GENUINE CORRELATIONS IN HADRONIC Z^0 DECAYS

E.A. DE WOLF
FOR THE OPAL COLLABORATION
CERN, European Organisation for Nuclear Research, CH-1211 Geneva 23, Switzerland
Physics Department, University of Antwerpen, B-2610 Antwerpen, Belgium
E-mail: Eddi.DeWolf@ua.ac.be

Correlations among hadrons with the same electric charge produced in Z^0 decays are studied using the high statistics data collected from 1991 through 1995 with the OPAL detector at LEP. Normalized factorial cumulants up to fourth order are used to measure genuine particle correlations as a function of the size of phase space domains in rapidity, azimuthal angle and transverse momentum. Some of the recently proposed algorithms to simulate Bose-Einstein effects, implemented in the Monte Carlo model Pythia, reproduce reasonably well the measured second- and higher-order correlations between particles with the same charge as well as those in all-charge particle multiplets.

1 Introduction

Correlations in momentum space between hadrons produced in high energy interactions have been extensively studied over many decades in different contexts.[1] Being a measure of event-to-event fluctuations of the number of hadrons in a phase space domain of size Δ, correlations provide detailed information on the hadronisation dynamics, complementary to that derived from inclusive single-particle distributions and global event-shape characteristics. The suggestion in[2] that multiparticle dynamics might possess (multi-)fractal properties or be "intermittent", emphasized the importance of studying correlations as a function of the size of domains in momentum space. A key ingredient for such studies is the normalized factorial moment and factorial cumulant technique. Unlike factorial moments, cumulants of order q are a direct measure of the stochastic interdependence among groups of exactly q particles emitted in the same phase space cell.[3−5] Therefore, they are well suited for the study of true or "genuine" correlations between hadrons and are particularly sensitive to Bose-Einstein correlations.

Two-particle Bose-Einstein correlations (BEC) have been observed in a wide range of multihadronic processes.[6] Such correlations were extensively studied at LEP.[7−9] Evidence for BEC among groups of more than two identical particles has also been reported.[10,11] The subject has acquired particular importance in connection with high-precision measurements of the W-boson mass at LEP-II.[12,13] For these, better knowledge of correlations in general is needed, as well as realistic Monte Carlo modelling of BEC.

The high statistics OPAL data collected at and near the Z^0 centre-of-mass energy have been used to measure cumulants for multiplets of particles with the same charge, hereafter referred to as "like-sign cumulants". They are compared to "all-charge" cumulants, corresponding to multiplets comprising particles of any (positive or negative) charge. The role of Bose-Einstein-type effects is studied, using recently proposed BEC algorithms[14] implemented in the Monte Carlo event generator Pythia for e^+e^- annihilation.[15] Proceeding beyond the usual analyses of two-particle correlations, we show that, at least within the framework of this model, a good description can be achieved of the factorial cumulants up to fourth order in one-, two- and three-dimensional phase space domains.

2 The method

To measure genuine multiparticle correlations in multi-dimensional phase space cells, we use the technique of normalized factorial cumulant moments, K_q, ("cumulants" for brevity) as proposed in.[5] The cumulants are computed as in a previous OPAL analysis.[16] A D-dimensional region of phase space is partitioned into M^D cells of equal size Δ. From the number of particles counted in each cell, n_m ($m = 1, \ldots, M^D$), event-averaged unnormalized factorial moments, $\langle n_m^{[q]} \rangle$, and unnormalized cumulants, $k_q^{(m)}$, are derived, using the relations given *e.g.* in[3]. For $q = 2, 3, 4$, one has

$$k_2^{(m)} = \langle n_m^{[2]} \rangle - \langle n_m \rangle^2, \tag{1}$$

$$k_3^{(m)} = \langle n_m^{[3]} \rangle - 3\,\langle n_m^{[2]} \rangle \langle n_m \rangle + 2\,\langle n_m \rangle^3 \tag{2}$$

$$k_4^{(m)} = \langle n_m^{[4]} \rangle - 4\,\langle n_m^{[3]} \rangle\,\langle n_m \rangle - 3\,\langle n_m^{[2]} \rangle^2 + 12\,\langle n_m^{[2]} \rangle\,\langle n_m \rangle^2 - 6\,\langle n_m \rangle^4. \tag{3}$$

Here, $\langle n^{[q]} \rangle = \langle n(n-1)\dots(n-q+1)\rangle$ and the brackets $\langle\cdot\rangle$ indicate that the average over all events is taken.

Normalized cumulants are calculated using the expression

$$K_q = (\mathcal{N})^q \bar{k}_q^{(m)} / \overline{N_m^{[q]}}. \tag{4}$$

As proposed in[17], this form is used to correct for statistical bias and non-uniformity of the single-particle spectra. Here, N_m is the number of particles in the mth cell summed over all $\mathcal{N}$ events in the sample, $N_m = \sum_{j=1}^{\mathcal{N}} (n_m)_j$. The horizontal bar indicates averaging over the M^D cells in each event, $(1/M^D)\sum_{m=1}^{M^D}$.

Here, data are presented for "all-charge" and for "like-sign" multiplets. For the former, the cell-counts n_m are determined using all charged particles in an event, irrespective of their charge. For the latter, the number of positive particles and the number of negative particles in a cell are counted separately. The corresponding cumulants are then averaged to obtain those for like-sign multiplets.

3 Experimental details

The analysis uses a sample of approximately 4.1×10^6 hadronic Z^0 decays collected from 1991 through 1995. The OPAL detector has been described in detail in.[18] The results presented are mainly based on the information from the central tracking chambers. The event selection criteria are based on the multihadronic selection algorithms described in.[16] Multihadron events were selected with at least 5 good tracks, a momentum imbalance (the magnitude of the vector sum of the momenta of all charged particles) of less than 0.4 $\sqrt{s}$ and the sum of the energies of all tracks (assumed to be pions) greater than 0.2 $\sqrt{s}$. In addition, the polar angle of the event sphericity axis, calculated using tracks and clusters had to satisfy $|\cos\theta_{\text{sph}}| < 0.7$ in order to accept only events well contained in the detector. A total of about 2.3×10^6 events were finally selected for further analysis.

The cumulant analysis is performed in the kinematic variables rapidity, y, azimuthal angle, Φ, and the transverse momentum variable, $\ln p_T$, all calculated with respect to the sphericity axis.

- Rapidity is defined as $y = 0.5\ln[(E+p_\parallel)/(E-p_\parallel)]$, with E and $p_\parallel$ the energy (assuming the pion mass) and longitudinal momentum of the particle, respectively. Only particles within the central rapidity region $-2.0 \leqslant y \leqslant 2.0$ were retained.
- In transverse momentum subspace, the logarithm of p_T is used to eliminate as much as possible the strong dependence of the cumulants on cell-size arising from the nearly exponential shape of the p_T^2-distribution. Only particles within the range $-2.4 \leqslant \ln(p_T) \leqslant 0.7$ (p_T in GeV/c) were used.
- The azimuthal angle Φ ($0 \leqslant \Phi < 2\pi$), is calculated with respect to the eigenvector of the momentum tensor having the smallest eigenvalue in the plane perpendicular to the sphericity axis.

4 Results

The fully corrected normalized cumulants K_q ($q = 2, 3, 4$) for all-charge and like-sign particle multiplets, calculated in two-dimensional $y\times\Phi$ (2D) and three-dimensional $y\times\Phi\times\ln p_T$ (3D) phase space cells, are displayed in Fig. 1. It is seen that positive genuine correlations among groups of two, three and four particles are present: $K_q > 0$. Cumulants in 2D and 3D continue to increase towards small phase space cells. Moreover, the 2D and 3D cumulants are of similar magnitude at fixed M, indicating that the contribution from correlations in transverse momentum is small. The like-sign cumulants increase faster and approach the all-charge ones at large M. As the cell-size becomes smaller, the rise of all-charge correlations is increasingly driven by that of like-sign multiplets.

The cumulant data have been compared with predictions of the Pythia Monte Carlo event generator (version 6.158) without and with Bose-Einstein effects. The model parameters, not related to BEC, were set at values obtained from a previous tune to OPAL data on event-shape and single-particle inclusive distributions.[19] In this tuning, BE-effects were not included.

To assess the importance of BE-type short-range correlations between identical particles, and their influence on all-charge cumulants, we concentrate on the algorithm BE_{32}, described in[14], using parameter values `PARJ(93)` $= 0.26$ GeV ($R = 0.76$ fm) and `PARJ(92)` $\equiv \lambda = 1.5$. Non-BEC related model-parameters were set at the following values: `PARJ(21)`=0.4 GeV, `PARJ(42)`=0.52 GeV^{-2}, `PARJ(81)`=0.25 GeV, `PARJ(82)`=1.9 GeV. We find that calculations with `PARJ(93)` in the range $0.2-0.3$ GeV, and the corresponding λ in the range $1.7-1.3$, still provide an acceptable description of the second-order like-sign cumulants.

The dashed lines in Fig. 1 are Pythia predictions for *like-sign* multiplets for the model without BEC. Model and data agree for small M (large phase space domains), indicating that the multiplicity distribution in those regions is well modelled. However, for larger M, the predicted cumulants are too small.

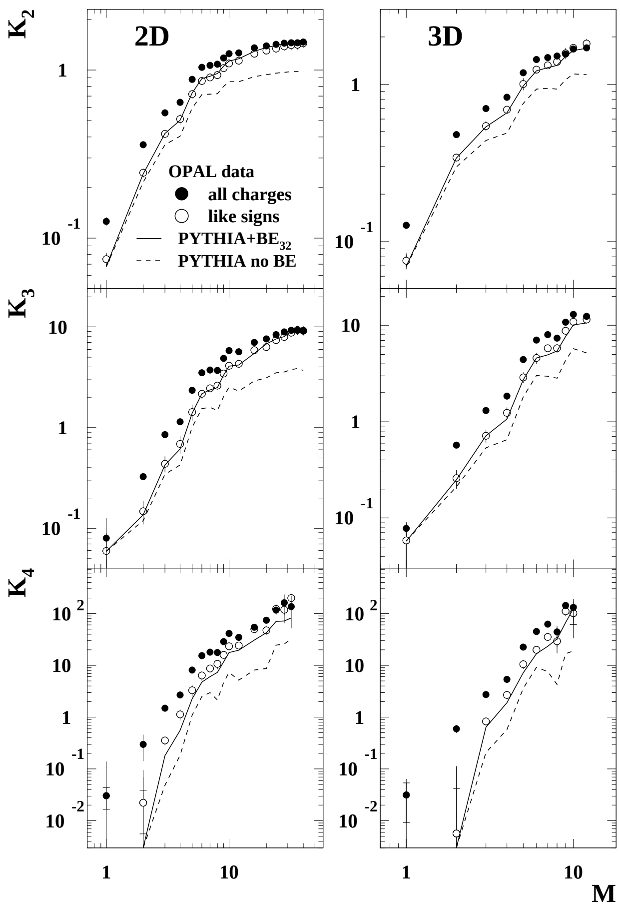

Figure 1. The cumulants K_q in two-dimensional $\Delta y \times \Delta\Phi$ (2D) and three-dimensional $\Delta y \times \Delta\Phi \times \Delta \ln p_T$ (3D) domains for all charged hadrons (solid symbols) and for multiplets of like-sign particles (open symbols), versus M. Where two error-bars are shown, inner ones are statistical, and outer ones are statistical and systematic errors added in quadrature. The lines connect Monte Carlo predictions from Pythia without BEC (dashed) and with BEC (full) simulated with algorithm BE_{32}[14] (see text).

The solid curves in Figs. 1 show predictions for *like-sign* multiplets using the BE_{32} algorithm. Inclusion of BEC leads to a very significant improvement of the data description. Also two-particle and higher order correlations in 1D rapidity space are well accounted for (not shown). The predicted 2D and 3D cumulants agree well with the data.

The 1D, 2D and 3D cumulants for particle pairs with the same charge are displayed in Fig. 2. Since BEC occur only when two identical mesons are close-by in all three phase space dimensions, projection onto lower-dimensional subspaces, such as rapidity and azimuthal angle, leads to considerable weakening of the effect. Nevertheless, the high precision of the data in Fig. 2 allows to demonstrate clear sensitivity to the presence or absence of BEC in the model.

Whereas the BE-algorithm used implements pair-wise BEC only, it is noteworthy (see Fig. 1) that the procedure also induces like-sign higher-order correlations of approximately correct magnitude. This seems to indicate that high-order cumulants are, to a large extent, determined by the second-order one.

To assess the sensitivity of the cumulants to variations in the BEC algorithms available in Pythia, we have further considered the algorithms BE_λ and BE_0.[14] Using the same parameter values as for BE_{32}, we observe that BE_λ slightly underestimates $K_2(y)$ and overestimates $K_2(\Phi)$ for like-sign pairs (Fig. 2), whereas the results coincide with those from BE_{32} in 2D and 3D. For all-charge multiplets (Fig. 3), the predicted cumulants generally fall below those for BE_{32}, except for K_3 and K_4 in 2D and 3D, where the differences are small. The differences with respect to BE_{32} are related to the different pair-correlation functions used in the algorithms. Although a different choice of the parameters R and λ may improve the agreement with the data, we have not attempted such fine-tuning.

We also considered the predictions based on the algorithm BE_0[14] (dash-dotted curves in the figures) for the same parameter values as quoted above. For like-sign pairs (Fig. 2), $K_2(y)$ and especially $K_2(\Phi)$ are overestimated. In contrast, all-charge higher-order cumulants differ little from those obtained with BE_{32}.

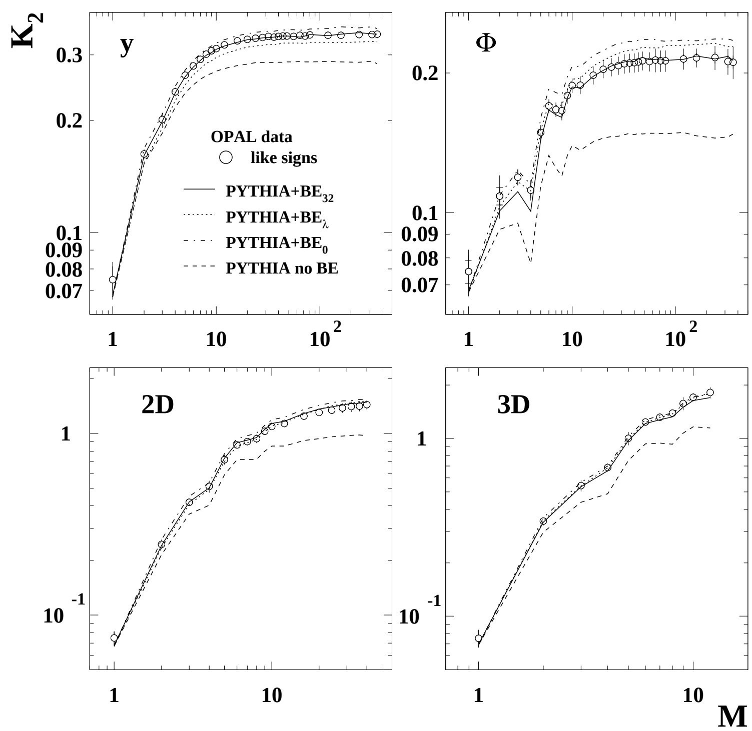

Figure 2. *The cumulants K_2 for like-sign pairs in one-dimensional domains of rapidity (y) and azimuthal angle (Φ), and in two-dimensional $\Delta y \times \Delta\Phi$ (2D) and three-dimensional $\Delta y \times \Delta\Phi \times \Delta \ln p_T$ (3D) domains versus M. The error-bars show statistical and systematic errors added in quadrature. The lines connect Monte Carlo predictions from* Pythia*, without BEC and with various Bose-Einstein algorithms*[14] *(see text).*

To summarize, a comparison with Pythia predictions shows that short-range correlations of the BE-type are needed, at least in this model, to reproduce the magnitude and the Δ-dependence of the cumulants for like-sign multiplets. This further leads to a much improved description of the cumulants for all-charge multiplets. Since Bose-Einstein correlations are a well-established phenomenon in multiparticle production, it is likely that the above conclusion has wider validity than the model from which it was derived.

The success of the Pythia model with BEC in predicting both the magnitude and domain-size dependence of cumulants, has led us to consider the inter-dependence of these quantities. Figure. 4 shows K_3 and K_4 in 2D and 3D, as a function of K_2. The 2D and 3D data for all-charge, as well as for like-sign multiplets follow approximately, within errors, the same functional dependence. The solid lines are a simple fit to the function $\ln K_q = a_q + r_q \ln K_2$. Figure 4 suggests that the *cumulants* of different orders obey simple so-called "hierarchical" relations, analogous to the Ochs-Wosiek relation, first established for *factorial moments*.[20] Interestingly, all-charge as well as like-sign multiplets are seen to follow, within errors, the same functional dependence.

Simple relations among the cumulants of different orders exist for certain probability distributions, such as the Negative Binomial distribution.[21] For this distribution, one has $K_q = (q-1)!\, K_2^{q-1}$ $(q = 3, 4, \ldots)$, showing that the cumulants are here solely determined by K_2. This relation, shown in Fig. 4 (dashed lines) does not describe the data, indicating that the multiplicity distribution of charged particles, and that of like-sign particles, deviates strongly from a Negative Binomial in small phase space domains.

The Ochs-Wosiek type of relation exhibited by the data in Fig. 4 may explain why the BE algorithms in Pythia generate higher-order correlations of (approximately) the correct magnitude.

5 Summary

A comparative study of like-sign and all-charge genuine correlations between two and more hadrons produced in e^+e^- annihilation at the Z^0 energy has been performed by OPAL using the high-statistics data on hadronic Z^0 decays recorded with the OPAL detector from 1991 through 1995. Normalized factorial cumulants were measured as a function of the domain size, Δ, in D-dimensional domains ($D = 1, 2, 3$) in rapidity, azimuthal angle and (the logarithm of) transverse momentum, defined in the event sphericity frame.

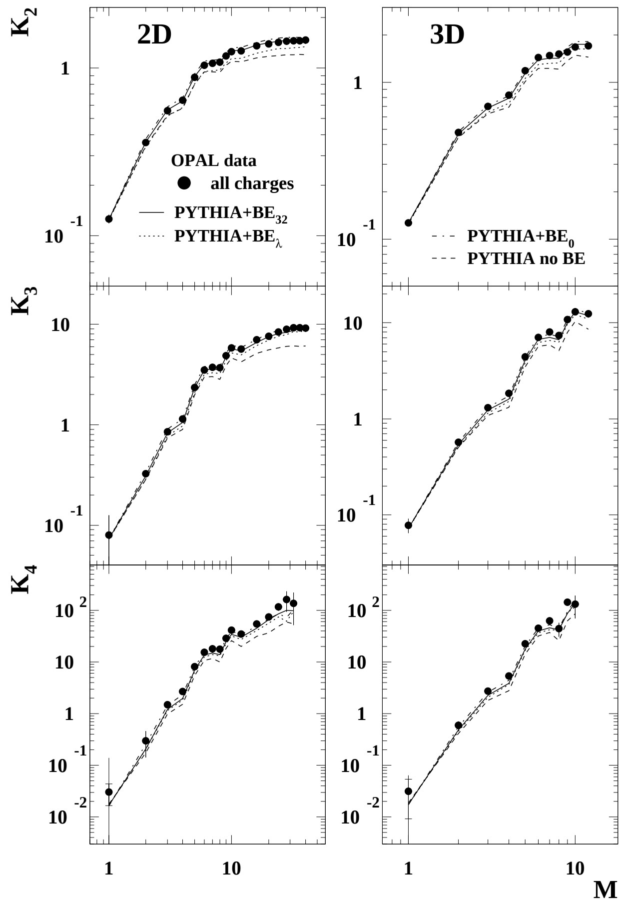

Figure 3. The cumulants K_q in two-dimensional $\Delta y \times \Delta\Phi$ (2D) and three-dimensional $\Delta y \times \Delta\Phi \times \Delta \ln p_T$ (3D) domains for all charged hadrons versus M. Where two error-bars are shown, inner ones are statistical, and outer ones are statistical and systematic errors added in quadrature. The lines connect Monte Carlo predictions from Pythia, without BEC and with various Bose-Einstein algorithms[14] (see text).

Both all-charge and like-sign multiplets show strong positive genuine correlations up to fourth order. The 2D and 3D cumulants K_3 and K_4, considered as a function of K_2, follow approximately a linear relation of the Ochs-Wosiek type: $\ln K_q \sim \ln K_2$, independent of D and the same for all-charge and for like-sign particle groups.

The Pythia model describes well dynamical fluctuations in large phase space domains. However, to achieve a more satisfactory data description, short-range correlations of the Bose-Einstein type between identical particles need to be included.

The Bose-Einstein model BE_{32} in Pythia is able to simultaneously account for the magnitude and Δ-dependence of like-sign as well as of all-charge cumulants. The models BE_0 and BE_λ, when using the same parameters as for BE_{32}, show reasonable agreement with the data. Although the algorithms implement pair-wise BEC only, surprisingly good agreement with the measured third- and fourth-order cumulants is observed.

References

1. E.A. De Wolf, I.M. Dremin and W. Kittel, Phys. Rep. **270** (1996) 1.
2. A. Białas and R. Peschanski, Nucl. Phys. **B273** (1986) 703; **B308** (1988) 857.
3. M.G. Kendall and A. Stuart, *The Advanced Theory of Statistics*, Vol. 1, C. Griffin and Co., London 1969.
4. A.H. Mueller, Phys. Rev. **D4** (1971) 150.
5. P. Carruthers and I. Sarcevic, Phys. Rev. Lett. **63** (1989) 1562; E.A. De Wolf, Acta Phys. Pol. **B21** (1990) 611.
6. R.M. Weiner, Phys. Rep. **327** (2000) 249.
7. OPAL Collaboration, P.D. Acton *et al.*, Phys. Lett. **B267** (1991) 143;
OPAL Collaboration, R. Akers *et al.*, Z. Phys. **C67** (1995) 389.
8. DELPHI Collaboration, P. Abreu *et al.*, Z. Phys. **C63** (1994) 17.

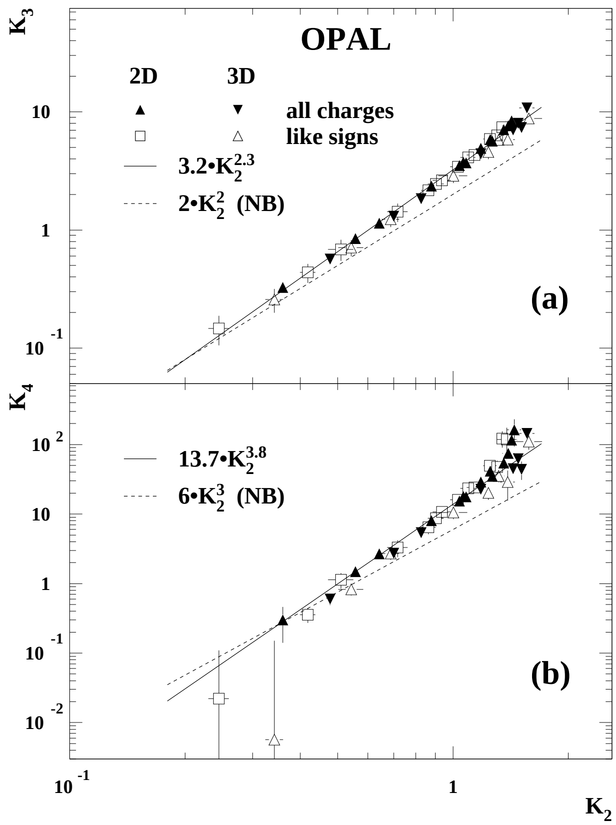

Figure 4. The Ochs-Wosiek plot in two-dimensional $\Delta y \times \Delta\Phi$ (2D) and three-dimensional $\Delta y \times \Delta\Phi \times \Delta \ln p_T$ (3D) domains for all charged hadrons (solid symbols) and for multiplets of like-sign particles (open symbols). The dashed line shows the function, $K_q = (q-1)!\, K_2^{q-1}$ $(q = 3, 4)$, valid for a Negative Binomial multiplicity distribution (NB) in each phase space cell. The solid line shows a fit to the relation $\ln K_q = a_q + r_q \ln K_2$.

9. L3 Collaboration, M. Acciarri *et al.*, Phys. Lett. **B458** (1999) 517;
DELPHI Collaboration, P. Abreu *et al.*, Phys. Lett. **B471** (2000) 460.
10. DELPHI Collaboration, P. Abreu *et al.*, Phys. Lett. **B355** (1995) 415;
OPAL Collaboration, K. Ackerstaff *et al.*, Eur. Phys. J. **C5** (1998) 239.
11. NA22 Collaboration, N.M. Agababyan *et al.*, Z. Phys. **C68** (1995) 229;
WA98 Collaboration, M.M. Aggarwal *et al.*, Phys. Rev. Lett. **85** (2000) 2895;
NA44 Collaboration, I.G. Bearden *et al.*, `nucl-ex/0102013`.
12. L. Lönnblad and T. Sjöstrand, Phys. Lett. **B351** (1995) 293.
13. V.A. Khoze and T. Sjöstrand, Eur. Phys. J. **C6** (1999) 271.
14. L. Lönnblad and T. Sjöstrand, Eur. Phys. J. **C2** (1998) 165 .
15. T. Sjöstrand, Comp. Phys. Comm. **82** (1994) 74 and LU-TP 00-30 (2000) [`hep-ph/0010017`].
16. OPAL Collaboration, G. Abbiendi *et al.*, Eur. Phys. J. **C11** (1999) 239.
17. K. Kadija and P. Seyboth, Z. Phys. **C61** (1994) 465.
18. OPAL Collaboration, K. Ahmet *et al.*, Nucl. Instr. Meth. **A305** (1991) 275;
O. Biebel *et al.*, Nucl. Instr. Meth. **A323** (1992) 169;
P.P. Allport *et al.*, Nucl. Instr. Meth. **A324** (1993) 34, **A346** (1994) 476.
19. OPAL Collaboration, G. Alexander *et al.*, Z. Phys. **C69** (1996) 543.
20. W. Ochs and J. Wosiek, Phys. Lett. **B214** (1988) 617;
W. Ochs, Z. Phys. **C50** (1991) 339.
21. For reviews see P. Carruthers and C.C. Shi, Int. J. Mod. Phys. **A2** (1987) 1447; G. Giacomelli, Int. J. Mod. Phys. **A5** (1990) 223.