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Consistent reduction of $N = 2 \rightarrow N = 1$ four dimensional supergravity coupled to matter

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Abstract: We analyze the constraints which follow both on the geometry and on the gauge sector for a consistent supergravity reduction of a general matter-coupled $N = 2$ supergravity theory in four dimensions. These constraints can be derived in an elegant way by looking at the fermionic sector of the theory.

In this note we analyze the constraints which arise from a consistent reduction of $N = 2$ matter coupled supergravity [1],[2],[3], with arbitrary gauging, to $N = 1$ standard matter coupled supergravity [4],[5].

This study may find applications in many physical situations, as partial supersymmetry breaking [6],[7], brane supersymmetry reduction [8],[9], string or M- theories in presence of H-fluxes [10] – [20].

The main reason why a consistent reduction gives non trivial constraints on the matter sector is due to the fact that the second gravitino must be consistently eliminated from the spectrum. This implies a condition:

$$\delta_{\epsilon_1} \psi_{2\mu} = 0|_{\epsilon_2=0} \quad (0.1)$$

which must be integrable.

In a general rigid supersymmetric theory the reduction $N = 2 \rightarrow N = 1$ would give no constraint in the number of matter multiplets, but only some restriction on their interactions. However this is not the case in local supersymmetry because the second gravitino multiplet generates some non linear couplings, which are required to be absent if a consistent reduction may occur.

A full derivation of these constraints, also in higher N theories, was given recently [21] by looking at the bosonic terms in the local supersymmetry variations of the fermions.

However, in the full-fledged $N = 2$ theory [3], the very same variations contain three fermion terms:

$$\delta f \sim f f \epsilon \quad (0.2)$$

other than the bosonic terms

$$\delta f \sim b \epsilon \quad (0.3)$$

(here f and b denote generic fermion and boson fields).

These terms, in the component formulation, have two different origins. They come either from supercovariantization of bosonic terms containing derivatives (such as all connection terms both of space-time and of the scalar σ -model) or by elimination of “auxiliary fields” (or, in superspace, by solving Bianchi identities of the gravitational multiplet coupled to the matter multiplets.) [3].

Note that these terms are crucial in the proof of local supersymmetry of the lagrangian, since they induce five-fermion terms in the supersymmetry variation of the lagrangian

$$\delta_{\epsilon} \mathcal{L} = f f f f \epsilon \quad (0.4)$$

which must vanish by Fierz identities since they are purely algebraic. The constraints on the three-fermion terms (0.2) are much simpler to analyze since fermions have simple transformation properties under the local symmetries of the theory. By close inspection of these terms one can indeed obtain a reduction of the matter sector which is precisely what is obtained, by supersymmetry, from the constraints on the reduced geometry analyzed in [21].

We further note that the fermionic terms in the supersymmetry variations do not depend on the gauging of the theory, so that the restriction on the terms coming from the gauging must be still analyzed separately and here we simply report their implications on the reduced $N = 1$ theory, as found in [21]. However, the consistency of the reduction in presence of gauging of any isometry of the scalar manifold reflects, by supersymmetry, in

the occurrence of generalized Yukawa interactions, i.e. fermion bilinear in the lagrangian. The consistent truncation requires that all such terms, which are linear in the fermions which are deleted, may not survive the reduction. This is seen to be a consequence of the performed reduction.

In this note we will analyze the reduction of the fermionic terms in the supersymmetry variations and in the lagrangian, showing that the results obtained are in perfect agreement with those found at the bosonic level in [21].

1 Fermionic contributions

The supersymmetry reduction $N = 2 \rightarrow N = 1$ is obtained by truncating the $N = 1$ spin 3/2 multiplet containing the second gravitino $\psi_{\mu 2}$ and the graviphoton.

Here and in the following we use the notations both for $N = 2$ and $N = 1$ supergravity as given in reference [22], the only differences being that we use here world indices $\mathcal{I}, \bar{\mathcal{I}} = 1, \dots, n_V$ and boldfaced gauge indices $\mathbf{\Lambda} = 0, 1, \dots, n_V$ for quantities in the $N = 2$ vector multiplets since we want to reserve the notation Λ and i, \bar{i} for the indices of the reduced $N = 1$ theory (see reference [21]).

Let us write down the complete supersymmetry transformation laws of the $N = 2$ theory, including 3-fermions terms [3]:

Supergravity transformation rules of the (left-handed) Fermi fields:

$$\begin{aligned} \delta \psi_{A\mu} &= \widehat{\nabla}_\mu \epsilon_A + \left[i g S_{AB} \eta_{\mu\nu} + \varepsilon_{AB} \left(T_{\mu\nu}^- + U_{\mu\nu}^+ \right) \right] \gamma^\nu \epsilon^B + \\ &+ \left(A_{\mu A}{}^B + \gamma_{\mu\nu} A_A{}^{\nu B} \right) \epsilon_B - \frac{1}{4} \left(\partial_{\mathcal{J}} \mathcal{K} \bar{\lambda}^{\mathcal{J}B} \epsilon_B - h.c. \right) \psi_{\mu A} + \\ &- \omega_{A|u}{}^B \left(\mathcal{U}^{\alpha C|u} \bar{\zeta}_\alpha \epsilon_C + \mathcal{U}_{\alpha C}^u \bar{\zeta}^\alpha \epsilon^C \right) \psi_{\mu B} \end{aligned} \quad (1.1)$$

$$\begin{aligned} \delta \lambda^{\mathcal{I}A} &= i \nabla_\mu z^{\mathcal{I}} \gamma^\mu \epsilon^A + G_{\mu\nu}^{-\mathcal{I}} \gamma^{\mu\nu} \epsilon_B \varepsilon^{AB} + g W^{\mathcal{I}AB} \epsilon_B + \\ &+ \frac{1}{4} \left(\partial_{\mathcal{J}} \mathcal{K} \bar{\lambda}^{\mathcal{J}B} \epsilon_B - h.c. \right) \lambda^{\mathcal{I}A} - \omega_{B|u}^A \left(\mathcal{U}^{\alpha C|u} \bar{\zeta}_\alpha \epsilon_C + \mathcal{U}_{\alpha C}^u \bar{\zeta}^\alpha \epsilon^C \right) \lambda^{\mathcal{I}B} + \\ &- \Gamma_{\mathcal{J}\mathcal{K}}^{\mathcal{I}} \bar{\lambda}^{\mathcal{K}B} \epsilon_B \lambda^{\mathcal{J}A} - i \lambda^{\mathcal{I}B} \psi_{\mu B} \gamma^\mu \epsilon^A + \frac{i}{2} g^{\mathcal{I}\bar{\mathcal{J}}} \varepsilon^{AB} C_{\bar{\mathcal{J}}\mathcal{K}\mathcal{L}} \bar{\lambda}_B^{\bar{\mathcal{K}}} \lambda_C^{\mathcal{L}} \epsilon_D \varepsilon^{CD} \end{aligned} \quad (1.2)$$

$$\begin{aligned} \delta \zeta_\alpha &= i \mathcal{U}_u^{B\beta} \nabla_\mu q^u \gamma^\mu \epsilon^A \varepsilon_{AB} C_{\alpha\beta} + g N_\alpha^A \epsilon_A + \\ &+ \frac{1}{4} \left(\partial_{\mathcal{J}} \mathcal{K} \bar{\lambda}^{\mathcal{J}B} \epsilon_B - h.c. \right) \zeta_\alpha - \Delta_{\alpha|u}{}^\beta \left(\mathcal{U}^{\gamma A|u} \bar{\zeta}_\gamma \epsilon_A + \mathcal{U}_{\gamma A}^u \bar{\zeta}^\gamma \epsilon^A \right) \zeta_\beta + \\ &- i \bar{\zeta}_\alpha \psi_{\mu A} \gamma^\mu \epsilon^A \end{aligned} \quad (1.3)$$

Supergravity transformation rules of the Bose fields:

$$\delta V_\mu^a = -i \bar{\psi}_{A\mu} \gamma^a \epsilon^A - i \bar{\psi}_\mu^A \gamma^a \epsilon_A \quad (1.4)$$

$$\begin{aligned} \delta A_\mu^\Lambda &= 2 \bar{L}^\Lambda \bar{\psi}_{A\mu} \epsilon_B \varepsilon^{AB} + 2 L^\Lambda \bar{\psi}_\mu^A \epsilon^B \varepsilon_{AB} \\ &+ i f_{\mathcal{I}}^\Lambda \bar{\lambda}^{\mathcal{I}A} \gamma_\mu \epsilon^B \varepsilon_{AB} + i \bar{f}_{\bar{\mathcal{I}}}^\Lambda \bar{\lambda}_A^{\bar{\mathcal{I}}} \gamma_\mu \epsilon_B \varepsilon^{AB} \end{aligned} \quad (1.5)$$

$$\delta z^{\mathcal{I}} = \bar{\lambda}^{\mathcal{I}A} \epsilon_A \quad (1.6)$$

$$\delta z^{\bar{\mathcal{I}}} = \bar{\lambda}_A^{\bar{\mathcal{I}}} \epsilon^A \quad (1.7)$$

$$\delta q^u = \mathcal{U}_{\alpha A}^u \left(\bar{\zeta}^\alpha \epsilon^A + C^{\alpha\beta} \epsilon^{AB} \bar{\zeta}_\beta \epsilon_B \right). \quad (1.8)$$

We have defined:

$$\widehat{\nabla}_\mu \epsilon_A = \mathcal{D}_\mu \epsilon_A + \widehat{\omega}_{\mu|A}{}^B \epsilon_B + \frac{i}{2} \widehat{\mathcal{Q}}_\mu \epsilon_A \quad (1.9)$$

where \mathcal{D} denotes the Lorentz covariant derivative (on the spinors, $\mathcal{D}_\mu = \partial_\mu - \frac{1}{4} \omega_\mu^{ab} \gamma_{ab}$), and the $SU(2)$ and $U(1)$ 1-form ‘‘gauged’’ connections are respectively given by:

$$\widehat{\omega}_A{}^B = \omega_A{}^B + g_{(\Lambda)} A^\Lambda P_\Lambda^x (\sigma^x)_A{}^B, \quad (1.10)$$

$$\widehat{\mathcal{Q}} = \mathcal{Q} + g_{(\Lambda)} A^\Lambda P_\Lambda^0, \quad (1.11)$$

$$\mathcal{Q} = -\frac{i}{2} \left(\partial_I \mathcal{K} dz^I - \partial_{\bar{I}} \mathcal{K} d\bar{z}^{\bar{I}} \right) \quad (1.12)$$

$\omega_A{}^B$, \mathcal{Q} are the $SU(2)$ and $U(1)$ composite connections of the ungauged theory. Moreover we have:

$$\nabla_\mu z^I = \partial_\mu z^I + g_{(\Lambda)} A_\mu^\Lambda k_\Lambda^I \quad (1.13)$$

$$\nabla_\mu q^u = \partial_\mu q^u + g_{(\Lambda)} A_\mu^\Lambda k_\Lambda^u \quad (1.14)$$

where k_Λ^I and k_Λ^u are the Killing vectors of the $N = 2$ special-Kähler manifold \mathcal{M}^{SK} and of the quaternionic manifold \mathcal{M}^Q respectively. They are related to the respective prepotentials by:

$$k_\Lambda^I = ig^{\bar{I}\bar{J}} \partial_{\bar{J}} P_\Lambda^0 \quad (1.15)$$

$$k_\Lambda^u = \frac{1}{6\lambda^2} \Omega^{x|vu} \nabla_v P_\Lambda^x; \quad \lambda = -1 \quad (1.16)$$

where Ω_{uv}^x is the $SU(2)$ -valued curvature of the connection $\omega_A{}^B$, λ is the scale of the quaternionic manifold which in our conventions is fixed to the value $\lambda = -1$ by supersymmetry (see ref. [3]). The prepotential P_Λ^0 satisfies:

$$P_\Lambda^0 L^\Lambda = P_\Lambda^0 \bar{L}^\Lambda = 0 \quad (1.17)$$

where L^Λ , together with its magnetic counterpart $M_\Lambda \equiv \mathcal{N}_{\Lambda\Sigma} L^\Sigma$, is the symplectic section of the $Sp(2n_V)$ flat bundle over \mathcal{M}^{SK} in terms of which the special-Kähler geometry is defined. Note that we use throughout the paper the definition $f_I^\Lambda = \nabla_I L^\Lambda = \partial_I L^\Lambda + \frac{1}{2} \partial_I \mathcal{K} L^\Lambda$. $T_{\mu\nu}^-$ appearing in the supersymmetry transformation law of the $N = 2$ left-handed gravitini is the ‘‘dressed’’ graviphoton defined as:

$$\begin{aligned} T_{\mu\nu}^- &\equiv 2i \text{Im} \mathcal{N}_{\Lambda\Sigma} L^\Sigma \left[F_{\mu\nu}^{\Lambda-} + (L^\Lambda \bar{\psi}_{A\mu} \psi_{B\nu} \epsilon^{AB} + \bar{L}^\Lambda \bar{\psi}^{A\mu} \psi^{B\nu} \epsilon_{AB} + \right. \\ &\quad \left. - i \bar{f}_{\bar{I}}^\Lambda \bar{\lambda}_A \gamma_{[\nu} \psi_{\mu]B} \epsilon^{AB})^- + \frac{1}{8} \nabla_\kappa f_{\bar{L}}^\Lambda \bar{\lambda}^{\kappa A} \gamma_{\mu\nu} \lambda^{\mathcal{L}B} \epsilon_{AB} - \frac{1}{4} L^\Lambda \mathbf{C}^{\alpha\beta} \bar{\zeta}_\alpha \gamma_{\mu\nu} \zeta_\beta \right] \end{aligned} \quad (1.18)$$

while

$$\begin{aligned} G_{\mu\nu}^{\bar{I}-} &\equiv -g^{\bar{I}\bar{J}} \text{Im} \mathcal{N}_{\Lambda\Sigma} \bar{f}_{\bar{J}}^\Sigma \left[F_{\mu\nu}^{\Lambda-} + (L^\Lambda \bar{\psi}_{A\mu} \psi_{B\nu} \epsilon^{AB} - i f_{\bar{K}}^\Lambda \bar{\lambda}^{\mathcal{K}A} \gamma_{[\nu} \psi_{\mu]B} \epsilon_{AB} + \right. \\ &\quad \left. - i \bar{f}_{\bar{I}}^\Lambda \bar{\lambda}_A \gamma_{[\nu} \psi_{\mu]B} \epsilon^{AB})^- + \frac{1}{8} \nabla_\kappa f_{\bar{L}}^\Lambda \bar{\lambda}^{\kappa A} \gamma_{\mu\nu} \lambda^{\mathcal{L}B} \epsilon_{AB} - \frac{1}{4} L^\Lambda \mathbf{C}^{\alpha\beta} \bar{\zeta}_\alpha \gamma_{\mu\nu} \zeta_\beta \right] \end{aligned} \quad (1.19)$$

are the “dressed” field strengths of the vectors inside the vector multiplets (the “minus” apex means taking the self-dual part.). The “auxiliary fields” $A_{\mu A}{}^B$ and $A'_{\mu A}{}^B$ are defined as:

$$A_{\mu A}{}^B = -\frac{i}{4}g_{\mathcal{I}\mathcal{J}} \left(\bar{\lambda}_A^{\mathcal{I}} \gamma^\mu \lambda^{\mathcal{J}B} - \delta_A^B \bar{\lambda}_C^{\mathcal{I}} \gamma^\mu \lambda^{\mathcal{J}C} \right) \quad (1.20)$$

$$A'_{\mu A}{}^B = \frac{i}{4}g_{\mathcal{I}\mathcal{J}} \left(\bar{\lambda}_A^{\mathcal{I}} \gamma^\mu \lambda^{\mathcal{J}B} - \frac{1}{2}\delta_A^B \bar{\lambda}_C^{\mathcal{I}} \gamma^\mu \lambda^{\mathcal{J}C} \right) - \frac{i}{4}\delta_A^B \zeta_\alpha \gamma^\mu \zeta^\alpha. \quad (1.21)$$

Moreover the fermionic shifts S_{AB} , $W^{\mathcal{I}AB}$ and N_α^A are given in terms of the prepotentials and Killing vectors of the quaternionic geometry as follows:

$$S_{AB} = i\frac{1}{2}P_{AB\Lambda} L^\Lambda \equiv i\frac{1}{2}P_\Lambda^x \sigma_{AB}^x L^\Lambda \quad (1.22)$$

$$W^{\mathcal{I}AB} = iP_\Lambda^{AB} g^{\mathcal{I}\mathcal{J}} f_{\mathcal{J}}^\Lambda + \epsilon^{AB} k_\Lambda^{\mathcal{I}} \bar{L}^\Lambda \quad (1.23)$$

$$N_\alpha^A = 2\mathcal{U}_{\alpha u}^A k_\Lambda^u \bar{L}^\Lambda \quad (1.24)$$

$$N_A^\alpha = -2\mathcal{U}_{Au}^\alpha k_\Lambda^u L^\Lambda \quad (1.25)$$

Since we are going to compare the $N = 2$ reduced theory with the standard $N = 1$ supergravity, we also quote the supersymmetry transformation laws of the latter theory [23],[4]. We have, up to 3-fermions terms:

$N = 1$ transformation laws

$$\delta\psi_{\bullet\mu} = \mathcal{D}_\mu \epsilon_\bullet + \frac{i}{2}\widehat{Q}_\mu \epsilon_\bullet + iL(z, \bar{z})\gamma_\mu \epsilon^\bullet \quad (1.26)$$

$$\delta\chi^i = i\left(\partial_\mu z^i + g_{(\Lambda)} A_\mu^\Lambda k_\Lambda^i\right)\gamma^\mu \epsilon_\bullet + N^i \epsilon_\bullet \quad (1.27)$$

$$\delta\lambda_\bullet^\Lambda = \mathcal{F}_{\mu\nu}^{(-)\Lambda} \gamma^{\mu\nu} \epsilon_\bullet + iD^\Lambda \epsilon_\bullet \quad (1.28)$$

$$\delta V_\mu^a = -i\psi_\bullet \gamma_\mu \epsilon^\bullet + \text{h.c.} \quad (1.29)$$

$$\delta A_\mu^\Lambda = i\frac{1}{2}\bar{\lambda}_\bullet^{\Lambda} \gamma_\mu \epsilon^\bullet + \text{h.c.} \quad (1.30)$$

$$\delta z^i = \bar{\chi}^i \epsilon_\bullet \quad (1.31)$$

where \widehat{Q} is defined in a way analogous to the $N = 2$ definition (1.11) and:

$$L(z, \bar{z}) = W(z) e^{\frac{1}{2}\mathcal{K}_{(1)}(z, \bar{z})}, \quad \nabla_{\bar{i}} L = 0 \quad (1.32)$$

$$N^i = 2g^{i\bar{j}} \nabla_{\bar{j}} \bar{L} \quad (1.33)$$

$$D^\Lambda = -2(\text{Im}f_{\Lambda\Sigma})^{-1} P_\Sigma(z, \bar{z}) \quad (1.34)$$

and $W(z)$, $\mathcal{K}_{(1)}(z, \bar{z})$, $P_\Sigma(z, \bar{z})$, $f_{\Lambda\Sigma}(z)$ are the superpotential, Kähler potential, Killing prepotential and vector kinetic matrix respectively [4], [23], [5]. Note that for the gravitino and gaugino fields we have denoted by a lower (upper) dot left-handed (right-handed) chirality. For the spinors of the chiral multiplets χ , instead, left-handed (right-handed) chirality is encoded via an upper holomorphic (antiholomorphic) world index (χ^i , $\chi^{\bar{i}}$).

Finally, we recall the equations defining special geometry:

$$\begin{aligned} D_i V &= U_i \\ D_i U_j &= iC_{ijk} g^{k\bar{k}} \bar{U}_{\bar{k}} \\ D_i U_{\bar{j}} &= g_{i\bar{j}} \bar{V} \\ D_i \bar{V} &= 0 \end{aligned} \quad (1.35)$$

where

$$V = (L^\Lambda, M_\Lambda), \quad U_i = D_i V = (f_i^\Lambda, h_{\Lambda i}) \quad \Lambda = 0, \dots, n; \quad (1.36)$$

$$M_\Lambda = \mathcal{N}_{\Lambda\Sigma} L^\Sigma, \quad h_{\Lambda i} = \overline{\mathcal{N}}_{\Lambda\Sigma} f_i^\Lambda \quad (1.37)$$

and $\mathcal{N}_{\Lambda\Sigma}$ is the kinetic vector matrix.

Gravitino reduction

To perform the truncation we set $A=1$ and 2 successively, putting $\psi_{2\mu} = \epsilon_2 = 0$, and we get from equation (1.1):

$$\begin{aligned} \delta \psi_{1\mu} &= \mathcal{D}_\mu \epsilon_1 - \widehat{\mathcal{Q}}_\mu \epsilon_1 - \widehat{\omega}_{\mu|1}{}^1 \epsilon_1 + i g S_{11} \eta_{\mu\nu} \gamma^\nu \epsilon^1 + (A_{\mu 1}{}^1 + \gamma_{\mu\nu} A_1^{\nu 1}) \epsilon^1 + \\ &- \omega_{1|u}{}^1 (\mathcal{U}^{\alpha 1|u} \overline{\zeta}_\alpha \epsilon_1 + \mathcal{U}_{\alpha 1}^u \overline{\zeta}^\alpha \epsilon^1) \psi_{\mu 1} - \frac{1}{4} (\partial_{\mathcal{J}} K \overline{\lambda}^{\mathcal{J} 1} \epsilon_1 - h.c.) \psi_{\mu 1} \end{aligned} \quad (1.38)$$

while, for consistency:

$$\begin{aligned} \delta \psi_{2\mu} \equiv 0 &= -\widehat{\omega}_{\mu|2}{}^1 \epsilon_1 + [i g S_{21} \eta_{\mu\nu} - (T_{\mu\nu}^- + U_{\mu\nu}^+)] \gamma^\nu \epsilon^1 + \\ &+ (A_{\mu 2}{}^1 + \gamma_{\mu\nu} A_2^{\nu 1}) \epsilon^1 - \omega_{2|u}{}^1 (\mathcal{U}^{\alpha 1|u} \overline{\zeta}_\alpha \epsilon_1 + \mathcal{U}_{\alpha 1}^u \overline{\zeta}^\alpha \epsilon^1) \psi_{\mu 1} \end{aligned} \quad (1.39)$$

Comparing (1.26) with (1.38), we learn that we must identify:

$$\psi_{1\mu} \equiv \psi_{\bullet\mu} \quad (1.40)$$

$$\epsilon_1 \equiv \epsilon_{\bullet}. \quad (1.41)$$

Furthermore, for a consistent truncation we must set to zero all the following structures:

$$T_{\mu\nu}^- = 0 \quad (1.42)$$

$$S_{21} = 0 \quad (1.43)$$

$$\widehat{\omega}_{\mu|2}{}^1 = 0 \quad (1.44)$$

$$\widehat{\omega}_{u|2}{}^1 \mathcal{U}_{\alpha 1}^u \overline{\zeta}^\alpha \epsilon^1 = 0 \quad (1.45)$$

$$U_{\mu\nu}^+ = -\frac{i}{4} \mathbb{C}_{\alpha\beta} \overline{\zeta}^\alpha \gamma_{\mu\nu} \zeta^\beta = 0 \quad (1.46)$$

$$A_{\mu 2}{}^1 = -\frac{i}{4} g_{\mathcal{I}\mathcal{J}} \overline{\lambda}_2^{\mathcal{I}} \gamma_\mu \lambda^{\mathcal{J} 1} = -A'_{\mu 2}{}^1 = 0 \quad (1.47)$$

We note that the expression “equal to zero” in (1.42) - (1.47) has to be intended in a weak sense, as a condition to be true on the reduced $N = 1$ theory.

Let us analyze in particular the constraints (1.42), (1.45), (1.46), (1.47) containing 3 fermions contributions.

We first consider the implications of these constraints on the hypermultiplet sector. Equations (1.46) and (1.42) impose to truncate out half of the hypermultiplets. Indeed, let us decompose the symplectic index $\alpha \rightarrow (I, \dot{I})$, so that we can write the symplectic matrix $\mathbb{C}_{\alpha\beta}$ as:

$$\begin{pmatrix} 0 & \mathbb{1}_{I\dot{I}} \\ -\mathbb{1}_{\dot{I}I} & 0 \end{pmatrix} \quad (1.48)$$

Then, equation (1.46) becomes:

$$\delta_{I\dot{I}} \overline{\zeta}^{\dot{I}} \gamma_{\mu\nu} \zeta^{\dot{I}} = 0 \quad (1.49)$$

which is an orthogonality condition between the set of $\{\zeta^I\}$ and $\{\zeta^{\dot{I}}\}$. A particular solution is to take $\zeta^I \neq 0$, and then:

$$\zeta^{\dot{I}} = 0, \quad (1.50)$$

that is at least half of the hypermultiplets have to be projected out in the truncation. More generally, we could decompose the indices as $I = (f, g)$; $\dot{I} = (\dot{f}, \dot{g})$ (with $f, \dot{f} = 1, \dots, k \leq n_H$; $g, \dot{g} = 1, \dots, n_H - k$) and, for $\zeta^f \neq 0$; $\zeta^{\dot{g}} \neq 0$ eq. (1.49) gives

$$\zeta^g = 0 ; \quad \zeta^{\dot{f}} = 0 \quad (1.51)$$

together with their scalar partners that, as we easily see when looking at the hyperini transformation laws, are respectively:

$$\mathcal{U}_u^{1g} dq^u = 0 ; \quad \mathcal{U}_u^{1\dot{f}} dq^u = 0 ; \quad (\mathcal{U}^{1g} = (\mathcal{U}^{2\dot{g}})^* ; \quad \mathcal{U}^{1\dot{f}} = (\mathcal{U}^{2f})^*). \quad (1.52)$$

However, by a symplectic rotation we can always choose a basis where $\dot{g} = 0$, $f = I$. As we will show in the following when looking at the hyperini transformation law reduction, there is no loss of generality by adopting the simpler choice (1.50) (that is $\dot{g} = 0$, $f = I$), as we will actually do in the following. Therefore in the rest of the paper we will treat the case $f = I$, where the only vielbein surviving on the submanifold $\mathcal{M}^{KH} \subset \mathcal{M}^Q$ are:

$$\mathcal{U}^{1I} = (\mathcal{U}^{2I})^* \quad (1.53)$$

while:

$$\mathcal{U}^{2I} = (\mathcal{U}^{1I})^* = 0. \quad (1.54)$$

Now we can make a choice of coordinates on the quaternionic manifold $q^u = (w^s, n^t)$, such that the n^t are the coordinates truncated out, and set, as a basis of vielbein for the submanifold spanned by the scalars of the surviving hypermultiplets:

$$P_I = P_{I_s} dw^s \equiv \sqrt{2} \mathcal{U}_{I1u} dq^u \quad (1.55)$$

$$\bar{P}_{\dot{I}} = P_{\dot{I}_{\bar{s}}} d\bar{w}^{\bar{s}} \equiv \sqrt{2} \mathcal{U}_{\dot{I}2u} dq^u. \quad (1.56)$$

With this position, equation (1.45) is now easy to interpret. It can be rewritten as:

$$\hat{\omega}_{u|2}{}^1 \mathcal{U}_{I1}^u \bar{\zeta}^I \epsilon^1 = \hat{\omega}_{\bar{s}|2}{}^1 \mathcal{U}_{\dot{I}1}^{\bar{s}} \bar{\zeta}^{\dot{I}} \epsilon^1 = 0 \quad (1.57)$$

which gives a condition on the component of the $SU(2)$ connection:

$$\hat{\omega}_{\bar{s}|2}{}^1 = 0. \quad (1.58)$$

This condition, obtained from the fermion-bilinear equation (1.45), coincides with the bosonic constraint (1.44) analyzed in reference [21]. Indeed eq. (1.44) is more properly written, in the appropriate basis, as:

$$\hat{\omega}_2{}^1|_{\mathcal{M}^{KH}} = \hat{\omega}_{\bar{s}|2}{}^1 d\bar{w}^{\bar{s}} = 0, \quad (1.59)$$

which is satisfied by (1.58). When looking at the explicit expression of the field-strength of the $SU(2)$ connection (whose component $\Omega_2{}^1$ has to be zero for consistency):

$$\Omega_2{}^1 \equiv d\omega_2{}^1 + \omega_2{}^A \wedge \omega_A{}^1 = i \lambda \mathcal{U}_{\alpha 2} \wedge \mathcal{U}^{\alpha 1} = i \lambda (\mathcal{U}_{I2} \wedge \mathcal{U}^{I1} + \mathcal{U}_{\dot{I}2} \wedge \mathcal{U}^{\dot{I}1}) = 0 \quad (1.60)$$

we see that it is automatically satisfied by the position (1.52).

The surviving $U(1)$ curvature $\Omega_1^1 = \Omega^3(\sigma^3)_1^1$ is instead different from zero and defines (one half) the Kähler 2-form on \mathcal{M}^{KH} , so that we may introduce complex coordinates w^s and Kähler metric such that $\Omega^3 = \frac{i}{2}g_{s\bar{s}}dw^s \wedge d\bar{w}^{\bar{s}}$ [21]. This does not exhaust the restrictions on the quaternionic manifold \mathcal{M}^Q , since, as we will see in the analysis of the fermionic sector of the hyperini transformation laws, extra constraints on the symplectic part of the quaternionic curvature have to be imposed.

Let us now come to the reduction of the $N = 2$ vector multiplets. To understand condition (1.47), let us observe that the n_V $N = 2$ vector multiplets $(A_\mu, \lambda^A, z)^{\mathcal{I}}$ ($\mathcal{I} = 1, \dots, n_V$) decompose to $N = 1$ chiral multiplets $(\lambda^1, z)^{\mathcal{I}}$ and $N = 1$ vector multiplets $(A_\mu, \lambda^2)^{\mathcal{I}}$ [21]. Let us suppose that in the reduction the number of chiral multiplets coming from $N = 2$ vector multiplets is $n_C \leq n_V$. We then have to decompose the indices $\mathcal{I} \rightarrow (i, \alpha)$, where $i = 1, \dots, n_C$ and $\alpha = n_C + 1, \dots, \mathcal{I}$, and the chiral multiplets are labeled as $(\lambda^1, z)^i$ (while $(\lambda^1, z)^\alpha = 0$).

Then eq. (1.47) can be rewritten as:

$$g_{i\bar{j}}\bar{\lambda}_2^{\bar{i}}\lambda^{j1} + g_{\bar{\alpha}j}\bar{\lambda}_2^{\bar{\alpha}}\lambda^{j1} = 0 \quad (1.61)$$

which is an orthogonality condition between the $N = 1$ chiral and vector multiplets coming from the $N = 2$ vector multiplets, satisfied for:

$$\lambda^{i2} = 0 \quad (1.62)$$

$$g_{i\bar{\alpha}} = 0. \quad (1.63)$$

The previous equations imply that if the $N = 1$ chiral multiplets have indices $i = 1, \dots, n_C \leq n_V$, then the $N = 1$ vector multiplets take the complementary indices $\alpha = \Lambda = 1, \dots, n'_V = n_V - n_C$. As a consequence, the scalar partners of the chiral fermions λ^{i1} span a Kähler manifold $\mathcal{M}_R \subset \mathcal{M}^{SK}$ of complex dimension n_C .

Furthermore, the three fermion terms in eq. (1.42) and (1.18) containing $N = 2$ gaugini impose conditions on the scalar sector of the theory. Indeed (1.42), (1.18) give:

$$\text{Im}\mathcal{N}_{\Lambda\Sigma}L^\Sigma\bar{f}_\alpha^\Lambda\bar{\lambda}_2^{\bar{\alpha}}\gamma_{[\nu}\psi_{\mu]1} = 0 \Rightarrow \text{Im}\mathcal{N}_{\Lambda\Sigma}L^\Sigma\bar{f}_\alpha^\Lambda = 0 \quad (1.64)$$

$$\text{Im}\mathcal{N}_{\Lambda\Sigma}L^\Sigma\nabla_i f_\alpha^\Lambda\bar{\lambda}^{i1}\gamma_{\mu\nu}\lambda^{\alpha 2} = 0 \Rightarrow \text{Im}\mathcal{N}_{\Lambda\Sigma}L^\Sigma\nabla_i f_\alpha^\Lambda = 0 \quad (1.65)$$

Equation (1.64) is an orthogonality condition between the set $\{L^\Lambda\}$ and the set $\{f_\alpha^\Lambda\}$. By decomposing the vector indices Λ as $\Lambda \rightarrow (\Lambda, X)$, (with $\Lambda = 1, \dots, n'_V$, $X = 0, 1, \dots, n_C = n_V - n'_V$) it becomes:

$$\text{Im}\mathcal{N}_{\Lambda\Sigma}L^\Sigma\bar{f}_\alpha^\Lambda + \text{Im}\mathcal{N}_{\Lambda X}L^X\bar{f}_\alpha^\Lambda + \text{Im}\mathcal{N}_{X\Lambda}L^\Lambda\bar{f}_\alpha^X + \text{Im}\mathcal{N}_{XY}L^Y\bar{f}_\alpha^X = 0 \quad (1.66)$$

A consistent solution of eq. (1.66) is easily found by setting:

$$L^\Lambda = 0 \quad (1.67)$$

$$f_\alpha^X = 0 \quad (1.68)$$

$$\text{Im}\mathcal{N}_{\Lambda X} = 0 \quad (1.69)$$

We observe that, since $\mathcal{N}_{\Lambda X}$ is anti-holomorphic on \mathcal{M}_R , equation (1.69) still allows a constant, purely real, term $\mathcal{N}_{\Lambda X} = C_{\Lambda X}$ that we do not discuss here.

With the same decomposition of indices the second equation (1.65) gives:

$$\text{Im}\mathcal{N}_{\Lambda\Sigma}L^\Sigma\nabla_i f_\alpha^\Lambda + \text{Im}\mathcal{N}_{\Lambda X}L^X\nabla_i f_\alpha^\Lambda + \text{Im}\mathcal{N}_{X\Lambda}L^\Lambda\nabla_i f_\alpha^X + \text{Im}\mathcal{N}_{XY}L^Y\nabla_i f_\alpha^X = 0 \quad (1.70)$$

which is satisfied (in a way consistent with (1.67) - (1.69)) with the further constraint:

$$\nabla_i f_\alpha^X = iC_{ij\alpha}f_j^X g^{j\bar{j}} = 0 \Rightarrow C_{ij\alpha} = 0 \quad (1.71)$$

where we have used the special geometry relation (1.35) defining $C_{\mathcal{I}JK}$. This solution tells us that the reduced manifold \mathcal{M}_R is a special-Kähler manifold with symplectic sections (L^X, M_X) . Indeed we have, recalling the differential identities satisfied by the symplectic sections of the $N = 2$ parent theory, that the 3 equations (1.67) - (1.69) induce on \mathcal{M}_R the special-Kähler structure with indices $\mathbf{\Lambda}$ restricted to X . Other possible solutions to equations (1.67) - (1.69) are not compatible with supersymmetry, as can be easily ascertained by looking at the bosonic sector (see [21]).

Given the conditions found above, let us now compute the reduction of the complete transformation laws for the spin one half fermions.

Hypermultiplets reduction

The $N = 2$ hyperini supersymmetry transformation law reduces to:

$$\begin{aligned} \delta \zeta_I &= i\mathcal{U}_u^{2j} \nabla_\mu q^u \gamma^\mu \epsilon^1 \delta_{IJ} + gN_I^1 \epsilon_1 + \\ &+ \frac{1}{4} \left(\partial_{\mathcal{J}} K \bar{\lambda}^{\mathcal{J}1} \epsilon_1 - h.c. \right) \zeta_I - \Delta_{I|u}{}^J \left(\mathcal{U}^{K1|u} \bar{\zeta}_K \epsilon_1 + \mathcal{U}_{K1}^u \bar{\zeta}^K \epsilon^1 \right) \zeta_J + \\ &- i\bar{\zeta}_I \psi_{\mu 1} \gamma^\mu \epsilon^1 \end{aligned} \quad (1.72)$$

while for consistency we have to impose:

$$\begin{aligned} \delta \zeta_j = 0 &= i\mathcal{U}_u^{2J} \nabla_\mu q^u \gamma^\mu \epsilon^1 \delta_{jJ} + gN_j^1 \epsilon_1 + \\ &- \Delta_{j|u}{}^J \left(\mathcal{U}^{K1|u} \bar{\zeta}_K \epsilon_1 + \mathcal{U}_{K1}^u \bar{\zeta}^K \epsilon^1 \right) \zeta_J + \\ &+ i\delta_{jI} \mathcal{U}_u^{J2} \nabla_\mu q^u \gamma^\mu \epsilon^1 \end{aligned} \quad (1.73)$$

We find therefore the consistency conditions:

$$\Delta_j{}^J = 0 \quad (1.74)$$

$$\mathcal{U}_u^{J2} \nabla_\mu q^u = 0 \quad (1.75)$$

$$N_j^1 = 0 \quad (1.76)$$

Eq. (1.74) reduces the holonomy of the quaternionic scalar manifold from $Sp(2n_H)$ to $U(n_H)$, a condition necessary for the validity of the truncation, since the manifold has to reduce to a Kähler-Hodge one.

We note that, if we had chosen the more general configuration (1.51), we had found instead of (1.74) the holonomy constraints:

$$\Delta_g{}^f = 0; \quad \Delta_g{}^{\dot{g}} = 0; \quad \Delta_j{}^{\dot{g}} = 0; \quad \Delta_j{}^f = 0 \quad (1.77)$$

Working out the curvatures associated to these components of the $Sp(2n_H)$ connection it is easy to see that they in fact reconstruct the full curvatures of the group $U(n_H)$, embedded

however into $Sp(2n_H)$ in a different way from the standard one related to the choice (1.53), (1.54). In group theoretical terms, if we set $f, \dot{f} = 1, \dots, k$; $g, \dot{g} = 1, \dots, n_H - k$, we find that the constraints (1.77) correspond to the decomposition:

$$Adj(Sp(2n_H)) \rightarrow Adj(U(k)) + Adj(U(n_H - k)) + 2(k, n_H - k) \quad (1.78)$$

Actually, in equation (1.77) we recognize that the r.h.s. is in fact the adjoint of $U(n_H)$, which is however decomposed with respect to its maximal subgroup $U(k) \times U(n_H - k)$. In the sequel we refer only to the simpler choice (1.50).

We stress the fact that the necessary condition (1.74) found above implies a further geometric constraint for the consistency of the truncation. Indeed, as it has been analyzed in [21] by using the Frobenius theorem, in order for the equations (1.54) and (1.74) to give a consistent truncation, the quaternionic manifold cannot be generic; in particular, the completely symmetric tensor $\Omega_{\alpha\beta\gamma\delta} \in Sp(2n_H)$, appearing in the $Sp(2n_H)$ curvature, must obey the following constraint:

$$\Omega_{ijkl} = 0. \quad (1.79)$$

Eq. (1.75) is automatically satisfied with the choice of basis (1.74). Indeed it means that the scalar partners of the ζ_i have to be truncated out, since they span the orthogonal complement to the retained submanifold:

$$\mathcal{U}_u^{J2} \nabla_\mu q^u |_{\mathcal{M}_{KH}} = \mathcal{U}_t^{J2} \nabla_\mu n^t |_{\mathcal{M}_{KH}} = 0. \quad (1.80)$$

We can now define chiral spinors with world indices:

$$\zeta^s \equiv \sqrt{2} P^{I,s} \zeta_I \quad (1.81)$$

and we find, for the transformation law of the ζ^s :

$$\begin{aligned} \delta \zeta^s &= i \nabla_\mu w^s \gamma^\mu \epsilon^1 + g N^s \epsilon_1 + \\ &+ \frac{i}{4} \left(\partial_J K \bar{\lambda}^{J1} \epsilon_1 - h.c. \right) \zeta^s - \Gamma_{s's''}^s \zeta^{s''} \bar{\zeta}^{s'} \epsilon_1 + \\ &- i \bar{\zeta}^s \psi_{\mu 1} \gamma^\mu \epsilon^1 \end{aligned} \quad (1.82)$$

with $N^s \equiv \sqrt{g} P^{I,s} N_I^1$ and $\Gamma_{s's''}^s = \delta_I^J P_{K,s'} \partial_{s''} P^{K,s} - P^{I,s} \Delta_{I|s'}^J P_{J,s''}$.

Vector multiplets reduction

Let us now consider the reduction of the gaugini transformation law. From eq. (1.2) we find:

$$\begin{aligned} \delta \lambda^{i1} &= i \nabla_\mu z^i \gamma^\mu \epsilon^1 + g W^{i11} \epsilon_1 + \\ &+ \frac{1}{4} \left(\partial_j K \bar{\lambda}^{j1} \epsilon_1 - h.c. \right) \lambda^{i1} - \left(\omega_{1|s}^1 \bar{\zeta}^s \epsilon_1 + \omega_{1|\bar{s}}^1 \bar{\zeta}^{\bar{s}} \epsilon^1 \right) \lambda^{i1} - \Gamma_{jk}^i \bar{\lambda}^{k1} \epsilon_1 \lambda^{j1} + \\ &- i \lambda^{i1} \psi_{\mu 1} \gamma^\mu \epsilon^1 - \frac{i}{2} g^{i\bar{j}} C_{\bar{j}\bar{\alpha}\bar{\beta}} \bar{\lambda}_2^{\bar{\alpha}} \lambda_2^{\bar{\beta}} \epsilon_1 \end{aligned} \quad (1.83)$$

$$\begin{aligned} \delta \lambda^{\alpha 2} &= -G_{\mu\nu}^{-\alpha} \gamma^{\mu\nu} \epsilon_1 + g W^{\alpha 21} \epsilon_1 + \\ &+ \frac{1}{4} \left(\partial_j K \bar{\lambda}^{j1} \epsilon_1 - h.c. \right) \lambda^{\alpha 2} - \left(\omega_{2|s}^2 \bar{\zeta}^s \epsilon_1 + \omega_{2|\bar{s}}^2 \bar{\zeta}^{\bar{s}} \epsilon^1 \right) \lambda^{\alpha 2} - \Gamma_{\beta i}^\alpha \bar{\lambda}^{i1} \epsilon_1 \lambda^{\beta 2} + \\ &+ \frac{i}{2} g^{\alpha\bar{\beta}} C_{\bar{\beta}\bar{i}\bar{\gamma}} \bar{\lambda}_1^{\bar{i}} \lambda_2^{\bar{\gamma}} \epsilon_1. \end{aligned} \quad (1.84)$$

while for consistency we have to impose:

$$\delta \lambda^{\alpha 1} = 0 = i \nabla_{\mu} z^{\alpha} \gamma^{\mu} \epsilon^1 + g W^{\alpha 11} \epsilon_1 - \Gamma_{ij}^{\alpha} \bar{\lambda}^{j1} \epsilon_1 \lambda^{i1} - \frac{i}{2} g^{\alpha \bar{\beta}} C_{\bar{\beta} \bar{\gamma} \bar{\delta}} \bar{\lambda}^{\bar{\gamma}} \lambda^{\bar{\delta}} \epsilon_1 \quad (1.85)$$

$$\delta \lambda^{i2} = 0 = -G_{\mu\nu}^{-i} \gamma^{\mu\nu} \epsilon_1 + g W^{i21} \epsilon_1 - \Gamma_{\alpha j}^i \bar{\lambda}^{j1} \epsilon_1 \lambda^{\alpha 2} + \frac{i}{2} g^{i \bar{j}} C_{\bar{j} \bar{k} \bar{\alpha}} \bar{\lambda}^{\bar{k}} \lambda^{\bar{\alpha}} \epsilon_1 \quad (1.86)$$

that implies that, on the reduced theory, the following quantities have to be zero:

$$\nabla_{\mu} z^{\alpha} = 0; \quad (1.87)$$

$$G_{\mu\nu}^{-i} \gamma^{\mu\nu} = 0; \quad (1.88)$$

$$W^{\alpha 11} = 0; \quad W^{i21} = 0 \quad (1.89)$$

$$\Gamma_{ij}^{\alpha} = g^{\alpha \bar{\beta}} \partial_i g_{j \bar{\beta}} = 0 \quad \Gamma_{\alpha j}^i g^{i \bar{k}} \partial_j g_{\alpha \bar{k}} = 0 \quad (1.90)$$

$$C_{\bar{j} \bar{k} \bar{\alpha}} = 0 \quad C_{\bar{\beta} \bar{\gamma} \bar{\delta}} = 0 \quad (1.91)$$

We analyze here the constraints coming from the 3-fermions terms. Equation (1.88) is automatically satisfied, on the reduced theory, because of equations (1.67) - (1.69) and (1.71). Equations (1.90) are also true on the sub-manifold since we have found in (1.63) that the mixed components of the metric are zero. Finally, equations (1.91) give constraints on the Special-Kahler manifold to be satisfied on the reduced sub-manifold \mathcal{M}_R . In particular, the first one $C_{\bar{j} \bar{k} \bar{\alpha}} = 0|_{\mathcal{M}_R}$ coincides with the already found condition (1.71), while $C_{\bar{\beta} \bar{\gamma} \bar{\delta}} = 0|_{\mathcal{M}_R}$ is a further constraint, due to supersymmetry, to be satisfied. We note that in particular it implies the following constraint on the curvature of the special manifold:

$$R_{\bar{\alpha} \bar{\beta} \bar{\gamma}}^{\bar{i}} = 0 \quad (1.92)$$

If we now define:

$$\chi^i \equiv \lambda^{i1}, \quad \lambda^{\Lambda} \equiv -2f_{\alpha}^{\Lambda} \lambda_{\bullet}^{\alpha} \quad (1.93)$$

and apply the special geometry relation:

$$C_{IJK} = f_I^{\Lambda} \partial_J \mathcal{N}_{\Lambda \Sigma} f_K^{\Sigma}, \quad (1.94)$$

which gives:

$$C_{i\alpha\beta} = f_{\alpha}^{\Lambda} \partial_i \mathcal{N}_{\Lambda \Sigma} f_{\beta}^{\Sigma}, \quad (1.95)$$

we can rewrite equations (1.83) and (1.84) as:

$$\begin{aligned} \delta \chi^i &= i \nabla_{\mu} z^i \gamma^{\mu} \epsilon_{\bullet} + g W^{i11} \epsilon_{\bullet} + \\ &+ \frac{1}{4} \left(\partial_j \mathcal{K} \bar{\chi}^j \epsilon_{\bullet} - h.c. \right) \chi^i - \left(\omega_{1|s} \bar{\zeta}^s \epsilon_{\bullet} + \omega_{1|\bar{s}} \zeta^{\bar{s}} \epsilon_{\bullet} \right) \chi^i - \Gamma_{jk}^i \bar{\chi}^k \epsilon_{\bullet} \chi^j + \\ &- i \chi^i \psi_{\mu \bullet} \gamma^{\mu} \epsilon_{\bullet} + \frac{i}{8} g^{i \bar{j}} \partial_{\bar{j}} \mathcal{N}_{\Lambda \Sigma} \bar{\lambda}^{\Lambda \bullet} \lambda^{\Sigma \bullet} \epsilon_{\bullet}. \end{aligned} \quad (1.96)$$

$$\begin{aligned} \delta \lambda_{\bullet}^{\Lambda} &= F_{\mu\nu}^{-\Lambda} \gamma^{\mu\nu} \epsilon_{\bullet} - 2 g f_{\alpha}^{\Lambda} W^{\alpha 21} \epsilon_{\bullet} + \\ &+ \frac{1}{4} \left(\partial_j \mathcal{K} \bar{\chi}^j \epsilon_{\bullet} - h.c. \right) \lambda_{\bullet}^{\Lambda} - \left(\omega_{2|s} \bar{\zeta}^s \epsilon_{\bullet} + \omega_{2|\bar{s}} \zeta^{\bar{s}} \epsilon_{\bullet} \right) \lambda_{\bullet}^{\Lambda} - \Gamma_{\beta i}^{\alpha} \bar{\chi}^i \epsilon_{\bullet} \lambda_{\bullet}^{\Lambda} + \\ &+ \frac{i}{4} (\text{Im} \mathcal{N})^{-1 \Lambda \Sigma} \left(\partial_{\bar{j}} \mathcal{N}_{\Sigma \Gamma} \bar{\chi}^{\bar{j}} \lambda^{\Gamma \bullet} + \partial_i \bar{\mathcal{N}}_{\Sigma \Gamma} \bar{\chi}^i \lambda_{\bullet}^{\Gamma} \right) \epsilon_{\bullet}. \end{aligned} \quad (1.97)$$

which have the form of the $N = 1$ transformation laws for chiral- and vector-multiplets fermions respectively.

We still have to identify the precise form of the bosonic quantities W^{i1} , $f_\alpha^\Lambda W^{\alpha 21}$ and $\mathcal{N}_{\Lambda\Sigma}$ in the reduced theory. This has been done in [21]. We just quote the main results here. For example, in order to retrieve the D -term of the $N = 1$ gaugino transformation, we have to identify:

$$-2g f_\alpha^\Lambda W^{\alpha 21} \equiv iD^\Lambda = i(\text{Im}\mathcal{N}^{-1})^{\Lambda\Sigma} (P_\Sigma^0 + P_\Sigma^3). \quad (1.98)$$

Moreover, in order to show that equation (1.97) is the correct $N = 1$ transformation law of the gauginos we have still to prove that $\mathcal{N}_{\Lambda\Sigma}$ is an antiholomorphic function of the scalar fields z^i , since the corresponding object of the $N = 2$ special geometry $\mathcal{N}_{\Lambda\Sigma}$ is not. For this purpose we observe that in $N = 2$ special geometry the following identity holds (at least when a $N = 2$ prepotential function exists)[24]:

$$\mathcal{N}_{\Lambda\Sigma} = \bar{F}_{\Lambda\Sigma} - 2i\bar{T}_\Lambda\bar{T}_\Sigma(L^\Gamma \text{Im}F_{\Gamma\Delta}L^\Delta) \quad (1.99)$$

where the matrix $F_{\Lambda\Sigma}$ is holomorphic and T_Λ is the so-called projector on the graviphoton [21], [24].

If we now reduce the indices $\Lambda\Sigma$ we find:

$$\mathcal{N}_{\Lambda\Sigma} = \bar{F}_{\Lambda\Sigma} - 2i\bar{T}_\Lambda\bar{T}_\Sigma(L^Z \text{Im}F_{ZW}L^W) \equiv \bar{F}_{\Lambda\Sigma} \quad (1.100)$$

since, as shown in [21], $T_\Lambda = 0$ is precisely the bosonic constraint derived from (1.42). Therefore $\mathcal{N}_{\Lambda\Sigma}$ is antiholomorphic and the D -term (1.98) becomes:

$$D^\Lambda \equiv 2i f_\alpha^\Lambda W^{\alpha 21} = -2(\text{Im}f^{-1}(z^i))^{\Lambda\Sigma} (P_\Sigma^0 + P_\Sigma^3). \quad (1.101)$$

where we have defined

$$F_{\Lambda\Sigma}(z^i) = \frac{1}{2}f_{\Lambda\Sigma}(z^i) \quad (1.102)$$

in order to match the normalization of the holomorphic kinetic matrix of the $N = 1$ theory appearing in equation (1.34).

The $N = 1$ transformation law of the gravitino with these notations takes the final form:

$$\begin{aligned} \delta\psi_{\bullet\mu} &= \mathcal{D}_\mu\epsilon_\bullet - \frac{i}{2}(\hat{\mathcal{Q}}_\mu - 2i\hat{\omega}_{\mu|1}{}^1)\epsilon_\bullet + igS_{11}\eta_{\mu\nu}\gamma^\nu\epsilon^\bullet + \\ &+ \frac{i}{4}\left[\text{Im}f_{\Lambda\Sigma}\bar{\lambda}^{\Lambda\bullet}\gamma_\mu\lambda_\bullet^\Sigma + \frac{1}{2}\gamma_{\mu\nu}\left(\bar{f}_{\Lambda\Sigma}\bar{\lambda}^{\Lambda\bullet}\gamma^\nu\lambda_\bullet^\Sigma + g_{i\bar{j}}\bar{\chi}^{\bar{j}}\gamma^\nu\chi^i + g_{s\bar{s}}\bar{\zeta}^{\bar{s}}\gamma^\nu\zeta^s\right)\right]\epsilon_\bullet + \\ &- \left(\omega_{1|s}{}^1\bar{\zeta}^s\epsilon_\bullet + \omega_{1|\bar{s}}{}^1\bar{\zeta}^{\bar{s}}\epsilon^\bullet\right)\psi_{\mu\bullet} - \frac{1}{4}\left(\partial_j\mathcal{K}\bar{\chi}^j\epsilon_\bullet - h.c.\right)\psi_{\mu\bullet} \end{aligned} \quad (1.103)$$

2 The gauging

As it was stressed in the introduction, the implications of the $N = 2 \rightarrow N = 1$ reduction on the gauging of the $N = 2$ theory cannot be obtained by looking only at the fermionic sectors, since the fermionic shifts are built up in terms of bosonic fields only. The analysis of the gauging has been thoroughly given in [21]. To make the paper self-contained, we just summarize here the conclusions, and in particular:

- The D -term of the $N = 1$ -reduced gaugino $\lambda^\Lambda = -2f_i^\Lambda \lambda^{i2}$ is:

$$D^\Lambda = W^{i21} = -2g_{(\Lambda)}(\text{Im}f)^{-1\Lambda\Sigma} \left(P_\Sigma^3(w^s) + P_\Sigma^0(z^i) \right) \quad (2.1)$$

- The $N = 1$ -reduced superpotential, that is the gravitino mass, is:

$$L(z, w) = \frac{i}{2}g_{(X)}L^X \left(P_X^1 - iP_X^2 \right) \quad (2.2)$$

and is a holomorphic function of its coordinates z^i and w^s .

- The fermion shifts of the $N = 1$ chiral spinors $\chi^i = \lambda^{i1}$ coming from the $N = 2$ gaugini are:

$$gW^{i11} \equiv N^i = 2g^{i\bar{j}}\nabla_{\bar{j}}\bar{L} \quad (2.3)$$

- The fermion shifts of the $N = 1$ chiral spinors ζ^s coming from $N = 2$ hypermultiplets are:

$$N^s = -4g_{(X)}k_X^t\bar{L}^X\mathcal{U}_t^{1i}\mathcal{U}_{2j}^s = 2g^{s\bar{s}}\nabla_{\bar{s}}\bar{L}. \quad (2.4)$$

- Only some components of the special-Kähler and quaternionic prepotentials and of the corresponding Killing vectors remain different from zero after the reduction, in particular we have:

$$P_X^0 = 0, \quad (2.5)$$

$$k_X^i = 0, \quad k_\Lambda^\alpha = 0 \quad (2.6)$$

$$k_\Lambda^i = ig^{i\bar{j}}\nabla_{\bar{j}}P_\Lambda^0 \neq 0. \quad (2.7)$$

and

$$P_X^3 = 0, \quad P_\Lambda^i = 0, \quad (i = 1, 2) \quad (2.8)$$

$$k_X^s = 0, \quad k_\Lambda^t = 0 \quad (2.9)$$

$$k_\Lambda^s = ig^{s\bar{s}}\nabla_{\bar{s}}P_\Lambda^3 \neq 0. \quad (2.10)$$

We note that, by using the quaternionic relation:

$$n_H P_\Lambda^x = -\frac{1}{2}\Omega_{uv}^x \nabla^u k_\Lambda^v \quad (2.11)$$

from equation (2.8) it follows:

$$n_H P_\Lambda^i = 0 = -\frac{1}{2}\Omega_{st}^i \left(\nabla^s k_\Lambda^t - \nabla^t k_\Lambda^s \right) \quad (2.12)$$

$$n_H P_X^3 = 0 = -\frac{1}{2}\Omega_{s\bar{s}}^3 \nabla^s k_X^{\bar{s}} + \Omega_{t\bar{t}}^3 \nabla^t k_X^{\bar{t}} \quad (2.13)$$

satisfied for:

$$\nabla^s k_\Lambda^t = 0; \quad \nabla^s k_X^{\bar{s}} = 0 \quad (2.14)$$

$$\nabla^t k_\Lambda^s = 0; \quad \nabla^t k_X^{\bar{t}} = 0. \quad (2.15)$$

Equations (2.14) follow from (2.9) for consistency of the reduction to the submanifold \mathcal{M}^{KH} . Equations (2.15) are instead further relations to be satisfied for the truncation. One can see for instance that the above relations do indeed hold in the model of reference [25] where the gauge group acts linearly on the coordinates of the scalar manifolds.

3 A closer look to consistency: Yukawa interactions

It is well known that, in order to have a consistent reduction, the solutions of the equations of motion of the reduced theory must be also solutions of the mother theory. This fact in particular implies that all terms in the lagrangian bilinear in the fermions, containing one retained and one truncated out fermion, must disappear in the reduction. Indeed, the corresponding field equations obtained by varying the lagrangian with respect to the truncated fermion would be inconsistent. Let us check that the bosonic quantities which are coefficients of these terms do indeed vanish in the reduction.

We will confine to analyze the ‘‘mass’’ terms of the $N = 2$ lagrangian, namely:

$$\begin{aligned} \mathcal{L}_{mass}^{N=2} &= g[2S_{AB}\bar{\psi}_\mu^A\gamma^{\mu\nu}\psi_\nu^B + ig_{I\bar{J}}W^{IAB}\bar{\lambda}_A^{\bar{J}}\gamma_\mu\psi_B^\mu + 2iN_\alpha^A\bar{\zeta}^\alpha\gamma_\mu\psi_A^\mu \\ &+ \mathcal{M}^{\alpha\beta}\bar{\zeta}_\alpha\zeta_\beta + \mathcal{M}^\alpha_{IB}\bar{\zeta}_\alpha\lambda^{IB} + \mathcal{M}_{IJAB}\bar{\lambda}^{IA}\lambda^{JB} + \text{h.c.}] \end{aligned} \quad (3.1)$$

where, besides the matrices S_{AB} , W^{IAB} , N_α^A defined in (1.22) - (1.25), there appear the mass matrices (see [22]):

$$\mathcal{M}^{\alpha\beta} = -\mathcal{U}_u^{\alpha A}\mathcal{U}_v^{\beta B}\varepsilon_{AB}\nabla^{[u}k_\Lambda^{v]}L^\Lambda = -\frac{1}{2}\mathcal{U}^{A\alpha|u}\nabla_u N_A^\beta \quad (3.2)$$

$$\mathcal{M}^\alpha_{IB} = -4\mathcal{U}_{Bu}^\alpha k_\Lambda^u f_I^\Lambda \quad (3.3)$$

$$\mathcal{M}_{ABIK} = \varepsilon_{AB}g_{I[\bar{J}K]}f_\Lambda^{\bar{J}}k_\Lambda^{\bar{K}} + \frac{1}{2}iP_{\Lambda AB}\nabla_I f_K^\Lambda \quad (3.4)$$

The gravitino mass term $S_{12}\bar{\psi}_\mu^1\gamma^{\mu\nu}\psi_\nu^2$ is automatically zero because of (1.43).

The term $g_{I\bar{J}}W^{IAB}\bar{\lambda}_A^{\bar{J}}\gamma_\mu\psi_B^\mu$ contains four potentially dangerous contributions, namely:

$$g_{i\bar{j}}W^{i12}\bar{\lambda}_1^{\bar{j}}\gamma_\mu\psi_2^\mu \quad (3.5)$$

$$g_{i\bar{j}}W^{i12}\bar{\lambda}_2^{\bar{j}}\gamma_\mu\psi_1^\mu \quad (3.6)$$

$$g_{\alpha\bar{\beta}}W^{\alpha 11}\bar{\lambda}_1^{\bar{\beta}}\gamma_\mu\psi_1^\mu \quad (3.7)$$

$$g_{\alpha\bar{\beta}}W^{\alpha 22}\bar{\lambda}_2^{\bar{\beta}}\gamma_\mu\psi_2^\mu \quad (3.8)$$

Looking at the expression (1.23) of W^{IAB} we see that W^{i12} is zero on the reduced theory, taking into account the constraints: $P_\Lambda^{12} \equiv P_\Lambda^3$, $f_i^\Lambda = 0$, $L^\Lambda \equiv L^X$, $k_X^i = 0$; furthermore, $W^{\alpha 11}$ and $W^{\alpha 22}$ are both zero due to the constraints: $P_\Lambda^{11} = (P_\Lambda^{22})^* \equiv P_X^1 - iP_X^2$, $f_\alpha^X = 0$.

Then, we need the terms $N_i^1\bar{\zeta}^i\gamma_\mu\psi_1^\mu$ and $N_i^2\bar{\zeta}^i\gamma_\mu\psi_2^\mu$ to be zero. And indeed, $N_i^1 = (N_I^2)^* = 0|_{\mathcal{M}_R}$ for consistency of the truncation of half hypermultiplets (see equation (1.76)).

Furthermore, the term $\mathcal{M}^{IJ}\bar{\zeta}_I\zeta_J$ must be zero, and this is satisfied if:

$$\mathcal{M}^{IJ} = \mathcal{U}_{\bar{s}}^{I1}\mathcal{U}_s^{J2}\nabla^{[\bar{s}}k_X^{s]}L^X - \mathcal{U}_{\bar{t}}^{I2}\mathcal{U}_t^{J1}\nabla^{[\bar{t}}k_X^{t]}L^X = 0. \quad (3.9)$$

Both terms are indeed zero as a consequence of (1.67) and (2.8),(2.9),(2.15).

From the mixing term $\mathcal{M}^\alpha_{IB}\bar{\zeta}_\alpha\lambda^{IB}$ we get the potentially inconsistent contributions:

$$\mathcal{M}^I_{\alpha 1}\bar{\zeta}_I\lambda^{\alpha 1}; \quad (3.10)$$

$$\mathcal{M}^I_{i 2}\bar{\zeta}_I\lambda^{i 2}; \quad (3.11)$$

$$\mathcal{M}^i_{i 1}\bar{\zeta}_i\lambda^{i 1}; \quad (3.12)$$

$$\mathcal{M}^i_{\alpha 2}\bar{\zeta}_i\lambda^{\alpha 2}; \quad (3.13)$$

All these quantities are indeed zero on the reduced theory, as can be ascertained by using in (3.2) the relations (1.53), (1.54), (1.67), (1.68) and (2.9).

Finally, the gaugino mass term $\mathcal{M}_{IJAB}\bar{\lambda}^{\bar{I}A}\lambda^{JB}$ contains the four contributions:

$$\mathcal{M}_{ij12}\bar{\lambda}^{i1}\lambda^{j2} \quad (3.14)$$

$$\mathcal{M}_{\alpha\beta12}\bar{\lambda}^{\alpha1}\lambda^{\beta2} \quad (3.15)$$

$$\mathcal{M}_{i\alpha11}\bar{\lambda}^{i1}\lambda^{\alpha1} \quad (3.16)$$

$$\mathcal{M}_{i\alpha22}\bar{\lambda}^{i2}\lambda^{\alpha2} \quad (3.17)$$

which have to be zero on the truncated theory. We find:

$$\mathcal{M}_{ij12} = g_{\bar{k}[i}f_{j]}^X k_{\bar{X}}^{\bar{k}} - \frac{i}{2}P_{\Lambda12}\nabla_i f_j^\Lambda \quad (3.18)$$

$$\mathcal{M}_{\alpha\beta12} = g_{\bar{\gamma}[\alpha}f_{\beta]}^\Lambda k_{\bar{\Lambda}}^{\bar{\gamma}} - \frac{i}{2}P_{\Lambda12}\nabla_\alpha f_\beta^\Lambda \quad (3.19)$$

$$\mathcal{M}_{i\alpha11} = -\frac{i}{4}P_{X11}\nabla_\alpha f_i^X \quad (3.20)$$

$$\mathcal{M}_{i\alpha22} = -\frac{i}{4}P_{X22}\nabla_\alpha f_i^X \quad (3.21)$$

Equations (3.18) – (3.21) are all satisfied as a consequence of (1.67), (1.68), (1.71), (1.91) and (2.6), (2.8).

Finally, coming to the reduction of the scalar potential of the $N = 2$ theory down to $N = 1$, we have that the $N = 2$ scalar potential, given by:

$$\mathcal{V}^{N=2} = \left(g_{\bar{I}\bar{J}}k_{\bar{\Lambda}}^{\bar{I}}k_{\bar{\Sigma}}^{\bar{J}} + 4h_{uv}k_{\bar{\Lambda}}^uk_{\bar{\Sigma}}^v\right)\bar{L}^\Lambda L^\Sigma + \left(-\frac{1}{2}(\text{Im}\mathcal{N}^{-1})^{\Lambda\Sigma} - \bar{L}^\Lambda L^\Sigma\right)P_\Lambda^x P_\Sigma^x - 3P_\Lambda^x P_\Sigma^x \bar{L}^\Lambda L^\Sigma \quad (3.22)$$

reduces to the standard form for the $N = 1$ scalar potential, written in terms of the covariantly holomorphic superpotential L as:

$$\mathcal{V}^{N=2\rightarrow N=1} = 4 \left[-3L\bar{L} + g^{i\bar{j}}\nabla_i L \nabla_{\bar{j}} \bar{L} + g^{s\bar{s}}\nabla_s L \nabla_{\bar{s}} \bar{L} + \frac{1}{16}\text{Im}f_{\Lambda\Sigma}D^\Lambda D^\Sigma\right] \quad (3.23)$$

The explicit proof is given in [21].

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