

Inflation

I.I. Tkachev^a

^aCERN Theory Division, CH-1211 Geneva 23, Switzerland.

Inflationary cosmology is reviewed. Particular attention is given to processes of creation.

1. Introduction

A typical universe should have had Planckian size, live Planckian time and contain 1 particle. Yet, the observable Universe contains 10^{90} particles in it and had survived 10^{65} Planckian times. Where does it all come from? In other words, why is the Universe so big, flat and old ($t > 10^{10}$ years), homogeneous and isotropic ($\delta T/T \sim 10^{-5}$), why does it contain so much entropy and does not contain unwanted relics? These puzzles of classical cosmology were solved with invention of Inflation [1].

1.1. Getting something for nothing

Stress-energy tensor $T^{\mu\nu}$ which drives the expansion of a homogeneous universe can be characterised by two parameters, energy density, $T_0^0 = \rho$, and pressure $T_j^i = -p\delta_j^i$. Conservation of energy and momentum $T^{\mu\nu}{}_{;\nu} = 0$ in an expanding Friedmann universe takes a simple form $\dot{\rho} + 3H(\rho + p) = 0$. Here H is a Hubble parameter, $H \equiv \dot{a}/a$, and $a(t)$ is a scale factor which describes the expansion of the Universe. Consider the stress-energy tensor $T_{\mu\nu}$ of a vacuum. Vacuum has to be Lorentz invariant, hence $T_\mu^\nu = V \delta_\mu^\nu$ and we find $p = -\rho$. Therefore, the energy of a vacuum stays constant despite the expansion. In this way, room for matter full of energy could have been created. It remains to find out how vacuum energy was converted into radiation at some later stage.

1.2. Horizon problem and the solution

The size of a causally connected region (horizon) scales in proportion to time, $R_H \propto t$. On the other hand, the physical size of a given patch grows in proportion to the scale factor, $R_P \propto a(t) \propto t^\gamma$. Exponent γ depends upon equation

of state, $\gamma = 1/2$ for radiation and $\gamma = 2/3$ for matter dominated expansion. In any case, for the “classical” Friedmann Universe $\gamma < 1$ and horizon expands faster than volume. Take the largest visible patch today. It follows that in the past it should have contained many causally disconnected regions and the question arises why the Universe is so homogeneous at large scales? This problem can be solved if during some period of time the volume had expanded faster than the horizon. During such a period the whole visible Universe can be inflated from one (“small”) causally connected region. Clearly, this happens if $\gamma > 1$, which means $\ddot{a} > 0$. Either of these two conditions can be used as definition of an inflationary regime.

Using the Einstein equation $\ddot{a} = -4\pi G a(\rho + 3p)/3$ we find that the inflationary stage is realized when $p < -\rho/3$. If $p = -\rho$ we have de Sitter metric and the Universe expands exponentially.

A crucial and testable prediction of Inflationary cosmology is a flat Universe, $\Omega = 1$ (as usual, $\Omega(t)$ is the ratio of current and critical energy densities, $\Omega(t) \equiv 8\pi G\rho/3H^2$). Indeed, Einstein equations can be cast into the form $\dot{a}^2(\Omega(t) - 1) = \dot{a}_0^2(\Omega_0 - 1)$. Accelerated expansion, $\ddot{a} > 0$, increases \dot{a} and therefore drives $\Omega(t)$ to 1.

The first model of inflation was de-facto suggested in [2]. De Sitter expansion appeared as a result of vacuum polarization effects in a one-loop order – too complicated to be sure that higher order corrections are unimportant.

1.3. Arranging for a vacuum

Consider $T_{\mu\nu}$ of a real scalar field ϕ

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\mathcal{L}$$

with the Lagrangian $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$. In a state when all derivatives of ϕ are negligible, $\partial_\mu\phi \simeq 0$, the stress-energy tensor of a scalar field is that of a vacuum, $T_{\mu\nu} \simeq V(\phi)g_{\mu\nu}$.

There are two basic ways to arrange $\phi \simeq \text{const}$ and hence to imitate the vacuum-like state.

1. The simplest possibility was suggested by A. Guth in his original paper [1]. Consider potential $V(\phi)$ which has a local minimum with a non-zero energy density separated from the true ground state by a potential barrier. The Universe will be trapped in the meta-stable minimum for a while and expansion will diminish all field gradients. Then the Universe enters a vacuum state. Subsequent phase transition into the true minimum ends inflation and creates the radiation phase. Today the model of Guth and its variants based on potential barriers is good for illustration purposes only. It did not stand up to observations since inhomogeneities which are created during the phase transition into the radiation phase are too large [3].

2. A. Linde was first to realise that things work in the simplest possible set up [4]. Consider potential

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2. \quad (1)$$

Equation of the field motion in expanding Universe is $\ddot{\phi} + 3H\dot{\phi} + m_\phi^2\phi = 0$. If $H \gg m$ the ‘‘friction’’ is too big and the field (almost) does not move. Therefore time derivatives in $T_{\mu\nu}$ can be neglected and inflation starts (in sufficiently homogeneous patch of the Universe). Hubble parameter in this case is given by $H \approx m\phi/M_{\text{Pl}}$ and we see that inflation starts if initial field value happen to satisfy $\phi > M_{\text{Pl}}$. During inflationary stage the field slowly rolls down the potential hill. This motion is very important in the theory of structure creation. Inflation ends when $\phi \sim M_{\text{Pl}}$. At this time field oscillations start around potential minimum and latter decay into radiation. In this way matter was likely created in our Universe.

2. Unified theory of creation

Small fluctuations of any field obey

$$\ddot{U}_k + [k^2 + m_{\text{eff}}^2(\tau)]U_k = 0. \quad (2)$$

Effective mass is time dependent here because of the expansion of the Universe. Because m_{eff} is time dependent, it is not possible to keep fluctuations in vacuum. If one arranges to put oscillators with momentum k into the vacuum, they will not be in vacuum at a latter time since this vacuum would correspond to wrong value of the field mass.

Some remarks are in order:

- Eq. (2) is valid for all particle species
- Equation looks that simple in conformal reference frame $ds^2 = a(\tau)^2 (d\tau^2 - dx^2)$. Everywhere in this chapter a ‘‘dot’’ means derivative with respect to τ .
- Of particular interest are ripples of space-time itself
 - curvature fluctuations (scalar fluctuations of the metric)
 - gravitons (tensor fluctuations of the metric)
- m_{eff} may be non-zero even for massless fields
 - graviton is the simplest example [5], $m_{\text{eff}}^2 = -\ddot{a}/a$
 - m_{eff}^2 for curvature fluctuations has similar structure [6] with a being replaced by $a\dot{\phi}/H$
- For conformally coupled, but massive scalar $m_{\text{eff}} = m_0 a(\tau)$

Creation was only possible because nature is not conformally-invariant. Otherwise $m_{\text{eff}} = 0$ and vacuum remains vacuum forever.

2.1. Sources of creation

Amplitudes U_k in Eq. (2) are quantum operators and a theory of creation reduces to the theory of Bogolyubov transformations or to a theory of particle creation in homogeneous time varying classical background. There are two important instances of such background in cosmology:

- Expansion of space-time, $a(\tau)$

- Motion of the inflaton field, $\phi(\tau)$

Both can be operational at any epoch of creation

- During inflation
- While the inflaton oscillates (reheating)

During inflation superhorizon size perturbations of metric are created which give seeds for Large Scale Structure (LSS) formation and eventually lead to formation of galaxies, the Solar system and all the rest which we can see around us. During reheating matter itself is created. Overall there are four different situations (two sources times two epochs). If coupling to the inflaton is not essential, the corresponding process will be called “pure gravitational creation” in what follows. Let me consider all four possibilities in turn, starting from

2.2. Gravitational creation of metric perturbations [7]

During inflation the motion of the inflaton field is slow, while the expansion of the Universe is fast. It follows that relevant cosmological scales encompass small $\Delta\phi$ interval. E.g. in the model Eq. (1) the whole visible Universe is inflated away while the inflaton field ϕ changes from $4M_{\text{Pl}}$ to M_{Pl} . Potential $V(\phi)$ should be relatively “flat” over this range of $\Delta\phi$ to maintain the inflationary regime. We may conclude that observables should essentially depend on a first few derivatives of V and the shape of the potential outside this region of $\Delta\phi$ is irrelevant. One may construct dimensionless quantities (slow roll parameters) out of potential derivatives

$$\begin{aligned}\epsilon &\equiv \frac{M_{\text{Pl}}^2}{16\pi} \left(\frac{V'}{V} \right)^2, \\ \eta &\equiv \frac{M_{\text{Pl}}^2}{8\pi} \frac{V''}{V}.\end{aligned}\quad (3)$$

The value of potential itself during this period is also relevant and defines the value of the Hubble parameter, $H^2(\phi) = 8\pi G V(\phi)/3$.

Solutions of Eq. (2) with vacuum initial conditions give for the power spectra of scalar (curvature) and tensor (gravity waves) perturbations

$$P(k)_S = \frac{1}{\pi\epsilon} \frac{H^2}{M_{\text{Pl}}^2},$$

$$P(k)_T = \frac{16}{\pi} \frac{H^2}{M_{\text{Pl}}^2}. \quad (4)$$

These spectra can be approximated as power law functions

$$\begin{aligned}P(k)_S &= P(k_0)_S \left(\frac{k}{k_0} \right)^{n-1}, \\ P(k)_T &= P(k_0)_T \left(\frac{k}{k_0} \right)^{n_T}.\end{aligned}\quad (5)$$

where scale dependence k enters via weak dependence of H on the current field value (current means here corresponding to the moment when the scale gets bigger than the horizon and evolution of the mode k freezes out). One expects spectra to be nearly scale invariant since the field (almost) does not move. Indeed, expanding in slow roll parameters one finds

$$\begin{aligned}n - 1 &= 2\eta - 6\epsilon, \\ n_T &= -2\epsilon.\end{aligned}\quad (6)$$

2.2.1. Consistency relation

According to Eqs. (4) the ratio of power in tensor to scalar perturbations is equal to 16ϵ . On the other hand the exponent of tensor perturbations is also proportional to ϵ , see Eq. (6). This gives “consistency relation”, $n_T \simeq -0.14r$, where the ratio of tensor to scalar power is expressed through the ratio of directly measurable respective contributions to quadrupole CMBR anisotropy, $r \equiv C^T/C^S$. Verification of the consistency relation should give major peace of evidence that the Inflation did happened.

2.2.2. Testing Inflation

Predictions of inflationary theory can be tested measuring CMBR anisotropy and power spectra of galaxies distribution. All tests completed so far are in agreement with predictions. Latest CMBR data [8] imply that the Universe is flat $\Omega_0 = 1.02 \pm 0.04$. The spectrum of perturbations is nearly scale invariant, $n_s = 0.97 \pm 0.1$, and Gaussian [9]. Overall normalisation of CMBR spectrum fixes inflaton parameters. In the simplest case of Eq. (1) $m_\phi \simeq 10^{13}$ GeV.

Consistency relation was not tested yet since it requires measurement of tensor perturbations. In forthcoming CMBR experiments tensor and

scalar modes may be disentangled. This can be done unambiguously because tensor and scalar perturbations contribute differently to the polarisation of CMBR (for a review see [10]) which hopefully will be measured.

Measured position of the first acoustic peak in CMBR not only tells us that the Universe is flat, but also that the isocurvature perturbations are ruled out as the primary block of structure formation. While during inflation both types of perturbations, curvature and isocurvature, can be produced, the models which do not involve inflation (e.g. network of cosmic strings) produce isocurvature fluctuations. Therefore this is an important test of inflation [11]. In principle, curvature perturbations may be mimicked by causal processes [12], but polarisation measurements will also give unambiguous proof that density perturbations are of superhorizon origin [13].

While the tensor mode is not measured yet, it is restricted by CMBR anisotropy measurements. Different models of inflation occupy well defined and different regions in the (r,n) parameter plane [14], see Eqs. (3), (6). Regions, specified for new [15], chaotic [4] and hybrid [16] inflationary models are shown in Fig. 1. Other inflationary models are also bounded to these regions. For example, “natural” inflation [17] fits the region of “new” inflationary model. Parameter space favoured by current CMBR and LSS measurements is also shown [18] as a shaded area (note however that size and shape of this region depends upon priors used). E.g., for the $V \propto \phi^p$ chaotic inflation model all this means that $p < 6$ at 99% c.l. [18], while the hybrid model of inflation is disfavoured.

2.3. Gravitational creation of matter

Let us consider the gravitational creation of matter. The source of creation is $m_{\text{eff}} = m_0 a(\tau)$. While matter is created in tiny amounts by this process, it is most effective for heaviest particles. If such superheavy particles do exist in nature, this process naturally leads to Superheavy Dark Matter (SDM) [19,20]. If these particles are unstable but long lived, they can explain [21] puzzling Ultra-High Energy Cosmic Ray (UHECR) events.

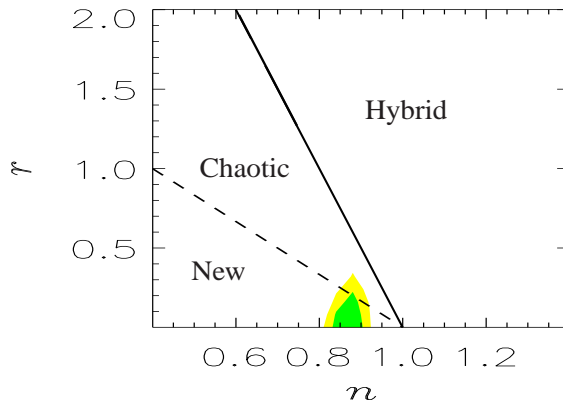


Figure 1. Different models of inflation predict n and r to be found in specified regions. Shaded area indicates parameter range favoured by CMBR and LSS measurements.

2.3.1. Friedmann cosmology

It is particle mass which couples quantum fluctuations to the background expansion and serves as the source of particle creation. Therefore we expect for the abundance of Super-Heavy particles

$$n_{\text{SH}} \propto m_{\text{SH}}^3 a^{-3}. \quad (7)$$

It is the expansion of the Universe which causes particle creation, therefore creation is most efficient at time τ_0 when $H \approx m_{\text{SH}}$, while (comoving) particle number is an adiabatic invariant at later times. We expect that coefficient in Eq. (7) should not be much smaller than unity if we normalize $a(\tau_0) = 1$. It follows that stable particles with $m_{\text{SH}} > 10^9$ GeV will overclose the Universe if there was Friedmann singularity in the past – Friedmann cosmology and SDM mutually exclude each other [20].

2.3.2. Inflationary cosmology

In inflationary cosmology there is no singularity and the Hubble constant is limited, $H \lesssim m_\phi$. Therefore, the production of particles with $m_{\text{SH}} > H \sim 10^{13}$ GeV is suppressed.

The present day ratio of the energy density in SH -particles to the critical energy density is shown in Fig. 2 [20] (see also [19]). Super-Heavy

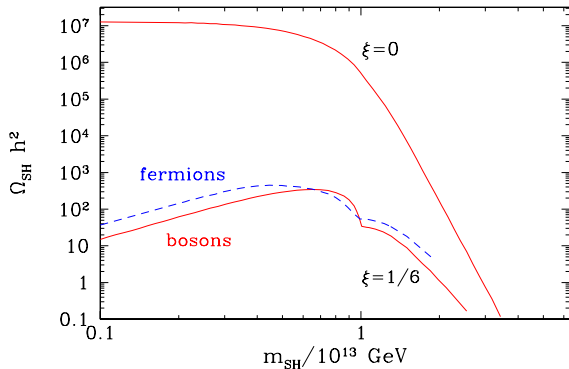


Figure 2. Present day density of gravitationally created Superheavy Dark Matter. Solid lines correspond to minimally or conformally coupled bosons, dotted line describes abundance of fermions.

particles with the mass $\text{few} \times 10^{13}$ GeV are excellent candidates for SDM and progenitors of UHECR. If it will be proven that UHECR are due to decays of SDM, it will mean that Friedmann expansion was preceded by some other epoch, likely by Inflation.

Consistency with observations requires $\Omega_{\text{SH}} h^2 \lesssim 0.3$. We see that light particles, $m_{\text{SH}} \ll 10^{13}$ GeV, are overproduced by many orders of magnitude, unless their coupling to curvature is conformal $\xi = 1/6$. This poses a serious danger [22] since supersymmetry and supergravity models predict many such particles, e.g. moduli, gravitino, etc. They may be dangerous relics, even if unstable, since they can easily survive to post-nucleosynthesis epoch and their decay products destroy ${}^4\text{He}$ and D nuclei by photodissociation [23].

2.4. Decay of the inflaton oscillations

While bosons and fermions are created equally efficiently by a pure gravitational mechanism, coupling to inflaton uncovers deep differences between them. Effective mass of a scalar X (interaction Lagrangian $L_{\text{int}} = \frac{1}{2}g^2\phi^2 X^2$) and of a fermion ψ (interaction Lagrangian $L_{\text{int}} = g\phi\bar{\psi}\psi$)

is given by the following expressions

$$\text{scalar } X : \quad m_{\text{eff}}^2 = m_X^2 + g^2\phi^2(t) \quad (8)$$

$$\text{fermion } \psi : \quad m_{\text{eff}} = m_\psi + g\phi(t). \quad (9)$$

Effective mass of a scalar X depends quadratically upon the inflaton field strength and therefore it is always larger than the bare mass m_X . In the case of fermions inflaton field strength enters linearly and effective mass can cross zero. Even superheavy fermions can be created easily during these moments of zero crossing [24]. (It is easier to create a light field and it is effective mass which counts at creation. At the end of the day it is bare mass which counts.)

Coupling g by itself is not relevant for the process of creation, g always comes in combination with inflaton field strength. To make dimensionless combination out of it we have to re-scale $g\phi$ by a typical time scale relevant for creation. In the present case this will be period of inflaton oscillations or inverse inflaton mass

$$g^2 \rightarrow q \equiv \frac{g^2\phi^2}{4m_\phi^2}. \quad (10)$$

Parameter q determines the strength of particle production caused by the oscillations of the inflaton field. It can be very large [25] even when g is small since $\phi^2/m_\phi^2 \approx 10^{12}$.

2.4.1. Matter creation: Bose versus Fermi

Bose-stimulation aids the process of creation of bosons. Occupation numbers grow exponentially with time, $n = e^{\mu t}$, which results in a fast, explosive decay of inflaton [25,26] and creates large classical fluctuations of all Bose-fields involved. This can have a number of observable consequences: non-thermal phase transitions [27,28], generation of a stochastic background of the gravitational waves [29], and a possibility for a novel mechanism of baryogenesis [30] are some examples.

Fortunately, the system in this regime of particle creation became classical and can be studied on a lattice [31–33].

On the other hand in the case of fermions $n \leq 1$ at all times because of the Pauli blocking. This may create an impression that the fermionic channel of inflaton decay is not important. This is a

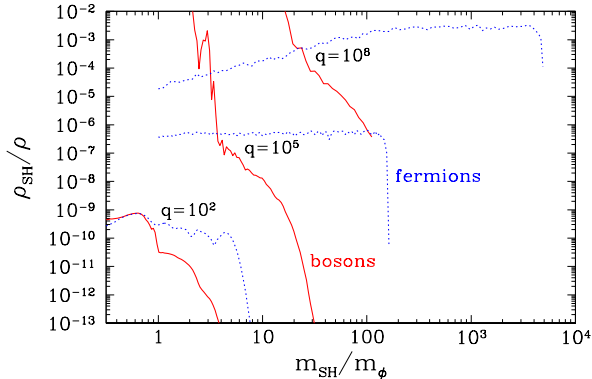


Figure 3. Effectiveness of Super-Heavy particle production. Solid lines production of fermions, dotted lines production of bosons.

false impression [34]. Production of fermions can be more efficient compared to bosons [24]. Vanishing effective mass allows for a larger Fermi-momentum of created particles and fermions can outnumber bosons.

A fraction of the initial energy density which goes to bosons and fermions respectively is plotted in Fig. 3 as a function of particle mass for several values of q . Indeed, superheavy fermions are more efficiently created compared to bosons at the same value of q .

2.4.2. Non-thermal phase transitions

Bosons of moderate or small mass (compared to inflaton), and sufficiently large coupling ($q > 10^4$ or $g^2 > 10^{-8}$ [35]) are produced explosively, their density grows exponentially. Field variances, $\langle X^2 \rangle$, can reach large values, larger than in a thermal equilibrium at the same energy density. These were calculated for different values of parameters in Ref. [31]. For one choice of m_X and q the time dependence of field variances is shown in Fig. 4. For smaller m_X and q variances can reach larger values. By the final moment of time shown in this figure inflaton zero mode had already decayed, but the system is still very far from thermal equilibrium.

In the theory of phase transitions the effective mass of the Higgs field in a medium is $m_H^2 = -\mu^2 + h^2 \langle X^2 \rangle$. If $\langle X^2 \rangle$ is sufficiently large a phase

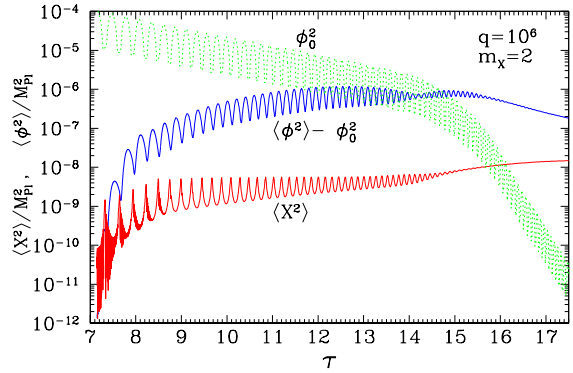


Figure 4. Late-time dependence of field variances and of inflaton zero mode ϕ_0 . Time and m_X are in units of m_ϕ . At $\tau = 0$ variances are in vacuum.

transition can occur. During preheating in the process of inflaton decay this can have especially interesting consequences. Symmetry restoration and subsequent breaking with possible formation of topological defects [36] can happen even in GUTs with large symmetry breaking scale.

2.5. Creation of matter during Inflation

While inflaton during Inflation moves slowly, it does move. This motion may lead to the creation of particles coupled to it. Most effectively and with observable consequences this occurs in the case of superheavy fermions. Features in the power spectrum of perturbations which may leave trace in CMBR and LSS appear at scales corresponding to zero crossings of effective mass. This can be a probe of Sub-Planckian particle content. Multiplet of N particles with $m_\psi \sim M_{Pl}$ and coupling $g > 0.2/N^{2/5}$ is detectable [37].

3. Conclusions

Inflationary cosmology is a beautiful theory. It inputs unknown and (largely) arbitrary initial conditions and replaces them with testable predictions. Even the simplest inflationary model, Eq. (1), reproduces all relevant features of observable Universe after adjustment of a single parameter, $m_\phi \simeq 10^{13}$ GeV.

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