Positioning Algorithms in the NOTTE Experiment

I. Ursu, A. Plaian, F . Ursu
"Elie Carafoli" National Institute for Aerospace Research
B-dul Iuliu Maniu 220, 77538 Bucharest 6, e-mail: iursu @ aero.incas.ro

Abstract

Two pointing algorithms for the position control are presented, one of them being preferred for implementation in the NOTTE experiment; this is a mechano - euristical algorithm. The second exploits the abilities of a predictor type Kalman estimator in a discrete time LQG synthesis, given the physical constraint of delayed incomplete state information in the NOTTE experimental set-up.

1 INTRODUCTION

The NOTTE (Neutrino Oscillations with Telescope during the Total Eclipse) Experiment was conceived as a co-operation between "Elie Carafoli" National Institute for Aerospace Research (ECNIAR), the Institute for Spatial Studies (ISS) -both from Bucharest- and the Universita degli Studi, Bologna. NOTTE was a complex airborne experiment, designed to prove some fundamental theories and hypotheses in elementary particle physics and astrophysics. ECNIAR's task in the NOTTE experiment was to construct the active control system to point the special measurement and recording apparatuses towards the center of the Eclipse. The system was designed to compensate for perturbations induced by the flying aircraft. Accordingly, an original large mass Controlled Position Mobile Platform (CPMP) with two angular degrees of freedom (DOF), driven by DC torque servo motors, was developed [1]. The two DOF are inertially and cinematically decoupled and counteract the rotations induced around two axes in a plane perpendicular to the direction of the Eclipse. ECNIAR also undertook the design of the control algorithm; the digital implementation and the sequence of taking and processing the image of the Eclipse in order to measure the angular position errors were made by ISS. This paper presents two of the algorithms developed to obtain the angular position control of the apparatus mounted on CPMP.

2 POINTING TYPE POSITIONING ALGORITHMS : TWO SOLUTIONS

2.1 Problem Setting

The pointing problem in position control refers to a control application where very precise positioning is the design goal. In the ideal case - i.e. in the absence of friction and inertial coupling - the CPMP could point with arbitrary accuracy. In reality, the CPMP with mounted on it special measurement and recording apparatus, cannot be perfectly isolated against the perturbations induced by the flying aircraft and transmitted by means of bearing friction and inertial effects. Although the necessary steps were taken to reduce these influences, only a control algorithm can face up to exactingness of ensuring position errors less than 10 angular minutes in absolute value. There are two major difficulties, which appear in the algorithm's conception. The first concerns the well-known theoretical complications derived from the implementation of an algorithm on a digital computer [2,3]; we have to choose between three well-established strategies: 1) analog design, followed by a digital implementation, 2) discretization of the plant's equation, followed by a discrete-time design and 3) sampled-data control approach. Unfortunately, a second difficulty of the experimental set-up (a significant τ-delay on angular position measurement (0.16*s*)), imposes a severe trade-off between the performance, fitness and proportions of different discrete time approaches. Only in this way, the control signals (torque pulses, of variable width and constant amplitude) can be sent up to DC servo motors with sampling time $\tau = 0.16$s (i.e., the same as time delay).

It is worth noting that several variants of such algorithms were considered, but are not presented here.

Given the CPMP architecture, the dynamics of the two axes are independent; thus, the same algorithm can be applied separately on each axis.

2.2 Mechano-Euristical Algorithm

This algorithm was performed in two steps. Firstly, by considering the equation

$$J\ddot{\theta} = u \tag{1}$$

with supposed known initial values of angular position θ and velocity $\dot{\theta}$ at the time $n\tau$, the control u is determined with intention of canceling, in the absence of external perturbations, the value of these parameters of the movement at the time $(n+1)\tau$ (where J is the moment of inertia of the considered axis). To ensure these requirements, the nonlinear control u consists of two constant torques of amplitude M, the first being applied on the interval $[n\tau, n\tau + t_1]$ to cancel the position $\theta_{n\tau+t_1}$ and the second being applied on the interval $[(n+1)\tau - t_2, (n+1)\tau]$ to cancel the velocity $\dot{\theta}_{n\tau+t_1}$. Tedious, but not very difficult calculus shows that, for a given M

$$t_1 = J|\theta + \dot{\theta}\tau| / \left(\tau M \operatorname{sgn}\left(\theta + \dot{\theta}\tau\right)\right), \; t_2 = J|\theta| / (\tau M).$$

Performing a torque pulse balance, a single torque pulse control is applied on the interval $[n\tau,\ n\tau + \tau_n]$, $\tau_n \le \tau$ (with generically $\theta_n := \theta(n\tau)$)

$$\tau_n = J|\theta_n - \theta_{n-1} + \dot{\theta}_n\tau| / (\tau M)$$
$$u_n := -M\ \mathrm{sgn}\left(\theta_n - \theta_{n-1} + \dot{\theta}_n\tau\right).$$

But, at time $n\tau$, only the position θ_{n-1} is known, resulting in incomplete, delayed measurement of the state. Now, as a second step, an euristical estimation of the state was considered, assuming small perturbation w added to u in (1) and small t_1 and t_2. To capture more information on system behaviour, control u is applied at the time ns, where the time period s is doubled (s = 2τ), to take into account the measured positions $\theta_{ns-\tau}$ and $\theta_{(n-1)s}$. Using the noted simplifying hypotheses, the state vector is estimated

$$\theta_{en} := \theta_e(ns) = 2\theta_{ns-\tau} - \theta_{(n-1)s}$$
$$\dot{\theta}_{en} := \dot{\theta}_e(ns) = \theta_{en} / s.$$

Finally, the structure of the control results in

$$\begin{aligned}\tau_n := \tau(ns) &= J|4\theta_{ns-\tau} - 2\theta_{(n-1)s} - \theta_{e(n-1)s}| / (sM) \\ &:= J|\gamma_n| / (sM), \quad \tau_n < s\end{aligned} \qquad (2)$$

$$u_n := u(ns) = \begin{cases} -M\ \mathrm{sgn}\,\gamma_n, & for\ t \in [ns, ns + \tau_n] \\ 0 & , for\ t \in [ns + \tau_n, (n+1)s] \end{cases}.$$

2.3 Discrete Time LQG Control Synthesis with Predictor-Type Kalman Estimator

There are three main reasons for choosing *LQG* algorithm of control with Kalman predictor: a) the efficiency of the *LQG* control in the presence of the deterministic step perturbation (white noise and step signals exhibit similar power spectral density characteristics); this perturbation is generally difficult to counteract, due to its nature of Coulomb friction; b) the nature of *LQG* norm (identical with the H_2 norm) which minimizes the entire area of the mean square error of the regulated output (in comparison with H_∞ norm minimization, which supposes only a min-max type of control of regulated output evolution); c) opportunity for the Kalman predictor to take into account the most recent measurements of the system position.

Given the linear decentralized equation of motion

$$J\ddot{\theta} = u + w \qquad (3)$$

the associated sampled type control discretized matrix equation with constant sampling interval is obtained in the form

$$\boldsymbol{x}(n+1) = \boldsymbol{A}\boldsymbol{x}(n) + \boldsymbol{B}u(n) + \boldsymbol{D}w(n) \qquad (3')$$

where generically $\boldsymbol{x}(n+1)$: $=\boldsymbol{x}((n+1)\tau)$ and the delay constant τ is the discrete sampling time. $\boldsymbol{x}$ represents the state vector, $\boldsymbol{x} = (\theta, \dot{\theta})^T$ (where superscript "T" indicates the transpose) and the structure of the introduced matrices is the following

$$\boldsymbol{A} = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} b\tau^2 / 2 \\ b\tau \end{bmatrix} = \boldsymbol{D},\ b := 1/J. \qquad (3'')$$

The measurement equation

$$y(n) = \boldsymbol{C}\boldsymbol{x}(n) + v(n) := y_1(n) + v(n),\ \boldsymbol{C} = [1\ 0] \qquad (4)$$

(only θ position information, with τ delay acquired, is available) and the stochastic discrete time cost function which is to be minimized

$$J = \lim_{N\to\infty} \frac{1}{N} E\{\bullet\}, \qquad (5)$$

$$\{\bullet\} := \left\{ \sum_{j=0}^{N} \left[\boldsymbol{x}^{\mathrm{T}}(n+j)\boldsymbol{Q_J}\,\boldsymbol{x}(n+j) + u^2(n+j)R_J \right] \right\}$$

(where $\boldsymbol{Q}_J = \boldsymbol{Q}_J^T \ge 0$ and R_J is a positive scalar) are added. The covariance matrices of the uncorrelated Gaussian white noises w and v are introduced thus

$$E\left\{ [w(t)\ v(t)]^T [w(\tau)\ v(\tau)] \right\} = \begin{bmatrix} Q_w & 0 \\ 0 & R_v \end{bmatrix} \delta(t - \tau). \qquad (6)$$

The solution to the problem (which holds in the following conditions: a) $(\boldsymbol{A}, \boldsymbol{B})$ and $\left(\boldsymbol{A}, \sqrt{\boldsymbol{D}Q_w\boldsymbol{D}^T}\right)$ stabilizable; b) $(\boldsymbol{C}, \boldsymbol{A})$ and $\left(\sqrt{\boldsymbol{Q}_J}, \boldsymbol{A}\right)$ detectable) is the well-known predictor Kalman estimator [4]

$$u(n) = -\boldsymbol{K}\,\hat{\boldsymbol{x}}(n / n-1) \qquad (7)$$

where

$$\begin{aligned}
\hat{\boldsymbol{x}}(n/n-1) &= (\boldsymbol{A} - \boldsymbol{M}\boldsymbol{C})\hat{\boldsymbol{x}}(n-1/n-2) + \\
&\quad \boldsymbol{B}u(n-1) + \boldsymbol{M}y(n-1) \\
\boldsymbol{K} &= (R_J + \boldsymbol{B^T X B})^{-1}\boldsymbol{B^T X A} \\
\boldsymbol{X} &= \boldsymbol{A}^T\boldsymbol{X}\boldsymbol{A} - \boldsymbol{A}^T\boldsymbol{X}\boldsymbol{B}(\boldsymbol{R_J} + \boldsymbol{B^T X B})^{-1}\boldsymbol{B^T X A} + \boldsymbol{Q}_J \\
\boldsymbol{M} &= \boldsymbol{AYC}^T(\boldsymbol{R}_J + \boldsymbol{CYC}^T)^{-1} \\
\boldsymbol{Y} &= \boldsymbol{AYA}^T - \boldsymbol{AYC}^T(\boldsymbol{R}_v + \boldsymbol{CYC}^T)^{-1}\boldsymbol{CYA}^T + \boldsymbol{Q}_w
\end{aligned} \qquad (8)$$

(The chosen weighting matrices were: $\boldsymbol{Q_J}$ = diag(10^6, 1) and R_J = 1). Writing the closed loop system

$$\begin{bmatrix} \boldsymbol{x}(n+1) \\ \hat{\boldsymbol{x}}(n+1/n) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & -\boldsymbol{BK} \\ \boldsymbol{MC} & \boldsymbol{A} - \boldsymbol{BK} - \boldsymbol{MC} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(n) \\ \hat{\boldsymbol{x}}(n/n-1) \end{bmatrix} + \begin{bmatrix} \boldsymbol{D} & 0 \\ 0 & \boldsymbol{M} \end{bmatrix} \begin{bmatrix} w \\ v \end{bmatrix}. \qquad (8)$$

from (8) the well-known complementary sensitivity T and sensitivity S functions [5] in the z - transform are obtained. Given the defining constraint

$$S + T = 1 \qquad (9)$$

we see that the problem of positioning control synthesis necessitates a trade-off between the precision requirement (i.e., a reduced influence about y_1 of the perturbations w and v, requiring simultaneously small T and S) *and robustness properties* (requiring an increased T).

3 CLOSURE

The vulnerable axis of the PCMP, from the viewpoint of control efficiency occurs with relatively small $J = 0.4 Kgm^2$ (as against $0.8 Kgm^2$). Coulomb friction type perturbation torque was limited by a special design of the DC servo motors with $M_c = 0.01 Nm$ and an active constant torque $M = 0.15 Nm$ was designed. The objective of the numerical experiments was to validate the algorithm's performance on an appropriate nonlinear model

$$J\ddot{\theta} + M_c \,\mathrm{sgn}\left(\dot{\theta} - \dot{\theta}_1\right) = u$$

This equation considers three types of perturbations induced by flying aircraft: step velocity $\theta_1 = \theta_v t$, sinusoidal velocity $\dot{\theta}_1 = \theta_s \sin(\omega t)$ and filtered white noise $\dot{\theta}_1$. The variable height control that was synthesized by discrete time LQG algorithm was easily adapted to a variable width pulse and constant height scheme, imposing the same position and velocity values at $(n\text{-}1)\tau$ and $n\tau$ as in scheme with

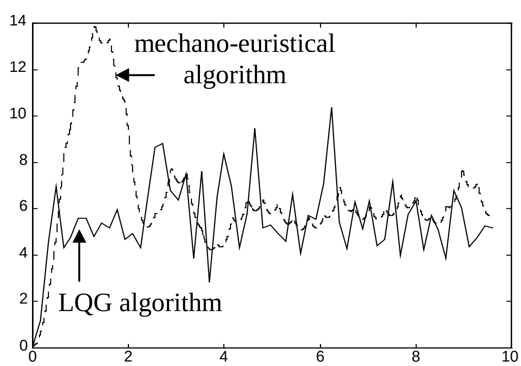

Fig. 1 Time history of controlled θ position (minutes) with measurement noise in the presence of step velocity

two applied torque pulses from section 2. 2; the values $t_1 = t_2$ were obtained in this way.

To demonstrate the algorithm's effect, Figs. 1 and 2 illustrate a time history of controlled position θ, with and without measurement noise respectively (a noise value of 2% of the imposed precision, $R_v = 0.34 \cdot 10^{-8} rad^2 s$, was used). θ_v was chosen to be $15 rad/s$, which is near a position step perturbation, $Q_w = 10^{-4} N^2 m^2 s$) and the best gain of control in terms of precision was retained. The obtained gains were then tuned on a testing set. In summary, the processing of the measurement recordings performed during pre-Eclipse airborne experiments confirmed a satisfactory working of the mechano-euristical algorithm; this was preferred for implementation.

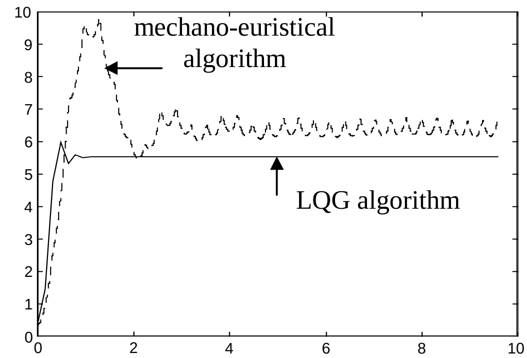

Fig. 2 Time history of controlled θ position (minutes) without measurement noise in the presence of step velocity

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