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Simulation of the Pion Decay Channel of a Neutrino Factory

E. B. Holzer

Abstract

In the pion decay channel of a neutrino factory the particles are transported in a solenoidal magnetic field, a 1.8 T constant field in the CERN reference scenario. Increasing the field strength decreases the transverse emittance of the decay muons. To define a lower limit on the achievable μ emittance, the reference scenario is compared with a study case where pions decay in a field of 20 T. This simulation shows that in any realistic scenario the μ emittance will only be reduced by less than 21% in each transverse plane as compared to the reference scenario, and at the expense of an increase in longitudinal emittance.

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1 Introduction and the Reference Scenario

In the CERN reference scenario for a neutrino factory [3, 4] intense beams of electron and muon neutrinos are produced by high energy muons decaying in the long straight sections of a muon decay ring. Those muons in turn are decay products of pions which are produced by a 4 MW beam of protons on a liquid mercury target. For pion capture, the target is inserted either in a magnetic horn or a 20 T solenoid. This is followed by the long (30 m) decay channel.

The pion production in the target, which serves as input distribution for this decay channel study, was simulated with the computer code MARS [1] for 10^6 protons at 2.2 GeV hitting the target instantaneously. The target is 26 cm long, has a radius of 0.75 cm and is tilted by 50 mrad with respect to the magnetic axis of a 20 T solenoid. The bore radius of the solenoid is 7.5 cm and hence pions with a transverse momentum, p_T, of less than about 250 MeV/c are captured. In the simulation, 44.5×10^3 π^+ and 1.2×10^3 μ^+ are produced in the forward direction. Of those 15.5×10^3 π^+ and 0.1×10^3 μ^+ fall into the longitudinal momentum range of 180 MeV/c $< p_z <$ 450 MeV/c. The 20 T field is tapered to 1.8 T over a distance of 2.02 meter according to $B_z(z) = \frac{B_0}{1+\alpha \cdot z}$ at $r = 0$, where $\alpha = 5$ m^{-1}. The following 31 m decay channel has a radius of 30 cm and a constant field of 1.8 T. The computer code ICOOL [2] was used to track the pions and muons from the end of the target to the end of the decay channel. Muon decay was switched off in these simulations.

2 Emittance Increase during the Decay Section and the Study Case

When pions decay to muons the transverse and longitudinal emittance increases. The amount of transverse emittance increase depends on the focusing of the channel. The effect of the additional divergence due to decay on the transverse emittance is smaller for bigger initial divergence of the pions and therefore higher B_z. The aim of this simulation is to define the achievable minimum transverse muon emittance. There are a number of other influences on the emittance. During a drift (or propagation in a constant longitudinal B-field) the longitudinal rms emittance increases due to weak relativistic effects. The longitudinal emittance of a highly divergent relativistic beam increases during drift (or propagation) because of the dependence of γ, the relativistic gamma factor, on the transverse velocity, v_T. The tapered solenoid transforms transverse momentum into longitudinal momentum, ideally without increasing the 6D emittance.

To quantify the correlation between the muon emittance and the B-field in the decay channel, an extreme scenario is chosen for comparison with the reference scenario. The pions decay in a long 20 T solenoid field, and tapering of the field from 20 to 1.8 T takes place 30 m after the target, rather than in the first 2 m. All realistic decay channel designs will have a lower magnetic field, hence this channel defines a lower limit for the achievable transverse muon emittance.

3 Results and Discussion

The transverse and longitudinal phase space of muons at the end of the decay channel is shown in figure 1. On the left is the reference scenario and the 20 T decay channel is on the right. Transversally an acceptance of 2.27 π cm rad (unnormalized) is indicated. It corresponds to 6 π cm rad normalized which is 4 times the acceptance of the recirculating accelerator downstream. (The CERN reference scenario foresees a reduction of ϵ_T by a factor of 4 in both planes, [5, 6].) Longitudinally, muons with a p_z in the range of 175 to 390 MeV/c are shown. Indicated is a cut on a time interval of 11.4 ns, half an RF period of 44 MHz (corresponding to the RF of the phase rotation and first muon cooling section [5, 6]).

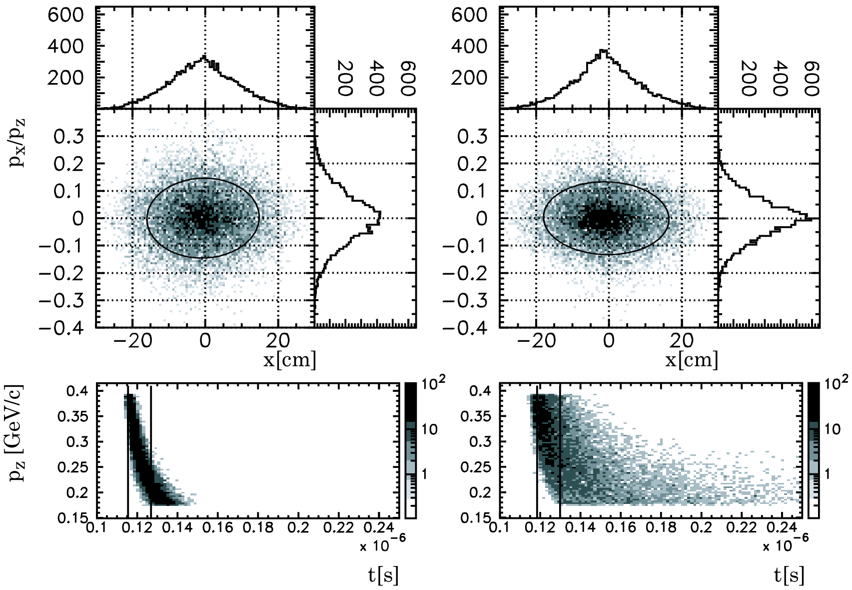

Figure 1: Transverse and longitudinal phase space of muons after pion decay in a field of 1.8 T (left) and 20 T (right).

Table 1 summarizes the normalized rms emittances of pions with 180 MeV/c $< p_z <$ 450 MeV/c, 10 cm after the target and of the decay muons from these pions 33 m after the target. The transverse rms emittance increase due to decay is 40.5$\%$ in the 1.8 T case and 11.1$\%$ in the 20 T case in both planes. The longitudinal rms emittance is 4.8 times bigger in the 20 T case than in the reference scenario.

The number of muons 33 m after the target within the transverse acceptance of 6 π cm rad, in the range of 175 MeV/c $< p_z <$ 390 MeV/c and a time interval of 11.4 ns is 4.7×10^3 for a decay channel at 1.8 T and 3.9×10^3 for a decay channel at 20 T.

ϵ [π cm rad]	ϵ_x	ϵ_y	ϵ_L	$\epsilon_x \cdot \epsilon_y \cdot \epsilon_L$
π (10 cm after target)	1.74	1.69	1.3	3.8
μ (decayed at 1.8 T)	2.38	2.44	49.4	287
μ (decayed at 20 T)	1.91	1.90	237	860

Table 1: Summary of normalized rms emittances. Pions are at 10 cm after the target and muons at the end of the decay channel (33 m after the target). Both channels have the same transmission.

The difference in longitudinal phase space in the two scenarios mainly originates from the travel time difference, Δt, over the same distance between two pions with the same p_z and a p_T of zero and maximum transverse momentum, $p_{T,max}$, respectively:

$$\Delta t = t|_{p_T=0} \cdot \left(\sqrt{1 + \frac{p_{T,max}^2}{E^2|_{p_T=0}}} - 1 \right) , \quad (1)$$

Where $t|_{p_T=0}$ and $E|_{p_T=0}$ are the flight time and energy of a pion with $p_T = 0$, and $p_{T,max}$ is a function of the B-field. In the approximation of adiabatic B-field changes, the invariance of the action integral leads to B/p_T^2 = const. The geometry at the target gives $p_{T,max}[MeV/c] \simeq$

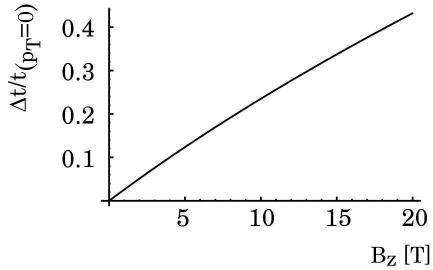

Figure 2: Relative travel time difference over a distance of 30 m between pions of the same p_z as a function of B.

$55.9 \cdot \sqrt{B_z[T]}$. Figure 2 shows $\Delta t/t|_{p_T=0}$ as a function of B_z for pions with a p_z of 200 MeV/c in this approximation. The function is nearly linear in the range of 0 T $< B_z <$ 20 T and reaches 43$\%$ at 20 T.

4 Conclusions

Rms emittances and the number of muons in a given acceptance (longitudinal acceptance of the first cooling section and 4 times the transverse acceptance of the recirculating linac) have been compared for the reference cooling channel and an extreme scenario. Higher B-fields decrease the transverse and increase the longitudinal emittance. In any realistic scenario (with a B-field less than 20 T) the transverse emittance will be decreased by less than 21$\%$ in each plane as compared to the reference scenario. The final choice of B_z will depend on the performance and acceptance of the following elements for cooling and acceleration. The number of muons in a given acceptance is higher for the lower value of B_z, but further simulations are necessary to determine the position of the maximum.

References

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