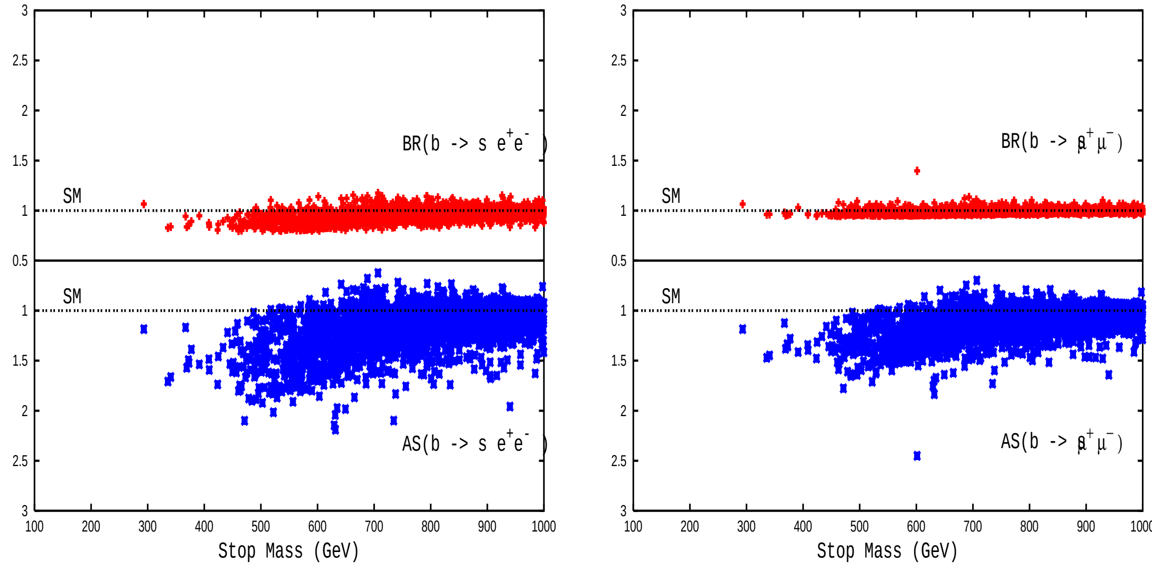

Figure 3: As in Fig. 1, but for a SUSY model with $U(2)$ flavour symmetry.

The definition of the parameters appearing in these matrices can be found in Ref.[14]. The important feature of the flavor structure of this model is the presence of a large mixing between the second and the third generation which would have significant effect on enhancing the BR and AS of $b \to s\, l^+ l^-$ decays. However, this mixing essentially enhances R_7 which means enhancing for the BR of $B \to X_s \gamma$ decay as well. Therefore imposing the $B \to X_s \gamma$ constraints this leads to a similar prediction to that we obtained with the previous model.

In Fig.3 we display the predictions of this model for BR and AS of $B \to X_s\, e^+ e^-$ and $B \to X_s\, \mu^+ \mu^-$ decays versus the lightest stop mass. As in the previous models we have considered, most of the parameter space (favored by the $B \to X_s \gamma$ and other constraints) leads to decreasing in the BR and increasing of AS. This is due to the fact that even for these models the major effect in the variation is due to R_7. The large enhancements in BR and AS, obtained in the other scenario with large and negative contributions to R_7, are not very likely to show up. This is mainly due to the constraints on the Higgs mass, which prevent stop masses to be lighter than 300 GeV.

Conclusions

We have analyzed the predictions for the inclusive semileptonic decays $B \to X_s\, l^+ l^-$ in different SUSY models. In particular, we have considered SUSY models with minimal flavor violation, non–degenerate A–terms and non–universal soft scalar and gaugino masses, and finally SUSY models with non–abelian symmetry that leads to a flavor structure for the soft scalar masses. We showed that in all these models the major effect on the variations of $B \to X_s\, l^+ l^-$ decays, with respect to their SM expectations, is due to the SUSY contributions to the magnetic dipole operator parametrized by R_7 (which also give the major contribution to the inclusive $B \to X_s \gamma$ decay). The SUSY contributions to the semileptonic operators is almost negligible.

We found that the general trend of our results, favoured by the CLEO $B \to X_s\gamma$ constraints and Higgs mass bound, is in decreasing the non-resonant BR and increasing the AS. Nevertheless, only non–universal models can have chances to get very large enhancements in BR and AS. In particular, in this case up to 3 and 2.5 time enhancements of BR and AS with respect to the SM expectations can be obtained in the electron and muon channel respectively.

Acknowledgements

We would like to thank A. De Andrea, G. Isidori, and A. Polosa for useful discussions. E.G. acknowledges the kind hospitality of the CERN Theory Division where part of this work has been done. The work of S.K. was supported by PPARC.

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