# Higgs-graviscalar mixing in type I string theory 

I. Antoniadis ${ }^{1 *}$, R. Sturani ${ }^{2}$<br>${ }^{1}$ CERN Theory Division, CH-1211 Geneva 23, Switzerland<br>${ }^{2}$ Physics Department, P.O. Box 9, FIN-00014 University of Helsinki, Finland


#### Abstract

We investigate the possibility of mixing between open and closed string excitations in D-brane models with the fundamental string scale at the TeV . The open string modes describe the Standard Model Higgs, while closed strings describe graviscalars living in the bulk. This provides a string setup for computing the Higgs-graviscalar mixing, that leads to a phenomenologically interesting invisible width of the Higgs in low scale quantum gravity models, as suggested previously by Giudice, Rattazzi and Wells.


[^0]
## 1 Introduction

An interesting possibility to address the gauge hierarchy problem and guarantee its stability is when the string scale lies in the TeV region $[1,2]$. In this work, we consider the scenario of large extra dimensions [3, 4] in the framework of perturbative type I string theory with the Standard Model localized on a collection of D-branes, in the bulk of $\delta$ extra large compact dimensions of submillimeter size $R$. Standard Model degrees of freedom are described by open strings ending on the D-branes, while gravity corresponds to closed strings that propagate also in the bulk.

In this framework we will study the mixing between brane fluctuations, or branons for short, and closed string modes, such as the graviton, graviphotons and the dilaton or other graviscalars. Since branons are generically gauge non singlets, such a mixing can arise from trilinear couplings of the form $\sigma^{2} h$, involving two open and one closed string modes that we denote $\sigma$ and $h$, respectively. Upon identifying $\sigma$ with the Standard Model Higgs scalar, the above coupling induces a Higgs-graviscalar mixing proportional to the Higgs vacuum expectation value (VEV). It has been suggested that this mixing leads to an invisible width of the Higgs that may be observable experimentally [5]. Indeed, since the Higgs is much heavier than the spacing of the bulk Kaluza-Klein (KK) modes, it would feel a coupling to a quasi-continuous tower of states, leading to a disappearance amplitude rather than to oscillations.

In the context of the effective field theory, the required trilinear coupling $\sigma^{2} h$ was postulated to emerge from an $\mathcal{R} \sigma^{2}$ term, where $\mathcal{R}$ is the curvature scalar formed by the pull-back metric on the D-brane world volume. Its coefficient $\xi$ cannot be fixed by the effective field theory and should be of order unity in order to obtain a visible effect. However, in the conformal case, one obtains a small value, $\xi=1 / 6$, dictated by the conformally coupled scalar in four dimensions.

In this work, we study the branon-bulk mixing in type I string theory and we compute in particular the trilinear coupling involving two open and one closed string states. Our results are obtained in supersymmetric theories but remain valid in non supersymmetric D-brane models, where supersymmetry is mainly broken only on the world-volume of some D-branes, located for instance on top of anti-orientifold planes [6]. More precisely, there are three possibilities for the Higgs field that we analyse separately.

In the first case, the Higgs scalar is identified with an excitation of an open string having both ends on the same collection of parallel D-branes (DirichletDirichlet or DD open strings in the transverse directions). To lowest order, the effective action can then be obtained by an appropriate truncation of an $N=4$ supersymmetric theory. In the abelian case, it is given by the BornInfeld action, depending on the pull-back of bulk fields on the D-brane world
volume. Expanding in normal coordinates one finds that although no $\mathcal{R} \sigma^{2}$ term is strictly speaking generated, there is a quadratic coupling of branons to the internal components of the Riemann tensor, $\mathcal{R}_{n \mu m}^{\mu} \sigma^{n} \sigma^{m}$, which induces a Higgsgraviscalar mixing. Moreover, there is an additional coupling of branons with the longitudinal component of a graviphoton in the bulk of the form $\sigma^{n} \sigma^{m} \partial_{n} \partial_{\mu} h_{m}^{\mu}$. The total effect in the invisible width can be summarized in terms of an effective parameter $\xi$ which is of order unity in the case of $\delta=2$ large transverse extra dimensions of (sub)millimeter size. The compatibility of this coupling with the conformal symmetry of D3-branes can be explained by analyzing the explicit form of the corresponding conformal transformations.

The second possibility is when the Higgs corresponds to an open string with one end on the Standard Model branes and the other end on another D-brane extended in the bulk (Neumann-Dirichlet or ND strings). In this case, the branon interactions do not emerge from a Born-Infeld action but can be extracted directly by evaluating the corresponding string amplitudes involving twist fields. An explicit computation of the 3 -point function shows that the branon coupling to the Riemann tensor now vanishes but it remains the mixing with the graviphoton. As a result, the invisible width is much smaller than in the previous case.

In the third case, the Higgs lives on a brane intersection, corresponding to an open string stretched between two orthogonal D-branes transverse to the large dimensions (ND string in non bulk directions). The Higgs-graviscalar mixing in this case vanishes.

The paper is organized as follows. In Section 2 we consider the first case where the Higgs is a DD state living on the brane and we derive the coupling between branons and closed string modes by expanding the Born-Infeld and Chern-Simons action [7]. In Section 3 we discuss the generalization to the non abelian case. In Section 4 we comment on the compatibility of the result obtained in the previous sections with the conformal symmetry of the D3-brane effective action in the $\alpha^{\prime} \rightarrow 0$ limit. In Section 5 we compute the disappearance amplitude for the Higgs. In Section 6 we extend our analysis to the cases where the Higgs emerges as an excitation of a ND open string, stretched between two orthogonal branes.

## 2 Branons' effective action

In the following we use capital Latin letters for 10-dimensional indices, lower case characters $(\mu \ldots \omega)$ for indices tangent to D-branes, $i \ldots n$ for directions orthogonal to the D-branes, the first part of Greek alphabet $(\alpha \ldots \delta)$ for spinor indices, and $a, b$ will be used for gauge indices. The metric signature is $(-,+, \ldots,+)$.

We start by considering the effective field theory on a single $\mathrm{D} p$-brane, i.e. with $U(1)$ gauge group, which is given by the sum of Born-Infeld and Chern-

Simons actions:

$$
\begin{align*}
& S_{B I}=-T_{p} \int d^{p+1} x e^{-\tilde{\Phi}} \sqrt{-\operatorname{det}\left(\tilde{G}_{\mu \nu}+\tilde{B}_{\mu \nu}+2 \pi \alpha^{\prime} F_{\mu \nu}\right)}  \tag{1}\\
& S_{C S}=\mu_{p} \int d^{p+1} x e^{\tilde{B}+2 \pi \alpha^{\prime} F} \wedge \sum_{p} \tilde{C}^{(p+1)}, \tag{2}
\end{align*}
$$

where $F_{\mu \nu}$ is the field strength of the abelian world-volume gauge field, and $T_{p}, \mu_{p}$ are the tension and Ramond-Ramond (R-R) charge of the $\mathrm{D} p$-brane. The closed string fields are the pull-back of the bulk fields to the D-brane world volume:

$$
\begin{align*}
& \tilde{G}_{\mu \nu}=G_{\mu \nu}+G_{m \mu} \partial_{\nu} \sigma^{m}+G_{m \nu} \partial_{\mu} \sigma^{m}+G_{m n} \partial_{\mu} \sigma^{m} \partial_{\nu} \sigma^{n}  \tag{3a}\\
& \tilde{B}_{\mu \nu}=B_{\mu \nu}+B_{m \mu} \partial_{\nu} \sigma^{m}-B_{m \nu} \partial_{\mu} \sigma^{m}+B_{m n} \partial_{\mu} \sigma^{m} \partial_{\nu} \sigma^{n}  \tag{3b}\\
& \tilde{\Phi}=\Phi  \tag{3c}\\
& \tilde{C}_{\mu_{0} \ldots \mu_{p}}^{(p+1)}=C_{\mu_{0} \ldots \mu_{p}}^{(p+1)}+\partial_{\mu_{0}} \sigma^{m} C_{m \mu_{1} \ldots \mu_{p}}^{(p+1)}+\partial_{\mu_{0}} \sigma^{m} \partial_{\mu_{1}} \sigma^{n} C_{m n \mu_{2} \ldots \mu_{p}}^{(p+1)}, \tag{3d}
\end{align*}
$$

where $G, B, \Phi$ and $C$ are the metric, two index antisymmetric tensor, dilaton and the R-R $(p+1)$-form potential, respectively. Here, we define the transverse coordinates of the brane as our $\sigma$ fields and an implicit antisymmetrization over indices $\mu_{0}, \mu_{1}, \ldots, \mu_{p}$ in (3d) is understood.

The tension and R - R charge of the brane can be computed either by T dualizing the tree level one-point closed string amplitude [8] or by one loop vacuum computation [9] and they are given in terms of the Yang-Mills coupling $g_{Y M}^{(p)}$ on the brane by

$$
\begin{equation*}
\mu_{p}=T_{p}=\left(2 \pi \alpha^{\prime} g_{Y M}^{(p)}\right)^{-2} \tag{4}
\end{equation*}
$$

We now recast the Born-Infeld action (1) into the Einstein frame, where the Ricci scalar in the bulk action is canonically normalized. This is obtained by rescaling to the Einstein metric $g_{M N}$ given by

$$
\begin{equation*}
g_{M N}=e^{-\Phi / 2} G_{M N} \tag{5}
\end{equation*}
$$

in terms of which the corresponding action $S_{B I}^{(E)}$ is

$$
\begin{equation*}
S_{B I}^{(E)}=-T_{p} \int d^{p+1} x e^{\frac{p-3}{4} \tilde{\Phi}} \sqrt{-\operatorname{det}\left(\tilde{g}_{\mu \nu}+e^{-\tilde{\Phi} / 2} \tilde{B}_{\mu \nu}+2 \pi \alpha^{\prime} e^{-\tilde{\Phi} / 2} F_{\mu \nu}\right)} \tag{6}
\end{equation*}
$$

No rescaling is needed for the Chern-Simons action as it is metric independent.
Expanding $S_{B I}^{(E)}+S_{C S}$ around a flat Minkowski space,

$$
\begin{align*}
g_{M N} & =\eta_{M N}+h_{M N}  \tag{7a}\\
B_{M N} & =b_{M N}  \tag{7b}\\
\Phi & =\phi \tag{7c}
\end{align*}
$$

one obtains [7]

$$
\begin{align*}
\mathcal{L}^{(1)}=- & T_{p}\left[\frac{(p-3)}{4} \phi+\frac{1}{2} h_{\mu}^{\mu}\right] \pm \mu_{p} C_{\mu_{0} \ldots \mu_{p}}^{(p+1)} \frac{\epsilon^{\mu_{0} \ldots \mu_{p}}}{(p+1)!},  \tag{8}\\
\mathcal{L}^{(2)}=- & T_{p}\left[\partial_{\mu} \sigma^{m} h^{\mu}{ }_{m}+\sigma^{m} \frac{\partial_{m} h_{\mu}^{\mu}}{2}+\frac{p-3}{4} \sigma^{m} \partial_{m} \phi+\pi \alpha^{\prime} b_{\mu \nu} F^{\mu \nu}\right] \\
\pm & \mu_{p}\left[\left(\sigma^{m} \partial_{m} C_{\mu_{0} \ldots \mu_{p}}^{(p+1)}+(p+1) \sigma^{m} \partial_{\mu_{0}} C_{m \mu_{1} \ldots \mu_{p}}^{(p+1)}\right) \frac{\epsilon^{\mu_{0} \ldots \mu_{p}}}{(p+1)!}+\right.  \tag{9}\\
& \left.\pi \alpha^{\prime} F \wedge C^{(p-1)}\right], \\
\mathcal{L}^{\left(3, N S^{2}\right)}= & \left\{\frac { 1 } { 2 g _ { Y M } ^ { 2 } } \left[\left(\partial_{\rho} A_{\mu} \partial^{\rho} A_{\nu}+\partial_{\mu} A^{\rho} \partial_{\nu} A_{\rho}-2 \partial_{\mu} A^{\rho} \partial_{\rho} A_{\nu}\right) h^{\mu \nu}-\right.\right. \\
& \left.\left(\partial_{\rho} A_{\sigma} \partial^{\rho} A^{\sigma}-\partial_{\rho} A^{\sigma} \partial_{\sigma} A^{\rho}\right) \frac{h_{\mu}^{\mu}}{2}-\left(\partial_{\rho} A_{\sigma} \partial^{\rho} A^{\sigma}-\partial_{\rho} A^{\sigma} \partial_{\sigma} A^{\rho}\right) \frac{p-7}{4} \phi\right] \\
& +\frac{T_{p}}{2}\left[\left(\partial_{\mu} \sigma_{m} \partial_{\nu} \sigma^{m}-\frac{1}{2} \eta_{\mu \nu} \partial_{\rho} \sigma_{m} \partial^{\rho} \sigma^{m}\right) h^{\mu \nu}-h_{m n} \partial_{\rho} \sigma^{m} \partial^{\rho} \sigma^{n}-\right.  \tag{10}\\
& \partial^{\rho} \sigma^{m} \partial_{\rho} \sigma_{m}\left(\frac{p-3}{4}\right) \phi-\sigma^{m} \sigma^{n} \partial_{m} \partial_{n}\left(\frac{p-3}{4} \phi+\frac{h_{\mu}^{\mu}}{2}\right)- \\
& \left.\left.2\left(\partial_{\mu} \sigma^{n}\right) \sigma^{m} \partial_{m} h_{n}^{\mu}-2 \pi \alpha^{\prime}\left(2 b_{\mu m} F^{\mu \nu} \partial_{\nu} \sigma^{m}+\sigma^{m} F_{\mu \nu} \partial_{m} b^{\mu \nu}\right)\right]\right\}, \\
\mathcal{L}_{b r}^{\left(3, R^{2}\right)}= & \pm \mu_{p}\left\{\frac{1}{2} \sigma^{m} \sigma^{n} \partial_{m} \partial_{n} C_{\mu_{0} \ldots \mu_{p}}^{(p+1)}+(p+1) \sigma^{m} \partial_{\mu_{0}} \sigma^{n} \partial_{m} C_{n \mu_{1} \ldots \mu_{p}}^{(p+1)}+\right. \\
& \frac{(p+1) p}{2} \partial_{\mu_{0}} \sigma^{m} \partial_{\mu_{1}} \sigma^{n} C_{m n \mu_{2} \ldots \mu_{p}}^{(p+1)}+\frac{(p+1) p}{2}\left(2 \pi \alpha^{\prime}\right) F_{\mu_{0} \mu_{1}} \sigma^{m} \partial_{m} C_{\mu_{2} \ldots \mu_{p}}^{(p-1)}+  \tag{11}\\
& \frac{(p+1) p(p-1)}{2}\left(2 \pi \alpha^{\prime}\right) F_{\mu_{0} \mu_{1}} \partial_{\mu_{2}} \sigma^{m} C_{m \mu_{3} \ldots \mu_{p}}^{(p-1)}+ \\
& \left.\frac{(p+1) p(p-1)(p-2)}{8}\left(2 \pi \alpha^{\prime}\right)^{2} F_{\mu_{0} \mu_{1}} F_{\mu_{2} \mu_{3}} C_{\mu_{4} \ldots \mu_{p}}^{(p-3)}\right\} \frac{\epsilon^{\mu_{0} \ldots \mu_{p}}}{(p+1)!},
\end{align*}
$$

where the $\pm$ signs correspond to the two choices of the D-branes R-R charge (branes or anti-branes) and $\epsilon^{\mu_{0} \ldots \mu_{p}}$ is the usual antisimmetric tensor density.

The non kinetic terms in the above expressions (with no spacetime derivative on $\sigma$ ) are obtained by retaining the terms up to quadratic level of the Taylor expansion

$$
\begin{equation*}
\left.\sum_{k=1}^{\infty} \frac{\left(\sigma^{m} \partial_{y^{m}}\right)^{k}}{k!}\left(e^{\frac{p-3}{4} \Phi} \sqrt{g} \mp C^{(p+1)}\right)\left(y^{n}\right)\right|_{y_{m}=0} \tag{12}
\end{equation*}
$$

This shows that the branons experience a non derivative interaction in a nontrivial background, which can be interpreted as a potential $V_{b r}$ for the position of the
brane

$$
\begin{equation*}
V_{b r} \equiv T_{p}\left(e^{\frac{p-3}{4} \Phi} \sqrt{g} \mp C^{(p+1)}\right) . \tag{13}
\end{equation*}
$$

We expect that in a supersymmetric background the Neveu-Schwarz NeveuSchwarz (NS-NS) and the Ramond Ramond (R-R) fields give mutually compensating contribution to the potential term: we shall check this fact in Section. 4, using the supergravity description of branes.

Let us consider now the trilinear Lagrangian (10). It corresponds to the closed string linearization of the following non linear Lagrangian, quadratic in NS open string modes:

$$
\begin{gather*}
\mathcal{L}^{N S}=-e^{\frac{p-3}{4} \Phi} \sqrt{g}\left[\frac{1}{4 g_{Y M}^{2}} F_{\mu \nu} F_{\rho \sigma} g^{\mu \rho} g^{\nu \sigma}+\frac{T_{p}}{2}\left(\nabla_{\mu} \sigma^{m} \nabla_{\nu} \sigma^{n} g^{\mu \nu} g_{m n}-\right.\right.  \tag{14}\\
\left.\left.\sigma^{m} \sigma^{n} \mathcal{R}_{m \mu n}^{\mu}+2 \pi \alpha^{\prime}\left(2 b_{m}^{\mu} F_{\mu \nu} \partial^{\nu} \sigma^{m}+F_{\mu \nu} \sigma^{m} \partial_{m} b^{\mu \nu}\right)\right)\right]
\end{gather*}
$$

where we introduced the (gravitational) covariant derivative over $\sigma$ fields

$$
\begin{equation*}
\nabla_{\mu} \sigma^{m}=\partial_{\mu} \sigma^{m}+\Gamma_{n \mu}^{m} \sigma^{n} \tag{15}
\end{equation*}
$$

The gravitational connection is given by

$$
\begin{equation*}
\Gamma_{n \mu}^{m}=\frac{1}{2} g^{m M}\left(g_{n M, \mu}+g_{M \mu, n}-g_{n \mu, M}\right) \tag{16}
\end{equation*}
$$

where column denotes differentiation as usual.
We thus found, besides the expected Yang-Mills kinetic terms, a potential of interaction between branons and the bulk closed string states. Note that the potential term in (14) vanishes in a trivial background; it generates interactions of $\sigma^{m}$ with higher KK modes of the bulk fields. The above results can be also obtained by a direct computation of corresponding on shell string amplitudes.

## 3 Non-abelian generalization

In the non-abelian case, we cannot rely on the Born-Infeld action to obtain the effective field theory. Instead, one can compute the relevant 3-point amplitude involving two branons and one closed string state. We concentrate below on the NS-NS sector. The amplitudes are given in terms of one kinematical invariant variable $t$, which is given in terms of the momenta of the open string excitations, $k_{2}, k_{3}$, and the momentum $k_{1}$ of the closed string state by

$$
\begin{array}{r}
\left(k_{2}+k_{3}\right)^{2}=2 k_{2} \cdot k_{3}=-t \\
k_{1}^{\mu} \cdot k_{1 \mu}=-k_{1}^{m} \cdot k_{1 m}=-t  \tag{17}\\
\left(k_{2}+k_{1}\right)^{2}=\left(k_{3}+k_{1}\right)^{2}=t,
\end{array}
$$

where the last product is understood to be over the full ten dimensional space, such that $k_{2,3}$ have non-vanishing components along the brane directions only, and $\sqrt{-t}$ is the KK mass of the closed string particle. The relevant amplitudes in our analysis are ${ }^{1}$ [10]

$$
\begin{align*}
A(h, \sigma, \sigma) & =i g_{c} \frac{2^{-\alpha^{\prime} t} \sqrt{\pi} \Gamma\left(1 / 2-\alpha^{\prime} t / 2\right)}{\Gamma\left(1-\alpha^{\prime} t / 2\right)} \operatorname{Tr}\left(t^{a} t^{b}\right)\left[2 i k_{2 \mu} \sigma^{a m} i k_{3 \nu} \sigma_{m}^{b} h^{\mu \nu}\right. \\
& -2 i k_{2 \mu} \sigma^{a m} i k_{3}^{\mu} \sigma^{b n} h_{m n}-4 i k_{2 \mu} \sigma^{a n} \sigma^{b m} i k_{1 m} h_{n}^{\mu}-  \tag{18a}\\
& \left.\sigma^{a m} \sigma^{b n} i k_{1 m} i k_{1 n} h_{\mu}^{\mu}-i k_{2 \mu} \sigma^{a m} i k_{3}^{\mu} \sigma_{m}^{b} h_{\nu}^{\nu}\left(1+\alpha^{\prime} t\right)^{-1}\right]+(2 \leftrightarrow 3) \\
A(\phi, \sigma, \sigma) & =i g_{c} \frac{2^{-\alpha^{\prime} t \sqrt{\pi} \Gamma\left(1 / 2-\alpha^{\prime} t / 2\right)}}{\Gamma\left(1-\alpha^{\prime} t / 2\right)} \operatorname{Tr}\left(t^{a} t^{b}\right)\left[\sigma^{a m} \sigma^{b n} i k_{1 m} i k_{1 n}(3-p)+\right.  \tag{18b}\\
& \left.i k_{2} \sigma^{a m} i k_{3} \sigma_{m}^{b}\left(\frac{4-p}{1+\alpha^{\prime} t}-1\right)\right] \times \frac{1}{2 \sqrt{2}} \phi+(2 \leftrightarrow 3)
\end{align*}
$$

which are in agreement with the Lagrangian (14) in the $U(1)$ case and justify its straightforward generalization to nontrivial Chan-Paton factors. Comparing the two expressions one must use the relations [9]

$$
\begin{equation*}
g_{c}=\kappa / 2 \pi \tag{19}
\end{equation*}
$$

and the following rescaling of the fields

$$
\begin{align*}
\sigma^{m} & \rightarrow \sigma^{m} /\left(2 \pi \alpha^{\prime} g_{Y M}\right),  \tag{20a}\\
h_{M N} & \rightarrow h_{M N} /(2 \kappa)  \tag{20b}\\
\phi & \rightarrow \sqrt{2} \phi /(2 \kappa) \tag{20c}
\end{align*}
$$

where $\kappa=\sqrt{8 \pi G_{N}}$ is the gravitational coupling.

## 4 Conformal invariance

It is known that the gauge field theory on a D3-brane is $\mathbf{N}=4$ supersymmetric, which is conformal invariant. It is also known that conformally coupled scalar fields in four dimensions exhibit a $\xi \mathcal{R} \sigma^{2}$ interaction with $\xi=1 / 6$ and $\mathcal{R}$ the four-dimensional Ricci scalar. One may be worried why the calculations exposed so far do not show the expected $\xi \mathcal{R} \sigma^{2}$ coupling for $p=3$, which is also a source of Higgs-graviscalar mixing. In fact we shall argue below that the conformal symmetry is realized in a rather different way. Moreover in Section 5 we shall show that the potential interaction we obtained in Eq. (14) gives rise still to a

[^1]disappearance amplitude for the Higgs that can be parametrized in terms of an effective $\xi$ that we shall compute.

Looking back to (14), we can check that conformal invariance of the gauge fields for $p=3$ (and no other values of $p$ ) is obtained in the usual way, using a conformal transformation on the brane

$$
\begin{equation*}
g_{\mu \nu} \rightarrow \hat{g}_{\mu \nu}=\Omega^{2} g_{\mu \nu}, \tag{21}
\end{equation*}
$$

with the dilaton $\Phi$ and the gauge field $A_{\mu}$ inert. For the scalar branons the situation is different as they also couple to graviphotons and graviscalars. Here we show that conformal invariance on a flat 3-brane is achieved if (21) is supplemented by

$$
\begin{equation*}
g_{m n} \rightarrow \hat{g}_{m n}=\Omega^{-2} g_{m n} \tag{22}
\end{equation*}
$$

with the branons unaltered. Indeed, it is easy to see that (21) and (22) applied together correspond to a conformal symmetry of the action derived from (14) for $p=3$ in a trivial background. Moreover in the presence of a R-R field, applying the transformations

$$
\begin{align*}
C_{\mu_{0} \ldots \mu_{p}} \rightarrow \hat{C}_{\mu_{0} \ldots \mu_{p}} & =\Omega^{4} C_{\mu_{0} \ldots \mu_{p}}  \tag{23a}\\
C_{m \mu_{1} \ldots \mu_{p}} \rightarrow \hat{C}_{m \mu_{1} \ldots \mu_{p}} & =\Omega^{2} C_{m \mu_{1} \ldots \mu_{p}}  \tag{23b}\\
C_{m n \mu_{2} \ldots \mu_{p}} \rightarrow \hat{C}_{m n \mu_{2} \ldots \mu_{p}} & =C_{m n \mu_{2} \ldots \mu_{p}} \tag{23c}
\end{align*}
$$

with $g_{\mu m}$ inert, one can show that the conformal symmetry is exact provided the background is chosen so that the potential (13) vanishes and that $\sqrt{g} h_{m}^{\mu_{0}}=$ $C_{m \mu_{1} \ldots \mu_{p}} \epsilon^{\mu_{0} \ldots \mu_{p}} / p$ !. Here, we dropped for simplicity the ( $p+1$ ) superscript on the R-R form $C^{(p+1)}$.

The effect of the $\xi \mathcal{R} \sigma^{2}$ term is thus replaced by other interaction which at the quadratic level becomes $\sigma^{m} \sigma^{n} \mathcal{R}^{\mu}{ }_{m \mu n}$. In relation to these effects, one may wonder about the argument in [11], where the $\mathbf{N}=4$ super Yang-Mills theory was considered on $S^{4}$ rather then on $\mathbb{R}^{4}$ and it was claimed that the flat direction of $\sigma$ is lifted by the curvature coupling $\mathcal{R} \sigma^{2}$, thus making the path integral to converge, at least if the metric is close enough to the one of a four sphere, which has $\mathcal{R}>0$. In our case the $\left(\sigma^{m} \partial_{m}\right)^{k}(\sqrt{g}+C)$ or its quadratic expansion may equally well do the job for the case of a four sphere embedded in a higher dimensional spacetime. It would be interesting to check this explicitly.

Amusingly enough, there is at least one case in which the potential (13) vanishes in a non-trivial way, thus preserving the conformal symmetry. It corresponds to the supergravity background induced by some parallel $\mathrm{D} p$-branes. The back-
ground is given, in the string frame, for $p<7$, by [12]

$$
\begin{align*}
& d s^{2}=H^{-\frac{1}{2}}\left(d x^{\mu} d x_{\mu}\right)+H^{\frac{1}{2}}\left(d y^{m} d y_{m}\right) \\
& e^{\Phi}=H^{\frac{p-3}{4}} \\
& C_{\mu_{0} \ldots \mu_{p}}=\epsilon_{\mu_{0} \ldots \mu_{p}} H^{-1}  \tag{24}\\
& H=1+\frac{Q}{7-p} \frac{1}{r^{7-p}}
\end{align*}
$$

where $r \equiv \sqrt{y^{m} y_{m}}$. The solution (24) depends on the parameter $Q$, with dimensions of $[\text { length }]^{7-p}$, defined by

$$
\begin{equation*}
\int_{\perp} d * d C^{(p+1)}=\int_{\partial \perp} * d C=S_{7-p} Q \tag{25}
\end{equation*}
$$

where $S_{7-p}$ is the volume of the $(7-p)$-dimensional sphere of unit radius. The classical parameter $Q$ is related to the microscopic string parameter $\mu_{p}$ by $^{2}$

$$
\begin{equation*}
Q=N \mu_{p} 2 \kappa^{2} \tag{26}
\end{equation*}
$$

where the integer $N$ counts the number of D -branes and the classical limit is recovered at $N \rightarrow \infty$. Using that $\mu_{p} \propto g_{o}^{-2}, \kappa \propto g_{c}$ and that $g_{o}^{2} \propto g_{c} \propto e^{\langle\Phi\rangle} \equiv g_{s}$ we have also

$$
\begin{equation*}
Q \sim N g_{s}\left(2 \pi \alpha^{\prime}\right)^{\frac{7-p}{2}} . \tag{27}
\end{equation*}
$$

In this supersymmetric background the potential felt by a stuck of $N^{\prime}$ test Dp-branes with tension and charge $T_{p}^{\text {test }}$ (assuming $N \gg N^{\prime}$ so that the test branes alter negligibly the background they are plunged into) is

$$
\begin{equation*}
V_{b r}=T_{p} H^{-1}-\mu_{p} H^{-1}=0 \tag{28}
\end{equation*}
$$

${ }^{2}$ This is derived using the terms in the supergravity action that involve the R-R form

$$
S=-\frac{1}{4 \kappa^{2}} \int d^{10} x(d C)^{2}-\mu_{p} \int_{\text {branes }} C .
$$

Thus, the equation of motion is

$$
\int_{\perp} d * d C=2 \kappa^{2} \sum_{\text {branes }} \mu_{p}
$$

which, compared to (25), gives (26).
where we used the BPS relation $T_{p}=\mu_{p}$. We also note that in this case the conformal transformations (21), (22) and (23) can be described at once by defining the conformal transformation of the function $H$

$$
\begin{equation*}
H(r) \rightarrow \hat{H}(r)=\Omega^{-4} H(r) \tag{29}
\end{equation*}
$$

If the background is non supersymmetric and cancellation (28) does not hold (as in the case of a test antibrane) conformal invariance is broken.

Finally, we check the limit of validity of the supergravity solution (24). We should have $\alpha^{\prime} R \ll 1$ for the curvature scale $R$, and the weak coupling condition $e^{\Phi} \ll 1$. In fact, on the background (24) we have

$$
\begin{aligned}
R & \sim \frac{H^{\prime 2}}{H^{5 / 2}}\left\{\begin{array}{l}
\xrightarrow{r \rightarrow 0} Q^{-1 / 2} r^{\frac{3-p}{2}} \\
\xrightarrow{r \rightarrow \infty}\left(Q / r^{8-p}\right)^{2}
\end{array}\right. \\
e^{\Phi} & =H^{\frac{p-3}{4}}\left\{\begin{array}{l}
\xrightarrow{r \rightarrow 0}\left(\frac{Q}{7-p}\right)^{\frac{p-3}{4}} r \frac{(p-7)(p-3)}{4} \\
\xrightarrow{r \rightarrow \infty} \\
1+\frac{p-3}{4(7-p)} \frac{Q}{r^{7-p}}
\end{array}\right.
\end{aligned}
$$

Thus, in the $r \rightarrow \infty$ limit the curvature vanishes and the coupling is bounded for every $p$, whereas in the $r \rightarrow 0$ limit both curvature and coupling blow up for $p>3$. However, in the $p \leq 3$ case, the curvature is bounded by $R_{\max }$

$$
\begin{equation*}
R_{\max } \propto Q^{-\frac{2}{7-p}} \tag{30}
\end{equation*}
$$

and thus, for $p \leq 3, \alpha^{\prime}$ corrections can be taken under control for any value of $r$ by sending $Q \rightarrow \infty$ in a way that $\alpha^{\prime} Q^{-2 /(7-p)} \rightarrow 0$ (for $p=3$, the $A d S_{5} \times S_{5}$ geometry is obtained in this way). If we plonge a brane into a nontrivial general background, relation (28) generally won't hold and a potential for the position of the brane will be generated.

Hence, everything appears to be consistent even in the absence of a $\xi \mathcal{R} \sigma^{2}$ term.

## 5 Higgs-graviscalar mixing

We shall now show how in our scenario a mixing may take place between branons and a graviscalar. The mixing is triggered by the trilinear coupling $\sigma^{2} h$ in (10) if $\sigma$ acquires an expectation value [5]. Before we analyse the mixing, we discuss first the abelian case of a single brane, where the graviphoton absorbs the branon and acquires a (localized) mass. For this purpose we need the expansion of the Born-Infeld action (6) at the quadratic level of the NS-NS closed string modes:

$$
\begin{equation*}
\mathcal{L}_{2 N S^{2}}=-T_{p}\left[\frac{1}{8}\left(h_{\mu}^{\mu}\right)^{2}-\frac{1}{4} h_{\mu \nu} h^{\mu \nu}+\frac{1}{2}\left(\frac{p-3}{4}\right)^{2} \phi^{2}+\frac{p-3}{8} \phi h_{\mu}^{\mu}+\frac{1}{4} b_{\mu \nu} b^{\mu \nu}\right], \tag{31}
\end{equation*}
$$

which can also be checked by computing the relative string scattering amplitudes [13].

We might have expected the appearance of a mass-term for the graviphoton as the presence of the brane breaks translational invariance. The graviphoton indeed becomes massive and eats the $U(1)$ part of the branons, but this is not manifest with the parametrization of the metric that we used, $g_{M N}=\eta_{M N}+h_{M N}$. Actually, with this parametrization, the field $h_{\mu m}$ is not the graviphoton unless one is restricted to the lowest order approximation. The graviphoton $V_{\mu}{ }^{m}$ is defined by parametrizing the metric in the following way

$$
d s^{2}=g_{M N}^{(10)} d x^{M} d x^{N}=g_{\mu \nu} d x^{\mu} d x^{\nu}+g_{m n}\left(d x^{m}+V_{\mu}^{m} d x^{\mu}\right)\left(d x^{n}+V_{\nu}{ }^{n} d x^{\nu}\right)
$$

or equivalently

$$
g_{M N}=\left(\begin{array}{cc}
g_{\mu \nu}+g_{m n} V_{\mu}^{m} V_{\nu}^{n} & V_{\mu n}  \tag{32}\\
V_{m \nu} & g_{m n}
\end{array}\right) .
$$

Then $V_{\mu}{ }^{m}$ can be identified with the graviphoton since the ten dimensional coordinate transformation

$$
x^{m} \rightarrow x^{m \prime}=x^{m}+\xi^{m}
$$

becomes equivalent to the gauge transformation

$$
V_{\mu}{ }^{m} \rightarrow V_{\mu}{ }^{m \prime}=V_{\mu}{ }^{m}+\partial_{\mu} \xi^{m}
$$

The resulting bulk kinetic terms for the dimensionally reduced theory, omitting the terms involving graviscalars and dilaton, is

$$
\begin{equation*}
\mathcal{L}_{\text {bulk }}=\frac{1}{2 \kappa^{2}} \sqrt{|g|}\left[\mathcal{R}^{(p+1)}-\frac{1}{4} g_{m n}\left(\partial_{\mu} V_{\nu}^{m}-\partial_{\nu} V_{\mu}^{m}\right)\left(\partial^{\mu} V^{\nu n}-\partial^{\nu} V^{\mu n}\right)\right] \tag{33}
\end{equation*}
$$

Expanding the Born-Infeld action (6) over the metric (32) we obtain, up to quadratic order in the fields

$$
\begin{array}{r}
\mathcal{L}_{2 N S^{2}}^{\prime}=-T_{p} \sqrt{|g|}\left[\frac{1}{2}\left(h_{\mu}^{\mu}+\frac{p-3}{2} \phi\right)+\frac{1}{2}\left(V_{\mu}^{m}+\partial_{\mu} \sigma^{m}\right)^{2}+\frac{1}{8}\left(h_{\mu}^{\mu}+\frac{p-3}{2} \phi\right)^{2}-\right.  \tag{34}\\
\left.\frac{1}{4} h_{\mu \nu} h^{\mu \nu}+\frac{1}{2} \sigma^{i} \partial_{i}\left(h_{\mu}^{\mu}+\frac{p-3}{2} \phi\right)+\frac{1}{4}\left(b_{\mu \nu}+2 \pi \alpha^{\prime} F_{\mu \nu}\right)^{2}\right]
\end{array}
$$

where we arranged the terms involving the branons and the graviphotons in a perfect square, showing that for each $m$ the $U(1)$ branon is eaten by the corresponding graviphoton which becomes massive [14]. Its mass $m_{g p}$ is given by ${ }^{3}$

$$
\begin{equation*}
m_{g p}^{2}=\frac{16 \pi T_{p}}{\left(M_{P l}\right)^{p-1}} \tag{35}
\end{equation*}
$$

[^2]where $M_{P l}$ is the lower dimensional Planck mass on the $p$-brane. This eating mechanism is T-dual of the mechanism which makes the antisymmetric tensor $b_{\mu \nu}$ massive by eating the $U(1)$ world-volume gauge field $A_{\mu}$ [15], triggered by the last term in (34). In fact, a massless two-index antisymmetric tensor in $(p+1)$ dimensions has $\frac{1}{2}(p-1)(p-2)$ components which absorbs the $(p-1)$ components of a gauge field through the last term of (34) to make a massive antisymmetric tensor field with $\frac{1}{2} p(p-1)$ components.

The terms quadratic in $h_{\mu}^{\mu}$ and $h_{\mu \nu}$ are due to the cosmological constant term associated to the brane tension. The additional interaction $\sigma^{i} \partial_{i}\left(h_{\mu}^{\mu}+(p-3) \phi / 2\right)$ can be interpreted as a mixing between the longitudinal mode of the graviphoton (involving only the branon with the identity $\mathbb{1}$ Chan-Paton factor) and the Kaluza-Klein excitations of the zero helicity part of the graviton and dilaton. Using canonically normalized fields, this mixing is given by

$$
\begin{equation*}
\mathcal{L}_{\text {mix }}^{\prime}=-\frac{1}{2 \sqrt{16 \pi}} m_{g p} M_{P l}^{\frac{p-1}{2}} \sigma^{i} \partial_{i}\left(h_{\mu}^{\mu}+\frac{p-3}{2} \phi\right) . \tag{36}
\end{equation*}
$$

We note that there is also a similar mixing with excitations of the R-R sector, whose amplitude is equal in magnitude and opposite in sign to the previous one. The equality of the magnitude of the NS-NS and R-R mixing contributions is not surprising, as unitarity relate these mixing amplitudes to the imaginary part of the one loop two branon point-function, which must vanish in a supersymmetric background.

We will now focus on the NS-NS sector. The mixing (36) vanishes on-shell unless $\sigma^{m}$ is massive for some direction $\bar{m}$ and involves only the identity $\mathbb{1}$ ChanPaton factor. For non trivial Chan-Paton factors, we start from the trilinear coupling we found in Sections 2, 3

$$
\begin{equation*}
\mathcal{L}=-\frac{1}{4} \sigma^{m} \sigma^{n} \partial_{m} \partial_{n}\left(h_{\mu}^{\mu}+\frac{p-3}{2} \phi\right)-\left(\partial_{\mu} \sigma^{n}\right) \sigma^{m} \partial_{m} h^{\mu}{ }_{n} \tag{37}
\end{equation*}
$$

and assuming that one of the branons gets a mass $m_{\sigma}$ and a non-vanishing VEV $v$, we substitute

$$
\begin{equation*}
\sigma^{\bar{m}}=v+\rho^{\bar{m}} \tag{38}
\end{equation*}
$$

and obtain the mixing term

$$
\begin{equation*}
\mathcal{L}_{m i x}=-\frac{1}{2} v \rho^{\bar{m}} \partial_{\bar{m}}^{2}\left(h_{\mu}^{\mu}+\frac{p-3}{2} \phi\right)+v \rho^{\bar{m}} \partial_{\mu} \partial_{\bar{m}} h_{\bar{m}}^{\mu} . \tag{39}
\end{equation*}
$$

Using next the contractions

$$
\begin{align*}
h_{\mu}^{\mu} h_{\nu}^{\nu}+\left(\frac{p-3}{2}\right)^{2} \phi \phi & =\frac{64 \pi}{M_{P l}^{p-1}} \frac{1}{k^{2}+m_{K K}^{2}}  \tag{40}\\
h_{\bar{m}}^{\mu} h_{\bar{n}}^{\nu} & =\frac{16 \pi}{M_{P l}^{p-1}} \frac{g^{\mu \nu} g_{\bar{m} \bar{n}}^{2}}{k^{2}+m_{K K}^{2}}  \tag{41}\\
h_{\mu}^{\mu} h_{\bar{m}}^{\nu} & =0 \tag{42}
\end{align*}
$$

and assuming that $m_{\sigma} \gg 1 / R$ (large extra-dimensions), so that the branons do not resolve the discreteness of the Kaluza-Klein spectrum, we obtain the following correction to the branon self-energy (in the notation of [5]):

$$
\begin{equation*}
\Sigma\left(p^{2}\right)=v^{2} \frac{16 \pi V}{M_{P l}^{p-1}} \int \frac{d^{\delta} k}{(2 \pi)^{\delta}} \frac{k_{\bar{m}}^{4}+k^{2} k_{\bar{m}}^{2}}{p^{2}+k^{2}+i \epsilon}, \tag{43}
\end{equation*}
$$

where $\delta$ is the number of large extra dimensions and $V$ their volume. $\Sigma$ contains the contribution from the insertion of KK modes in the branon propagator, which reads:

$$
\begin{equation*}
G_{\sigma}\left(p^{2}\right)=-\frac{1}{p^{2}+m_{\sigma}^{2}+\Sigma\left(p^{2}\right)+i \epsilon} . \tag{44}
\end{equation*}
$$

The imaginary part of $\Sigma$ above is related to the decay amplitude $\Gamma$ of the branon. Using

$$
\lim _{\epsilon \rightarrow 0} \operatorname{Im}\left[\frac{1}{x+i \epsilon}\right]=\pi \delta(x)
$$

and the type I relation for the theory on a $p+1$ brane in ten dimensions [16]

$$
\begin{equation*}
M_{P l}^{p-1}=\frac{2}{\alpha_{Y M}^{2}} M_{s}^{5-p} \bar{V}(2 \pi)^{p-3}, \tag{45}
\end{equation*}
$$

with $\bar{V}$ the reduced volume defined by

$$
\bar{V} \equiv M_{s}^{9-p} \Pi_{i=p+1}^{9} R_{i},
$$

and $M_{s}$ the string scale, we have

$$
\begin{equation*}
\Gamma=\frac{1}{m_{\sigma}} \operatorname{Im}\left[\Sigma\left(p^{2}=m_{\sigma}^{2}\right)\right]=\frac{4 \pi \alpha_{Y M}^{2}}{(2 \pi)^{p-3}} \frac{a(\delta)}{2} m_{\sigma} \pi \frac{v^{2}}{M_{s}^{5-p}}\left(\frac{m_{\sigma}}{M_{s}}\right)^{\delta} S_{\delta-1} \tag{46}
\end{equation*}
$$

where $S_{\delta-1}$ is the volume of the $(\delta-1)$-dimensional sphere of unit radius and we defined

$$
\begin{equation*}
a(\delta) \equiv \frac{3}{\delta(\delta+2)}+\frac{1}{\delta}=\frac{\delta+5}{\delta(\delta+2)} . \tag{47}
\end{equation*}
$$

Equation (46) is the same with the expression that appears in [5], using $M_{D}^{3 p+\delta-7}=$ $(2 \pi)^{p-3} M_{s}^{p+\delta-1} /\left(4 \pi \alpha_{Y M}^{2}\right)$ and identifying $\kappa \xi$ with $\sqrt{a(\delta) / 2}$. The two terms in the expression (47) of $a(\delta)$ correspond to the contributions from the mixing with the graviscalars (first term) and with the graviphotons (second term).

Thus, we see that despite the absence of a $\xi \sigma^{2} \mathcal{R}$ term in the effective action, a mixing can nevertheless take place with an effective $\xi$ given by

$$
\begin{equation*}
\xi=\sqrt{\frac{\delta+5}{6 \delta(\delta-1)}} \tag{48}
\end{equation*}
$$

This mixing becomes maximal for the case of $\delta=2$ large extra dimensions, where $\xi=\sqrt{7 / 12} \simeq 0.76$, leading to a possible observable invisible width for the Higgs [5]. For $\delta>2$, the effective $\xi$ decreases and varies between $\xi \simeq 0.47$ for $\delta=3$ and $\xi \simeq 1 / 4$ for $\delta=6$.

## 6 Higgs on branes intersection

In this Section, we study the case where the Higgs lives on a branes intersection, corresponding to an open string with mixed Neumann-Dirichlet (ND) boundary conditions in four internal directions. We will distinguish two subcases, depending on whether one of the two orthogonal branes extend (partly) in the bulk of large extra dimensions.

We thus consider the coupling between two ND open string modes and a closed string NS-NS state. We will consider first the oriented theory. As we cannot use now the Born-Infeld action, a string calculation is the only way to compute this coupling.

The kinematics of the problem is the same with the one described in (17). The vertex operator for a NS open string state $\chi$, with one end on D5-branes and the other end on D9-branes, is (in the ( -1 )-ghost picture, $\varphi$ is the superghost field):

$$
\begin{equation*}
V_{59}^{(-1)}=g_{o} t_{a a^{\prime}} \chi_{\alpha} e^{-\varphi} \Delta S^{\alpha} e^{i k X} \tag{49}
\end{equation*}
$$

where the Chan-Paton factor index $a\left(a^{\prime}\right)$ transforms in the (anti-)fundamental of the D5(9)-branes gauge group. The operator $\Delta$ is the product of twist fields associated to the four internal coordinates with mixed ND boundary conditions, $S^{\alpha}$ is the corresponding spin field, and $\chi_{\alpha}$ selects the internal spinor helicity. This vertex operator has the same expression as the left (supersymmetric) part of the vertex for a massless heterotic twisted state of a $\mathbb{Z}_{2}$ orbifold [18]. For a 95 state, one has the same operator with $\chi_{\alpha}$ replaced by $(\bar{\chi})^{\alpha} \equiv\left(\chi_{\alpha}\right)^{\dagger}$ and $S^{\alpha}$ replaced by
$S_{\alpha}$. The NS-NS closed string state vertex operator (in the 0-ghost picture) is

$$
\begin{align*}
& V_{2 N S^{2}}^{(0,0)}(\zeta, k)= \\
& \quad-\frac{2 g_{c}}{\alpha^{\prime}} \zeta_{M N}\left(i \partial X^{M}+\frac{\alpha^{\prime}}{2} k \cdot \psi \psi^{M}\right) e^{i k X}\left(i \bar{\partial} \tilde{X}^{N}+\frac{\alpha^{\prime}}{2} k \cdot \tilde{\psi} \tilde{\psi}^{N}\right) e^{i k \tilde{X}}, \tag{50}
\end{align*}
$$

where $X^{M}$ denote the bosonic coordinates and $\psi^{M}$ their (2d) fermionic superpartners.

The relevant correlators between the twist field $\Delta$ and $X$ is (for left-movers) [17]:

$$
\begin{equation*}
\frac{\left\langle\Delta\left(z_{1}\right) \Delta\left(z_{2}\right) X_{L}^{M}\left(z_{3}\right) X_{L}^{N}\left(z_{4}\right)\right\rangle}{\left\langle\Delta\left(z_{1}\right) \Delta\left(z_{2}\right)\right\rangle}=-\frac{\alpha^{\prime}}{2} \eta^{M N} \ln \left[\frac{1-\sqrt{\frac{z_{13} z_{24}}{z_{14} z_{23}}}}{1+\sqrt{\frac{z_{13} z_{24}}{z_{14} z_{23}}}}\right], \tag{51}
\end{equation*}
$$

where $z_{i}$ denote the corresponding world-sheet positions. For right-movers, the correlator is the same provided one substitutes $L, z_{i}$ with $R, \bar{z}_{i}$. The correlator between two $\Delta$ 's is

$$
\begin{equation*}
\left\langle\Delta\left(z_{1}\right) \Delta\left(z_{2}\right)\right\rangle=\frac{1}{\left(z_{1}-z_{2}\right)^{1 / 2}} \tag{52}
\end{equation*}
$$

Note that the normalization coefficient $g_{o}$ (it is understood $g_{o}$ for $p=5$ ) in front of the vertex operator (49) is the same with the normalization of untwisted open string states. This can be checked by comparing the $\chi^{2} A_{\mu}^{2}$ amplitude and the exchange interaction $\chi A \chi \chi A \chi$ which leads to internal propagation of a ND state. The $\chi_{\alpha}$ field carries an ${ }^{\text {ind }}$ dex which labels the spinor representation of the internal $S O(4)$ and the GSO projection forces it to be a Weyl spinor. Hence, it has two helicity states forming the fundamental representation of $S U(2)$ (usually called $\left.S U(2)_{R}\right)$, rather then the full $S O(4)$. This representation is pseudoreal, two-dimensional in the complex sense and four-dimensional when viewed over the real numbers. In the oriented theory the two $\chi$ 's correspond to the two independent excitations described by 59 and 95 states which together make up the bosonic content of an $\mathbf{N}=1$ hypermultiplet in six dimensions.

In the unoriented theory, we expect just one complex boson (the bosonic content of half of a hypermultiplet) as 59 and 95 modes are correlated. This is obtained [19] via a projection which involves $S U(2)_{R}$ as well as the 5 -brane gauge index, being the gauge group $S p(k)$. The projection is a reality condition which can be applied to the spinor $\chi$ as the representation $(\mathbf{2 k}, \mathbf{2})$ of $S p(k) \times S U(2)_{R}$ is real (the $\mathbf{2 k}$ of $S p(k)$ being also pseudoreal).

The relevant correlators involving the spinor fields $S^{\alpha}$ in 4 internal dimensions
are [20]:

$$
\begin{gather*}
\left\langle S_{\alpha}\left(z_{1}\right) S^{\beta}\left(z_{2}\right) \psi^{M} \psi^{N}\left(z_{3}\right)\right\rangle=-\frac{1}{2}\left(\Gamma^{M N}\right)_{\alpha}^{\beta} \frac{z_{12}^{1 / 2}}{z_{31} z_{32}}  \tag{53a}\\
\left\langle S_{\alpha}\left(z_{1}\right) S^{\beta}\left(z_{2}\right) \psi^{M}\left(z_{3}\right) \psi^{N}\left(z_{4}\right)\right\rangle=\frac{1}{2}\left(z_{32} z_{42} z_{31} z_{41}\right)^{-1 / 2} z_{12}^{-1 / 2} z_{34}^{-1} \times \\
{\left[\delta^{M N} \delta_{\alpha}^{\beta}\left(z_{32} z_{41}+z_{31} z_{42}\right)-\left(\Gamma^{M N}\right)_{\alpha}^{\beta} z_{12} z_{34}\right]}  \tag{53b}\\
\left\langle S_{\alpha}\left(z_{1}\right) S^{\beta}\left(z_{2}\right) \psi^{M} \psi^{N}\left(z_{3}\right) \psi^{R} \psi^{S}\left(z_{4}\right)\right\rangle=-\frac{\delta_{\alpha}^{\beta}}{z_{12}} \frac{\eta^{M R} \eta^{N S}-\eta^{M S} \eta^{N R}}{z_{34}^{2}}+  \tag{53c}\\
\left(\eta^{M R} \Gamma^{N S}+\eta^{N S} \Gamma^{M R}-\eta^{M S} \Gamma^{N R}-\eta^{N R} \Gamma^{M S}\right)_{\alpha}^{\beta}\left(2 z_{34}\right)^{-1}\left(z_{32} z_{42} z_{31} z_{41}\right)^{-1 / 2},
\end{gather*}
$$

where $-i \Gamma_{M N} / 2=-i\left[\Gamma_{M}, \Gamma_{N}\right] / 4$ is the Lorentz generator in the spinor representation. This correlators can be used to compute the 3-point amplitude, which in the $\alpha^{\prime} t \rightarrow 0$ limit becomes:

$$
\begin{align*}
& A_{2 N D, N S^{2}}=2 i g_{c} \pi\left[\left(-k_{2} \cdot k_{3} \eta^{\mu \nu}+k_{2}^{\mu} k_{3}^{\nu}+k_{3}^{\mu} k_{2}^{\nu}+\frac{2}{\pi t}\left(k_{3}^{\mu} k_{2}^{\nu}-k_{3}^{\nu} k_{2}^{\mu}\right)\right) \delta_{\alpha}^{\beta} \zeta_{\mu \nu}\right.  \tag{54}\\
& \left.+\frac{k_{1 r}}{4}\left(\left(\Gamma^{r m}\right)_{\alpha}^{\beta}\left(k_{2}-k_{3}\right)^{\nu} \zeta_{m \nu}+\left(\Gamma^{r n}\right)_{\alpha}^{\beta}\left(k_{2}-k_{3}\right)^{\mu} \zeta_{\mu n}\right)\right]\left(\chi_{\alpha}^{2}\right)^{\dagger} \chi_{\beta}^{3}
\end{align*}
$$

where $\chi^{2,3}$ is the $\chi$-field with momentum $k_{2,3}$. The amplitude ${ }^{4}$ above displays a pole term in $t$ due to the $\chi \chi A A b$ exchange interaction that has to be subtracted in order to extract the contact terms. Using (19) and after the usual rescaling (20b) one obtains the trilinear Lagrangian:

$$
\begin{gather*}
\mathcal{L}_{2 N D, N S^{2}}=-\frac{1}{2}\left[-\partial_{\mu} \bar{\chi} \partial_{\nu} \chi h^{\mu \nu}+\partial \bar{\chi} \partial \chi\left(\frac{h_{\mu}^{\mu}}{2}+\frac{p-3}{4} \phi\right)+\right.  \tag{55}\\
\left.\frac{1}{4} \partial_{n} h_{\mu m}\left(\partial^{\mu} \bar{\chi} \Gamma^{m n} \chi-\bar{\chi} \Gamma^{m n} \partial^{\mu} \chi\right)\right] .
\end{gather*}
$$

No contact interaction with $b_{\mu \nu}$ is found, neither a potential coupling to the internal components of the Riemann tensor, as in the untwisted DD case we studied in Sections 2, 3. However, besides the standard kinetic terms we find still a coupling of the ND open string modes to the KK excitations of the graviphoton, arising through the spin connection in the gravitational covariant derivative

$$
\begin{equation*}
\nabla_{\mu}^{g r} \chi=\partial_{\mu} \chi+\frac{1}{4} \omega_{\mu}^{m n} \Gamma_{m n} \chi \tag{56}
\end{equation*}
$$

[^3]Here, $\omega_{\mu}{ }^{m n}$ is the standard spin connection (with one index parallel and two orthogonal to the D5-brane) which is given in terms of the vielbein $e_{\mu}^{a}$ by

$$
\omega_{\mu}^{m n}=\frac{1}{2} e^{\nu m}\left(\partial_{\mu} e_{\nu}^{n}-\partial_{\nu} e_{\mu}^{n}\right)-\frac{1}{2} e^{\nu n}\left(\partial_{\mu} e_{\nu}^{m}-\partial_{\nu} e_{\mu}^{m}\right)-\frac{1}{2} e^{\rho m} e^{\sigma n}\left(\partial_{\rho} e_{\sigma i}-\partial_{\sigma} e_{\rho i}\right) e_{\mu}^{i}
$$

whose first order expansion around flat space, $g_{M N}=\eta_{M N}+h_{M N}$, is

$$
\begin{equation*}
\omega_{\mu}{ }^{m n}=h_{\mu}{ }^{[m, n]} . \tag{57}
\end{equation*}
$$

The connection part of the covariant derivative is completed by gauge terms to make the full covariant derivative

$$
\begin{equation*}
\nabla_{\mu} \chi=\partial_{\mu} \chi+\frac{1}{4} \omega_{\mu}^{m n} \Gamma_{m n} \chi+\left(i g_{5} A_{\mu}-i g_{9} A_{\mu}^{\prime}\right) \chi, \tag{58}
\end{equation*}
$$

where $A_{\mu}\left(A_{\mu}^{\prime}\right)$ is the D 5 (D9) world-volume gauge field with gauge coupling $g_{5}$ $\left(g_{9}\right)$. Finally, open string excitations $\sigma$ and $\chi$ have also non-derivative ( $D$-terms) interactions [9]

$$
\begin{equation*}
\mathcal{L}_{D}=-\frac{g_{Y M}^{2}}{4}\left(\left[\sigma^{m}, \sigma^{n}\right]-\frac{i}{2} \bar{\chi} \Gamma^{m n} \chi\right)^{2} \tag{59}
\end{equation*}
$$

in the normalization of (55).
We consider now the possibility of mixing between $\chi$ and closed string modes. In the case where the Higgs, identified with $\chi$, lives on an intersection of two orthogonal branes, both transverse to the submillimeter bulk (e.g. D3 and D7, or D5 and D5'), no mixing is generated between $\chi$ and closed string states. On the other hand, in the case where one of the two orthogonal branes extends in the bulk, a mixing is induced, as can be seen from the effective Lagrangian (55), between $\chi$ and the longitudinal component of the corresponding graviphoton in the bulk. As in the DD case, in order to obtain a quadratic coupling between closed and open string states, one of the $\chi$ 's must acquire a non-vanishing vacuum expectation value.

Note that a VEV of $\chi$ along a supersymmetric flat direction, i.e. when the $D$-term (59) vanishes, gives rise to the well-known Higgs branch which provides a string realization of a non-abelian soliton [19] that we are not interested in here. We consider instead a real vacuum expectation value for $\chi$, with non-vanishing $D$ term that breaks supersymmetry, and we study the effective field theory obtained by expanding around the $\operatorname{VEV} v, \chi_{1}=v+\chi_{1}^{\prime}$, where $\chi_{1}$ is one of the two complex bosons. Dropping the prime from $\chi_{1}^{\prime}$ and assuming the ND conditions to be along the directions $\hat{6} \ldots \hat{9}$ (orthogonal to the 5 -brane), we have up to quadratic order in $\chi$ and $\sigma$ :

$$
\begin{gather*}
\mathcal{L}_{D}^{\prime}=-\frac{g_{Y M}^{2}}{4}\left[v^{4}+4 v^{3} \operatorname{Re}\left(\chi_{1}\right)+v^{2}\left(4\left(\operatorname{Re} \chi_{1}\right)^{2}+2 \chi_{1}^{\dagger} \chi_{1}+3 \chi_{2}^{\dagger} \chi_{2}\right)+\right.  \tag{60}\\
\left.+v^{2}\left(\left[\sigma^{\hat{6}}, \sigma^{\hat{9}}\right]+\left[\sigma^{\hat{7}}, \sigma^{\hat{8}}\right]\right)\right]
\end{gather*}
$$

where fields are canonically normalized. Note the appearance of a cosmological constant and of a tadpole for $\chi_{1}$. This is anyway only an effective approach and other potential terms may be generated when supersymmetry is broken.

On the other hand, $\chi$ can also obtain a mass in a supersymmetric way, avoiding the cosmological constant and tadpole-like terms, as in (60). This is achieved by turning on a Wilson line for the gauge fields with polarization parallel to the 5 -branes, or if we T-dualize, by separating lower and higher dimensional branes by giving an expectation value to one (or some) of the branons orthogonal to both branes. This corresponds to moving in the so-called Coulomb branch of the theory.

The $\chi_{1}$ expectation value determines the mixing terms between the $\chi$ field and the corresponding graviphoton

$$
\begin{equation*}
\mathcal{L}_{m i x}=-\frac{1}{4} v\left(\partial_{[\hat{6}} h_{\hat{9}] \mu}+\partial_{[\hat{\gamma}} h_{\hat{8}] \mu}\right) \partial^{\mu} \operatorname{Im} \chi_{1} \tag{61}
\end{equation*}
$$

Using (41), one finds for the bosonic field $\operatorname{Im} \chi_{1}$,

$$
\begin{equation*}
\Sigma_{\chi}\left(p^{2}\right)=\frac{v^{2}}{8} \frac{8 \pi V}{M_{P l}^{p-1}} \int \frac{d^{\delta} k}{(2 \pi)^{\delta}} \frac{k^{2} k_{\bar{m}}^{2}}{p^{2}+k^{2}} \tag{62}
\end{equation*}
$$

and consequently, using (45), one finds the following invisible width

$$
\begin{equation*}
\Gamma_{\chi}=\frac{1}{m_{\chi}} \operatorname{Im}\left[\Sigma\left(p^{2}=m_{\chi}^{2}\right)\right]=\frac{4 \pi \alpha_{Y M}^{2}}{(2 \pi)^{p-3}} \frac{\pi}{32 \delta} \frac{v^{2}}{M_{s}^{5-p}} m_{\chi}\left(\frac{m_{\chi}}{M_{s}}\right)^{\delta} S_{\delta-1} . \tag{63}
\end{equation*}
$$

Hence, the resulting effective parameter $\xi$ in this case reads

$$
\begin{equation*}
\xi=\frac{1}{4} \sqrt{\frac{\delta+2}{6 \delta(\delta-1)}}, \tag{64}
\end{equation*}
$$

which is significantly smaller than in the DD case, studied in Section 5. Indeed, the highest value obtained for $\delta=2$ is $\xi \simeq 1 / 7$. Due to the fact that the graviphoton and its KK tower form a quasi-continuum set of states, this result is not altered if we consider unoriented type I models in which an orbifold projection takes the zero mode of the graviphoton out of the spectrum.

In conclusion, in this work, we investigated the possibility of mixing between the Higgs, identified as an open string excitation, and closed string states from the bulk (graviscalars), when the fundamental string scale is in the TeV region. We found that such a mixing can occur, leading to a possible observable invisible decay width of the Higgs, only when the Higgs lives on the Standard Model world-brane and corresponds to a DD open string with both ends on parallel D-branes.

As we mentioned in the introduction, although our analysis was done in the context of supersymmetric type I theory, our results remain valid in non supersymmetric D-brane models where supersymmetry is broken (mainly) in the open string sector, using appropriate combinations of branes with (anti)-orientifolds that preserve different amount of supersymmetries. The reason is that in these cases, the effective action can be obtained by a corresponding truncation of a supersymmetric action.

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[^0]:    *On leave from Ecole Polytechnique CPHT, UMR du CNRS 7644, F-91128 Palaiseau

[^1]:    ${ }^{1}$ The dilaton is defined by a polarization tensor $\zeta_{M N}$ of the closed string vertex operator of the form $\zeta_{M N}=\phi\left(\eta_{M N}-l_{M} k_{N}-k_{M} l_{N}\right) / \sqrt{8}$, where $k$ is its momentum and $l$ a vector satisfying $l^{2}=0, k l=1$.

[^2]:    ${ }^{3}$ In our analysis we implicitly assumed that the Kaluza-Klein scale $1 / R \gg m_{g p}$ otherwise bulk terms may induce mixing and mass terms of comparable strength to (35).

[^3]:    ${ }^{4}$ Note that in contrast to the $S O(3,1)$ case, for Euclidean $S O(4)$ spinors, the quantity $\chi^{\dagger} \xi$ is scalar provided $\chi$ and $\xi$ have the same chirality.

