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NOVEL METHOD FOR MULTI-TURN EXTRACTION: TRAPPING CHARGED PARTICLES IN ISLANDS OF PHASE SPACE

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Abstract

A novel method for multi-turn extraction from a circular particle accelerator is presented. It is based on trapping particles into islands of phase space generated by nonlinear resonances. By appropriate use of nonlinear elements (sextupoles, octupoles), stable islands can be created at small amplitude in phase space. By varying the linear tune, it is shown how particles can be trapped inside these islands. The particles can then be coherently transported towards higher amplitudes for extraction. Results of numerical simulations are presented and discussed in detail.

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Charged particles can be extracted from a circular machine by two different methods: i) fast extraction, ii) slow extraction. In the first case, the whole beam is ejected from the machine in one turn by means of fast dipole (*kicker*) and septum magnets. This technique can be used for transferring the beam either to a subsequent machine or towards a target for physics experiments. The latter method, is based on the effect of a third-order resonance [1, 2]: unstable motion generated by the separatrix joining the three unstable fixed points increases the particles' amplitude. This technique allows beam extraction over many machine turns (typically many thousands), and it is only used when delivering beam to a target for physics experiments.

In some special cases, an intermediate extraction mode called multi-turn extraction is needed. This is the case of the transfer between the CERN Proton Synchrotron (PS) and the Super Proton Synchrotron (SPS). The two machines have different circumferences, satisfying the relation $C_{SPS} = 11 C_{PS}$. Hence, to fill the SPS completely one would require ten fast-extracted pulses from the PS (the empty gap in the SPS is needed for the rise-time of the injection kicker). If the filling time has to be minimised, then the solution consists of extracting the beam over a few turns, instead of a single one. In practise, a five-turn extraction was proposed, allowing the SPS to be filled with only two PS pulses. Such an approach is called Continuous Transfer (CT) [3]. In Fig. 1 the layout of the extraction elements is shown together with the horizontal normalised phase space. Just before extraction, the horizontal tune Q_H is set to the value

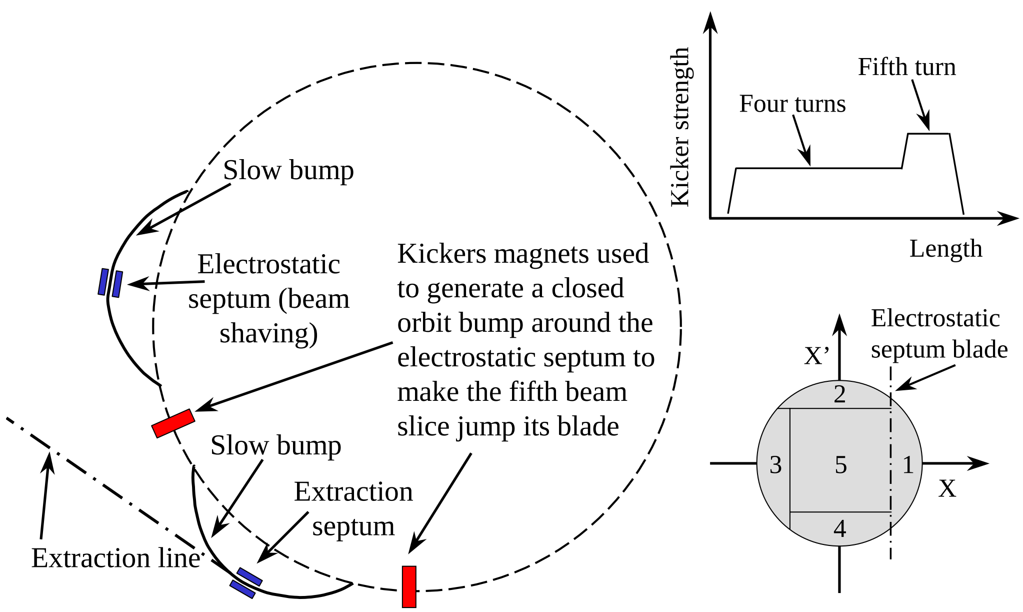

Figure 1: Principle of the CT extraction from the PS machine. The schematics of the extraction elements is shown on the left, while in the upper part the kicker strength as a function of time is shown. In the right part the normalised phase space is depicted.

6.25 and the closed orbit is modified so that the blade of an electrostatic septum intercepts the beam. Because of the value of the horizontal tune, four slices are shaved off the main core and extracted as a continuous ribbon over four turns. The central part is extracted last, during the fifth turn, by changing the beam trajectory so as to jump over the septum blade. In addition to the septum used to slice the beam, a kicker and a magnetic septum are used for the extraction. Another interesting property of such an approach is that the transverse emittance of the extracted beam is decreased by a factor five with respect to that of the circulating one. However, a number of drawbacks are present, namely: i) beam losses, especially at the electrostatic septum, are an intrinsic and unavoidable characteristic of this extraction process. They amount to about $10 - 15\ \%$ of the total beam intensity; ii) the extracted slices do not match the natural foliation of phase space into circles. This generates a betatronic mismatch, i.e. the equivalent horizontal optical parameters for each slice differ with respect to the nominal ones. It can be shown that such a difference can be as large as a factor of two. This, in turn, generates emittance blow-up in the receiving machine; iii) the extracted slices have different transverse emittance. Such a difference can be as large as a factor of two.

These points mean that the CT extraction is not very suitable for the planned CERN Neutrino to Gran Sasso (CNGS) beam [4]. This is a high-intensity proton beam: in its nominal version, the PS should deliver two pulses of more than 3×10^{13} protons each to the SPS. To increase the rate of good events for physics, thus reducing the data-taking time, efforts are being made to study a possible intensity upgrade by a factor of about two. In the new scenario, the beam losses related to the present scheme of the five-turn CT would not be acceptable, and the properties of the extracted beam (matching of phase space structure and transverse emittance) would not allow an efficient injection into the SPS.

A novel approach has thus been proposed, based on the use of stable nonlinear resonances. In this scheme, nonlinear elements such as sextupoles and octupoles are used to generate stable islands in transverse phase space. Then, by varying the horizontal tune, particles can be selectively trapped in the islands by adiabatic capture: some will remain in the phase space area around the origin, while others will migrate to the stable islands. As a result, the beam is split into a number of parts in transverse phase space, determined by the order of the resonance used, without any *mechanical* action. Finally, it is possible to move the particles trapped inside the islands towards higher amplitudes. This increases the separation between the different slices so that enough room is available for the beam to jump over a septum blade with almost no particles lost.

The principle of this novel approach has been tested using a simple model representing the horizontal betatron motion in a circular machine under the influence of sextupole and octupole magnets. The motion in the vertical plane can be safely neglected for our purposes. The nonlinear magnets are assumed to be at the same location in the ring and are represented in the single-kick approximation (see Ref. [5] for more details). Under these assumptions, the one-turn transfer map can be expressed as $\mathbf{x}_{n+1} = \mathbf{M}_n(\mathbf{x}_n)$, or, more explicitly

$$\begin{pmatrix} x_{n+1} \\ {x'}_{n+1} \end{pmatrix} = R(2\,\pi\,\nu_n) \begin{pmatrix} x_n \\ {x'}_n + {x^2}_n + \kappa\, {x^3}_n \end{pmatrix}, \tag{1}$$

where (x, x') are obtained from the Courant-Snyder [6] co-ordinates (X, X') by means of the non-symplectic transformations

$$x = \frac{K_2\ {\beta_H}^{3/2}}{2}\ X \qquad x' = \frac{K_2\ {\beta_H}^{3/2}}{2}\ X', \qquad \text{with} \qquad K_l = \frac{L}{B_0\,\rho}\,\frac{\partial^l B_y}{\partial x^l}, \tag{2}$$

K_2 (K_3) being the integrated sextupole (octupole) gradient, L the length of the nonlinear element, B_y the vertical component of the magnetic field and $B_0\,\rho$ the magnetic rigidity of the charged particle. Here, β_H is the value of the horizontal beta-function at the sextupole location. $R(2\,\pi\,\nu_n)$ is a 2×2 rotation matrix of angle ν_n (the fractional part of Q_H)

$$R(2\,\pi\,\nu_n) = \begin{pmatrix} \cos 2\,\pi\,\nu_n & \sin 2\,\pi\,\nu_n \\ -\sin 2\,\pi\,\nu_n & \cos 2\,\pi\,\nu_n \end{pmatrix} \tag{3}$$

and κ is expressed as

$$\kappa = \frac{2}{3} \frac{K_3}{\beta_H \ {K_2}^2}. \tag{4}$$

The map (1) is actually a time-dependent system through the linear tune. The importance of introducing a time-dependence is twofold. Firstly, it allows varying the phase space topology, thus creating and moving the islands. Secondly, it allows trapping particles inside the islands, which is the necessary condition for the proposed scheme to work efficiently. In fact, a classical time-independent 2D system always has an invariant of motion, namely a scalar quantity satisfying $\mathcal{I}(\mathbf{x}_n) = \mathcal{I}(\mathbf{x}_0)$. This is certainly the case for the linearised version of the map (1) ($\mathcal{I}(\mathbf{x}) = x^2 + x'^2$), and even in the nonlinear case, an approximate invariant can be constructed, e.g. by using the *normal form* approach [5]. This implies that a particle outside an island would never be able to jump across the separatrix, as this would mean changing the value of the invariant $\mathcal{I}$ for that particle. A slow variation of the linear tune, adiabatic with respect to the time scale introduced by the betatron oscillations, allows particles to cross the separatix and be trapped inside the newly-created islands.

In the analysis reported here, the function representing the tune variation is shown in Fig. 2. In the first part, the linear tune is decreased linearly from its initial value of 0.252 to

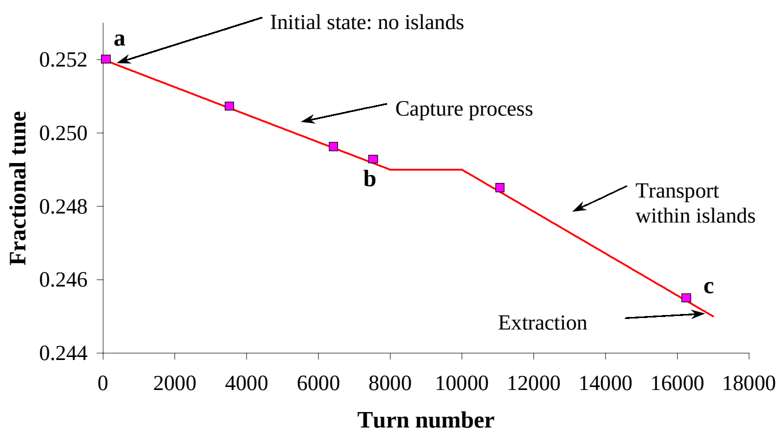

Figure 2: Linear tune ν as a function of turn number n used for the model (1). The points on the curve labelled with **a**, **b**, and **c** correspond to the values of ν_n used to generate the phase space portraits shown in Figs 3, and 4 (left) and (right) respectively.

0.249. During this part, the capture process takes place. Then a zero-slope part follows, used to allow the beam to filament after capture, to match better the phase space topology. Finally, a second linear decrease to the value 0.245 is performed which allows the islands to be moved towards higher amplitudes before extraction.

The different stages of this novel extraction are shown in Figs. 3, 4 (left) and (right), evaluated by the computer code *GIOTTO* [7]. Each orbit consists of a set of points generated by using the map (1) where, for this special purpose, the linear tune has been kept constant, namely $\mathbf{x}_j = \mathrm{M}^j(\mathbf{x}_0), \ 1 \leq j \leq 2 \times 10^3$, and different orbits correspond to different initial conditions $\mathbf{x}_0$. The coefficient κ has the same value for all the cases, namely $\kappa = -2$.

In Fig. 3, the phase portrait for $\nu = 0.252$ is shown. The phase space is foliated in closed curves and no chain of islands is visible, apart from an eleventh-order one near the dynamic aperture. This configuration corresponds to the initial state where the beam is located around

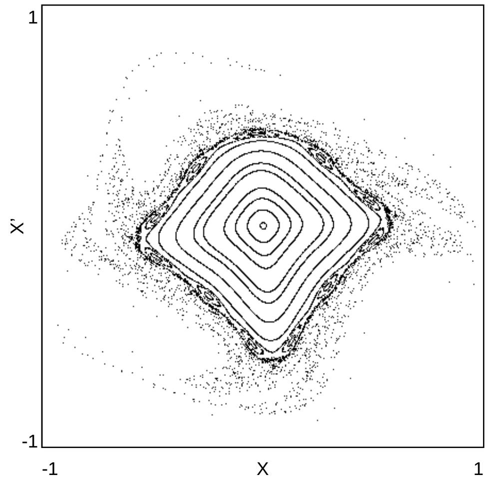

Figure 3: Phase space portrait of the map (1) for $\nu = 0.252$.

the origin and the dynamics is quasi-linear.

In Fig. 4 (left) the topology of the phase space is shown for $\nu = 0.2492$. A chain of four islands, corresponding to a stable fourth-order resonance, is clearly visible near the origin. An important feature is that at higher amplitude, the phase space is again foliated into closed curves until the last regular curve that corresponds to the border of the dynamic aperture. Under these conditions, it is possible to split the beam into five slices: one around the origin, and four inside the stable islands. The closed curve outside the chain acts as a barrier, preventing particles from moving towards higher amplitudes.

Finally, Fig. 4 (right) represents the phase space topology for $\nu = 0.2453$. The four islands are still present, but their amplitude is increased. This means that particles trapped inside islands can be transported towards the outside of phase space.

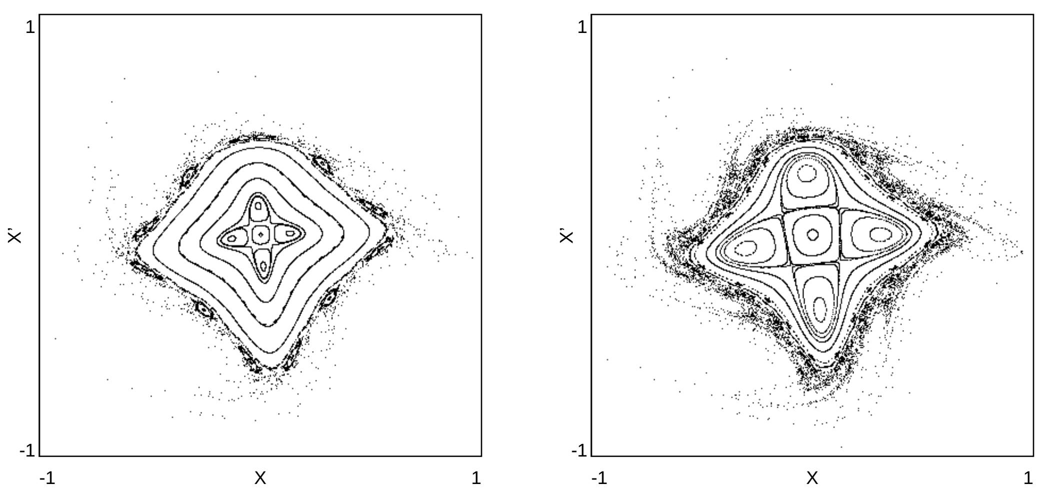

Figure 4: Phase space portrait of the map (1) for $\nu = 0.2492$ (left) and for $\nu = 0.2453$ (right).

The whole process has been simulated numerically by using the model (1) with $\kappa = -2$, while the tune ν_n is varied according to the curve shown in Fig. 2. A set of Gaussian-distributed initial conditions has been generated, and its evolution under the dynamics induced by the map (1) is shown in Fig. 5. The Coulomb interaction between the charged particles is not taken into account in these simulations. The trapping process is clearly visible in the picture: it generates five beam slices, well-separated at the end of the process. No particle is lost during the trapping phase, nor when the islands are moved. The five slices have rather similar surfaces, but also their shape matches the phase space topology very well, making the five parts similar as far as transverse properties are concerned. It is worthwhile pointing out that the first four extracted beam slices have exactly the same emittance, as their shape is dictated by the same phase space structure, i.e. the island along the positive x axis. In this respect, the novel approach proves to be superior to the present CT extraction mode.

The distribution functions $\rho(x)$ for the different beam slices at the end of the trapping and transport process are shown in Fig. 6. They are obtained from the results of the numerical simulations by summing all the contributions from the angular variable x'. They are all Gaussian-like, as is the initial beam distribution, thus showing that the shape is almost preserved throughout the whole process. Additional numerical simulations have been carried out to test the sensitivity of the proposed scheme to the functional dependence of the tune ν on n and on the strength of the nonlinearity κ. Power-laws $\nu_n = \nu_a + (\nu_b - \nu_a)(\frac{n-n_0}{n_1})^m, n_0 \leq n \leq n_0 + n_1$ have been simulated with $1 \leq m \leq 3$: no major effects in the properties of the captured beam were observed, provided the tune variation is slow when the islands approach the centre of phase space. As far as the dependence on κ is concerned, numerical simulations confirm that the results do not change when κ is varied by ± 20 %.

The results of the numerical simulations presented in this paper, although rather encouraging, are only the first step towards a well-established technique for multi-turn extraction based on island trapping. Some points need to be investigated in more detail. Firstly, the question concerning space-charge effects. It has to be stressed that the method presented in this paper is completely general. However, for high-intensity beams (as is the case of the application discussed in the introduction), one could question whether the proposed approach would work. The space-charge forces generate a tune shift (coherent and incoherent) according to the well-known formulae (see Refs. [8, 9] for more details):

$$\begin{aligned}
\Delta\nu_{inc} &= -\frac{2\, r_p}{e\, c}\frac{R\, I_0}{\beta^3\gamma^3} < \beta_H > \left[\frac{\epsilon_{sc}}{b^2}\frac{1}{B_f} + \frac{\epsilon_1}{h^2}\left(\beta^2\gamma^2 + \frac{1}{B_f}\right)\right] \\
\Delta\nu_{coh} &= -\frac{2\, r_p}{e\, c}\frac{R\, I_0}{\beta^3\gamma^3} < \beta_H > \left[\frac{\beta^2\gamma^2}{h^2}\epsilon_1 + \frac{\xi_1}{h^2}\frac{1}{B_f}\right],
\end{aligned} \tag{5}$$

where r_p, e, c, β, γ are the classical proton radius, electron charge, speed of light, and relativistic factors respectively. Here, $R, < \beta_H >$, I_0, and B_f represent the machine radius, the average beta-function, the average beam current, and the bunching factor (ratio of average to peak current). Finally, $\epsilon_{sc}, \epsilon_1, \xi_1$ are the Laslett coefficients: they depend on the beam size and vacuum chamber size. The coherent tune shift can be neglected, as it represents a tune offset of the beam as a whole. By assuming Gaussian beams, total intensity of 3×10^{13}, and typical PS machine parameters, $\Delta\nu_{inc}$ is either -2.8×10^{-3} or -2.4×10^{-4} depending on the momentum of the extracted beam, i.e. 14 or 26 GeV/*c* respectively. For 26 GeV/*c*, the space-charge tune shift is negligible on the scale of tune variation needed for the adiabatic capture. For 14 GeV/*c*, the space-charge tune shift, which is about $1/3$ of the total tune variation needed to capture the beam, can no longer be neglected. However, it has to be stressed that $\Delta\nu_{inc}$ represents the maximum value for the tune shift. Such a maximum is achieved near the beam centre, while particles at higher amplitude experience a smaller tune shift. The adiabatic trapping is actually performed by capturing particles at higher amplitudes: those near the beam centre are left

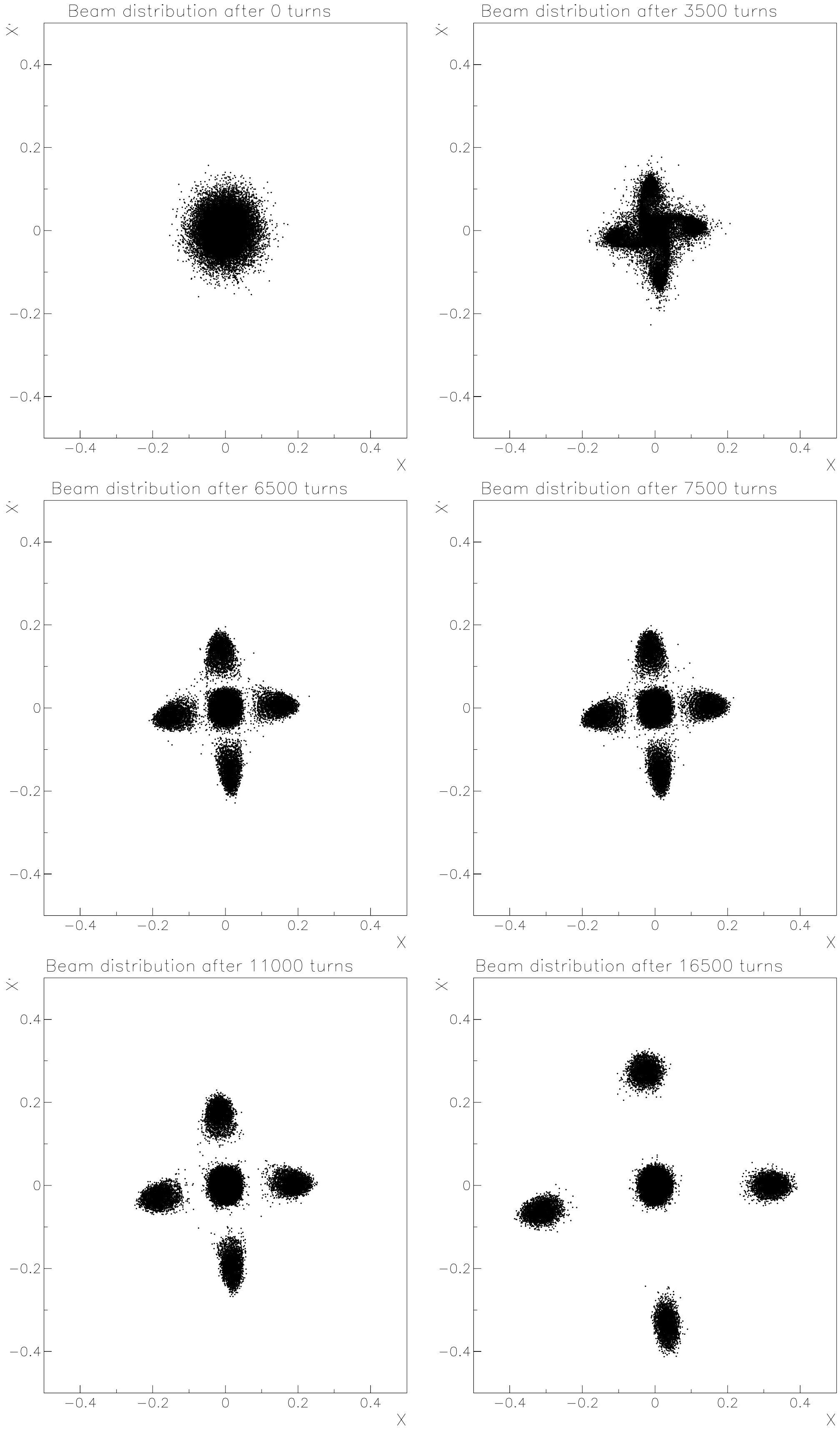

Figure 5: Evolution of the beam distribution during the trapping process. The different plots correspond to tune values represented by solid squares in Fig. 2. Each plot represents 2.25×10^4 points. The initial Gaussian distribution is centred on zero and has $\sigma = 0.04$.

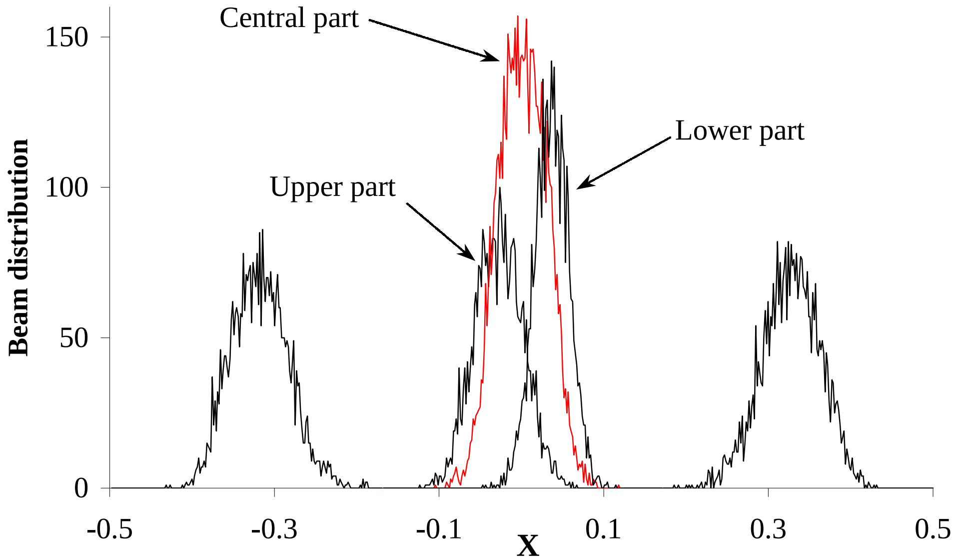

Figure 6: Beam distribution function $\rho(x)$ at the end of the capture and transport process, for all five beam slices. It is obtained by summing all the contributions from the angular variable x'.

unperturbed. Hence, it seems reasonable to assume that the process should not be very much affected, even in this case.

Other points also need clarifying. The quantitative relationship between the transverse emittance of the different slices and the island parameters should be found. Also, the link between the way ν_n is varied and the trapping efficiency should be quantified, to optimise the adiabatic capture. The analysis of these two points would allow a good control of the transverse properties of the extracted beam to be achieved. Finally, the stability of this method by introducing full 4D betatron motion as well as chromaticity effects should be checked in detail.

Experiments using the PS proton beam are already planned to verify the practical feasibility of this novel extraction method.

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