

hep-ph/0112113

CERN-TH/2001-339

UMN-TH-2032/01

TPI-MINN-01/50

Calculations of Neutralino-Stop Coannihilation in the CMSSM

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Abstract

We present detailed calculations of the $\tilde{\chi}_1^0 \tilde{t}_1$ coannihilation channels that have the largest impact on the relic $\tilde{\chi}$ density in the constrained minimal supersymmetric extension of the Standard Model (CMSSM), in which scalar masses m_0 , gaugino masses $m_{1/2}$ and the trilinear soft supersymmetry-breaking parameters A_0 are each assumed to be universal at some input grand unification scale. The most important $\tilde{t}_1 \tilde{t}_1^*$ and $\tilde{t}_1 \tilde{t}_1$ annihilation channels are also calculated, as well as $\tilde{t}_1 \tilde{\ell}$ coannihilation channels. We illustrate the importance of these new coannihilation calculations when A_0 is relatively large. While they do not increase the range of $m_{1/2}$ and hence m_χ allowed by cosmology, these coannihilation channels do open up new ‘tails’ of parameter space extending to larger values of m_0 .

CERN-TH/2001-339

November 2001

1 Introduction

A favoured candidate for cold dark matter is the lightest supersymmetric particle (LSP), which is generally thought to be the lightest neutralino $\tilde{\chi}$ [1] in the minimal supersymmetric extension of the Standard Model (MSSM). It is common to focus attention on the constrained MSSM (CMSSM), in which all the soft supersymmetry-breaking scalar masses m_0 are required to be equal at an input supersymmetric GUT scale, as are the gaugino masses $m_{1/2}$ and the trilinear soft supersymmetry-breaking parameters A_0 . These assumptions yield well-defined relations between the various sparticle masses, and correspondingly more definite predictions for the relic abundance $\Omega_{\tilde{\chi}} h^2$ and observable signatures. This paper is devoted to relic-abundance calculations including coannihilations of the lightest neutralino $\tilde{\chi}$ with \tilde{t}_1 , the lighter supersymmetric partner of the top quark [2].

The range $0.1 < \Omega_{\tilde{\chi}} h^2 < 0.3$ is generally thought to be preferred by astrophysics and cosmology [3]. Lower values of $\Omega_{\tilde{\chi}} h^2$ might be possible if there is some other source of cold dark matter, but higher values are incompatible with observation. The regions of the $m_{1/2}, m_0$ plane where the relic density falls within the preferred range $0.1 < \Omega_{\tilde{\chi}} h^2 < 0.3$ have generally been divided into four generic parts. There is a ‘bulk’ region at moderate $m_{1/2}$ and m_0 [1]. Then, extending to larger $m_{1/2}$, there is a ‘tail’ of the parameter space where the LSP $\tilde{\chi}$ is almost degenerate with the next-to-lightest supersymmetric particle (NLSP), which is in this region the $\tilde{\tau}_1$, the lighter supersymmetric partner of the τ lepton. Along this ‘tail’, efficient coannihilations [4, 5, 6] keep $\Omega_{\tilde{\chi}} h^2$ down in the preferred range, even for larger values of $m_{\tilde{\chi}}$ [7, 8, 9, 10]. At larger m_0 , close to the boundary where electroweak symmetry breaking is no longer possible, there is the ‘focus-point’ region where the LSP has a larger Higgsino component and $m_{\tilde{\chi}}$ is small enough for $\Omega_{\tilde{\chi}} h^2$ to be acceptable [11]. Finally, extending to larger $m_{1/2}$ and m_0 at intermediate values of $m_{1/2}/m_0$, there may be a ‘funnel’ of CMSSM parameter space where rapid direct-channel annihilations via the poles of the heavier Higgs bosons A and H keep $\Omega_{\tilde{\chi}} h^2$ in the preferred range [12, 13].

In this paper, we emphasize the significance of coannihilation of the LSP $\tilde{\chi}$ with \tilde{t}_1 , the lighter supersymmetric partner of the t quark [2]. This mechanism opens up another ‘tail’ of parameter space, this time extending to larger values of m_0 . It is not relevant for the small values of A_0 considered in previous coannihilation calculations [8, 9, 10], but may be important for large A_0 , as we demonstrate in this paper. Coannihilations of $\tilde{\chi}$ with \tilde{t}_1 are important when the latter is the NLSP, just as $\tilde{\chi}\tilde{\tau}_1$ coannihilations are important when the $\tilde{\tau}_1$ is the NLSP. In the latter case, one must also consider coannihilations with the \tilde{e}_1 and $\tilde{\mu}_1$, which are not much heavier than the $\tilde{\tau}_1$ [7, 8, 9, 10]. There are also regions of

CMSSM parameter space where both the \tilde{t}_1 and $\tilde{\tau}_1$ are close in mass to the LSP $\tilde{\chi}$, and $\tilde{t}_1\tilde{\tau}_1$ coannihilations must also be considered. We present here detailed calculations of the matrix elements and cross sections for all the leading $\tilde{\chi}\tilde{t}_1$ and $\tilde{t}_1\tilde{\ell}$ coannihilation processes, and illustrate their importance for $\Omega_{\tilde{\chi}}h^2$ in some instances in the CMSSM when $A_0 \neq 0$.

The structure of the paper is as follows. In Section 2 we recall some important features of LSP relic-density calculations in general, and coannihilations in particular. Then, in Section 3 we compare the relative magnitudes of the $\tilde{\chi}\tilde{\chi}$, $\tilde{\chi}\tilde{t}_1$, $\tilde{t}_1\tilde{t}_1^{(*)}$ and $\tilde{t}_1\tilde{\ell}^{(*)}$ processes for some specific choices of the CMSSM parameters. Section 4 provides an overview of the implications of $\tilde{\chi}\tilde{t}_1$ coannihilation and related processes for the regions of the $m_{1/2}, m_0$ plane allowed by the constraint $0.1 < \Omega_{\tilde{\chi}}h^2 < 0.3$ for various choices of the other CMSSM parameters. Relevant details of our calculations of the matrix elements are contained in an Appendix.

2 Formalism for Annihilation and Coannihilation

The density of neutralino relics left over from the early Universe may be determined relatively simply in terms of relevant annihilation cross sections, using the Boltzmann rate equation to determine a freeze-out density. The relic density subsequently scales with the inverse of the comoving volume, and hence with the entropy density. In the MSSM framework discussed here, since neutralinos are Majorana fermions, the S -wave annihilation cross sections into fermion-antifermion pairs are suppressed by the masses of the final-state fermions, and it is therefore necessary to compute P -wave contributions to the cross sections [1].

The rate equation for a stable particle with density n is

$$\frac{dn}{dt} = -3\frac{\dot{R}}{R}n - \langle\sigma v_{\text{rel}}\rangle(n^2 - n_{\text{eq}}^2), \quad (1)$$

where n_{eq} is the equilibrium number density and $\langle\sigma v_{\text{rel}}\rangle$ is the thermally averaged product of the annihilation cross section σ and the relative velocity v_{rel} . In the early Universe, we can write $\dot{R}/R = (8\pi G_N \rho/3)^{1/2}$, where $\rho = \pi^2 g(T)T^4/30$ is the energy density in radiation and $g(T)$ is the number of relativistic degrees of freedom. Conservation of the entropy density $s = 2\pi^2 h(T)T^4/45$ implies that $\dot{R}/R = -\dot{T}/T - h'\dot{T}/3h$ where $h' \equiv dh/dT$. Generally, we have $h(T) \approx g(T)$. Defining $x \equiv T/m$ and $q \equiv n/T^3 h$, we can write

$$\frac{dq}{dx} = m \left(\frac{4\pi^3}{45} G_N g\right)^{-1/2} \left(h + \frac{1}{3} m x h'\right) \langle\sigma v_{\text{rel}}\rangle (q^2 - q_{\text{eq}}^2). \quad (2)$$

The effect of the h' term was discussed in detail in [14], and is most important when the mass m is between 2 and 10 GeV. Since we only consider neutralinos that are significantly

more massive, we neglect it below (though it is not neglected in our calculations). In the case of the MSSM, freeze-out occurs when $x \sim 1/20$, and the final relic density is determined by integrating (1) down to $x = 0$, and is given by

$$\rho_{\tilde{\chi}} = mq(0)h(0)T_0^3. \quad (3)$$

When coannihilations are important, there are several relevant particle species i , each with different mass, number density n_i and equilibrium number density $n_{\text{eq},i}$. Even in such a situation [4], the rate equation (1) still applies, provided n is interpreted as the total number density,

$$n \equiv \sum_i n_i, \quad (4)$$

n_{eq} as the total equilibrium number density,

$$n_{\text{eq}} \equiv \sum_i n_{\text{eq},i}, \quad (5)$$

and the effective annihilation cross section as

$$\langle \sigma_{\text{eff}} v_{\text{rel}} \rangle \equiv \sum_{ij} \frac{n_{\text{eq},i} n_{\text{eq},j}}{n_{\text{eq}}^2} \langle \sigma_{ij} v_{\text{rel}} \rangle. \quad (6)$$

In (2), m is now understood as the mass of the lightest particle under consideration. For $T \lesssim m_i$, the equilibrium number density of each species is given by [14, 15]

$$\begin{aligned} n_{\text{eq},i} &= g_i \int \frac{d^3 p}{(2\pi)^3} e^{-E/T} \\ &= \frac{g_i m_i^2 T}{2\pi^2} K_2(m_i/T), \\ &= g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} \exp(-m_i/T) \left(1 + \frac{15T}{8m_i} + \dots \right), \end{aligned} \quad (7)$$

where g_i is a spin and color degeneracy factor and $K_2(x)$ is a modified Bessel function. We make the approximation of Boltzmann statistics for the annihilating particles, which is excellent in practice.

We now recall how to compute $\langle \sigma_{12} v_{\text{rel}} \rangle$ for the process $1+2 \rightarrow 3+4$ in an efficient manner. Suppose we have determined the squared transition matrix element $|\mathcal{T}|^2$ (summed over final spins and averaged over initial spins) and expressed it as a function of the Mandelstam variables s, t, u . We then express $|\mathcal{T}|^2$ in terms of s and the scattering angle θ_{CM} in the center-of-mass frame, as described in [8]. We now define

$$\begin{aligned} w(s) &\equiv \frac{1}{4} \int \frac{d^3 p_3}{(2\pi)^3 E_3} \frac{d^3 p_4}{(2\pi)^3 E_4} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) |\mathcal{T}|^2 \\ &= \frac{1}{32\pi} \frac{p_3(s)}{s^{1/2}} \int_{-1}^{+1} d \cos \theta_{\text{CM}} |\mathcal{T}|^2. \end{aligned} \quad (8)$$

In terms of $w(s)$, the total annihilation cross section $\sigma_{12}(s)$ is given by $\sigma_{12}(s) = w(s)/s^{1/2}p_1(s)$ ¹.

The above analysis is exact. To reproduce the usual partial wave expansion, we expand $|\mathcal{T}|^2$ in powers of $p_1(s)/m_1$. The odd powers vanish upon integration over θ_{CM} , while the zeroth- and second-order terms correspond to the usual S and P waves, respectively. Each factor of $p_1(s)$ is accompanied by a factor of $\cos\theta_{\text{CM}}$, so we have

$$\int_{-1}^{+1} d\cos\theta_{\text{CM}} |\mathcal{T}|^2 = \left(|\mathcal{T}|_{\cos\theta_{\text{CM}} \rightarrow +1/\sqrt{3}}^2 + |\mathcal{T}|_{\cos\theta_{\text{CM}} \rightarrow -1/\sqrt{3}}^2 \right) + \mathcal{O}(p_1^4). \quad (9)$$

We can therefore evaluate the S - and P -wave contributions to $w(s)$ simply by evaluating $|\mathcal{T}|^2$ at two different values of $\cos\theta_{\text{CM}}$; no integrations are required.

The proper procedure for thermal averaging has been discussed in [14, 15] for the case of $m_1 = m_2$, and in [16, 6] for the case of $m_1 \neq m_2$, so we do not discuss it in detail here. One finds

$$\langle \sigma_{12} v_{\text{rel}} \rangle = a_{12} + b_{12} x + \mathcal{O}(x^2), \quad (10)$$

where $x \equiv T/m_1$ (assuming $m_1 < m_2$). In our case, we extract a_{12} and b_{12} from the transition amplitudes listed in the Appendix for each final state. We set $x = 0$ to get a_{12} , and then compute b_{12} by setting x to a numerical value small enough to render the $\mathcal{O}(x^2)$ terms negligible. We then compute a_{eff} and b_{eff} by performing the sum over initial states as in (6), and integrate the rate equation (2) numerically to obtain the relic LSP abundance. To a fair approximation, the relic density can simply be written as [1, 4]

$$\Omega h^2 \approx \frac{10^9 \text{ GeV}^{-1}}{g_f^{1/2} M_{\text{pl}} (a_{\text{eff}} + b_{\text{eff}} x_f / 2) x_f}, \quad (11)$$

where the freeze-out temperature $T_f \sim m_{\tilde{\chi}}/20$, and g_f is the number of relativistic degrees of freedom at T_f .

This implies that the ratio of relic densities computed with and without coannihilations is approximately

$$R \equiv \frac{\Omega^0}{\Omega} \approx \left(\frac{\hat{\sigma}_{\text{eff}}}{\hat{\sigma}_0} \right) \left(\frac{x_f}{x_f^0} \right), \quad (12)$$

where $\hat{\sigma} \equiv a + bx/2$ and sub- and superscripts 0 denote quantities computed ignoring coannihilations. The ratio $x_f^0/x_f \approx 1 + x_f^0 \ln(g_{\text{eff}} \sigma_{\text{eff}}/g_1 \sigma_0)$, where $g_{\text{eff}} \equiv \sum_i g_i (m_i/m_1)^{3/2} e^{-(m_i-m_1)/T}$. For the case where the \tilde{t}_1 and $\tilde{\chi}$ are almost degenerate, $g_{\text{eff}} \approx \sum_i g_i = 8$ and $x_f^0/x_f \approx 1.2$.

¹Our $w(s)$ is also the same as $w(s)$ in [14, 16, 7], which is written as $W/4$ in [6].

3 Coannihilation Rates for \tilde{t}_1 in the MSSM

We now use the above formalism to estimate the relative importance of the $\tilde{t}_1\tilde{\chi}$ coannihilation processes, $\tilde{t}_1\tilde{t}_1^*$ and $\tilde{t}_1\tilde{t}_1$ annihilations calculated in the Appendix. We also take into account the $\tilde{\chi}\tilde{\ell}$ coannihilations calculated previously [7, 8] and, for completeness, include the $\tilde{t}_1\tilde{\ell}$ and $\tilde{t}_1\tilde{\ell}^*$ coannihilations also calculated in the Appendix.

To compute the effective annihilation cross sections for light sparticles in the MSSM, we allow the index i in (4) to run over $\tilde{t}_1, \tilde{t}_1^*, \tilde{\tau}_1, \tilde{\tau}_1^*, \tilde{e}_R, \tilde{e}_R^*, \tilde{\mu}_R$ and $\tilde{\mu}_R^*$, as well as $\tilde{\chi}$. The following is the change in σ_{eff} compared with [8], where 49 of the σ_{ij} in (6) were already included:

$$\Delta\sigma_{\text{eff}} = 2(\sigma_{\tilde{t}_1\tilde{t}_1} + \sigma_{\tilde{t}_1\tilde{t}_1^*})r_{\tilde{t}_1}^2 + 4\sigma_{\tilde{\chi}\tilde{t}_1}r_{\tilde{\chi}}r_{\tilde{t}_1} + 8(\sigma_{\tilde{t}_1\tilde{e}_R} + \sigma_{\tilde{t}_1\tilde{e}_R^*})r_{\tilde{t}_1}r_{\tilde{e}_R} + 4(\sigma_{\tilde{t}_1\tilde{\tau}_1} + \sigma_{\tilde{t}_1\tilde{\tau}_1^*})r_{\tilde{t}_1}r_{\tilde{\tau}_1} \quad (13)$$

where $r_i \equiv n_{\text{eq},i}/n_{\text{eq}}$. We have taken the \tilde{e}_R and $\tilde{\mu}_R$ (but not the $\tilde{\tau}$) to be degenerate in mass, thus accounting for the 81 possible initial state combinations. Note that we have summed over color states in the cross sections amplitudes listed in the Appendix, and we have taken the stop degeneracy factor $g_{\tilde{t}} = 3$. We list in Table 1 the sets of initial and final states for which we compute the annihilation cross sections, using the transition amplitudes given in the Appendix. We use q to denote the four light quarks, which we have taken to be massless.

Table 1: Initial and Final States for \tilde{t}_1 Annihilation and Coannihilation Processes

Initial State	Final States
$\tilde{t}_1\tilde{t}_1^*$	$gg, \gamma g, Zg, t\bar{t}, b\bar{b}, q\bar{q}, gh, gH, Zh, ZH, ZA, W^\pm H^\mp, hh, hH, HH, AA, hA, HA, H^+H^-$
$\tilde{t}_1\tilde{t}_1$	$t\bar{t}$
$\tilde{\chi}\tilde{t}_1$	$tq, tZ, bW^+, tH, th, tA, bH^+$
$\tilde{t}_1\tilde{\ell}$	$t\ell, b\nu$
$\tilde{t}_1\tilde{\ell}^*$	$t\bar{\ell}$

In the CMSSM, the diagonal entries of the squark mass matrix tend to pick up large contributions from the gaugino masses, $m_{LL,RR}^2 \ni \mathcal{O}(6)m_{1/2}^2$, thus making the squarks heavier than the neutralinos. The off-diagonal entry for an up-type squark ²

$$m_{LR}^2 = -m_q(A_q + \mu \cot \beta) \quad (14)$$

can, however, be large, particularly for the stops, or for sbottoms at large $\tan \beta$ ³. When A_t is sufficiently large, the lighter stop, \tilde{t}_1 , can become degenerate with (or lighter than)

²Note here our sign convention for A_q .

³For down-type squarks, the factor $\cot \beta$ in (14) is replaced by $\tan \beta$.

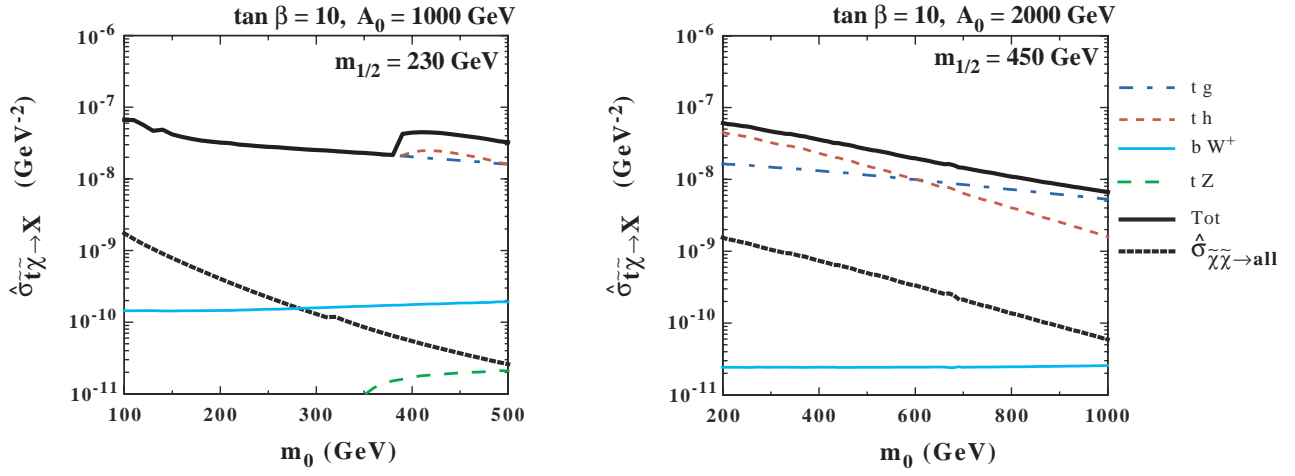


Figure 1: *The separate contributions to the $\tilde{\chi}\tilde{t}_1$ coannihilation cross sections $\hat{\sigma} \equiv a + \frac{1}{2}bx$ for $x = T/m_{\tilde{\chi}} = 1/23$ as functions of m_0 for (a) $m_{1/2} = 230$ GeV, $A_0 = 1000$ GeV and (b) $m_{1/2} = 450$ GeV, $A_0 = 2000$ GeV. Also shown are the total cross section and, for comparison, the much smaller total cross section for $\tilde{\chi}\tilde{\chi}$ annihilation.*

the neutralino. Thus, it is when A_0 is large that we expect $\tilde{\chi}\tilde{t}_1$ coannihilations to become important.

It is important to distinguish between the effective low-energy parameters A_t , etc., and the high-energy input parameter A_0 , which are related through the running of the renormalization-group equations. For example, for $\tan\beta = 10$ and $m_0 = 300$ GeV, $\tilde{t}_1\tilde{\chi}$ coannihilations are important when $m_{1/2} = 200, 450,$ and 670 GeV and $A_0 = 1000, 2000,$ and 3000 GeV, but these values correspond to $A_t \simeq 565, 1200,$ and 1700 GeV respectively. Furthermore, the values of A for the light squarks are different and typically larger than A_t .

We display in Fig. 1 numerical values of the contributions to $\hat{\sigma} \equiv a + bx/2$ (see (11)) in $\tilde{\chi}\tilde{t}_1$ coannihilation, for the representative values $x = 1/23, \tan\beta = 10, \mu > 0$ and (a) $m_{1/2} = 230$ GeV, $A_0 = 1000$ GeV and (b) $m_{1/2} = 450$ GeV, $A_0 = 2000$ GeV as functions of m_0 . For comparison, the total cross section for $\tilde{\chi}\tilde{\chi}$ annihilation to all final states is also shown, as a thick dotted line. We see that the $\tilde{\chi}\tilde{t}_1 \rightarrow tg$ and th coannihilation cross sections dominate by large factors over the total $\tilde{\chi}\tilde{\chi}$ annihilation cross section, suggesting that they may have a greater importance than that suggested by simply comparing Boltzmann suppression factors. The feature in Fig. 1(a) at $m_0 \sim 400$ GeV is due to the threshold for the production of th final states. At smaller values of m_0 , this final state is kinematically forbidden.

Fig. 2 displays similar plots for $\tilde{t}_1\tilde{t}_1^*$ annihilation, for the same parameter choices as in Fig. 1. In this case, the dominant $\tilde{t}_1\tilde{t}_1^*$ annihilation cross sections are into gg and hh , and even

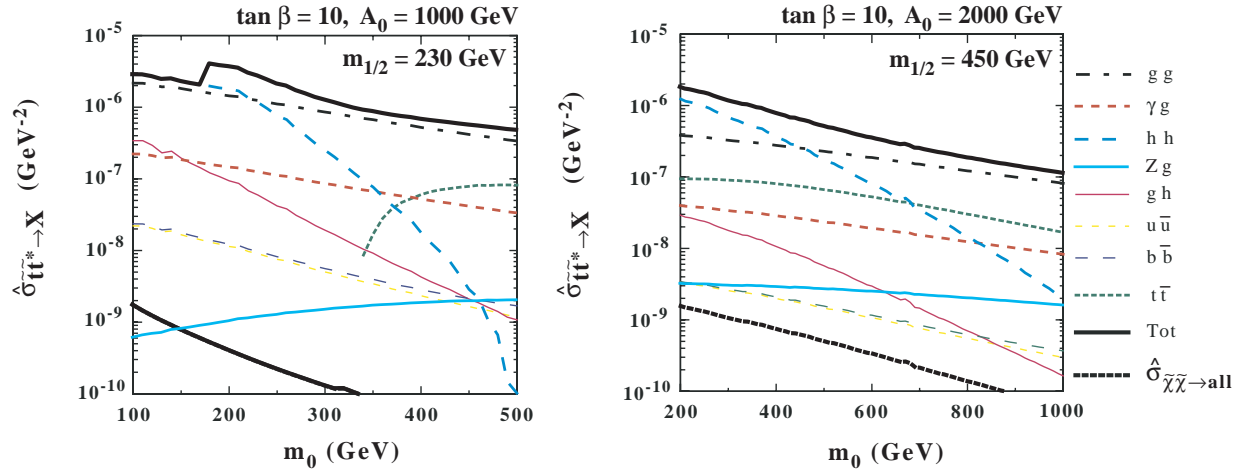


Figure 2: The separate contributions to the $\tilde{t}_1\tilde{t}_1^*$ annihilation cross sections $\hat{\sigma} \equiv a + \frac{1}{2}bx$ for $x = T/m_{\tilde{\chi}} = 1/23$, as functions of m_0 for (a) $m_{1/2} = 230$ GeV, $A_0 = 1000$ GeV and (b) $m_{1/2} = 450$ GeV, $A_0 = 2000$ GeV. Also shown are the total cross section and, for comparison, the much smaller total cross section for $\tilde{\chi}\tilde{\chi}$ annihilation.

subdominant cross sections such as γg , Zg , gh and the various quark-antiquark channels are far larger than the total $\tilde{\chi}\tilde{\chi}$ annihilation cross section. Once again, when $A_0 = 1000$ GeV we see thresholds, in this case corresponding to hh production at $m_0 \sim 180$ GeV and $t\bar{t}$ production at $m_0 \sim 330$ GeV.

The $\tilde{t}_1\tilde{t}_1^*$ annihilation cross sections shown in Fig. 3 show that the cross section for annihilation into the $t\bar{t}$ final state, when it is kinematically open, is also far larger than the total $\tilde{\chi}\tilde{\chi}$ annihilation cross section.

A complete study of coannihilation effects must include not only the $\tilde{\chi}\tilde{\ell}$ processes considered previously [7, 8], and the $\tilde{\chi}\tilde{t}_1$ processes considered above, but also $\tilde{\ell}_1\tilde{t}_1$ coannihilations. Accordingly, the final set of coannihilation cross sections we present are those for $\tilde{\ell}_1\tilde{t}_1$ and $\tilde{\ell}_1^*\tilde{t}_1^*$, shown in Fig. 4. We see that, when (a) $A_0 = 1000$ GeV, the $t\tau$, $t\bar{\tau}$ and $b\nu_e$ final states are the most important, followed by te , $b\nu_\tau$ and $t\bar{e}$, whereas (b) the $t\bar{\tau}$ and $t\bar{e}$ final states are relatively much less important when $A_0 = 2000$ GeV. In all panels of Fig. 4, there are coannihilation cross sections much greater than the total $\tilde{\chi}\tilde{\chi}$ annihilation cross section, which is also plotted.

The basic reason for the relatively small magnitude of the $\tilde{\chi}\tilde{\chi}$ annihilation cross section is that it is dominated by the P -wave suppressed cross sections for $\tilde{\chi}\tilde{\chi}$ annihilation to fermion pairs. This was also the basic reason why $\tilde{\chi}\tilde{\ell}$ coannihilation processes were previously found to be so important [7, 8, 9, 10].

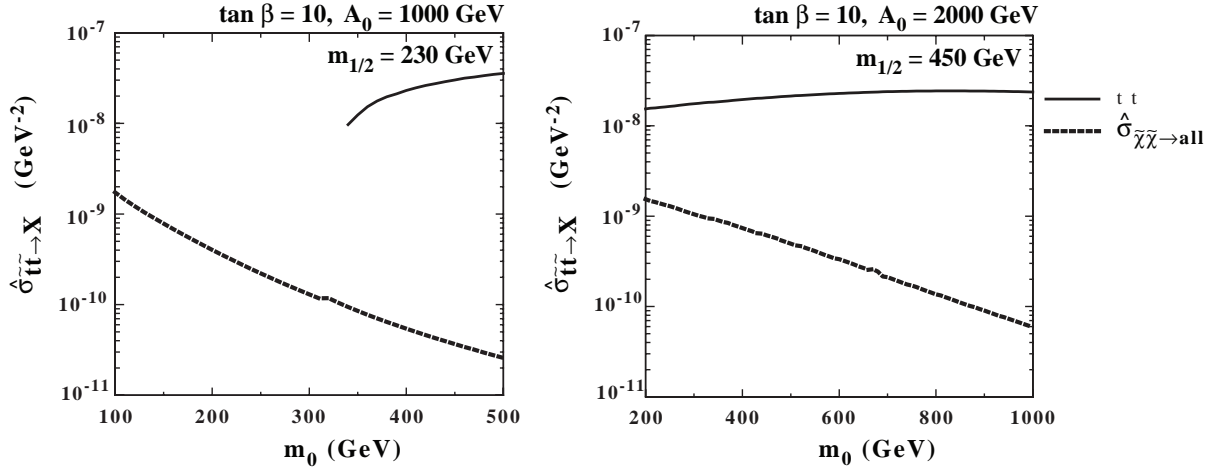


Figure 3: The $\tilde{t}_1\tilde{t}_1 \rightarrow tt$ annihilation cross sections $\hat{\sigma} \equiv a + \frac{1}{2}bx$ for $x = T/m_{\tilde{\chi}} = 1/23$, as functions of m_0 for (a) $m_{1/2} = 230$ GeV, $A_0 = 1000$ GeV and (b) $m_{1/2} = 450$ GeV, $A_0 = 2000$ GeV. Also shown, for comparison, is the much smaller total cross section for $\tilde{\chi}\tilde{\chi}$ annihilation.

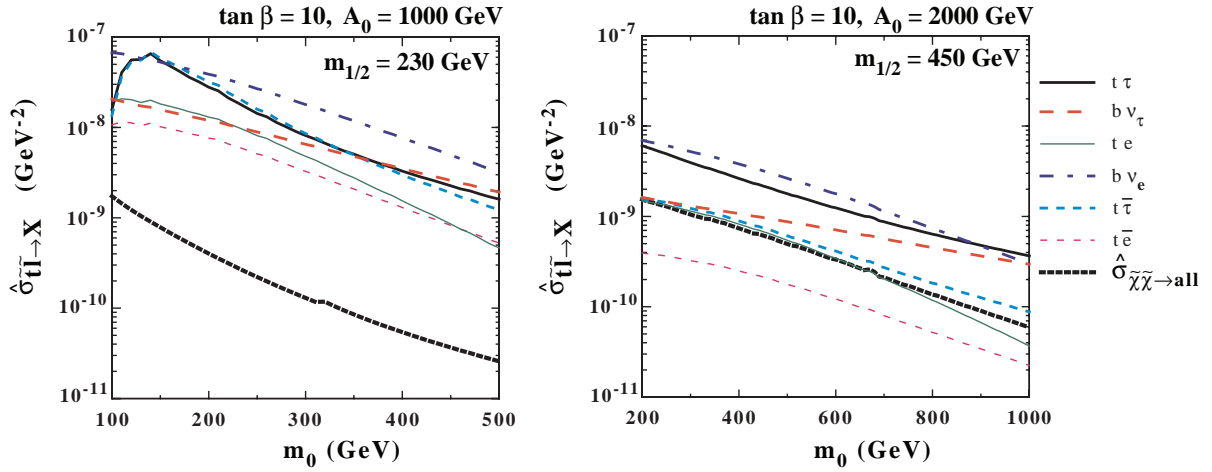


Figure 4: The separate contributions to the $\tilde{\ell}_1\tilde{t}_1$ coannihilation cross sections $\hat{\sigma} \equiv a + \frac{1}{2}bx$ for $x = T/m_{\tilde{\chi}} = 1/23$, as functions of m_0 for (a) $m_{1/2} = 230$ GeV, $A_0 = 1000$ GeV and (b) $m_{1/2} = 450$ GeV, $A_0 = 2000$ GeV. Also shown, for comparison, is the much smaller total cross section for $\tilde{\chi}\tilde{\chi}$ annihilation.

The contributions of the various annihilation channels to σ_{eff} are weighted by the relative abundances of the $\tilde{\chi}$, \tilde{t}_1 and $\tilde{\ell}_1$. For a stop degenerate with the $\tilde{\chi}$, $\tilde{t}_1\tilde{t}_1^*$ annihilation, $\tilde{t}_1\tilde{t}_1$ annihilation and $\tilde{\chi}\tilde{t}_1$ coannihilation are clearly the dominant contributions to $\Delta\sigma_{\text{eff}}$, and hence to σ_{eff} in (6), and the final neutralino relic density is greatly reduced. As the stops become heavier than the neutralinos, their number densities are exponentially suppressed and the stop contributions to σ_{eff} become less important. Fig. 5 shows the sizes of the separate contributions to $\hat{\sigma}_{\text{eff}}$ from $\tilde{\chi}\tilde{\chi}$ annihilation, $\tilde{t}_1\tilde{\chi}$ coannihilation and $\tilde{t}_1\tilde{t}_1^*$, $\tilde{t}_1\tilde{t}_1$ annihilations (combined), as functions of the mass difference between the \tilde{t}_1 and $\tilde{\chi}$. In Fig. 5, we have fixed $m_0 = 300$ GeV, $\tan\beta = 10$, $\mu > 0$, $A_0 =$ (a) 1000 and (b) 2000 GeV, and computed $\hat{\sigma}_{\text{eff}}$ for varying $m_{1/2}$, which amounts to varying the fractional mass difference $\Delta M \equiv (m_{\tilde{t}_1} - m_{\tilde{\chi}})/m_{\tilde{\chi}}$. For these choices, the stau mass, $m_{\tilde{\tau}_1}$, is much larger than $m_{\tilde{\chi}}$. The thin dotted lined is the value of $\hat{\sigma}$ that one would compute if one ignored all coannihilation contributions ⁴. Note that, in the case of close degeneracy between the $\tilde{\chi}$ and \tilde{t}_1 , the $\tilde{t}_1\tilde{t}_1$ and $\tilde{t}_1\tilde{t}_1^*$ annihilations dominate $\hat{\sigma}_{\text{eff}}$. However, since these contributions are suppressed by two powers of $n_{\text{eq},\tilde{t}_1}$, they drop rapidly with ΔM , and neutralino-stop coannihilation takes over. For $A_0 = 2000$ GeV, this occurs at $\Delta M \gtrsim 0.18$. This contribution in turn falls with one power of $n_{\text{eq},\tilde{t}_1}$, and $\tilde{\chi}\tilde{\chi}$ annihilation re-emerges as the dominant reaction for $\Delta M \gtrsim 0.25$. When $\Delta M \gtrsim 0.35$, \tilde{t}_1 coannihilation effects can be neglected. In We see the presence of kinematic thresholds also in Fig. 5. In panel (a), we see the threshold for producing a single top quark in $\tilde{t}\tilde{\chi}$ coannihilations, and in panel (b) we see the threshold for producing a $t\bar{t}$ pair in $\tilde{\chi}\tilde{\chi}$ annihilations.

4 Implications of \tilde{t}_1 Coannihilations for the Region of CMSSM Parameter Space Favoured by Cosmology

We now explore the consequences of \tilde{t}_1 coannihilations for the region of CMSSM parameter space in which $0.1 < \Omega_{\tilde{\chi}} h^2 < 0.3$, as favoured by cosmology. We display in Fig. 6 the $(m_{1/2}, m_0)$ planes for $\tan\beta = 10$ and $\mu > 0$, for the different values of $A_0 =$ (a) 0, (b) 1000, (c) 2000 and (d) 3000 GeV. The very dark (red) shaded regions have $m_{\tilde{\tau}_1}$ or $m_{\tilde{t}_1} < m_{\tilde{\chi}}$, and hence are excluded by the very stringent bounds on charged dark matter [1]. The light (turquoise) shaded regions correspond to the preferred relic-density range $0.1 < \Omega_{\tilde{\chi}} h^2 < 0.3$. The dark (green) shaded regions are those excluded by measurements of $b \rightarrow s\gamma$. The intermediate (pink) shaded regions in panels (a) and (b) are those favoured by the BNL

⁴This differs from $\hat{\sigma}_{\tilde{\chi}\tilde{\chi}}$ because of the number-density weighting factor.

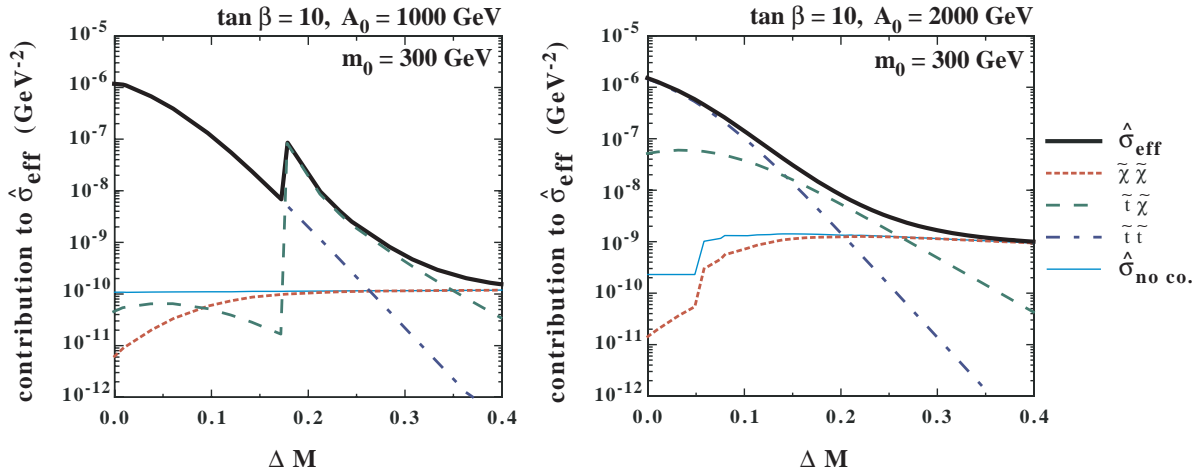


Figure 5: *The separate contributions to the total effective cross sections $\hat{\sigma}_{\text{eff}}$ for $x = T/m_{\tilde{\chi}} = 1/23$, as functions of $\Delta M \equiv (m_{\tilde{\tau}_R} - m_{\tilde{\chi}})/m_{\tilde{\chi}}$, obtained by varying $m_{1/2}$, with (a) $A_0 = 1000$ GeV and (b) $A_0 = 2000$ GeV, both for $\tan\beta = 10$, $\mu > 0$ and $m_0 = 300$ GeV.*

measurement of $g_\mu - 2$ at the $2\text{-}\sigma$ level [17]. The (black) dashed line in panel (a) is the contour $m_{\tilde{\chi}^\pm} = 104$ GeV, corresponding to the kinematic range of LEP 2, and the (red) dotted line is the contour $m_h = 114$ GeV, as evaluated using `FeynHiggs` [18], corresponding to the LEP lower limit on the mass of the Higgs boson. These contours are also glimpsed in the other panels.

Panel (a) of Fig. 6 is indistinguishable from analogous plots shown previously [8, 12]: \tilde{t}_1 coannihilations are not important when $A_0 = 0$. Panel (b) shows how \tilde{t}_1 coannihilation generates a ‘tail’ of allowed CMSSM parameter space, extending the ‘bulk’ region of the $(m_{1/2}, m_0)$ plane up as far as $m_0 \sim 450$ GeV. The boundary of the region where $m_{\tilde{t}_1} < m_{\tilde{\chi}}$, and hence the $\tilde{t}_1\tilde{\chi}$ coannihilation region, slopes to the left as m_0 is increased, since we have fixed A_0 over the plane. As m_0 is increased, the impact of the A -dependent off-diagonal term is diminished and the light stop is heavier. To compensate for this, one must decrease $m_{1/2}$ to obtain the necessary degree of degeneracy between \tilde{t}_1 and $\tilde{\chi}$. This effect is also seen in panels (c) and (d) of Fig. 6.

In the particular case of $A_0 = 1000$ GeV shown in panel (b) of Fig. 6, the \tilde{t}_1 coannihilation tail happens to be excluded by $b \rightarrow s\gamma$. However, this is not the case for (c) $A_0 = 2000$ GeV and (d) $A_0 = 3000$ GeV, where the \tilde{t}_1 coannihilation tail extends up to $m_0 \sim 900$ and beyond 1350 GeV, respectively. For (b) $A_0 = 2000$ GeV, the region favoured by $g_\mu - 2$ is hidden on the left, inside the excluded region. However, around $A_0 = 1500$ GeV there is a $\tilde{\chi}\tilde{t}_1$ coannihilation region that satisfies both the $b \rightarrow s\gamma$ and $g_\mu - 2$ constraints.

The cosmologically-favoured region is broadened significantly when $t\bar{t}$ production is kine-

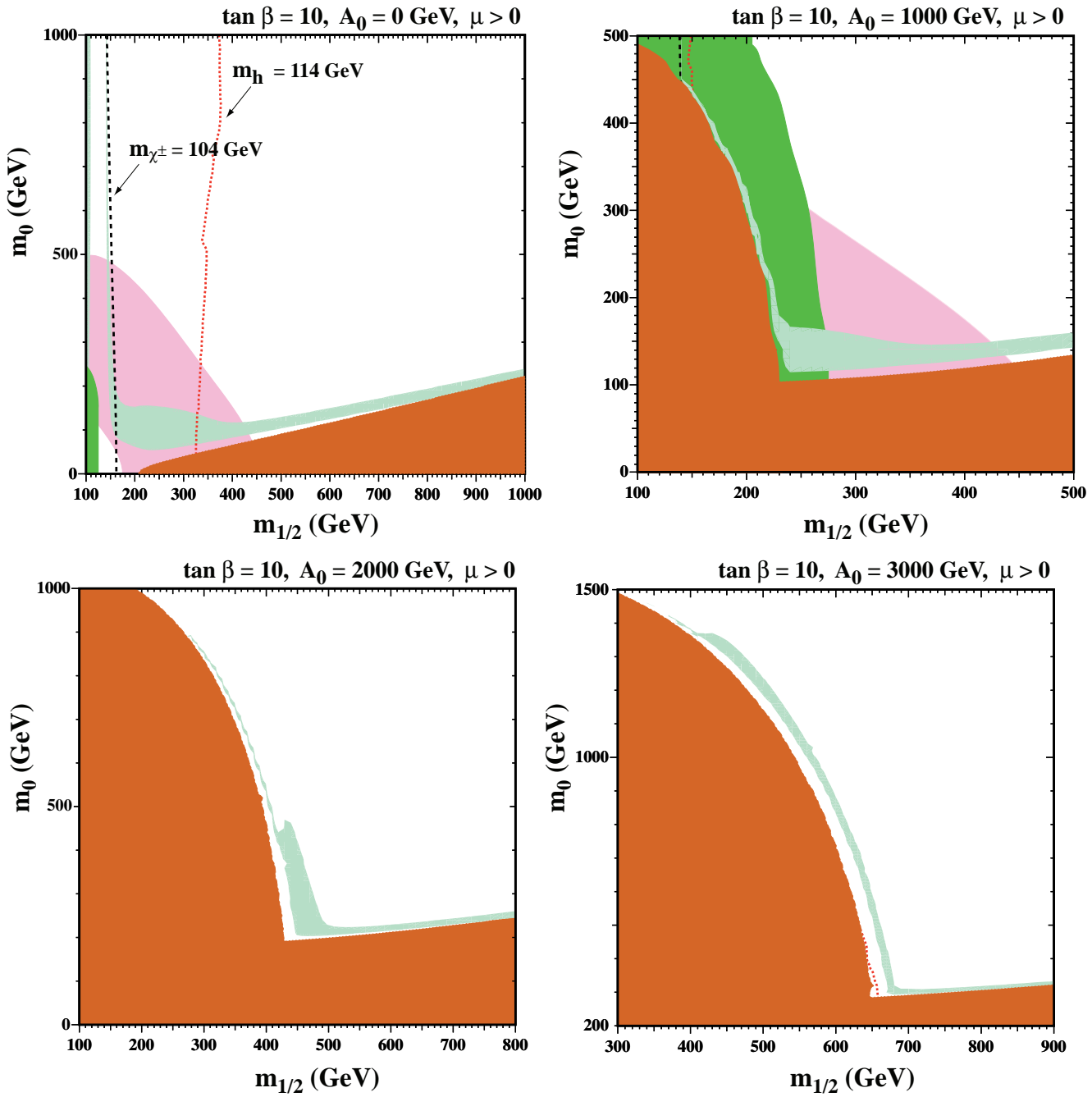


Figure 6: The $(m_{1/2}, m_0)$ planes for $\tan \beta = 10$, $\mu > 0$ and $A_0 = (a) 0$, (b) 1000, (c) 2000 and (d) 3000 GeV. The very dark (red) shaded regions are excluded because the LSP is the \tilde{t}_1 or the $\tilde{\tau}_1$. The dark shaded regions are excluded by $b \rightarrow s\gamma$. The intermediate shaded regions are favoured by $g_\mu - 2$ at the $2\text{-}\sigma$ level. The light (turquoise) shaded regions are those favoured by cosmology, with $0.1 \leq \Omega_{\tilde{\chi}} h^2 \leq 0.3$ after the inclusion of coannihilation effects. The (black) dashed lines are the contours $m_{\tilde{\chi}^\pm} = 104$ GeV, and the (red) dotted lines are the contours $m_h = 114$ GeV.

matically allowed in $\tilde{\chi}\tilde{\chi}$ annihilation. The t-channel stop exchange contribution to this process is significantly enhanced when \tilde{t}_1 is relatively light: $m_{\tilde{t}_1} \simeq m_{\tilde{\chi}}$. This feature can be seen in Fig. 6(c) and Fig. 6(d). The $\tilde{t}_1\tilde{\ell}$ coannihilations are only important in the corner area near the instep of the dark (red) shaded region where $m_{\tilde{\tau}_1} \simeq m_{\tilde{t}_1} \simeq m_{\tilde{\chi}}$.

Fig. 7 shows analogous $(m_{1/2}, m_0)$ planes for the larger values $\tan\beta = 20$ and 30 , both again for $\mu > 0$. We see in panel (a) that the \tilde{t}_1 coannihilation tail extends up to $m_0 \sim 900$ GeV when $\tan\beta = 20$ and $A_0 = 2000$ GeV, beyond the region forbidden by $b \rightarrow s\gamma$. We then see in panel (b) that the \tilde{t}_1 coannihilation tail extends beyond $m_0 \sim 1400$ GeV when $\tan\beta = 20$ and $A_0 = 3000$ GeV. A similar portion of the \tilde{t}_1 coannihilation tail that is consistent with $b \rightarrow s\gamma$ is also visible in panel (c), for $\tan\beta = 20$ and $A_0 = 2000$ GeV, where $m_0 \sim 900$ GeV. For fixed A_0 , as $\tan\beta$ is increased the $b \rightarrow s\gamma$ constraint becomes more severe.

The \tilde{t}_1 coannihilation tails do not increase the allowed range of $m_{1/2}$, as was the case for the $\tilde{\ell}$ coannihilation tail [7, 8, 9, 10], but they do add a significant filament to the CMSSM region allowed by experiment and favoured by cosmology.

In the above illustrative examples, we have chosen the sign of A_0 to be the same as that of μ , with the effect of maximizing the stop off-diagonal mass terms, and hence minimizing $m_{\tilde{t}_1}$. If the sign of A_0 is switched while keeping $\mu > 0$, an analogous $\tilde{\chi}\tilde{t}_1$ coannihilation region is found only at even larger negative A_0 . However, the m_h constraint is more severe for $A_0 < 0$, and also for $\mu < 0$, excluding the region of parameter space of interest in the $\tilde{\chi}\tilde{t}_1$ coannihilation context.

We have not discussed in this paper the potential constraint imposed by the absence of colour and charge breaking (CCB) vacua, or at least the suppression of transitions to CCB vacua. This constraint would restrict the value of A_0 relative to m_0 [19]. If the CCB constraint is imposed, some of the parameter space for small m_0 will be excluded, but there will still be regions at high m_0 where $\tilde{\chi}\tilde{t}_1$ coannihilation is crucial. The issue of compatibility with the potential $g_\mu - 2$ constraint would then arise. Since raising m_0 suppresses the neutralino-proton elastic scattering cross section, $\tilde{\chi}\tilde{t}_1$ coannihilation would tend to lower the neutralino direct detection rate for the same range of $m_{\tilde{\chi}}$.

5 Conclusions and Open Issues

We have documented in this paper the potential importance of $\tilde{\chi}\tilde{t}_1$ coannihilation in delineating the preferred domain of CMSSM parameter space for $A_0 \neq 0$. In this paper, we have only scratched the surface of this subject, whose higher-dimensional parameter space merits

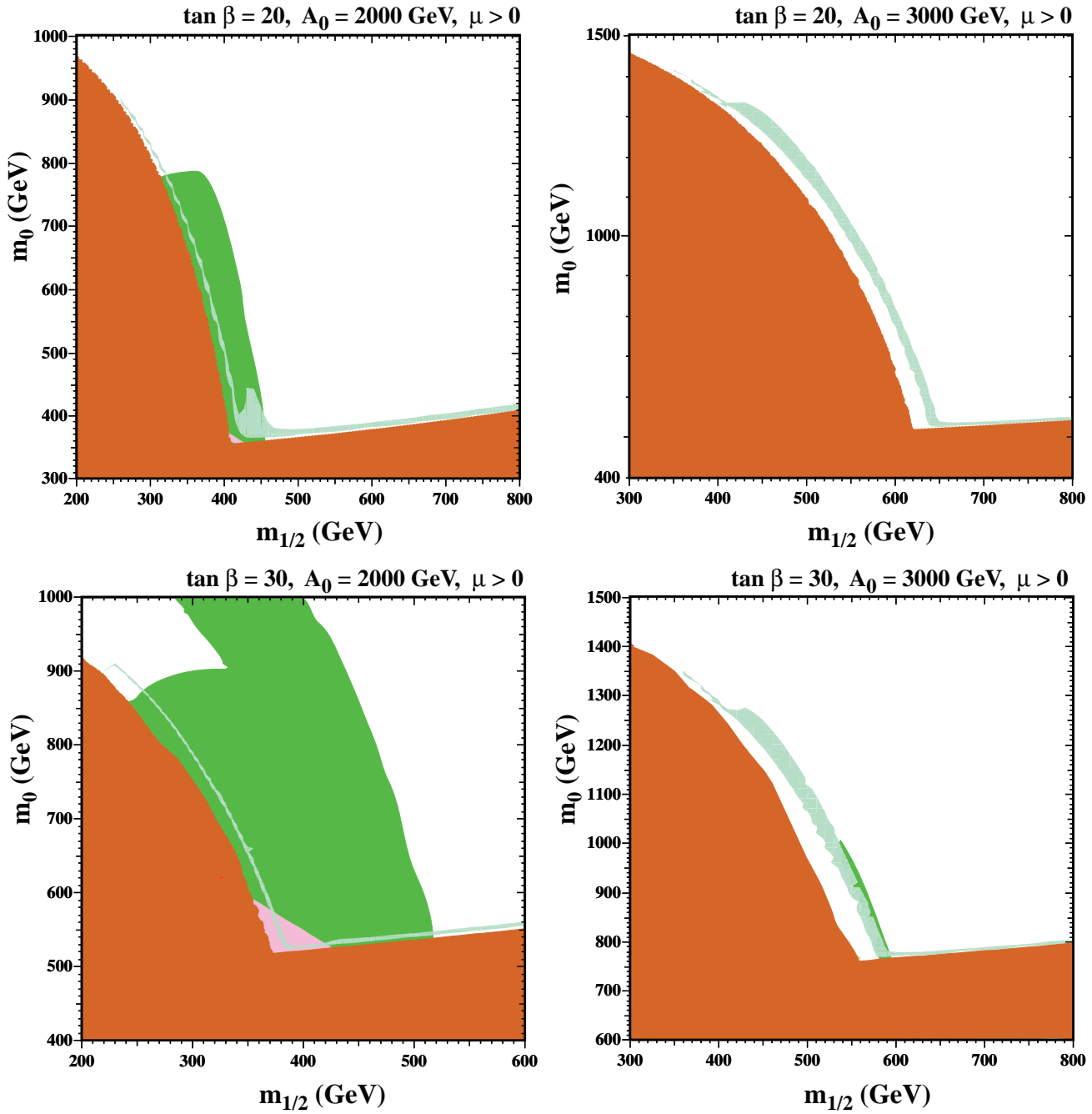


Figure 7: Notation as for Fig. 6. Panels (a,b) are for $\tan \beta = 20, \mu > 0$, and panels (c,d) are for $\tan \beta = 30, \mu > 0$. Panels (a,c) are for $A_0 = 2000 \text{ GeV}$, and panels (b,d) for $A_0 = 3000 \text{ GeV}$.

more detailed exploration. The Appendix provides details of the diagrammatic calculations that should be sufficient for our results to be verified and used by other authors. Although applied in the context of the CMSSM, our results may also be used in more general MSSM contexts. However, other coannihilation processes are also important in other regions of the general MSSM parameter space. For example, in the CMSSM the sbottom mass is generally larger than the stop mass even for large $\tan\beta$. However, if one allow non-degeneracy in the scalar soft breaking mass term, a sbottom NLSP becomes possible [10]. A complete calculation of the LSP relic density in the MSSM requires a careful discussion of all such coannihilation possibilities.

Acknowledgments

We thank Toby Falk for many related discussions. The work of K.A.O. and Y.S. was supported in part by DOE grant DE-FG02-94ER-40823.

Appendix

We calculate the tree contributions to the coannihilation and annihilation amplitudes that are leading order in α_s , $\alpha_t \equiv \lambda_t^2/4\pi$ and/or $\alpha_b \equiv \lambda_b^2/4\pi$. We keep terms that are of the same order in $\sqrt{\alpha_s\alpha_{em}}$, $\sqrt{\alpha_t\alpha_{em}}$, $\sqrt{\alpha_b\alpha_{em}}$, $\sqrt{\alpha_s\alpha_W}$, $\sqrt{\alpha_t\alpha_W}$ and/or $\sqrt{\alpha_b\alpha_W}$, but neglect most terms in α_{em} and/or α_W . This approximation is motivated by the relatively large magnitudes of α_s and α_t , and the fact that $\alpha_b \sim \alpha_t$ at large $\tan\beta$. We have however included s-channel Z , and γ exchange for $\tilde{t}_1\tilde{t}_1^* \rightarrow t\bar{t}$ and checked that they make negligible contribution to the total cross section and therefore neglect these channels in the remaining amplitudes.

Couplings

Here we list couplings used in the calculation. Factors not written, such as i , γ 's, QCD generators and momentum, are taken into account in the calculation of amplitudes squared, which are listed subsequently.

$$\begin{aligned}
C_{g-g-g} &= -g_s \\
C_{\tilde{t}_1-\tilde{t}_1-g} &= -g_s \\
C_{\tilde{t}_1-\tilde{t}_1-g-g} &= g_s^2 \\
C_{\tilde{t}_1-\tilde{t}_1-\gamma} &= -2e/3 \\
C_{\tilde{t}_1-\tilde{t}_1-\gamma-g} &= 4eg_s/3 \\
C_{\tilde{t}_1-\tilde{t}_1-Z} &= \frac{-g_2}{\cos\theta_W}((-2/3)\sin^2\theta_W + \cos^2\theta_t/2) \\
C_{\tilde{t}_1-\tilde{t}_2-Z} &= \frac{g_2}{\cos\theta_W}(1/2)\cos\theta_t\sin\theta_t \\
C_{\tilde{t}_1-\tilde{t}_1-Z-g} &= \frac{2g_s g_2}{\cos\theta_W}((1/2)\cos^2\theta_t - (2/3)\sin^2\theta_W) \\
C_{q-q-g} &= -g_s \\
C_{\tilde{t}_1-t-\tilde{g}}^L &= \sqrt{2}g_s\sin\theta_t \\
C_{\tilde{t}_1-t-\tilde{g}}^R &= -\sqrt{2}g_s\cos\theta_t \\
C_{\tilde{t}_1-\tilde{t}_1-h} &= \frac{g_2 m_Z}{\cos\theta_W} \left(\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W \right) \cos^2\theta_t + \frac{2}{3}\sin^2\theta_W \sin^2\theta_t \right) \sin(\alpha + \beta) \\
&\quad - \frac{g_2 m_t^2 \cos\alpha}{m_W \sin\beta} - \frac{g_2 m_t (\mu \sin\alpha - A_t \cos\alpha)}{m_W \sin\beta} \sin\theta_t \cos\theta_t \\
C_{\tilde{t}_1-\tilde{t}_2-h} &= \frac{g_2 m_Z}{\cos\theta_W} \left(\frac{2}{3}\sin^2\theta_W - \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W \right) \right) \cos\theta_t \sin\theta_t \sin(\alpha + \beta) \\
&\quad - \frac{g_2 m_t (\mu \sin\alpha - A_t \cos\alpha)}{2 m_W \sin\beta} (\cos^2\theta_t - \sin^2\theta_t)
\end{aligned}$$

$$\begin{aligned}
C_{\tilde{t}_1-\tilde{t}_1-H} &= -\frac{g_2 m_Z}{\cos \theta_W} \left(\left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \cos^2 \theta_t + \frac{2}{3} \sin^2 \theta_W \sin^2 \theta_t \right) \cos(\alpha + \beta) \\
&\quad - \frac{g_2 m_t^2 \sin \alpha}{m_W \sin \beta} + \frac{g_2 m_t (\mu \cos \alpha + A_t \sin \alpha)}{m_W \sin \beta} \sin \theta_t \cos \theta_t \\
C_{\tilde{t}_1-\tilde{t}_2-H} &= -\frac{g_2 m_Z}{\cos \theta_W} \left(\frac{2}{3} \sin^2 \theta_W - \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \right) \sin \theta_t \cos \theta_t \cos(\alpha + \beta) \\
&\quad + \frac{g_2 m_t (\mu \cos \alpha + A_t \sin \alpha)}{2 m_W \sin \beta} (\cos^2 \theta_t - \sin^2 \theta_t) \\
C_{\tilde{t}_1-\tilde{t}_1-A} &= 0 \\
C_{\tilde{t}_1-\tilde{t}_2-A} &= \frac{g_2 m_t}{2 m_W} (\mu - A_t \cot \beta) \\
C_{t-t-\gamma} &= -2e/3 \\
C_{f-f-Z}^L &= \frac{g_2}{\cos \theta_W} (Q_f \sin^2 \theta_W - T_{3f}) \\
C_{f-f-Z}^R &= \frac{g_2}{\cos \theta_W} Q_f \sin^2 \theta_W \\
C_{t-t-h} &= -\frac{g_2 m_t \cos \alpha}{2 m_W \sin \beta} \\
C_{t-t-H} &= -\frac{g_2 m_t \sin \alpha}{2 m_W \sin \beta} \\
C_{t-t-A} &= \frac{g_2 m_t \cot \beta}{2 m_W} \\
C_{b-b-h} &= \frac{g_2 m_b \sin \alpha}{2 m_W \cos \beta} \\
C_{b-b-H} &= -\frac{g_2 m_b \cos \alpha}{2 m_W \cos \beta} \\
C_{\tilde{t}_1-b-\chi_i^+}^L &= \frac{g_2 m_b}{\sqrt{2} m_W \cos \beta} U_{i2} \cos \theta_t \\
C_{\tilde{t}_1-b-\chi_i^+}^R &= -g_2 V_{i1} \cos \theta_t + \frac{g_2 m_t}{\sqrt{2} m_W \sin \beta} V_{i2} \sin \theta_t \\
C_{\tilde{\ell}_1-\nu-\chi_i^+}^L &= 0 \\
C_{\tilde{\ell}_1-\nu-\chi_i^+}^R &= \frac{g_2 m_\ell}{\sqrt{2} m_W \cos \beta} U_{i2} \sin \theta_\ell - g_2 U_{i1} \cos \theta_\ell \\
C_{h-h-h} &= -\frac{3 g_2 m_Z}{2 \cos \theta_W} \cos 2\alpha \sin(\beta + \alpha) \\
C_{h-h-H} &= -\frac{g_2 m_Z}{2 \cos \theta_W} (2 \sin 2\alpha \sin(\beta + \alpha) - \cos(\beta + \alpha) \cos 2\alpha) \\
C_{h-H-H} &= \frac{g_2 m_Z}{2 \cos \theta_W} (2 \sin 2\alpha \cos(\beta + \alpha) + \sin(\beta + \alpha) \cos 2\alpha) \\
C_{H-H-H} &= -\frac{3 g_2 m_Z}{2 \cos \theta_W} \cos 2\alpha \cos(\beta + \alpha)
\end{aligned}$$

$$\begin{aligned}
C_{h-A-A} &= -\frac{g_2 m_Z}{2 \cos \theta_w} \cos 2\beta \sin(\beta + \alpha) \\
C_{H-A-A} &= \frac{g_2 m_Z}{2 \cos \theta_w} \cos 2\beta \cos(\beta + \alpha) \\
C_{h-H^+-H^-} &= -g_2(m_W \sin(\beta - \alpha) + m_Z/(2 \cos \theta_w) \cos(2\beta) \sin(\beta + \alpha)) \\
C_{H-H^+-H^-} &= -g_2(m_W \cos(\beta - \alpha) - m_Z/(2 \cos \theta_w) \cos(2\beta) \cos(\beta + \alpha)) \\
C_{h-Z-A} &= g_2 \cos(\alpha - \beta)/(2 \cos \theta_w) \\
C_{H-Z-A} &= g_2 \sin(\alpha - \beta)/(2 \cos \theta_w) \\
C_{h-W^+-H^-} &= \frac{g_2}{2} \cos(\alpha - \beta) \\
C_{H-W^+-H^-} &= \frac{g_2}{2} \sin(\alpha - \beta) \\
C_{\tilde{t}_1-\tilde{t}_1-h-h} &= \frac{g_2^2}{2} \left(\frac{\cos 2\alpha}{\cos^2 \theta_w} \left(\left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) \cos^2 \theta_t + \frac{2}{3} \sin^2 \theta_w \sin^2 \theta_t \right) - \frac{m_t^2 \cos^2 \alpha}{m_W^2 \sin^2 \beta} \right) \\
C_{\tilde{t}_1-\tilde{t}_1-h-H} &= \frac{g_2^2 \sin 2\alpha}{2} \left(\left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) \sec^2 \theta_w \cos^2 \theta_t + \frac{2}{3} \tan^2 \theta_w \sin^2 \theta_t \right. \\
&\quad \left. - \frac{m_t^2}{2 m_W^2 \sin^2 \beta} \right) \\
C_{\tilde{t}_1-\tilde{t}_1-H-H} &= \frac{g_2^2}{2} \left(\frac{-\cos 2\alpha}{\cos^2 \theta_w} \left(\left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) \cos^2 \theta_t + \frac{2}{3} \sin^2 \theta_w \sin^2 \theta_t \right) - \frac{m_t^2 \sin^2 \alpha}{m_W^2 \sin^2 \beta} \right) \\
C_{\tilde{t}_1-\tilde{t}_1-A-A} &= \frac{g_2^2}{2} \left(\frac{\cos 2\beta}{\cos^2 \theta_w} \left(\left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) \cos^2 \theta_t + \frac{2}{3} \sin^2 \theta_w \sin^2 \theta_t \right) - \frac{m_t^2}{m_W^2 \tan^2 \beta} \right) \\
C_{\tilde{t}_1-\tilde{t}_1-H^+-H^-} &= \frac{g_2^2 \cos 2\beta}{2} \left(\left(-1 + \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) \sec^2 \theta_w \right) \cos^2 \theta_t + \frac{2}{3} \tan^2 \theta_w \sin^2 \theta_t \right) \\
&\quad - \frac{g_2^2}{2 m_W^2} (m_b^2 \tan^2 \beta \cos^2 \theta_t + m_t^2 \cot^2 \beta \sin^2 \theta_t) \\
C_{\tilde{t}_1-\tilde{b}_1-H^+} &= -\frac{g_2 m_W}{\sqrt{2}} \left(\sin 2\beta - \frac{m_b^2 \tan \beta + m_t^2 \cot \beta}{m_W^2} \right) \cos \theta_t \cos \theta_b + \frac{g_2}{\sqrt{2} m_W} \\
&\quad \times (m_t m_b (\tan \beta + \cot \beta) \sin \theta_t \sin \theta_b - m_b (A_b \tan \beta - \mu) \cos \theta_t \sin \theta_b \\
&\quad - m_t (A_t \cot \beta - \mu) \sin \theta_t \cos \theta_b) \\
C_{\tilde{t}_1-\tilde{b}_2-H^+} &= \frac{g_2 m_W}{\sqrt{2}} \left(\sin 2\beta - \frac{m_b^2 \tan \beta + m_t^2 \cot \beta}{m_W^2} \right) \cos \theta_t \sin \theta_b + \frac{g_2}{\sqrt{2} m_W} \\
&\quad \times (m_t m_b (\tan \beta + \cot \beta) \sin \theta_t \cos \theta_b - m_b (A_b \tan \beta - \mu) \cos \theta_t \cos \theta_b \\
&\quad + m_t (A_t \cot \beta - \mu) \sin \theta_t \sin \theta_b) \\
C_{\tilde{t}_1-\tilde{b}_1-W^+} &= \frac{-g_2}{\sqrt{2}} \cos \theta_t \cos \theta_b \\
C_{\tilde{t}_1-\tilde{b}_2-W^+} &= \frac{g_2}{\sqrt{2}} \cos \theta_t \sin \theta_b
\end{aligned}$$

$$\begin{aligned}
C_{i_1-t-\tilde{\chi}_i^0}^L &= \frac{-g_2}{\sqrt{2}} \left(\cos \theta_t (N_{i2} + \frac{\tan \theta_w}{3} N_{i1}) + \sin \theta_t \frac{m_t N_{i4}}{m_W \sin \beta} \right) \\
C_{i_1-t-\tilde{\chi}_i^0}^R &= \frac{-g_2}{\sqrt{2}} \left(\cos \theta_t \frac{m_t N_{i4}}{m_W \sin \beta} - \frac{4}{3} \sin \theta_t \tan \theta_w N_{i1} \right) \text{sign}(m_{\tilde{\chi}_i^0}) \\
C_{i_2-t-\tilde{\chi}_i^0}^L &= \frac{-g_2}{\sqrt{2}} \left(-\sin \theta_t (N_{i2} + \frac{\tan \theta_w}{3} N_{i1}) + \cos \theta_t \frac{m_t N_{i4}}{m_W \sin \beta} \right) \\
C_{i_2-t-\tilde{\chi}_i^0}^R &= \frac{-g_2}{\sqrt{2}} \left(-\sin \theta_t \frac{m_t N_{i4}}{m_W \sin \beta} - \frac{4}{3} \cos \theta_t \tan \theta_w N_{i1} \right) \text{sign}(m_{\tilde{\chi}_i^0}) \\
C_{b_1-b-\tilde{\chi}_i^0}^L &= \frac{-g_2}{\sqrt{2}} \left(-\cos \theta_b (N_{i2} - \frac{\tan \theta_w}{3} N_{i1}) + \sin \theta_b \frac{m_b N_{i3}}{m_W \cos \beta} \right) \\
C_{b_1-b-\tilde{\chi}_i^0}^R &= \frac{-g_2}{\sqrt{2}} \left(\cos \theta_b \frac{m_b N_{i3}}{m_W \cos \beta} + \frac{2}{3} \sin \theta_b \tan \theta_w N_{i1} \right) \text{sign}(m_{\tilde{\chi}_i^0}) \\
C_{b_2-b-\tilde{\chi}_i^0}^L &= \frac{-g_2}{\sqrt{2}} \left(\sin \theta_b (N_{i2} - \frac{\tan \theta_w}{3} N_{i1}) + \cos \theta_b \frac{m_b N_{i3}}{m_W \cos \beta} \right) \\
C_{b_2-b-\tilde{\chi}_i^0}^R &= \frac{-g_2}{\sqrt{2}} \left(-\sin \theta_b \frac{m_b N_{i3}}{m_W \cos \beta} + \frac{2}{3} \cos \theta_b \tan \theta_w N_{i1} \right) \text{sign}(m_{\tilde{\chi}_i^0}) \\
C_{\tilde{\ell}_1-\ell-\tilde{\chi}_i^0}^L &= \frac{-g_2}{\sqrt{2}} \left(-\cos \theta_\ell (N_{i2} + \tan \theta_w N_{i1}) + \sin \theta_\ell \frac{m_\ell N_{i3}}{m_W \cos \beta} \right) \\
C_{\tilde{\ell}_1-\ell-\tilde{\chi}_i^0}^R &= \frac{-g_2}{\sqrt{2}} \left(\cos \theta_\ell \frac{m_\ell N_{i3}}{m_W \cos \beta} + 2 \sin \theta_\ell \tan \theta_w N_{i1} \right) \text{sign}(m_{\tilde{\chi}_i^0}) \\
C_{\tilde{\chi}_i^0-\tilde{\chi}_j^0-h}^L &= \frac{g_2}{2} [(N_{i3}(N_{j2} - N_{j1} \tan \theta_w) + N_{j3}(N_{i2} - N_{i1} \tan \theta_w)) \text{sign}(m_{\tilde{\chi}_i^0}) \sin \alpha \\
&\quad + (N_{i4}(N_{j2} - N_{j1} \tan \theta_w) + N_{j4}(N_{i2} - N_{i1} \tan \theta_w)) \cos \alpha] \\
C_{\tilde{\chi}_i^0-\tilde{\chi}_j^0-h}^R &= \frac{g_2}{2} [(N_{j3}(N_{i2} - N_{i1} \tan \theta_w) + N_{i3}(N_{j2} - N_{j1} \tan \theta_w)) \text{sign}(m_{\tilde{\chi}_j^0}) \sin \alpha \\
&\quad + (N_{j4}(N_{i2} - N_{i1} \tan \theta_w) + N_{i4}(N_{j2} - N_{j1} \tan \theta_w)) \cos \alpha] \\
C_{\tilde{\chi}_i^0-\tilde{\chi}_j^0-H}^L &= -\frac{g_2}{2} [(N_{i3}(N_{j2} - N_{j1} \tan \theta_w) + N_{j3}(N_{i2} - N_{i1} \tan \theta_w)) \text{sign}(m_{\tilde{\chi}_i^0}) \\
&\quad \times \cos \alpha - (N_{i4}(N_{j2} - N_{j1} \tan \theta_w) + N_{j4}(N_{i2} - N_{i1} \tan \theta_w)) \sin \alpha] \\
C_{\tilde{\chi}_i^0-\tilde{\chi}_j^0-H}^R &= -\frac{g_2}{2} [(N_{j3}(N_{i2} - N_{i1} \tan \theta_w) + N_{i3}(N_{j2} - N_{j1} \tan \theta_w)) \text{sign}(m_{\tilde{\chi}_j^0}) \\
&\quad \times \cos \alpha - (N_{j4}(N_{i2} - N_{i1} \tan \theta_w) + N_{i4}(N_{j2} - N_{j1} \tan \theta_w)) \sin \alpha] \\
C_{\tilde{\chi}_i^0-\tilde{\chi}_j^0-A}^L &= \frac{g_2}{2} [(N_{i3}(N_{j2} - N_{j1} \tan \theta_w) + N_{j3}(N_{i2} - N_{i1} \tan \theta_w)) \text{sign}(m_{\tilde{\chi}_i^0}) \\
&\quad \times \sin \beta - (N_{i4}(N_{j2} - N_{j1} \tan \theta_w) + N_{j4}(N_{i2} - N_{i1} \tan \theta_w)) \cos \beta] \\
C_{\tilde{\chi}_i^0-\tilde{\chi}_j^0-A}^R &= -\frac{g_2}{2} [(N_{j3}(N_{i2} - N_{i1} \tan \theta_w) + N_{i3}(N_{j2} - N_{j1} \tan \theta_w)) \text{sign}(m_{\tilde{\chi}_j^0}) \\
&\quad \times \sin \beta - (N_{j4}(N_{i2} - N_{i1} \tan \theta_w) + N_{i4}(N_{j2} - N_{j1} \tan \theta_w)) \cos \beta]
\end{aligned}$$

$$\begin{aligned}
C_{t-b-H^+}^L &= \frac{g_2}{\sqrt{2} m_W} m_b \tan \beta \\
C_{t-b-H^+}^R &= \frac{g_2}{\sqrt{2} m_W} m_t \cot \beta \\
C_{\tilde{\chi}_i^0-\tilde{\chi}_j^- -H^+}^L &= -g_2 \left(N_{i3} U_{j1} - (N_{i2} + N_{i1} \tan \theta_w) U_{j2}/\sqrt{2} \right) \text{sign}(m_{\tilde{\chi}_i^0}) \sin \beta \\
C_{\tilde{\chi}_i^0-\tilde{\chi}_j^- -H^+}^R &= -g_2 \left(N_{i4} V_{j1} + (N_{i2} + N_{i1} \tan \theta_w) V_{j2}/\sqrt{2} \right) \cos \beta \\
C_{\tilde{\chi}_i^0-\tilde{\chi}_j^0-Z}^L &= \frac{g_2}{4 \cos \theta_w} \text{sign}(m_{\tilde{\chi}_j^0}) (N_{i4} N_{j4} - N_{i3} N_{j3}) \\
C_{\tilde{\chi}_i^0-\tilde{\chi}_j^0-Z}^R &= -C_{\tilde{\chi}_i^0-\tilde{\chi}_j^0-Z}^L \\
C_{\tilde{\chi}_i^0-\tilde{\chi}_j^- -W^+}^L &= g_2 (N_{i2} V_{j1} - N_{i4} V_{j2}/\sqrt{2}) \\
C_{\tilde{\chi}_i^0-\tilde{\chi}_j^- -W^+}^R &= g_2 (N_{i2} U_{j1} + N_{i3} U_{j2}/\sqrt{2})
\end{aligned}$$

Squared Amplitudes

Below is the list of the amplitudes squared. Note that, for identical-particle final states, one needs to divide them by two when performing the momentum integrations.

$$\tilde{t}_1 \tilde{t}_1^* \longrightarrow gg$$

- I. s-channel gluon annihilation
- II. t-channel \tilde{t}_1 exchange
- III. u-channel \tilde{t}_1 exchange
- IV. point interaction

$$\begin{aligned}
f_1 &= C_{\tilde{t}_1-\tilde{t}_1-g} C_{g-g-g} \\
f_2 &= (C_{\tilde{t}_1-\tilde{t}_1-g})^2 \\
f_3 &= (C_{\tilde{t}_1-\tilde{t}_1-g})^2 \\
f_4 &= C_{\tilde{t}_1-\tilde{t}_1-g-g} \\
\mathcal{T}_I \times \mathcal{T}_I &= (12/9)(4(m_{\tilde{t}_1}^2 - s) + (5/2)(u - t)^2)/s^2 \\
\mathcal{T}_{II} \times \mathcal{T}_{II} &= (16/27)(2m_{\tilde{t}_1}^2 + 2t)^2/(t - m_{\tilde{t}_1}^2)^2 \\
\mathcal{T}_{III} \times \mathcal{T}_{III} &= (16/27)(2m_{\tilde{t}_1}^2 + 2u)^2/(u - m_{\tilde{t}_1}^2)^2 \\
\mathcal{T}_{IV} \times \mathcal{T}_{IV} &= (28/27)(4) \\
\mathcal{T}_I \times \mathcal{T}_{II} &= 0
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}_I \times \mathcal{T}_{III} &= 0 \\
\mathcal{T}_I \times \mathcal{T}_{IV} &= 0 \\
\mathcal{T}_{II} \times \mathcal{T}_{III} &= (-2/27)(4m_{\tilde{t}_1}^2 - s)^2 / ((u - m_{\tilde{t}_1}^2)(t - m_{\tilde{t}_1}^2)) \\
\mathcal{T}_{II} \times \mathcal{T}_{IV} &= (-14/27)(6m_{\tilde{t}_1}^2 - 2u - (5/2)s) / (t - m_{\tilde{t}_1}^2) \\
\mathcal{T}_{III} \times \mathcal{T}_{IV} &= (-14/27)(6m_{\tilde{t}_1}^2 - 2t - (5/2)s) / (u - m_{\tilde{t}_1}^2) \\
|\mathcal{T}|^2 &= f_1^2 \mathcal{T}_I \times \mathcal{T}_I + f_2^2 \mathcal{T}_{II} \times \mathcal{T}_{II} + f_3^2 \mathcal{T}_{III} \times \mathcal{T}_{III} + f_4^2 \mathcal{T}_{IV} \times \mathcal{T}_{IV} + 2f_1 f_2 \mathcal{T}_I \times \mathcal{T}_{II} + \\
&\quad 2f_1 f_3 \mathcal{T}_I \times \mathcal{T}_{III} + 2f_1 f_4 \mathcal{T}_I \times \mathcal{T}_{IV} + 2f_2 f_3 \mathcal{T}_{II} \times \mathcal{T}_{III} + 2f_2 f_4 \mathcal{T}_{II} \times \mathcal{T}_{IV} + \\
&\quad 2f_3 f_4 \mathcal{T}_{III} \times \mathcal{T}_{IV}
\end{aligned} \tag{A1}$$

$$\tilde{t}_1 \tilde{t}_1^* \longrightarrow \gamma g$$

I. t-channel \tilde{t}_1 exchange

II. u-channel \tilde{t}_1 exchange

III. point interaction

$$\begin{aligned}
f_1 &= C_{\tilde{t}_1 - \tilde{t}_1 - g} C_{\tilde{t}_1 - \tilde{t}_1 - \gamma} \\
f_2 &= C_{\tilde{t}_1 - \tilde{t}_1 - g} C_{\tilde{t}_1 - \tilde{t}_1 - \gamma} \\
f_3 &= C_{\tilde{t}_1 - \tilde{t}_1 - \gamma - g} \\
\mathcal{T}_I \times \mathcal{T}_I &= (4/9)(2m_{\tilde{t}_1}^2 + 2t)^2 / (t - m_{\tilde{t}_1}^2)^2 \\
\mathcal{T}_{II} \times \mathcal{T}_{II} &= (4/9)(2m_{\tilde{t}_1}^2 + 2u)^2 / (u - m_{\tilde{t}_1}^2)^2 \\
\mathcal{T}_{III} \times \mathcal{T}_{III} &= (4/9)(4) \\
\mathcal{T}_I \times \mathcal{T}_{II} &= (4/9)(4m_{\tilde{t}_1}^2 - s)^2 / ((t - m_{\tilde{t}_1}^2)(u - m_{\tilde{t}_1}^2)) \\
\mathcal{T}_I \times \mathcal{T}_{III} &= (-4/9)(6m_{\tilde{t}_1}^2 - (5/2)s - 2u) / (t - m_{\tilde{t}_1}^2) \\
\mathcal{T}_{II} \times \mathcal{T}_{III} &= (-4/9)(6m_{\tilde{t}_1}^2 - (5/2)s - 2t) / (u - m_{\tilde{t}_1}^2) \\
|\mathcal{T}|^2 &= f_1^2 \mathcal{T}_I \times \mathcal{T}_I + f_2^2 \mathcal{T}_{II} \times \mathcal{T}_{II} + f_3^2 \mathcal{T}_{III} \times \mathcal{T}_{III} + 2f_1 f_2 \mathcal{T}_I \times \mathcal{T}_{II} + 2f_1 f_3 \mathcal{T}_I \times \mathcal{T}_{III} + \\
&\quad 2f_2 f_3 \mathcal{T}_{II} \times \mathcal{T}_{III}
\end{aligned} \tag{A2}$$

$$\tilde{t}_1 \tilde{t}_1^* \longrightarrow Zg$$

I. t-channel \tilde{t}_1 exchange

II. u-channel \tilde{t}_1 exchange

III. point interaction

$$\begin{aligned}
f_1 &= C_{\tilde{t}_1-\tilde{t}_1-Z} C_{\tilde{t}_1-\tilde{t}_1-g} \\
f_2 &= C_{\tilde{t}_1-\tilde{t}_1-Z} C_{\tilde{t}_1-\tilde{t}_1-g} \\
f_3 &= C_{\tilde{t}_1-\tilde{t}_1-Z-g} \\
\mathcal{T}_I \times \mathcal{T}_I &= (4/9)(2m_{\tilde{t}_1}^2 - m_Z^2 + 2t - (m_{\tilde{t}_1}^2 - t)^2/m_Z^2)(2m_{\tilde{t}_1}^2 + 2t)/(t - m_{\tilde{t}_1}^2)^2 \\
\mathcal{T}_{II} \times \mathcal{T}_{II} &= (4/9)(2m_{\tilde{t}_1}^2 - m_Z^2 + 2u - (u - m_{\tilde{t}_1}^2)^2/m_Z^2)(2m_{\tilde{t}_1}^2 + 2u)/(u - m_{\tilde{t}_1}^2)^2 \\
\mathcal{T}_{III} \times \mathcal{T}_{III} &= 12/9 \\
\mathcal{T}_I \times \mathcal{T}_{II} &= (4/9)(4m_{\tilde{t}_1}^2 - s - (u - m_{\tilde{t}_1}^2)(m_{\tilde{t}_1}^2 - t)/m_Z^2)(4m_{\tilde{t}_1}^2 - m_Z^2 - s) \\
&\quad /((t - m_{\tilde{t}_1}^2)(u - m_{\tilde{t}_1}^2)) \\
\mathcal{T}_I \times \mathcal{T}_{III} &= -(4/9)(6m_{\tilde{t}_1}^2 + (3/2)m_Z^2 - 2u - (5/2)s \\
&\quad - (m_{\tilde{t}_1}^2 - t)((1/2)s - (3/2)m_Z^2 - m_{\tilde{t}_1}^2 + u)/m_Z^2)/(t - m_{\tilde{t}_1}^2) \\
\mathcal{T}_{II} \times \mathcal{T}_{III} &= -(4/9)(6m_{\tilde{t}_1}^2 + (3/2)m_Z^2 - 2t - (5/2)s \\
&\quad - (m_{\tilde{t}_1}^2 - u)((1/2)s - (3/2)m_Z^2 - m_{\tilde{t}_1}^2 + t)/m_Z^2)/(u - m_{\tilde{t}_1}^2) \\
|\mathcal{T}|^2 &= f_1^2 \mathcal{T}_I \times \mathcal{T}_I + f_2^2 \mathcal{T}_{II} \times \mathcal{T}_{II} + f_3^2 \mathcal{T}_{III} \times \mathcal{T}_{III} + 2f_1 f_2 \mathcal{T}_I \times \mathcal{T}_{II} + 2f_1 f_3 \mathcal{T}_I \times \mathcal{T}_{III} + \\
&\quad 2f_2 f_3 \mathcal{T}_{II} \times \mathcal{T}_{III}
\end{aligned} \tag{A3}$$

$$\tilde{t}_1 \tilde{t}_1^* \longrightarrow t\bar{t}$$

I. s-channel gluon annihilation

II. t-channel gluino exchange

III. s-channel h annihilation

IV. s-channel H annihilation

V. t-channel $\tilde{\chi}_{(1,2,3,4)}^0$ exchange

VI. s-channel Z annihilation

VII. s-channel γ annihilation

$$\begin{aligned}
f_1 &= C_{\bar{t}_1 - \bar{t}_1 - g} C_{t - t - g} \\
f_{2L} &= \left(C_{\bar{t}_1 - t - \bar{g}}^L \right)^2 \\
f_{2R} &= \left(C_{\bar{t}_1 - t - \bar{g}}^R \right)^2 \\
f_3 &= C_{\bar{t}_1 - \bar{t}_1 - h} C_{t - t - h} \\
f_4 &= C_{\bar{t}_1 - \bar{t}_1 - H} C_{t - t - H} \\
J_i &= (C_{\bar{t}_1 - t - \bar{\chi}_i^0}^L + C_{\bar{t}_1 - t - \bar{\chi}_i^0}^R) / 2 \\
K_i &= (C_{\bar{t}_1 - t - \bar{\chi}_i^0}^L - C_{\bar{t}_1 - t - \bar{\chi}_i^0}^R) / 2 \\
f_{6c} &= C_{\bar{t}_1 - \bar{t}_1 - Z} (C_{t - t - Z}^R + C_{t - t - Z}^L) / 2 \\
f_{6d} &= C_{\bar{t}_1 - \bar{t}_1 - Z} (C_{t - t - Z}^R - C_{t - t - Z}^L) / 2 \\
f_{7c} &= C_{\bar{t}_1 - \bar{t}_1 - \gamma} C_{t - t - \gamma} \\
f_{7d} &= 0 \\
\mathcal{T}_I \times \mathcal{T}_I &= (2/9)(2s(s - 4m_{\bar{t}_1}^2) - 2(u - t)^2) / s^2 \\
\mathcal{T}_{II} \times \mathcal{T}_{II} &= (2/9)((f_{2L}^2 + f_{2R}^2)((2m_{\bar{t}_1}^2 - s)t - (m_{\bar{t}_1}^2 - m_{\bar{g}}^2 - t)^2) - 4\sqrt{f_{2L}f_{2R}}(f_{2L} + f_{2R}) \\
&\quad \times m_t m_{\bar{g}} (m_{\bar{t}_1}^2 - m_{\bar{g}}^2 - t) + 2f_{2L}f_{2R}(m_{\bar{g}}^2(s - 4m_{\bar{t}_1}^2) - 2m_{\bar{t}_1}^2 t)) / (t - m_{\bar{g}}^2)^2 \\
\mathcal{T}_{III} \times \mathcal{T}_{III} &= (2(s - 4m_{\bar{t}_1}^2)) / (s - m_{\bar{h}}^2)^2 \\
\mathcal{T}_{IV} \times \mathcal{T}_{IV} &= (2(s - 4m_{\bar{t}_1}^2)) / (s - m_H^2)^2 \\
\mathcal{T}_V \times \mathcal{T}_V &= 2(-m_{\bar{t}_1}^4 (4 J_i J_j K_i K_j + J_i^2 (J_j^2 + K_j^2) + K_i^2 (J_j^2 + K_j^2)) + 4m_{\bar{\chi}_i^0} m_{\bar{\chi}_j^0} (J_i^2 - K_i^2) \\
&\quad \times (J_j^2 - K_j^2) m_{\bar{t}_1}^2 - 2m_{\bar{t}_1}^2 (4 J_i J_j K_i K_j + J_i^2 (J_j^2 + K_j^2) + K_i^2 (J_j^2 + K_j^2)) m_{\bar{t}_1}^2 \\
&\quad + 4m_{\bar{\chi}_j^0} (J_i^2 + K_i^2) (J_j^2 - K_j^2) m_{\bar{t}_1}^3 + 4m_{\bar{\chi}_i^0} (J_i^2 - K_i^2) (J_j^2 + K_j^2) m_{\bar{t}_1}^3 \\
&\quad + 3(4 J_i J_j K_i K_j + J_i^2 (J_j^2 + K_j^2) + K_i^2 (J_j^2 + K_j^2)) m_{\bar{t}_1}^4 - m_{\bar{\chi}_i^0} m_{\bar{\chi}_j^0} (J_i^2 - K_i^2) \\
&\quad \times (J_j^2 - K_j^2) s + m_{\bar{t}_1}^2 (4 J_i J_j K_i K_j + J_i^2 (J_j^2 + K_j^2) + K_i^2 (J_j^2 + K_j^2)) s \\
&\quad - m_{\bar{\chi}_j^0} (J_i^2 + K_i^2) (J_j^2 - K_j^2) m_t s - m_{\bar{\chi}_i^0} (J_i^2 - K_i^2) (J_j^2 + K_j^2) m_t s \\
&\quad - (4 J_i J_j K_i K_j + J_i^2 (J_j^2 + K_j^2) + K_i^2 (J_j^2 + K_j^2)) m_{\bar{t}_1}^2 s + m_{\bar{t}_1}^2 (4 J_i J_j K_i K_j \\
&\quad + J_i^2 (J_j^2 + K_j^2) + K_i^2 (J_j^2 + K_j^2)) t + m_{\bar{\chi}_j^0} (J_i^2 + K_i^2) (J_j^2 - K_j^2) m_t t \\
&\quad + m_{\bar{\chi}_i^0} (J_i^2 - K_i^2) (J_j^2 + K_j^2) m_t t + 2(-4 J_i J_j K_i K_j + J_i^2 (J_j^2 + K_j^2) \\
&\quad + K_i^2 (J_j^2 + K_j^2)) m_{\bar{t}_1}^2 t + (4 J_i J_j K_i K_j + J_i^2 (J_j^2 + K_j^2) + K_i^2 (J_j^2 + K_j^2)) m_{\bar{t}_1}^2 t
\end{aligned}$$

$$\begin{aligned}
& +m_{\tilde{t}_1}^2(4J_iJ_jK_iK_j + J_i^2(J_j^2 + K_j^2) + K_i^2(J_j^2 + K_j^2))u - m_{\tilde{\chi}_j^0}(J_i^2 + K_i^2) \\
& \times (J_j^2 - K_j^2)m_tu - m_{\tilde{\chi}_i^0}(J_i^2 - K_i^2)(J_j^2 + K_j^2)m_tu - (4J_iJ_jK_iK_j \\
& + J_i^2(J_j^2 + K_j^2) + K_i^2(J_j^2 + K_j^2))m_t^2u - (4J_iJ_jK_iK_j + J_i^2(J_j^2 + K_j^2) \\
& + K_i^2(J_j^2 + K_j^2))tu)/((m_{\tilde{\chi}_i^0}^2 - t)(-m_{\tilde{\chi}_j^0}^2 + t)) \\
\mathcal{T}_{VI} \times \mathcal{T}_{VI} &= (2(16f_{6d}^2m_{\tilde{t}_1}^2m_t^2 - 4f_{6c}^2m_{\tilde{t}_1}^2s - 4f_{6d}^2m_{\tilde{t}_1}^2s - 4f_{6d}^2m_t^2s + f_{6c}^2s^2 + f_{6d}^2s^2 - f_{6c}^2t^2 \\
& - f_{6d}^2t^2 + 2f_{6c}^2tu + 2f_{6d}^2tu - f_{6c}^2u^2 - f_{6d}^2u^2))/(m_Z^2 - s)^2 \\
\mathcal{T}_{VII} \times \mathcal{T}_{VII} &= (2(16f_{7d}^2m_{\tilde{t}_1}^2m_t^2 - 4f_{7c}^2m_{\tilde{t}_1}^2s - 4f_{7d}^2m_{\tilde{t}_1}^2s - 4f_{7d}^2m_t^2s + f_{7c}^2s^2 + f_{7d}^2s^2 - f_{7c}^2t^2 \\
& - f_{7d}^2t^2 + 2f_{7c}^2tu + 2f_{7d}^2tu - f_{7c}^2u^2 - f_{7d}^2u^2))/s^2 \\
\mathcal{T}_I \times \mathcal{T}_{II} &= (2/27)((f_{2L} + f_{2R})((t - u)(m_{\tilde{t}_1}^2 - m_t^2 - t) - (1/2)s(4m_{\tilde{t}_1}^2 - s + t - u)) \\
& + 4\sqrt{f_{2L}f_{2R}}m_{\tilde{g}}m_t(t - u))/(s(t - m_{\tilde{g}}^2)) \\
\mathcal{T}_I \times \mathcal{T}_{III} &= 0 \\
\mathcal{T}_I \times \mathcal{T}_{IV} &= 0 \\
\mathcal{T}_I \times \mathcal{T}_V &= 0 \\
\mathcal{T}_I \times \mathcal{T}_{VI} &= 0 \\
\mathcal{T}_I \times \mathcal{T}_{VII} &= 0 \\
\mathcal{T}_{II} \times \mathcal{T}_{III} &= 0 \\
\mathcal{T}_{II} \times \mathcal{T}_{IV} &= 0 \\
\mathcal{T}_{II} \times \mathcal{T}_V &= 0 \\
\mathcal{T}_{II} \times \mathcal{T}_{VI} &= 0 \\
\mathcal{T}_{II} \times \mathcal{T}_{VII} &= 0 \\
\mathcal{T}_{III} \times \mathcal{T}_{IV} &= 2(s - 4m_t^2)/((s - m_h^2)(s - m_H^2)) \\
\mathcal{T}_{III} \times \mathcal{T}_V &= 2f_3((J_i^2 + K_i^2)(-4m_t^3 + m_t s - m_t t + m_t u) + (J_i^2 - K_i^2)(-4m_t^2 m_{\tilde{\chi}_i^0} + m_{\tilde{\chi}_i^0} s)) \\
& /((m_h^2 - s)(m_{\tilde{\chi}_i^0}^2 - t)) \\
\mathcal{T}_{III} \times \mathcal{T}_{VI} &= (4(f_3 f_{6c} m_t t - f_3 f_{6c} m_t u))/((m_h^2 - s)(-m_Z^2 + s)) \\
\mathcal{T}_{III} \times \mathcal{T}_{VII} &= (4(f_3 f_{7c} m_t t - f_3 f_{7c} m_t u))/((m_h^2 - s)s) \\
\mathcal{T}_{IV} \times \mathcal{T}_V &= 2f_4((J_i^2 + K_i^2)(-4m_t^3 + m_t s - m_t t + m_t u) + (J_i^2 - K_i^2)(-4m_t^2 m_{\tilde{\chi}_i^0} + m_{\tilde{\chi}_i^0} s)) \\
& /((m_H^2 - s)(m_{\tilde{\chi}_i^0}^2 - t))
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}_{IV} \times \mathcal{T}_{VI} &= (4(f_4 f_{6c} m_t t - f_4 f_{6c} m_t u)) / ((m_H^2 - s)(-m_Z^2 + s)) \\
\mathcal{T}_{IV} \times \mathcal{T}_{VII} &= (4(f_4 f_{7c} m_t t - f_4 f_{7c} m_t u)) / ((m_H^2 - s)s) \\
\mathcal{T}_V \times \mathcal{T}_{VI} &= (-16f_{6d} K_i J_i m_{i_1}^2 m_t^2 - 16f_{6d} J_i K_i m_{i_1}^2 m_t^2 - 4f_{6c} J_i^2 m_{i_1}^2 s + 4f_{6d} K_i J_i m_{i_1}^2 s \\
&\quad + 4f_{6d} J_i K_i m_{i_1}^2 s - 4f_{6c} K_i^2 m_{i_1}^2 s + 4f_{6d} K_i J_i m_t^2 s + 4f_{6d} J_i K_i m_t^2 s + f_{6c} J_i^2 s^2 \\
&\quad - f_{6d} K_i J_i s^2 - f_{6d} J_i K_i s^2 + f_{6c} K_i^2 s^2 - 4f_{6c} J_i^2 m_t^2 t - 4f_{6c} K_i^2 m_t^2 t \\
&\quad - 4f_{6c} J_i^2 m_t m_{\tilde{\chi}_i^0} t + 4f_{6c} K_i^2 m_t m_{\tilde{\chi}_i^0} t - f_{6c} J_i^2 t^2 + f_{6d} K_i J_i t^2 + f_{6d} J_i K_i t^2 \\
&\quad - f_{6c} K_i^2 t^2 + 4f_{6c} J_i^2 m_t^2 u + 4f_{6c} K_i^2 m_t^2 u + 4f_{6c} J_i^2 m_t m_{\tilde{\chi}_i^0} u \\
&\quad - 4f_{6c} K_i^2 m_t m_{\tilde{\chi}_i^0} u + 2f_{6c} J_i^2 t u - 2f_{6d} K_i J_i t u - 2f_{6d} J_i K_i t u + 2f_{6c} K_i^2 t u \\
&\quad - f_{6c} J_i^2 u^2 + f_{6d} K_i J_i u^2 + f_{6d} J_i K_i u^2 - f_{6c} K_i^2 u^2) / ((m_Z^2 - s)(m_{\tilde{\chi}_i^0}^2 - t)) \\
\mathcal{T}_V \times \mathcal{T}_{VII} &= -((-4f_{7c} J_i^2 m_{i_1}^2 s - 4f_{7c} K_i^2 m_{i_1}^2 s + f_{7c} J_i^2 s^2 + f_{7c} K_i^2 s^2 - 4f_{7c} J_i^2 m_t^2 t \\
&\quad - 4f_{7c} K_i^2 m_t^2 t - 4f_{7c} J_i^2 m_t m_{\tilde{\chi}_i^0} t + 4f_{7c} K_i^2 m_t m_{\tilde{\chi}_i^0} t - f_{7c} J_i^2 t^2 - f_{7c} K_i^2 t^2 \\
&\quad + 4f_{7c} J_i^2 m_t^2 u + 4f_{7c} K_i^2 m_t^2 u + 4f_{7c} J_i^2 m_t m_{\tilde{\chi}_i^0} u - 4f_{7c} K_i^2 m_t m_{\tilde{\chi}_i^0} u \\
&\quad + 2f_{7c} J_i^2 t u + 2f_{7c} K_i^2 t u - f_{7c} J_i^2 u^2 - f_{7c} K_i^2 u^2) / (s(m_{\tilde{\chi}_i^0}^2 - t)) \\
\mathcal{T}_{VI} \times \mathcal{T}_{VII} &= (2(-16f_{6d} f_{7d} m_{i_1}^2 m_t^2 + 4f_{6c} f_{7c} m_{i_1}^2 s + 4f_{6d} f_{7d} m_{i_1}^2 s + 4f_{6d} f_{7d} m_t^2 s \\
&\quad - f_{6c} f_{7c} s^2 - f_{6d} f_{7d} s^2 + f_{6c} f_{7c} t^2 + f_{6d} f_{7d} t^2 - 2f_{6c} f_{7c} t u - 2f_{6d} f_{7d} t u \\
&\quad + f_{6c} f_{7c} u^2 + f_{6d} f_{7d} u^2)) / ((m_Z^2 - s)s) \\
|\mathcal{T}|^2 &= f_1^2 \mathcal{T}_I \times \mathcal{T}_I + \mathcal{T}_{II} \times \mathcal{T}_{II} + f_3^2 \mathcal{T}_{III} \times \mathcal{T}_{III} + f_4^2 \mathcal{T}_{IV} \times \mathcal{T}_{IV} + \sum_{i,j=1}^4 \mathcal{T}_V \times \mathcal{T}_V + \mathcal{T}_{VI} \times \mathcal{T}_{VI} + \\
&\quad \mathcal{T}_{VII} \times \mathcal{T}_{VII} + 2f_1 \mathcal{T}_I \times \mathcal{T}_{II} + 2f_1 f_3 \mathcal{T}_I \times \mathcal{T}_{III} + 2f_1 f_4 \mathcal{T}_I \times \mathcal{T}_{IV} + 2f_1 \sum_{i=1}^4 \mathcal{T}_I \times \mathcal{T}_V + \\
&\quad 2f_1 \mathcal{T}_I \times \mathcal{T}_{VI} + 2f_1 \mathcal{T}_I \times \mathcal{T}_{VII} + 2f_3 \mathcal{T}_{II} \times \mathcal{T}_{III} + 2f_4 \mathcal{T}_{II} \times \mathcal{T}_{IV} + 2 \sum_{i=1}^4 \mathcal{T}_{II} \times \mathcal{T}_V + \\
&\quad 2\mathcal{T}_{II} \times \mathcal{T}_{VI} + 2\mathcal{T}_{II} \times \mathcal{T}_{VII} + 2f_3 f_4 \mathcal{T}_{III} \times \mathcal{T}_{IV} + 2f_3 \sum_{i=1}^4 \mathcal{T}_{III} \times \mathcal{T}_V + 2\mathcal{T}_{III} \times \mathcal{T}_{VI} + \\
&\quad 2\mathcal{T}_{III} \times \mathcal{T}_{VII} + 2f_4 \sum_{i=1}^4 \mathcal{T}_{IV} \times \mathcal{T}_V + 2\mathcal{T}_{IV} \times \mathcal{T}_{VI} + 2\mathcal{T}_{IV} \times \mathcal{T}_{VII} + 2\mathcal{T}_V \times \mathcal{T}_{VI} + \\
&\quad 2\mathcal{T}_V \times \mathcal{T}_{VII} + 2\mathcal{T}_{VI} \times \mathcal{T}_{VII}
\end{aligned} \tag{A4}$$

$$\tilde{t}_1 \tilde{t}_1^* \longrightarrow b \bar{b}$$

I. s-channel h annihilation

II. s-channel H annihilation

III. t-channel $\tilde{\chi}_{(1,2)}^+$ exchange

IV. s-channel gluon annihilation

$$\begin{aligned}
f_1 &= C_{\tilde{t}_1 - \tilde{t}_1 - h} C_{b-b-h} \\
f_2 &= C_{\tilde{t}_1 - \tilde{t}_1 - H} C_{b-b-H} \\
f_{3LL}^i &= C_{\tilde{t}_1 - b - \tilde{\chi}_i^+}^L C_{\tilde{t}_1 - b - \tilde{\chi}_i^+}^L \\
f_{3LR}^i &= C_{\tilde{t}_1 - b - \tilde{\chi}_i^+}^L C_{\tilde{t}_1 - b - \tilde{\chi}_i^+}^R \\
f_{3RL}^i &= C_{\tilde{t}_1 - b - \tilde{\chi}_i^+}^R C_{\tilde{t}_1 - b - \tilde{\chi}_i^+}^L \\
f_{3RR}^i &= C_{\tilde{t}_1 - b - \tilde{\chi}_i^+}^R C_{\tilde{t}_1 - b - \tilde{\chi}_i^+}^R \\
f_4 &= C_{\tilde{t}_1 - \tilde{t}_1 - g} C_{b-b-g} \\
\mathcal{T}_I \times \mathcal{T}_I &= 2(s - 4m_b^2)/(s - m_h^2)^2 \\
\mathcal{T}_{II} \times \mathcal{T}_{II} &= 2(s - 4m_b^2)/(s - m_H^2)^2 \\
\mathcal{T}_{III} \times \mathcal{T}_{III} &= ((f_{3RR}^i f_{3RR}^j + f_{3LL}^i f_{3LL}^j)(t(2m_b^2 - s) - (m_{\tilde{t}_1}^2 - m_b^2 - t)^2) + (f_{3RL}^i f_{3LR}^j \\
&\quad + f_{3LR}^i f_{3RL}^j) m_{\tilde{\chi}_i^+} m_{\tilde{\chi}_j^+} (s - 2m_b^2) - (f_{3RR}^i f_{3LR}^j + f_{3LL}^i f_{3RL}^j + f_{3LL}^i f_{3LR}^j + \\
&\quad f_{3RR}^i f_{3RL}^j) m_{\tilde{\chi}_j^+} m_b (m_{\tilde{t}_1}^2 - m_b^2 - t) - (f_{3RL}^i f_{3RR}^j + f_{3LR}^i f_{3LL}^j + f_{3LR}^i f_{3RR}^j + \\
&\quad f_{3RL}^i f_{3LL}^j) m_{\tilde{\chi}_i^+} m_b (m_{\tilde{t}_1}^2 - m_b^2 - t) - (f_{3LL}^i f_{3RR}^j + f_{3RR}^i f_{3LL}^j) m_b^2 2t \\
&\quad - (f_{3LR}^i f_{3LR}^j + f_{3RL}^i f_{3RL}^j) 2m_b^2 m_{\tilde{\chi}_i^+} m_{\tilde{\chi}_j^+}) / ((t - m_{\tilde{\chi}_i^+}^2)(t - m_{\tilde{\chi}_j^+}^2)) \\
\mathcal{T}_{IV} \times \mathcal{T}_{IV} &= (2/9)(2s(s - 4m_{\tilde{t}_1}^2) - 2(u - t)^2)/s^2 \\
\mathcal{T}_I \times \mathcal{T}_{II} &= 2(s - 4m_b^2)/((s - m_H^2)(s - m_h^2)) \\
\mathcal{T}_I \times \mathcal{T}_{III} &= ((f_{3LL}^i + f_{3RR}^i) 2m_b(t + m_b^2 - m_{\tilde{t}_1}^2) + (f_{3RL}^i + f_{3LR}^i) m_{\tilde{\chi}_i^+} (s - 4m_b^2)) \\
&\quad / ((s - m_h^2)(t - m_{\tilde{\chi}_i^+}^2)) \\
\mathcal{T}_I \times \mathcal{T}_{IV} &= 0 \\
\mathcal{T}_{II} \times \mathcal{T}_{III} &= ((f_{3LL}^i + f_{3RR}^i) 2m_b(t + m_b^2 - m_{\tilde{t}_1}^2) + (f_{3RL}^i + f_{3LR}^i) m_{\tilde{\chi}_i^+} (s - 4m_b^2)) \\
&\quad / ((s - m_H^2)(t - m_{\tilde{\chi}_i^+}^2)) \\
\mathcal{T}_{II} \times \mathcal{T}_{IV} &= 0
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}_{\text{III}} \times \mathcal{T}_{\text{IV}} &= 0 \\
|\mathcal{T}|^2 &= f_1^2 \mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{I}} + f_2^2 \mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{II}} + \sum_{i,j=1}^2 \mathcal{T}_{\text{III}} \times \mathcal{T}_{\text{III}} + f_4^2 \mathcal{T}_{\text{IV}} \times \mathcal{T}_{\text{IV}} + 2f_1 f_2 \mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{II}} + \\
& 2f_1 \sum_{i=1}^2 \mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{III}} + 2f_1 f_4 \mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{IV}} + 2f_2 \sum_{i=1}^2 \mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{III}} + 2f_2 f_4 \mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{IV}} + \\
& 2f_4 \sum_{i=1}^2 \mathcal{T}_{\text{III}} \times \mathcal{T}_{\text{IV}}
\end{aligned} \tag{A5}$$

$$\tilde{t}_1 \tilde{t}_1^* \longrightarrow q\bar{q}, \quad q = u, d, s, c$$

I. s-channel gluon annihilation

$$\begin{aligned}
f_1 &= C_{\tilde{t}_1 - \tilde{t}_1 - g} C_{q - q - g} \\
\mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{I}} &= (2/9)(2s(s - 4m_{\tilde{t}_1}^2) - 2(u - t)^2)/s^2 \\
|\mathcal{T}|^2 &= f_1^2 \mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{I}}
\end{aligned} \tag{A6}$$

$$\tilde{t}_1 \tilde{t}_1^* \longrightarrow gH \quad [gh]$$

I. t-channel \tilde{t}_1 exchange

II. u-channel \tilde{t}_1 exchange

$$\begin{aligned}
f_1 &= C_{\tilde{t}_1 - \tilde{t}_1 - g} C_{\tilde{t}_1 - \tilde{t}_1 - H} [C_{\tilde{t}_1 - \tilde{t}_1 - g} C_{\tilde{t}_1 - \tilde{t}_1 - h}] \\
f_2 &= C_{\tilde{t}_1 - \tilde{t}_1 - g} C_{\tilde{t}_1 - \tilde{t}_1 - H} [C_{\tilde{t}_1 - \tilde{t}_1 - g} C_{\tilde{t}_1 - \tilde{t}_1 - h}] \\
\mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{I}} &= (-4/9)(2(m_{\tilde{t}_1}^2 + t))/(t - m_{\tilde{t}_1}^2)^2 \\
\mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{II}} &= (-4/9)(2(m_{\tilde{t}_1}^2 + u))/(u - m_{\tilde{t}_1}^2)^2 \\
\mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{II}} &= (-4/9)(6m_{\tilde{t}_1}^2 - u - t - 2s)/((t - m_{\tilde{t}_1}^2)(u - m_{\tilde{t}_1}^2)) \\
|\mathcal{T}|^2 &= f_1^2 \mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{I}} + f_2^2 \mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{II}} + 2f_1 f_2 \mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{II}}
\end{aligned} \tag{A7}$$

$$\tilde{t}_1 \tilde{t}_1^* \longrightarrow Zh \quad [ZH]$$

I. t-channel $\tilde{t}_{(1,2)}$ exchange

II. u-channel $\tilde{t}_{(1,2)}$ exchange

$$\begin{aligned}
f_1^i &= C_{\tilde{t}_1-\tilde{t}_i-Z} C_{\tilde{t}_1-\tilde{t}_i-h} \\
f_2^i &= C_{\tilde{t}_1-\tilde{t}_i-Z} C_{\tilde{t}_1-\tilde{t}_i-h} \\
\mathcal{T}_I \times \mathcal{T}_I &= -(2m_{\tilde{t}_1}^2 + 2t - m_Z^2 - (m_{\tilde{t}_1}^2 - t)^2/m_Z^2)/((t - m_{\tilde{t}_i}^2)(t - m_{\tilde{t}_j}^2)) \\
\mathcal{T}_{II} \times \mathcal{T}_{II} &= -(2m_{\tilde{t}_1}^2 + 2u - m_Z^2 - (m_{\tilde{t}_1}^2 - u)^2/m_Z^2)/((u - m_{\tilde{t}_i}^2)(u - m_{\tilde{t}_j}^2)) \\
\mathcal{T}_I \times \mathcal{T}_{II} &= -(4m_{\tilde{t}_1}^2 - m_h^2 - s - (m_{\tilde{t}_1}^2 - t)(u - m_{\tilde{t}_i}^2)/m_Z^2)/((t - m_{\tilde{t}_i}^2)(u - m_{\tilde{t}_j}^2)) \\
|\mathcal{T}|^2 &= \sum_{i=1}^2 \sum_{j=1}^2 [f_1^i f_1^j \mathcal{T}_I \times \mathcal{T}_I + f_2^i f_2^j \mathcal{T}_{II} \times \mathcal{T}_{II} + 2f_1^i f_2^j \mathcal{T}_I \times \mathcal{T}_{II}] \tag{A8}
\end{aligned}$$

$$\tilde{t}_1 \tilde{t}_1^* \longrightarrow ZA$$

I. s-channel h annihilation

II. s-channel H annihilation

III. t-channel \tilde{t}_2 exchange

IV. u-channel \tilde{t}_2 exchange

$$\begin{aligned}
f_1 &= C_{\tilde{t}_1-\tilde{t}_1-h} C_{h-Z-A} \\
f_2 &= C_{\tilde{t}_1-\tilde{t}_1-H} C_{H-Z-A} \\
f_3 &= C_{\tilde{t}_1-\tilde{t}_2-Z} C_{\tilde{t}_1-\tilde{t}_2-A} \\
f_4 &= C_{\tilde{t}_1-\tilde{t}_2-Z} C_{\tilde{t}_1-\tilde{t}_2-A} \\
\mathcal{T}_I \times \mathcal{T}_I &= ((s - m_A^2)^2/m_Z^2 - (2s - m_Z^2 + 2m_A^2))/(s - m_h^2)^2 \\
\mathcal{T}_{II} \times \mathcal{T}_{II} &= ((s - m_A^2)^2/m_Z^2 - (2s - m_Z^2 + 2m_A^2))/(s - m_H^2)^2 \\
\mathcal{T}_{III} \times \mathcal{T}_{III} &= ((m_{\tilde{t}_1}^2 - t)^2/m_Z^2 - (2m_{\tilde{t}_1}^2 + 2t))/(t - m_{\tilde{t}_2}^2)^2 \\
\mathcal{T}_{IV} \times \mathcal{T}_{IV} &= ((m_{\tilde{t}_1}^2 - u)^2/m_Z^2 - (2m_{\tilde{t}_1}^2 + 2u))/(u - m_{\tilde{t}_2}^2)^2 \\
\mathcal{T}_I \times \mathcal{T}_{II} &= ((s - m_A^2)^2/m_Z^2 - (2s - m_Z^2 + 2m_A^2))/((s - m_H^2)(s - m_h^2)) \\
\mathcal{T}_I \times \mathcal{T}_{III} &= ((s - m_A^2)(m_{\tilde{t}_1}^2 - t)/m_Z^2 - m_{\tilde{t}_1}^2 + 2m_A^2 - u)/((s - m_h^2)(t - m_{\tilde{t}_2}^2)) \\
\mathcal{T}_I \times \mathcal{T}_{IV} &= ((s - m_A^2)(m_{\tilde{t}_1}^2 - u)/m_Z^2 - m_{\tilde{t}_1}^2 + 2m_A^2 - t)/((s - m_h^2)(u - m_{\tilde{t}_2}^2)) \\
\mathcal{T}_{II} \times \mathcal{T}_{III} &= ((s - m_A^2)(m_{\tilde{t}_1}^2 - t)/m_Z^2 - m_{\tilde{t}_1}^2 + 2m_A^2 - u)/((s - m_H^2)(t - m_{\tilde{t}_2}^2)) \\
\mathcal{T}_{II} \times \mathcal{T}_{IV} &= ((s - m_A^2)(m_{\tilde{t}_1}^2 - u)/m_Z^2 - m_{\tilde{t}_1}^2 + 2m_A^2 - t)/((s - m_H^2)(u - m_{\tilde{t}_2}^2))
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}_{\text{III}} \times \mathcal{T}_{\text{IV}} &= ((u - m_{\tilde{t}_1}^2)(m_{\tilde{t}_1}^2 - t)/m_Z^2 - (4m_{\tilde{t}_1}^2 - m_A^2 - s))/((t - m_{\tilde{t}_2}^2)(u - m_{\tilde{t}_2}^2)) \\
|\mathcal{T}|^2 &= f_1^2 \mathcal{T}_I \times \mathcal{T}_I + f_2^2 \mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{II}} + f_3^2 \mathcal{T}_{\text{III}} \times \mathcal{T}_{\text{III}} + f_4^2 \mathcal{T}_{\text{IV}} \times \mathcal{T}_{\text{IV}} + 2f_1 f_2 \mathcal{T}_I \times \mathcal{T}_{\text{II}} + \\
&\quad 2f_1 f_3 \mathcal{T}_I \times \mathcal{T}_{\text{III}} + 2f_1 f_4 \mathcal{T}_I \times \mathcal{T}_{\text{IV}} + 2f_2 f_3 \mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{III}} + 2f_2 f_4 \mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{IV}} + \\
&\quad 2f_3 f_4 \mathcal{T}_{\text{III}} \times \mathcal{T}_{\text{IV}}
\end{aligned} \tag{A9}$$

$$\tilde{t}_1 \tilde{t}_1^* \longrightarrow W^+ H^- \quad [H^+ W^-]$$

I. s-channel h annihilation

II. s-channel H annihilation

III. t-channel $\tilde{b}_{(1,2)}$ exchange

$$\begin{aligned}
f_1 &= C_{\tilde{t}_1 - \tilde{t}_1 - h} C_{h - W^+ - H^-} \\
f_2 &= C_{\tilde{t}_1 - \tilde{t}_1 - H} C_{H - W^+ - H^-} \\
f_3^i &= C_{\tilde{t}_1 - \tilde{b}_i - W^+} C_{\tilde{t}_1 - \tilde{b}_i - H^+} \\
\mathcal{T}_I \times \mathcal{T}_I &= ((s - m_{H^+}^2)^2/m_W^2 - (2s - m_W^2 + 2m_{H^+}^2))/(s - m_h^2)^2 \\
\mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{II}} &= ((s - m_{H^+}^2)^2/m_W^2 - (2s - m_W^2 + 2m_{H^+}^2))/(s - m_H^2)^2 \\
\mathcal{T}_{\text{III}} \times \mathcal{T}_{\text{III}} &= ((m_{\tilde{t}_1}^2 - t)^2/m_W^2 - 2(m_{\tilde{t}_1}^2 + t))/((t - m_{\tilde{b}_i}^2)(t - m_{\tilde{b}_j}^2)) \\
\mathcal{T}_I \times \mathcal{T}_{\text{II}} &= ((s - m_{H^+}^2)^2/m_W^2 - (2s - m_W^2 + 2m_{H^+}^2))/((s - m_h^2)(s - m_H^2)) \\
\mathcal{T}_I \times \mathcal{T}_{\text{III}} &= ((s - m_{H^+}^2)(m_{\tilde{t}_1}^2 - t)/m_W^2 - (m_{\tilde{t}_1}^2 + 2m_{H^+}^2 - u))/((s - m_h^2)(t - m_{\tilde{b}_i}^2)) \\
\mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{III}} &= ((s - m_{H^+}^2)(m_{\tilde{t}_1}^2 - t)/m_W^2 - (m_{\tilde{t}_1}^2 + 2m_{H^+}^2 - u))/((s - m_H^2)(t - m_{\tilde{b}_i}^2)) \\
|\mathcal{T}|^2 &= f_1^2 \mathcal{T}_I \times \mathcal{T}_I + f_2^2 \mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{II}} + \sum_{i=1}^2 \sum_{j=1}^2 f_3^i f_3^j \mathcal{T}_{\text{III}} \times \mathcal{T}_{\text{III}} + 2f_1 f_2 \mathcal{T}_I \times \mathcal{T}_{\text{II}} + \\
&\quad 2 \sum_{i=1}^2 f_1 f_3^i \mathcal{T}_I \times \mathcal{T}_{\text{III}} + 2 \sum_{i=1}^2 f_2 f_3^i \mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{III}}
\end{aligned} \tag{A10}$$

$$\tilde{t}_1 \tilde{t}_1^* \longrightarrow HH \quad [hh] \quad [hH]$$

I. t-channel $\tilde{t}_{(1,2)}$ exchange

II. u-channel $\tilde{t}_{(1,2)}$ exchange

III. point interaction

IV. s-channel h annihilation

V. s-channel H annihilation

$$\begin{aligned}
f_1^i &= C_{\tilde{t}_1-\tilde{t}_i-H}^2 [C_{\tilde{t}_1-\tilde{t}_i-h}^2] [C_{\tilde{t}_1-\tilde{t}_i-h} C_{\tilde{t}_1-\tilde{t}_i-H}] \\
f_2^i &= C_{\tilde{t}_1-\tilde{t}_i-H}^2 [C_{\tilde{t}_1-\tilde{t}_i-h}^2] [C_{\tilde{t}_1-\tilde{t}_i-h} C_{\tilde{t}_1-\tilde{t}_i-H}] \\
f_3 &= C_{\tilde{t}_1-\tilde{t}_1-H-H} [C_{\tilde{t}_1-\tilde{t}_1-h-h}] [C_{\tilde{t}_1-\tilde{t}_1-h-H}] \\
f_4 &= C_{\tilde{t}_1-\tilde{t}_1-h} C_{h-H-H} [C_{\tilde{t}_1-\tilde{t}_1-h} C_{h-h-h}] [C_{\tilde{t}_1-\tilde{t}_1-h} C_{h-h-H}] \\
f_5 &= C_{\tilde{t}_1-\tilde{t}_1-H} C_{H-H-H} [C_{\tilde{t}_1-\tilde{t}_1-H} C_{h-h-H}] [C_{\tilde{t}_1-\tilde{t}_1-H} C_{h-H-H}] \\
\mathcal{T}_I \times \mathcal{T}_I &= 1/((t - m_{\tilde{t}_i}^2)(t - m_{\tilde{t}_j}^2)) \\
\mathcal{T}_{II} \times \mathcal{T}_{II} &= 1/((u - m_{\tilde{t}_i}^2)(u - m_{\tilde{t}_j}^2)) \\
\mathcal{T}_{III} \times \mathcal{T}_{III} &= 1 \\
\mathcal{T}_{IV} \times \mathcal{T}_{IV} &= 1/(s - m_h^2)^2 \\
\mathcal{T}_V \times \mathcal{T}_V &= 1/(s - m_H^2)^2 \\
\mathcal{T}_I \times \mathcal{T}_{II} &= 1/((t - m_{\tilde{t}_i}^2)(u - m_{\tilde{t}_j}^2)) \\
\mathcal{T}_I \times \mathcal{T}_{III} &= -1/(t - m_{\tilde{t}_i}^2) \\
\mathcal{T}_I \times \mathcal{T}_{IV} &= 1/((t - m_{\tilde{t}_i}^2)(s - m_h^2)) \\
\mathcal{T}_I \times \mathcal{T}_V &= 1/((t - m_{\tilde{t}_i}^2)(s - m_H^2)) \\
\mathcal{T}_{II} \times \mathcal{T}_{III} &= -1/(u - m_{\tilde{t}_i}^2) \\
\mathcal{T}_{II} \times \mathcal{T}_{IV} &= 1/((u - m_{\tilde{t}_i}^2)(s - m_h^2)) \\
\mathcal{T}_{II} \times \mathcal{T}_V &= 1/((u - m_{\tilde{t}_i}^2)(s - m_H^2)) \\
\mathcal{T}_{III} \times \mathcal{T}_{IV} &= -1/(s - m_h^2) \\
\mathcal{T}_{III} \times \mathcal{T}_V &= -1/(s - m_H^2) \\
\mathcal{T}_{IV} \times \mathcal{T}_V &= 1/((s - m_H^2)(s - m_h^2)) \\
|\mathcal{T}|^2 &= \sum_{i,j=1}^2 \left(f_1^i f_1^j \mathcal{T}_I \times \mathcal{T}_I + f_2^i f_2^j \mathcal{T}_{II} \times \mathcal{T}_{II} + 2 f_1^i f_2^j \mathcal{T}_I \times \mathcal{T}_{II} \right) + 2 \sum_{i=1}^2 \left(f_1^i f_3 \mathcal{T}_I \times \mathcal{T}_{III} + \right. \\
&\quad \left. f_1^i f_4 \mathcal{T}_I \times \mathcal{T}_{IV} + f_1^i f_5 \mathcal{T}_I \times \mathcal{T}_V + f_2^i f_3 \mathcal{T}_{II} \times \mathcal{T}_{III} + f_2^i f_4 \mathcal{T}_{II} \times \mathcal{T}_{IV} + f_2^i f_5 \mathcal{T}_{II} \times \mathcal{T}_V \right) + \\
&\quad f_3^2 \mathcal{T}_{III} \times \mathcal{T}_{III} + f_4^2 \mathcal{T}_{IV} \times \mathcal{T}_{IV} + f_5^2 \mathcal{T}_V \times \mathcal{T}_V + 2 f_3 f_4 \mathcal{T}_{III} \times \mathcal{T}_{IV} + \\
&\quad 2 f_3 f_5 \mathcal{T}_{III} \times \mathcal{T}_V + 2 f_4 f_5 \mathcal{T}_{IV} \times \mathcal{T}_V
\end{aligned} \tag{A11}$$

$$\tilde{t}_1 \tilde{t}_1^* \longrightarrow AA$$

I. t-channel \tilde{t}_2 exchange

II. u-channel \tilde{t}_2 exchange

III. point interaction

IV. s-channel h annihilation

V. s-channel H annihilation

$$f_1 = C_{\tilde{t}_1 - \tilde{t}_2 - A}^2$$

$$f_2 = C_{\tilde{t}_1 - \tilde{t}_2 - A}^2$$

$$f_3 = C_{\tilde{t}_1 - \tilde{t}_1 - A - A}$$

$$f_4 = C_{\tilde{t}_1 - \tilde{t}_1 - h} C_{h - A - A}$$

$$f_5 = C_{\tilde{t}_1 - \tilde{t}_1 - H} C_{H - A - A}$$

$$\mathcal{T}_I \times \mathcal{T}_I = 1/(t - m_{\tilde{t}_2}^2)^2$$

$$\mathcal{T}_{II} \times \mathcal{T}_{II} = 1/(u - m_{\tilde{t}_2}^2)^2$$

$$\mathcal{T}_{III} \times \mathcal{T}_{III} = 1$$

$$\mathcal{T}_{IV} \times \mathcal{T}_{IV} = 1/(s - m_h^2)^2$$

$$\mathcal{T}_V \times \mathcal{T}_V = 1/(s - m_H^2)^2$$

$$\mathcal{T}_I \times \mathcal{T}_{II} = 1/((t - m_{\tilde{t}_2}^2)(u - m_{\tilde{t}_2}^2))$$

$$\mathcal{T}_I \times \mathcal{T}_{III} = 1/(t - m_{\tilde{t}_2}^2)$$

$$\mathcal{T}_I \times \mathcal{T}_{IV} = -1/((t - m_{\tilde{t}_2}^2)(s - m_h^2))$$

$$\mathcal{T}_I \times \mathcal{T}_V = -1/((t - m_{\tilde{t}_2}^2)(s - m_H^2))$$

$$\mathcal{T}_{II} \times \mathcal{T}_{III} = 1/(u - m_{\tilde{t}_2}^2)$$

$$\mathcal{T}_{II} \times \mathcal{T}_{IV} = -1/((u - m_{\tilde{t}_2}^2)(s - m_h^2))$$

$$\mathcal{T}_{II} \times \mathcal{T}_V = -1/((t - m_{\tilde{t}_2}^2)(s - m_H^2))$$

$$\mathcal{T}_{III} \times \mathcal{T}_{IV} = -1/(s - m_h^2)$$

$$\mathcal{T}_{III} \times \mathcal{T}_V = -1/(s - m_H^2)$$

$$\mathcal{T}_{IV} \times \mathcal{T}_V = 1/((s - m_h^2)(s - m_H^2))$$

$$|\mathcal{T}|^2 = f_1^2 \mathcal{T}_I \times \mathcal{T}_I + f_2^2 \mathcal{T}_{II} \times \mathcal{T}_{II} + f_3^2 \mathcal{T}_{III} \times \mathcal{T}_{III} + f_4^2 \mathcal{T}_{IV} \times \mathcal{T}_{IV} + f_5^2 \mathcal{T}_V \times \mathcal{T}_V +$$

$$\begin{aligned}
& 2f_1f_2\mathcal{T}_I\times\mathcal{T}_{II} + 2f_1f_3\mathcal{T}_I\times\mathcal{T}_{III} + 2f_1f_4\mathcal{T}_I\times\mathcal{T}_{IV} + 2f_1f_5\mathcal{T}_I\times\mathcal{T}_V + \\
& 2f_2f_3\mathcal{T}_{II}\times\mathcal{T}_{III} + 2f_2f_4\mathcal{T}_{II}\times\mathcal{T}_{IV} + 2f_2f_5\mathcal{T}_{II}\times\mathcal{T}_V + 2f_3f_4\mathcal{T}_{III}\times\mathcal{T}_{IV} + \\
& 2f_3f_5\mathcal{T}_{III}\times\mathcal{T}_V + 2f_4f_5\mathcal{T}_{IV}\times\mathcal{T}_V
\end{aligned} \tag{A12}$$

$$\tilde{t}_1\tilde{t}_1^* \longrightarrow hA \quad [HA]$$

I. t-channel \tilde{t}_2 exchange

II. u-channel \tilde{t}_2 exchange

$$\begin{aligned}
f_1 &= C_{\tilde{t}_1-\tilde{t}_2-h} C_{\tilde{t}_1-\tilde{t}_2-A} [C_{\tilde{t}_1-\tilde{t}_2-H} C_{\tilde{t}_1-\tilde{t}_2-A}] \\
f_2 &= C_{\tilde{t}_1-\tilde{t}_2-h} C_{\tilde{t}_1-\tilde{t}_2-A} [C_{\tilde{t}_1-\tilde{t}_2-H} C_{\tilde{t}_1-\tilde{t}_2-A}] \\
\mathcal{T}_I\times\mathcal{T}_I &= 1/(t - m_{\tilde{t}_2}^2)^2 \\
\mathcal{T}_{II}\times\mathcal{T}_{II} &= 1/(u - m_{\tilde{t}_2}^2)^2 \\
\mathcal{T}_I\times\mathcal{T}_{II} &= 1/((t - m_{\tilde{t}_2}^2)(u - m_{\tilde{t}_2}^2)) \\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_I\times\mathcal{T}_I + f_2^2\mathcal{T}_{II}\times\mathcal{T}_{II} + 2f_1f_2\mathcal{T}_I\times\mathcal{T}_{II}
\end{aligned} \tag{A13}$$

$$\tilde{t}_1\tilde{t}_1^* \longrightarrow H^+H^-$$

I. t-channel $\tilde{b}_{(1,2)}$ exchange

II. point interaction

III. s-channel h

IV. s-channel H

$$\begin{aligned}
f_1^i &= (C_{\tilde{t}_1-\tilde{b}_i-H^+})^2 \\
f_2 &= C_{\tilde{t}_1-\tilde{t}_1-H^+-H^-} \\
f_3 &= C_{\tilde{t}_1-\tilde{t}_1-h} C_{h-H^+-H^-} \\
f_4 &= C_{\tilde{t}_1-\tilde{t}_1-H} C_{H-H^+-H^-} \\
\mathcal{T}_I\times\mathcal{T}_I &= 1/((t - m_{\tilde{b}_i}^2)(t - m_{\tilde{b}_j}^2)) \\
\mathcal{T}_{II}\times\mathcal{T}_{II} &= 1 \\
\mathcal{T}_{III}\times\mathcal{T}_{III} &= 1/(s - m_h^2)^2
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}_{\text{IV}} \times \mathcal{T}_{\text{IV}} &= 1/(s - m_H^2)^2 \\
\mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{II}} &= -1/(t - m_{b_i}^2) \\
\mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{III}} &= 1/((s - m_h^2)(t - m_{b_i}^2)) \\
\mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{IV}} &= 1/((s - m_H^2)(t - m_{b_i}^2)) \\
\mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{III}} &= -1/(s - m_h^2) \\
\mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{IV}} &= -1/(s - m_H^2) \\
\mathcal{T}_{\text{III}} \times \mathcal{T}_{\text{IV}} &= 1/((s - m_h^2)(s - m_H^2)) \\
|\mathcal{T}|^2 &= \sum_{i,j=1}^2 f_1^i f_1^j \mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{I}} + f_2^2 \mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{II}} + f_3^2 \mathcal{T}_{\text{III}} \times \mathcal{T}_{\text{III}} + f_4^2 \mathcal{T}_{\text{IV}} \times \mathcal{T}_{\text{IV}} + 2 \sum_{i=1}^2 f_1^i f_2 \mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{II}} + \\
& 2 \sum_{i=1}^2 f_1^i f_3 \mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{III}} + 2 \sum_{i=1}^2 f_1^i f_4 \mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{IV}} + 2 f_2 f_3 \mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{III}} + 2 f_2 f_4 \mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{IV}} + \\
& 2 f_3 f_4 \mathcal{T}_{\text{III}} \times \mathcal{T}_{\text{IV}} \tag{A14}
\end{aligned}$$

$$\tilde{t}_1 \tilde{t}_1 \longrightarrow tt$$

I. t-channel gluino exchange

II. u-channel gluino exchange

III. t-channel $\chi_{(1,2,3,4)}^0$ exchange

IV. u-channel $\chi_{(1,2,3,4)}^0$ exchange

$$\begin{aligned}
f_{1L} &= f_{2L} \equiv f_L = (C_{\tilde{t}_1-t-\tilde{g}}^L)^2 \\
f_{1R} &= f_{2R} \equiv f_R = (C_{\tilde{t}_1-t-\tilde{g}}^R)^2 \\
J_i &= (C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^L + C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^R)/2 \\
K_i &= (C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^L - C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^R)/2 \\
\mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{I}} &= (2/9)((f_L^2 + f_R^2)m_{\tilde{g}}^2(s - 2m_t^2) - f_L f_R(4m_t^2 m_{\tilde{g}}^2 + 2st + 2(m_{\tilde{t}_1}^2 - m_t^2 - t)^2) \\
& + \sqrt{f_L f_R} (f_L + f_R) 4 m_t m_{\tilde{g}}(t + m_t^2 - m_{\tilde{t}_1}^2))/(t - m_{\tilde{g}}^2)^2 \\
\mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{II}} &= (2/9)((f_L^2 + f_R^2)m_{\tilde{g}}^2(s - 2m_t^2) - f_L f_R(4m_t^2 m_{\tilde{g}}^2 + 2su + 2(m_{\tilde{t}_1}^2 - m_t^2 - u)^2) \\
& + \sqrt{f_L f_R} (f_L + f_R) 4 m_t m_{\tilde{g}}(u + m_t^2 - m_{\tilde{t}_1}^2))/(u - m_{\tilde{g}}^2)^2 \\
\mathcal{T}_{\text{III}} \times \mathcal{T}_{\text{III}} &= (-2(4 J_i J_j K_i K_j m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s + K_i^2 (J_j^2 (-m_{\tilde{t}_1}^4 - 2m_{\tilde{t}_1}^2 m_t^2 + 3m_t^4 - 4m_t^3 m_{\tilde{\chi}_i^0} \\
& + 4m_t^3 m_{\tilde{\chi}_j^0} - 4m_t^2 m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} + m_{\tilde{t}_1}^2 s - m_t^2 s + m_t m_{\tilde{\chi}_i^0} s - m_t m_{\tilde{\chi}_j^0} s + m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s
\end{aligned}$$

$$\begin{aligned}
& +m_{\tilde{t}_1}^2 t + 3m_t^2 t - m_t m_{\tilde{\chi}_i^0} t + m_t m_{\tilde{\chi}_j^0} t + m_{\tilde{t}_1}^2 u - m_t^2 u + m_t m_{\tilde{\chi}_i^0} u - m_t m_{\tilde{\chi}_j^0} u \\
& -tu) + K_j^2(m_{\tilde{t}_1}^4 + 2m_{\tilde{t}_1}^2 m_t^2 - 3m_t^4 + 4m_t^3 m_{\tilde{\chi}_i^0} + 4m_t^3 m_{\tilde{\chi}_j^0} - 4m_t^2 m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} \\
& -m_{\tilde{t}_1}^2 s + m_t^2 s - m_t m_{\tilde{\chi}_i^0} s - m_t m_{\tilde{\chi}_j^0} s + m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s - m_{\tilde{t}_1}^2 t - 3m_t^2 t + m_t m_{\tilde{\chi}_i^0} t \\
& +m_t m_{\tilde{\chi}_j^0} t - m_{\tilde{t}_1}^2 u + m_t^2 u - m_t m_{\tilde{\chi}_i^0} u - m_t m_{\tilde{\chi}_j^0} u + tu)) + J_i^2(K_j^2(-m_{\tilde{t}_1}^4 \\
& -2m_{\tilde{t}_1}^2 m_t^2 + 3m_t^4 + 4m_t^3 m_{\tilde{\chi}_i^0} - 4m_t^3 m_{\tilde{\chi}_j^0} - 4m_t^2 m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} + m_{\tilde{t}_1}^2 s - m_t^2 s \\
& -m_t m_{\tilde{\chi}_i^0} s + m_t m_{\tilde{\chi}_j^0} s + m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s + m_{\tilde{t}_1}^2 t + 3m_t^2 t + m_t m_{\tilde{\chi}_i^0} t - m_t m_{\tilde{\chi}_j^0} t \\
& +m_{\tilde{t}_1}^2 u - m_t^2 u - m_t m_{\tilde{\chi}_i^0} u + m_t m_{\tilde{\chi}_j^0} u - tu) + J_j^2(m_{\tilde{t}_1}^4 + 2m_{\tilde{t}_1}^2 m_t^2 - 3m_t^4 \\
& -4m_t^3 m_{\tilde{\chi}_i^0} - 4m_t^3 m_{\tilde{\chi}_j^0} - 4m_t^2 m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} - m_{\tilde{t}_1}^2 s + m_t^2 s + m_t m_{\tilde{\chi}_i^0} s + m_t m_{\tilde{\chi}_j^0} s \\
& +m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s - m_{\tilde{t}_1}^2 t - 3m_t^2 t - m_t m_{\tilde{\chi}_i^0} t - m_t m_{\tilde{\chi}_j^0} t - m_{\tilde{t}_1}^2 u + m_t^2 u \\
& +m_t m_{\tilde{\chi}_i^0} u + m_t m_{\tilde{\chi}_j^0} u + tu)))/((m_{\tilde{\chi}_i^0}^2 - t)(-m_{\tilde{\chi}_j^0}^2 + t))
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}_{\text{IV}} \times \mathcal{T}_{\text{IV}} = & (-2(4 J_i J_j K_i K_j m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s + J_i^2(K_j^2(-m_{\tilde{t}_1}^4 - 2m_{\tilde{t}_1}^2 m_t^2 + 3m_t^4 + 4m_t^3 m_{\tilde{\chi}_i^0} \\
& -4m_t^3 m_{\tilde{\chi}_j^0} - 4m_t^2 m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} + m_{\tilde{t}_1}^2 s - m_t^2 s - m_t m_{\tilde{\chi}_i^0} s + m_t m_{\tilde{\chi}_j^0} s + m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s \\
& +m_{\tilde{t}_1}^2 t - m_t^2 t - m_t m_{\tilde{\chi}_i^0} t + m_t m_{\tilde{\chi}_j^0} t + m_{\tilde{t}_1}^2 u + 3m_t^2 u + m_t m_{\tilde{\chi}_i^0} u - m_t m_{\tilde{\chi}_j^0} u - tu) \\
& +J_j^2(m_{\tilde{t}_1}^4 + 2m_{\tilde{t}_1}^2 m_t^2 - 3m_t^4 - 4m_t^3 m_{\tilde{\chi}_i^0} - 4m_t^3 m_{\tilde{\chi}_j^0} - 4m_t^2 m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} - m_{\tilde{t}_1}^2 s \\
& +m_t^2 s + m_t m_{\tilde{\chi}_i^0} s + m_t m_{\tilde{\chi}_j^0} s + m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s - m_{\tilde{t}_1}^2 t + m_t^2 t + m_t m_{\tilde{\chi}_i^0} t + m_t m_{\tilde{\chi}_j^0} t \\
& -m_{\tilde{t}_1}^2 u - 3m_t^2 u - m_t m_{\tilde{\chi}_i^0} u - m_t m_{\tilde{\chi}_j^0} u + tu)) + K_i^2(J_j^2(-m_{\tilde{t}_1}^4 - 2m_{\tilde{t}_1}^2 m_t^2 \\
& +3m_t^4 - 4m_t^3 m_{\tilde{\chi}_i^0} + 4m_t^3 m_{\tilde{\chi}_j^0} - 4m_t^2 m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} + m_{\tilde{t}_1}^2 s - m_t^2 s + m_t m_{\tilde{\chi}_i^0} s \\
& -m_t m_{\tilde{\chi}_j^0} s + m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s + m_{\tilde{t}_1}^2 t - m_t^2 t + m_t m_{\tilde{\chi}_i^0} t - m_t m_{\tilde{\chi}_j^0} t + m_{\tilde{t}_1}^2 u + 3m_t^2 u \\
& -m_t m_{\tilde{\chi}_i^0} u + m_t m_{\tilde{\chi}_j^0} u - tu) + K_j^2(m_{\tilde{t}_1}^4 + 2m_{\tilde{t}_1}^2 m_t^2 - 3m_t^4 + 4m_t^3 m_{\tilde{\chi}_i^0} \\
& +4m_t^3 m_{\tilde{\chi}_j^0} - 4m_t^2 m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} - m_{\tilde{t}_1}^2 s + m_t^2 s - m_t m_{\tilde{\chi}_i^0} s - m_t m_{\tilde{\chi}_j^0} s + m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s \\
& -m_{\tilde{t}_1}^2 t + m_t^2 t - m_t m_{\tilde{\chi}_i^0} t - m_t m_{\tilde{\chi}_j^0} t - m_{\tilde{t}_1}^2 u - 3m_t^2 u + m_t m_{\tilde{\chi}_i^0} u + m_t m_{\tilde{\chi}_j^0} u \\
& +tu)))/((m_{\tilde{\chi}_i^0}^2 - u)(-m_{\tilde{\chi}_j^0}^2 + u))
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{II}} = & (-2/27)(f_L f_R (2(m_{\tilde{t}_1}^2 - m_t^2 - u)(m_{\tilde{t}_1}^2 - m_t^2 - t) - 4(s - 2m_t^2)(m_{\tilde{t}_1}^2 - m_t^2)) \\
& +\sqrt{f_L f_R} (f_L + f_R) m_t m_{\tilde{g}} (t - u) + (f_L^2 + f_R^2) m_{\tilde{g}}^2 (s - 2m_t^2) - 4m_t^2 t f_L f_R \\
& -m_t m_{\tilde{g}} \sqrt{f_L f_R} (f_L + f_R) (t + u) - 4m_t^2 m_{\tilde{g}}^2 f_L f_R) / ((t - m_{\tilde{g}}^2)(u - m_{\tilde{g}}^2))
\end{aligned}$$

$$\mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{III}} = 0$$

$$\mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{IV}} = 0$$

$$\mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{III}} = 0$$

$$\mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{IV}} = 0$$

$$\begin{aligned} \mathcal{T}_{\text{III}} \times \mathcal{T}_{\text{IV}} = & (8 J_i J_j K_i K_j m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s + J_i^2 (-(K_j^2 (2m_{\tilde{t}_1}^4 + 4m_{\tilde{t}_1}^2 m_t^2 - 6m_t^4 - 8m_t^3 m_{\tilde{\chi}_i^0} \\ & + 8m_t^3 m_{\tilde{\chi}_j^0} + 8m_t^2 m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} + 2m_{\tilde{t}_1}^2 s + 2m_t^2 s + 2m_t m_{\tilde{\chi}_i^0} s - 2m_t m_{\tilde{\chi}_j^0} s \\ & - 2m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s - s^2 - 2m_{\tilde{t}_1}^2 t - 2m_t^2 t + 2m_t m_{\tilde{\chi}_i^0} t + 2m_t m_{\tilde{\chi}_j^0} t + t^2 - 2m_{\tilde{t}_1}^2 u \\ & - 2m_t^2 u - 2m_t m_{\tilde{\chi}_i^0} u - 2m_t m_{\tilde{\chi}_j^0} u + u^2)) + J_j^2 (2m_{\tilde{t}_1}^4 + 4m_{\tilde{t}_1}^2 m_t^2 - 6m_t^4 \\ & - 8m_t^3 m_{\tilde{\chi}_i^0} - 8m_t^3 m_{\tilde{\chi}_j^0} - 8m_t^2 m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} + 2m_{\tilde{t}_1}^2 s + 2m_t^2 s + 2m_t m_{\tilde{\chi}_i^0} s + 2m_t m_{\tilde{\chi}_j^0} s \\ & + 2m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s - s^2 - 2m_{\tilde{t}_1}^2 t - 2m_t^2 t + 2m_t m_{\tilde{\chi}_i^0} t - 2m_t m_{\tilde{\chi}_j^0} t + t^2 - 2m_{\tilde{t}_1}^2 u \\ & - 2m_t^2 u - 2m_t m_{\tilde{\chi}_i^0} u + 2m_t m_{\tilde{\chi}_j^0} u + u^2)) + K_i^2 (K_j^2 (2m_{\tilde{t}_1}^4 + 4m_{\tilde{t}_1}^2 m_t^2 - 6m_t^4 \\ & + 8m_t^3 m_{\tilde{\chi}_i^0} + 8m_t^3 m_{\tilde{\chi}_j^0} - 8m_t^2 m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} + 2m_{\tilde{t}_1}^2 s + 2m_t^2 s - 2m_t m_{\tilde{\chi}_i^0} s - 2m_t m_{\tilde{\chi}_j^0} s \\ & + 2m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s - s^2 - 2m_{\tilde{t}_1}^2 t - 2m_t^2 t - 2m_t m_{\tilde{\chi}_i^0} t + 2m_t m_{\tilde{\chi}_j^0} t + t^2 - 2m_{\tilde{t}_1}^2 u \\ & - 2m_t^2 u + 2m_t m_{\tilde{\chi}_i^0} u - 2m_t m_{\tilde{\chi}_j^0} u + u^2) - J_j^2 (2m_{\tilde{t}_1}^4 + 4m_{\tilde{t}_1}^2 m_t^2 - 6m_t^4 \\ & + 8m_t^3 m_{\tilde{\chi}_i^0} - 8m_t^3 m_{\tilde{\chi}_j^0} + 8m_t^2 m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} + 2m_{\tilde{t}_1}^2 s + 2m_t^2 s - 2m_t m_{\tilde{\chi}_i^0} s + 2m_t m_{\tilde{\chi}_j^0} s \\ & - 2m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s - s^2 - 2m_{\tilde{t}_1}^2 t - 2m_t^2 t - 2m_t m_{\tilde{\chi}_i^0} t - 2m_t m_{\tilde{\chi}_j^0} t + t^2 - 2m_{\tilde{t}_1}^2 u \\ & - 2m_t^2 u + 2m_t m_{\tilde{\chi}_i^0} u + 2m_t m_{\tilde{\chi}_j^0} u + u^2)))/((m_{\tilde{\chi}_i^0}^2 - t)(m_{\tilde{\chi}_j^0}^2 - u)) \end{aligned}$$

$$\begin{aligned} |\mathcal{T}|^2 = & \mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{I}} + \mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{II}} + \sum_{i,j=1}^4 (\mathcal{T}_{\text{III}} \times \mathcal{T}_{\text{III}} + \mathcal{T}_{\text{IV}} \times \mathcal{T}_{\text{IV}}) + 2\mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{II}} + \\ & 2 \sum_{i=1}^4 (\mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{III}} + \mathcal{T}_{\text{I}} \times \mathcal{T}_{\text{IV}} + \mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{III}} + \mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{IV}}) + 2 \sum_{i,j=1}^4 \mathcal{T}_{\text{III}} \times \mathcal{T}_{\text{IV}} \end{aligned} \quad (\text{A15})$$

$$\tilde{\chi} \tilde{t}_1 \longrightarrow tg$$

I. s-channel t annihilation

II. t-channel \tilde{t}_1 exchange

$$\begin{aligned} f_{1L} &= C_{\tilde{t}_1-t-\chi_1^0}^L C_{t-t-g} \\ f_{1R} &= C_{\tilde{t}_1-t-\chi_1^0}^R C_{t-t-g} \\ f_{2L} &= C_{\tilde{t}_1-t-\chi_1^0}^L C_{\tilde{t}_1-\tilde{t}_1-g} \\ f_{2R} &= C_{\tilde{t}_1-t-\chi_1^0}^R C_{\tilde{t}_1-\tilde{t}_1-g} \end{aligned}$$

$$\begin{aligned}
\mathcal{T}_I \times \mathcal{T}_I &= (-4/6)((f_{1L}^2 + f_{1R}^2)2((3m_t^2 - s)(s + m_{\tilde{\chi}}^2 - m_{\tilde{t}_1}^2) + (s - m_t^2)(m_{\tilde{\chi}}^2 + m_t^2 - t)) \\
&\quad - 8f_{1L}f_{1R}m_t m_{\tilde{\chi}}(s + m_t^2))/(s - m_t^2)^2 \\
\mathcal{T}_{II} \times \mathcal{T}_{II} &= (-4/6)(2(t + m_{\tilde{t}_1}^2)((f_{2L}^2 + f_{2R}^2)(m_{\tilde{\chi}}^2 + m_t^2 - t) - 4m_{\tilde{\chi}}m_t f_{2L}f_{2R}))/ (t - m_{\tilde{t}_1}^2)^2 \\
\mathcal{T}_I \times \mathcal{T}_{II} &= (-4/6)((f_{1L}f_{2L} + f_{1R}f_{2R})/2((m_{\tilde{\chi}}^2 + m_t^2 - t)(s - 2m_{\tilde{\chi}}^2 + 2m_{\tilde{t}_1}^2 + m_t^2) \\
&\quad - (s + m_t^2)(2s - 3m_{\tilde{\chi}}^2 - 2m_{\tilde{t}_1}^2 + u) + (2m_{\tilde{t}_1}^2 + 3m_t^2 - 2u - s)(s + m_{\tilde{\chi}}^2 - m_{\tilde{t}_1}^2) \\
&\quad + 2m_t^2(2s - 3m_{\tilde{\chi}}^2 - 2m_{\tilde{t}_1}^2 + u)) - (f_{1L}f_{2R} + f_{1R}f_{2L})m_t m_{\tilde{\chi}}(4m_{\tilde{t}_1}^2 + 4m_t^2 \\
&\quad - 2m_{\tilde{\chi}}^2 - 2u))/((s - m_t^2)(t - m_{\tilde{t}_1}^2)) \\
|\mathcal{T}|^2 &= \mathcal{T}_I \times \mathcal{T}_I + \mathcal{T}_{II} \times \mathcal{T}_{II} + 2\mathcal{T}_I \times \mathcal{T}_{II} \tag{A16}
\end{aligned}$$

$$\tilde{\chi}\tilde{t}_1 \longrightarrow tZ$$

I. u-channel $\tilde{\chi}_{(1,2,3,4)}^0$ exchange

$$\begin{aligned}
f_{LL}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^L C_{\tilde{\chi}_1^0-\tilde{\chi}_i^0-Z}^L \\
f_{LR}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^L C_{\tilde{\chi}_1^0-\tilde{\chi}_i^0-Z}^R \\
f_{RL}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^R C_{\tilde{\chi}_1^0-\tilde{\chi}_i^0-Z}^L \\
f_{RR}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^R C_{\tilde{\chi}_1^0-\tilde{\chi}_i^0-Z}^R \\
\mathcal{T}_I \times \mathcal{T}_I &= (1/2)((f_{LL}^i f_{LL}^j + f_{RR}^i f_{RR}^j)((u - m_{\tilde{t}_1}^2 + m_t^2)(u + m_{\tilde{\chi}}^2 - m_Z^2) - (m_{\tilde{\chi}}^2 + m_t^2 - t)u) \\
&\quad + ((m_{\tilde{\chi}}^2 + m_Z^2 - u)/m_Z^2)((m_{\tilde{\chi}}^2 - m_Z^2 - u)(u - m_{\tilde{t}_1}^2 + m_t^2) - (s - m_t^2 - m_Z^2)u)) \\
&\quad + (f_{LR}^i f_{RR}^j + f_{RL}^i f_{LL}^j)m_t m_{\tilde{\chi}_i^0}((u + m_{\tilde{\chi}}^2 - m_Z^2) + (1/m_Z^2)((m_{\tilde{\chi}}^2 + m_Z^2 - u) \\
&\quad \times (m_{\tilde{\chi}}^2 - m_Z^2 - u))) + (f_{LL}^i f_{RL}^j + f_{RR}^i f_{LR}^j)m_t m_{\tilde{\chi}_j^0}((u + m_{\tilde{\chi}}^2 - m_Z^2) \\
&\quad + (1/m_Z^2)((m_{\tilde{\chi}}^2 + m_Z^2 - u)(m_{\tilde{\chi}}^2 - m_Z^2 - u))) \\
&\quad + (f_{LR}^i f_{LR}^j + f_{RL}^i f_{RL}^j)m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}((t + m_{\tilde{\chi}}^2 - m_t^2) \\
&\quad + (1/m_Z^2)((m_{\tilde{\chi}}^2 + m_Z^2 - u)(s - m_t^2 - m_Z^2))) \\
&\quad + (f_{LL}^i f_{RR}^j + f_{RR}^i f_{LL}^j)6m_t m_{\tilde{\chi}}u + (f_{LL}^i f_{LR}^j + f_{RR}^i f_{RL}^j)3m_{\tilde{\chi}_j^0}(u + m_t^2 - m_{\tilde{t}_1}^2)m_{\tilde{\chi}} \\
&\quad + (f_{LR}^i f_{LL}^j + f_{RL}^i f_{RR}^j)3m_{\tilde{\chi}_i^0}(u + m_t^2 - m_{\tilde{t}_1}^2)m_{\tilde{\chi}} \\
&\quad + (f_{LR}^i f_{RL}^j + f_{RL}^i f_{LR}^j)6m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}m_t m_{\tilde{\chi}})/((t - m_{\tilde{\chi}_i^0}^2)(t - m_{\tilde{\chi}_j^0}^2)) \\
|\mathcal{T}|^2 &= \sum_{i,j=1}^4 \mathcal{T}_I \times \mathcal{T}_I \tag{A17}
\end{aligned}$$

$$\tilde{\chi} \tilde{t}_1 \longrightarrow bW^+$$

I. u-channel $\tilde{\chi}_{(1,2)}^+$ exchange

$$\begin{aligned}
f_{LL}^i &= C_{\tilde{t}_1-b-\tilde{\chi}_i^+}^L C_{\tilde{\chi}_1^0-\tilde{\chi}_i^- -W^+}^L \\
f_{LR}^i &= C_{\tilde{t}_1-b-\tilde{\chi}_i^+}^L C_{\tilde{\chi}_1^0-\tilde{\chi}_i^- -W^+}^R \\
f_{RL}^i &= C_{\tilde{t}_1-b-\tilde{\chi}_i^+}^R C_{\tilde{\chi}_1^0-\tilde{\chi}_i^- -W^+}^L \\
f_{RR}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^+}^R C_{\tilde{\chi}_1^0-\tilde{\chi}_i^- -W^+}^R \\
\mathcal{T}_I \times \mathcal{T}_I &= (1/2)((f_{LL}^i f_{LL}^j + f_{RR}^i f_{RR}^j)((u - m_{\tilde{t}_1}^2 + m_b^2)(u + m_{\tilde{\chi}}^2 - m_W^2) - (m_{\tilde{\chi}}^2 + m_b^2 - t)u) \\
&\quad + ((m_{\tilde{\chi}}^2 + m_W^2 - u)/m_W^2)((m_{\tilde{\chi}}^2 - m_W^2 - u)(u - m_{\tilde{t}_1}^2 + m_b^2) - (s - m_b^2 - m_W^2)u)) \\
&\quad + (f_{LR}^i f_{RR}^j + f_{RL}^i f_{LL}^j) m_b m_{\tilde{\chi}_i^+} ((u + m_{\tilde{\chi}}^2 - m_W^2) \\
&\quad + (1/m_W^2)((m_{\tilde{\chi}}^2 + m_W^2 - u)(m_{\tilde{\chi}}^2 - m_W^2 - u))) \\
&\quad + (f_{LL}^i f_{RL}^j + f_{RR}^i f_{LR}^j) m_b m_{\tilde{\chi}_j^+} ((u + m_{\tilde{\chi}}^2 - m_W^2) \\
&\quad + (1/m_W^2)((m_{\tilde{\chi}}^2 + m_W^2 - u)(m_{\tilde{\chi}}^2 - m_W^2 - u))) \\
&\quad + (f_{LR}^i f_{LR}^j + f_{RL}^i f_{RL}^j) m_{\tilde{\chi}_i^+} m_{\tilde{\chi}_j^+} ((t + m_{\tilde{\chi}}^2 - m_b^2) \\
&\quad + (1/m_W^2)((m_{\tilde{\chi}}^2 + m_W^2 - u)(s - m_b^2 - m_W^2))) \\
&\quad + (f_{LL}^i f_{RR}^j + f_{RR}^i f_{LL}^j) 6m_b m_{\tilde{\chi}} u + (f_{LL}^i f_{LR}^j + f_{RR}^i f_{RL}^j) 3m_{\tilde{\chi}_j^+} (u + m_b^2 - m_{\tilde{t}_1}^2) m_{\tilde{\chi}} \\
&\quad + (f_{LR}^i f_{LL}^j + f_{RL}^i f_{RR}^j) 3m_{\tilde{\chi}_i^+} (u + m_b^2 - m_{\tilde{t}_1}^2) m_{\tilde{\chi}} \\
&\quad + (f_{LR}^i f_{RL}^j + f_{RL}^i f_{LR}^j) 6m_{\tilde{\chi}_i^+} m_{\tilde{\chi}_j^+} m_b m_{\tilde{\chi}} / ((t - m_{\tilde{\chi}_i^+}^2)(t - m_{\tilde{\chi}_j^+}^2)) \\
|\mathcal{T}|^2 &= \sum_{i,j=1}^2 \mathcal{T}_I \times \mathcal{T}_I \tag{A18}
\end{aligned}$$

$$\tilde{\chi} \tilde{t}_1 \longrightarrow tH \quad [th]$$

I. s-channel t annihilation

II. t-channel $\tilde{t}_{(1,2)}$ exchange

III. u-channel $\chi_{(1,2,3,4)}^0$ exchange

$$\begin{aligned}
f_{1L} &= C_{\tilde{t}_1-t-\tilde{\chi}_1^0}^L C_{t-t-H} \quad [C_{\tilde{t}_1-t-\tilde{\chi}_1^0}^L C_{t-t-h}] \\
f_{1R} &= C_{\tilde{t}_1-t-\tilde{\chi}_1^0}^R C_{t-t-H} \quad [C_{\tilde{t}_1-t-\tilde{\chi}_1^0}^R C_{t-t-h}]
\end{aligned}$$

$$\begin{aligned}
f_{2L}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_1}^L C_{\tilde{t}_1-\tilde{t}_1-H} [C_{\tilde{t}_1-t-\tilde{\chi}_1}^L C_{\tilde{t}_1-\tilde{t}_1-h}] \\
f_{2R}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_1}^R C_{\tilde{t}_1-\tilde{t}_1-H} [C_{\tilde{t}_1-t-\tilde{\chi}_1}^R C_{\tilde{t}_1-\tilde{t}_1-h}] \\
f_{3LL}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i}^L C_{\tilde{\chi}_1^0-\tilde{\chi}_i^0-H} [C_{\tilde{t}_1-t-\tilde{\chi}_i}^L C_{\tilde{\chi}_1^0-\tilde{\chi}_i^0-h}] \\
f_{3LR}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i}^L C_{\tilde{\chi}_1^0-\tilde{\chi}_i^0-H} [C_{\tilde{t}_1-t-\tilde{\chi}_i}^L C_{\tilde{\chi}_1^0-\tilde{\chi}_i^0-h}] \\
f_{3RL}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i}^R C_{\tilde{\chi}_1^0-\tilde{\chi}_i^0-H} [C_{\tilde{t}_1-t-\tilde{\chi}_i}^R C_{\tilde{\chi}_1^0-\tilde{\chi}_i^0-h}] \\
f_{3RR}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i}^R C_{\tilde{\chi}_1^0-\tilde{\chi}_i^0-H} [C_{\tilde{t}_1-t-\tilde{\chi}_i}^R C_{\tilde{\chi}_1^0-\tilde{\chi}_i^0-h}] \\
\mathcal{T}_I \times \mathcal{T}_I &= (1/2)((f_{1L}^2 + f_{1R}^2)((s + m_t^2 - m_{H[h]}^2)(s + m_{\tilde{\chi}}^2 - m_{\tilde{t}_1}^2) - (s - m_t^2)(m_{\tilde{\chi}}^2 + m_{\tilde{t}_1}^2 - t) + 2m_t^2(s + m_{\tilde{\chi}}^2 - m_{\tilde{t}_1}^2)) - f_{1L}f_{1R}4m_{\tilde{\chi}}m_t(2s + 2m_t^2 - m_{H[h]}^2))/(s - m_t^2)^2 \\
\mathcal{T}_{II} \times \mathcal{T}_{II} &= (1/2)((f_{2L}^i f_{2L}^j + f_{2R}^i f_{2R}^j)(m_{\tilde{\chi}}^2 + m_{\tilde{t}_1}^2 - t) - 2m_t m_{\tilde{\chi}}(f_{2L}^i f_{2R}^j + f_{2R}^i f_{2L}^j)) / ((t - m_{\tilde{t}_1}^2)(t - m_{\tilde{t}_j}^2)) \\
\mathcal{T}_{III} \times \mathcal{T}_{III} &= (1/2)((f_{3LR}^i f_{3LR}^j + f_{3RL}^i f_{3RL}^j)((m_{\tilde{t}_1}^2 - m_{\tilde{t}_1}^2 + u)(m_{\tilde{\chi}}^2 - m_{H[h]}^2 + u) - u(m_{\tilde{\chi}}^2 + m_{\tilde{t}_1}^2 - t)) + (f_{3LL}^i f_{3LL}^j + f_{3RR}^i f_{3RR}^j)m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}(m_{\tilde{\chi}}^2 + m_{\tilde{t}_1}^2 - t) - (f_{3LR}^i f_{3LL}^j + f_{3RL}^i f_{3RR}^j)m_{\tilde{\chi}_j^0}(m_{\tilde{t}_1}^2 - m_{\tilde{t}_1}^2 + u) - (f_{3LL}^i f_{3LR}^j + f_{3RR}^i f_{3RL}^j) \times m_{\tilde{\chi}_i^0}m_{\tilde{\chi}}(m_{\tilde{t}_1}^2 - m_{\tilde{t}_1}^2 + u) + (f_{3RL}^i f_{3LL}^j + f_{3LR}^i f_{3RR}^j)m_t m_{\tilde{\chi}_j^0}(m_{\tilde{\chi}}^2 - m_{H[h]}^2 + u) + (f_{3RR}^i f_{3LR}^j + f_{3LL}^i f_{3RL}^j)m_t m_{\tilde{\chi}_i^0}(m_{\tilde{\chi}}^2 - m_{H[h]}^2 + u) - (f_{3RL}^i f_{3LR}^j + f_{3LR}^i f_{3RL}^j)2m_t m_{\tilde{\chi}}u - (f_{3RR}^i f_{3LL}^j + f_{3LL}^i f_{3RR}^j)2m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}m_t m_{\tilde{\chi}}) / ((u - m_{\tilde{\chi}_i^0}^2)(u - m_{\tilde{\chi}_j^0}^2)) \\
\mathcal{T}_I \times \mathcal{T}_{II} &= (1/2)((f_{1L}f_{2L}^i + f_{1R}f_{2R}^i)m_t(s + 2m_{\tilde{\chi}}^2 + m_{\tilde{t}_1}^2 - m_{\tilde{t}_1}^2 - t) - (f_{1R}f_{2L}^i + f_{1L}f_{2R}^i)m_{\tilde{\chi}}(s + 3m_{\tilde{t}_1}^2 - m_{H[h]}^2))/((s - m_{\tilde{t}_1}^2)(t - m_{\tilde{t}_i}^2)) \\
\mathcal{T}_I \times \mathcal{T}_{III} &= (1/2)((f_{1L}f_{3RL}^i + f_{1R}f_{3LR}^i)((s + m_{\tilde{\chi}}^2 - m_{\tilde{t}_1}^2)(u + m_{\tilde{t}_1}^2 - m_{\tilde{t}_1}^2) - (m_{\tilde{\chi}}^2 + m_{\tilde{t}_1}^2 - t)(m_{\tilde{t}_1}^2 + m_{\tilde{\chi}}^2 - m_{\tilde{t}_1}^2 - m_{H[h]}^2) + (u + m_{\tilde{\chi}}^2 - m_{H[h]}^2)(s + m_{\tilde{t}_1}^2 - m_{H[h]}^2) + 2m_t(u + m_{\tilde{\chi}}^2 - m_{H[h]}^2)) + (f_{1L}f_{3LL}^i + f_{1R}f_{3RR}^i)m_{\tilde{\chi}_i^0}m_t(s + 2m_{\tilde{\chi}}^2 + m_{\tilde{t}_1}^2 - m_{\tilde{t}_1}^2 - t) - (f_{1L}f_{3LR}^i + f_{1R}f_{3RL}^i)m_{\tilde{\chi}}m_t(u + 2m_{\tilde{t}_1}^2 + m_{\tilde{\chi}}^2 - 2m_{\tilde{t}_1}^2 - m_{H[h]}^2) - (f_{1L}f_{3RR}^i + f_{1R}f_{3LL}^i)m_{\tilde{\chi}}m_{\tilde{\chi}_i^0}(s + 3m_{\tilde{t}_1}^2 - m_{H[h]}^2))/((s - m_{\tilde{t}_1}^2)(u - m_{\tilde{\chi}_i^0}^2)) \\
\mathcal{T}_{II} \times \mathcal{T}_{III} &= (1/2)((f_{2R}^j f_{3LR}^i + f_{2L}^j f_{3RL}^i)m_t(u + m_{\tilde{\chi}}^2 - m_{H[h]}^2) + (f_{2L}^j f_{3LL}^i + f_{2R}^j f_{3RR}^i) \times m_{\tilde{\chi}_i^0}(m_{\tilde{\chi}}^2 + m_{\tilde{t}_1}^2 - t) - (f_{2L}^j f_{3LR}^i + f_{2R}^j f_{3RL}^i)m_{\tilde{\chi}}(u + m_{\tilde{t}_1}^2 - m_{\tilde{t}_1}^2) - (f_{2R}^j f_{3LL}^i + f_{2L}^j f_{3RR}^i)2m_{\tilde{\chi}}m_t m_{\tilde{\chi}_i^0})/((t - m_{\tilde{t}_j}^2)(u - m_{\tilde{\chi}_i^0}^2))
\end{aligned}$$

$$\begin{aligned}
|\mathcal{T}|^2 &= \mathcal{T}_I \times \mathcal{T}_I + \sum_{i,j=1}^2 \mathcal{T}_{II} \times \mathcal{T}_{II} + \sum_{i,j=1}^4 \mathcal{T}_{III} \times \mathcal{T}_{III} + 2 \sum_{i=1}^2 \mathcal{T}_I \times \mathcal{T}_{II} + 2 \sum_{i=1}^4 \mathcal{T}_I \times \mathcal{T}_{III} + \\
&2 \sum_{j=1}^2 \sum_{i=1}^4 \mathcal{T}_{II} \times \mathcal{T}_{III}
\end{aligned} \tag{A19}$$

$$\tilde{\chi} \tilde{t}_1 \longrightarrow tA$$

I. s-channel t annihilation

II. t-channel \tilde{t}_2 exchange

III. u-channel $\chi_{(1,2,3,4)}^0$ exchange

$$\begin{aligned}
f_{1L} &= C_{\tilde{t}_1-t-\tilde{\chi}_1^0}^L C_{t-t-A} \\
f_{1R} &= C_{\tilde{t}_1-t-\tilde{\chi}_1^0}^R C_{t-t-A} \\
f_{2L} &= C_{\tilde{t}_2-t-\tilde{\chi}_1^0}^L C_{\tilde{t}_1-\tilde{t}_2-A} \\
f_{2R} &= C_{\tilde{t}_2-t-\tilde{\chi}_1^0}^R C_{\tilde{t}_1-\tilde{t}_2-A} \\
f_{3LL}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^L C_{\tilde{\chi}_1^0-\tilde{\chi}_i^0-A} \\
f_{3LR}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^L C_{\tilde{\chi}_1^0-\tilde{\chi}_i^0-A} \\
f_{3RL}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^R C_{\tilde{\chi}_1^0-\tilde{\chi}_i^0-A} \\
f_{3RR}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^R C_{\tilde{\chi}_1^0-\tilde{\chi}_i^0-A} \\
\mathcal{T}_I \times \mathcal{T}_I &= (1/2)((f_{1L}^2 + f_{1R}^2)((s - m_t^2 - m_A^2)(s + m_{\tilde{\chi}}^2 - m_{\tilde{t}_1}^2) - (s - m_t^2)(m_{\tilde{\chi}}^2 + m_t^2 - t)) \\
&\quad - f_{1L} f_{1R} 4 m_{\tilde{\chi}} m_t m_A^2)/(s - m_t^2)^2 \\
\mathcal{T}_{II} \times \mathcal{T}_{II} &= (1/2)((f_{2L} f_{2L} + f_{2R} f_{2R})(m_{\tilde{\chi}}^2 + m_t^2 - t) - 2 m_t m_{\tilde{\chi}}(f_{2L} f_{2R} + f_{2R} f_{2L})) \\
&\quad /((t - m_{\tilde{t}_2}^2)^2) \\
\mathcal{T}_{III} \times \mathcal{T}_{III} &= (1/2)((f_{3LR}^i f_{3LR}^j + f_{3RL}^i f_{3RL}^j)((m_t^2 - m_{\tilde{t}_1}^2 + u)(m_{\tilde{\chi}}^2 - m_A^2 + u) - u(m_{\tilde{\chi}}^2 + m_t^2 \\
&\quad - t)) + (f_{3LL}^i f_{3LL}^j + f_{3RR}^i f_{3RR}^j) m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} (m_{\tilde{\chi}}^2 + m_t^2 - t) - (f_{3LR}^i f_{3LL}^j \\
&\quad + f_{3RL}^i f_{3RR}^j) m_{\tilde{\chi}} m_{\tilde{\chi}_j^0} (m_t^2 - m_{\tilde{t}_1}^2 + u) - (f_{3LL}^i f_{3LR}^j + f_{3RR}^i f_{3RL}^j) m_{\tilde{\chi}_i^0} m_{\tilde{\chi}} (m_t^2 \\
&\quad - m_{\tilde{t}_1}^2 + u) + (f_{3RL}^i f_{3LL}^j + f_{3LR}^i f_{3RR}^j) m_t m_{\tilde{\chi}_j^0} (m_{\tilde{\chi}}^2 - m_A^2 + u) + (f_{3RR}^i f_{3LR}^j \\
&\quad + f_{3LL}^i f_{3RL}^j) m_t m_{\tilde{\chi}_i^0} (m_{\tilde{\chi}}^2 - m_A^2 + u) - (f_{3RL}^i f_{3LR}^j + f_{3LR}^i f_{3RL}^j) 2 m_t m_{\tilde{\chi}} u \\
&\quad - (f_{3RR}^i f_{3LL}^j + f_{3LL}^i f_{3RR}^j) 2 m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} m_t m_{\tilde{\chi}})/((u - m_{\tilde{\chi}_i^0}^2)(u - m_{\tilde{\chi}_j^0}^2)) \\
\mathcal{T}_I \times \mathcal{T}_{II} &= (1/2)((f_{2R} f_{1R} - f_{2L} f_{1L}) m_t (u - m_{\tilde{\chi}}^2 - m_A^2) + (f_{2L} f_{1R} - f_{2R} f_{1L}) m_{\tilde{\chi}} (s - m_t^2)
\end{aligned}$$

$$\begin{aligned}
& -m_A^2)) / ((s - m_t^2)(t - m_{t_2}^2)) \\
\mathcal{T}_I \times \mathcal{T}_{III} &= (1/2)((f_{3LR}^i f_{1R} - f_{3RL}^i f_{1L})((m_{\tilde{\chi}}^2 + m_t^2 - t)(m_t^2 + m_{\tilde{\chi}}^2 - m_{t_1}^2 - m_A^2) \\
& - (s - m_t^2 - m_A^2)(u + m_{\tilde{\chi}}^2 - m_A^2) - (u + m_t^2 - m_{t_1}^2)(s + m_{\tilde{\chi}}^2 - m_{t_1}^2))/2 \\
& + (f_{3RR}^i f_{1R} - f_{3LL}^i f_{1L}) m_{\tilde{\chi}_i^0} m_t (u - m_A^2 - m_{\tilde{\chi}}^2) + (f_{3LR}^i f_{1L} - f_{3RL}^i f_{1R}) \\
& \times m_t m_{\tilde{\chi}} (u - m_{\tilde{\chi}}^2 + m_A^2) + (f_{3RR}^i f_{1L} - f_{3LL}^i f_{1R}) m_{\tilde{\chi}_i^0} m_{\tilde{\chi}} (m_t^2 + m_A^2 - s)) \\
& / ((s - m_t^2)(u - m_{\tilde{\chi}_i^0}^2)) \\
\mathcal{T}_{II} \times \mathcal{T}_{III} &= (1/2)((f_{2R} f_{3LR}^i + f_{2L} f_{3RL}^i) m_t (u + m_{\tilde{\chi}}^2 - m_A^2) + (f_{2L} f_{3LL}^i + f_{2R} f_{3RR}^i) \\
& \times m_{\tilde{\chi}_i^0} (m_{\tilde{\chi}}^2 + m_t^2 - t) - (f_{2L} f_{3LR}^i + f_{2R} f_{3RL}^i) m_{\tilde{\chi}} (u + m_t^2 - m_{t_1}^2) \\
& - (f_{2R} f_{3LL}^i + f_{2L} f_{3RR}^i) 2 m_{\tilde{\chi}} m_t m_{\tilde{\chi}_i^0}) / ((t - m_{t_2}^2)(u - m_{\tilde{\chi}_i^0}^2)) \\
|\mathcal{T}|^2 &= \mathcal{T}_I \times \mathcal{T}_I + \mathcal{T}_{II} \times \mathcal{T}_{II} + \sum_{i,j=1}^4 \mathcal{T}_{III} \times \mathcal{T}_{III} + 2\mathcal{T}_I \times \mathcal{T}_{II} + 2 \sum_{i=1}^4 \mathcal{T}_I \times \mathcal{T}_{III} + 2 \sum_{i=1}^4 \mathcal{T}_{II} \times \mathcal{T}_{III} \quad (\text{A20})
\end{aligned}$$

$$\tilde{\chi} \tilde{t}_1 \longrightarrow bH^+$$

I. s-channel t annihilation

II. t-channel $\tilde{\chi}_{(1,2)}^+$ exchange

III. u-channel $\tilde{b}_{(1,2)}$ exchange

$$\begin{aligned}
f_{1LL} &= C_{t-b-H^+}^L C_{\tilde{t}_1-t-\tilde{\chi}_1^0}^L \\
f_{1LR} &= C_{t-b-H^+}^L C_{\tilde{t}_1-t-\tilde{\chi}_1^0}^R \\
f_{1RL} &= C_{t-b-H^+}^R C_{\tilde{t}_1-t-\tilde{\chi}_1^0}^L \\
f_{1RR} &= C_{t-b-H^+}^R C_{\tilde{t}_1-t-\tilde{\chi}_1^0}^R \\
f_{2LL}^i &= C_{\tilde{t}_1-b-\tilde{\chi}_i^+}^L C_{\tilde{\chi}_1^0-\tilde{\chi}_i^+-H^+}^L \\
f_{2LR}^i &= C_{\tilde{t}_1-b-\tilde{\chi}_i^+}^L C_{\tilde{\chi}_1^0-\tilde{\chi}_i^+-H^+}^R \\
f_{2RL}^i &= C_{\tilde{t}_1-b-\tilde{\chi}_i^+}^R C_{\tilde{\chi}_1^0-\tilde{\chi}_i^+-H^+}^L \\
f_{2RR}^i &= C_{\tilde{t}_1-b-\tilde{\chi}_i^+}^R C_{\tilde{\chi}_1^0-\tilde{\chi}_i^+-H^+}^R \\
f_{3L}^i &= C_{\tilde{b}_i-b-\tilde{\chi}_1^0}^L C_{\tilde{t}_1-\tilde{b}_i-H^+} \\
f_{3R}^i &= C_{\tilde{b}_i-b-\tilde{\chi}_1^0}^R C_{\tilde{t}_1-\tilde{b}_i-H^+} \\
\mathcal{T}_I \times \mathcal{T}_I &= (1/2)((f_{1LR}^2 + f_{1RL}^2)((s + m_b^2 - m_{H^+}^2)(s + m_{\tilde{\chi}}^2 - m_{t_1}^2) - s(m_{\tilde{\chi}}^2 + m_b^2 - u))
\end{aligned}$$

$$\begin{aligned}
& +(f_{1LL}^2 + f_{1RR}^2)m_t^2(m_{\tilde{\chi}}^2 + m_b^2 - u) - 2(f_{1LL}f_{1LR} + f_{1RR}f_{1RL})m_{\tilde{\chi}}m_t \\
& \times (s + m_b^2 - m_{H^+}^2) + 2(f_{1LR}f_{1RR} + f_{1LL}f_{1RL})m_b m_t (s + m_{\tilde{\chi}}^2 - m_{\tilde{t}_1}^2) \\
& - 4(f_{1LR}f_{1RL})m_b m_{\tilde{\chi}} s - 4f_{1LL}f_{1RR}m_t^2 m_{\tilde{\chi}} m_b / (s - m_t^2)^2 \\
\mathcal{T}_{II} \times \mathcal{T}_{II} & = (1/2)((f_{2RR}^i f_{2RR}^j + f_{2LL}^i f_{2LL}^j)((t + m_b^2 - m_{\tilde{t}_1}^2)(t + m_{\tilde{\chi}}^2 - m_{H^+}^2) - t(m_{\tilde{\chi}}^2 + m_b^2 \\
& - u)) + (f_{2RL}^i f_{2RL}^j + f_{2LR}^i f_{2LR}^j)m_{\tilde{\chi}_i^+} m_{\tilde{\chi}_j^+} (m_{\tilde{\chi}}^2 + m_b^2 - u) - (f_{2RR}^i f_{2RL}^j \\
& + f_{2LL}^i f_{2LR}^j)m_{\tilde{\chi}} m_{\tilde{\chi}_j^+} (t + m_b^2 - m_{\tilde{t}_1}^2) - (f_{2RL}^i f_{2RR}^j + f_{2LR}^i f_{2LL}^j)m_{\tilde{\chi}_i^+} m_{\tilde{\chi}} \\
& \times (t - m_b^2 - m_{\tilde{t}_1}^2) + (f_{2LL}^i f_{2RL}^j + f_{2RR}^i f_{2LR}^j)m_b m_{\tilde{\chi}_j^+} (t + m_{\tilde{\chi}}^2 - m_{H^+}^2) \\
& + (f_{2LR}^i f_{2RR}^j + f_{2RL}^i f_{2LL}^j)m_b m_{\tilde{\chi}_i^+} (t + m_{\tilde{\chi}}^2 - m_{H^+}^2) - (f_{2LL}^i f_{2RR}^j + f_{2RR}^i f_{2LL}^j) \\
& \times 2m_b m_{\tilde{\chi}} t - (f_{2LR}^i f_{2RL}^j + f_{2RL}^i f_{2LR}^j)2m_b m_{\tilde{\chi}} m_{\tilde{\chi}_i^+} m_{\tilde{\chi}_j^+} \\
& / ((t - m_{\tilde{\chi}_i^+}^2)(t - m_{\tilde{\chi}_j^+}^2)) \\
\mathcal{T}_{III} \times \mathcal{T}_{III} & = (1/2)((f_{3L}^i f_{3L}^j + f_{3R}^i f_{3R}^j)(m_{\tilde{\chi}}^2 + m_b^2 - u) - 2m_{\tilde{\chi}} m_b (f_{3L}^i f_{3R}^j + f_{3R}^i f_{3L}^j)) \\
& / ((u - m_{\tilde{b}_i}^2)(u - m_{\tilde{b}_j}^2)) \\
\mathcal{T}_I \times \mathcal{T}_{II} & = (1/2)((f_{2RR}^i f_{1LR} + f_{2LL}^i f_{1RL})(1/2)((t + m_b^2 - m_{\tilde{t}_1}^2)(s + m_{\tilde{\chi}}^2 - m_{\tilde{t}_1}^2) - (m_{\tilde{\chi}}^2 \\
& + m_b^2 - u)(m_{\tilde{\chi}}^2 + m_b^2 - m_{\tilde{t}_1}^2 - m_{H^+}^2) + (s + m_b^2 - m_{H^+}^2)(t + m_{\tilde{\chi}}^2 - m_{H^+}^2)) \\
& + (f_{2RL}^i f_{1LL} + f_{2LR}^i f_{1RR})m_{\tilde{\chi}_i^+} m_t (m_{\tilde{\chi}}^2 + m_b^2 - u) - (f_{2RR}^i f_{1LL} + f_{2LL}^i f_{1RR}) \\
& \times m_{\tilde{\chi}} m_t (t + m_b^2 - m_{\tilde{t}_1}^2) - (f_{2RL}^i f_{1LR} + f_{2LR}^i f_{1RL})m_{\tilde{\chi}_i^+} m_{\tilde{\chi}} (s + m_b^2 - m_{H^+}^2) \\
& + (f_{2LL}^i f_{1LL} + f_{2RR}^i f_{1RR})m_b m_t (t + m_{\tilde{\chi}}^2 - m_{H^+}^2) + (f_{2LR}^i f_{1LR} + f_{2RL}^i f_{1RL}) \\
& \times m_b m_{\tilde{\chi}_i^+} (s + m_{\tilde{\chi}}^2 - m_{\tilde{t}_1}^2) - (f_{2LL}^i f_{1LR} + f_{2RR}^i f_{1RL})m_b m_{\tilde{\chi}} (m_{\tilde{\chi}}^2 + m_b^2 - m_{\tilde{t}_1}^2 \\
& - m_{H^+}^2) - (f_{2LR}^i f_{1LL} + f_{2RL}^i f_{1RR})2m_b m_{\tilde{\chi}} m_t m_{\tilde{\chi}_i^+} / ((s - m_t^2)(t - m_{\tilde{\chi}_i^+}^2)) \\
\mathcal{T}_I \times \mathcal{T}_{III} & = (1/2)((f_{1LR}f_{3R}^i + f_{1RL}f_{3L}^i)m_b (s + m_{\tilde{\chi}}^2 - m_{\tilde{t}_1}^2) + (f_{1LL}f_{3L}^i + f_{1RR}f_{3R}^i)m_t \\
& \times (m_{\tilde{\chi}}^2 + m_b^2 - u) - (f_{1LR}f_{3L}^i + f_{1RL}f_{3R}^i)m_{\tilde{\chi}} (s + m_b^2 - m_{H^+}^2) - (f_{1LL}f_{3R}^i \\
& + f_{1RR}f_{3L}^i)2m_t m_b m_{\tilde{\chi}}) / ((u - m_{\tilde{b}_i}^2)(s - m_t^2)) \\
\mathcal{T}_{II} \times \mathcal{T}_{III} & = (1/2)((f_{2LR}^j f_{3R}^i + f_{2RL}^j f_{3L}^i)m_b (t + m_{\tilde{\chi}}^2 - m_{H^+}^2) + (f_{2LL}^j f_{3L}^i + f_{2RR}^j f_{3R}^i) \\
& \times m_{\tilde{\chi}_j^+} (m_{\tilde{\chi}}^2 + m_b^2 - u) - (f_{2LR}^j f_{3L}^i + f_{2RL}^j f_{3R}^i)m_{\tilde{\chi}} (t + m_b^2 - m_{\tilde{t}_1}^2) \\
& - (f_{2LL}^j f_{3R}^i + f_{2RR}^j f_{3L}^i)2m_{\tilde{\chi}_j^+} m_b m_{\tilde{\chi}}) / ((u - m_{\tilde{b}_i}^2)(t - m_{\tilde{\chi}_j^+}^2)) \\
|\mathcal{T}|^2 & = \mathcal{T}_I \times \mathcal{T}_I + \sum_{i,j=1}^2 \mathcal{T}_{II} \times \mathcal{T}_{II} + \sum_{i,j=1}^2 \mathcal{T}_{III} \times \mathcal{T}_{III} + 2 \sum_{i=1}^2 \mathcal{T}_I \times \mathcal{T}_{II} + 2 \sum_{i=1}^2 \mathcal{T}_I \times \mathcal{T}_{III} +
\end{aligned}$$

$$2 \sum_{i,j=1}^2 \mathcal{T}_{\text{II}} \times \mathcal{T}_{\text{III}} \quad (\text{A21})$$

$$\tilde{t}_1 \tilde{\ell} \longrightarrow t \ell$$

I. t-channel $\tilde{\chi}_{(1,2,3,4)}^0$ exchange

$$\begin{aligned}
f_{LL}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^L C_{\tilde{\ell}_1-\ell-\tilde{\chi}_i^0}^L \\
f_{LR}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^L C_{\tilde{\ell}_1-\ell-\tilde{\chi}_i^0}^R \\
f_{RL}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^R C_{\tilde{\ell}_1-\ell-\tilde{\chi}_i^0}^L \\
f_{RR}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^R C_{\tilde{\ell}_1-\ell-\tilde{\chi}_i^0}^R \\
\mathcal{T}_1 \times \mathcal{T}_1 &= ((f_{LR}^i f_{RL}^j + f_{RL}^i f_{LR}^j) ((m_{\tilde{t}_1}^2 - m_t^2 - t)(t + m_\ell^2 - m_{\tilde{\ell}_1}^2) - t(s - m_t^2 - m_\ell^2)) \\
&\quad + (f_{LL}^i f_{LL}^j + f_{RR}^i f_{RR}^j) m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} (s - m_t^2 - m_\ell^2) \\
&\quad - (f_{LR}^i f_{LL}^j + f_{RL}^i f_{RR}^j) m_\ell m_{\tilde{\chi}_j^0} (m_{\tilde{t}_1}^2 - m_t^2 - t) \\
&\quad - (f_{LL}^i f_{RL}^j + f_{RR}^i f_{LR}^j) m_\ell m_{\tilde{\chi}_i^0} (m_{\tilde{t}_1}^2 - m_t^2 - t) \\
&\quad + (f_{RL}^i f_{LL}^j + f_{LR}^i f_{RR}^j) m_t m_{\tilde{\chi}_j^0} (t + m_\ell^2 - m_{\tilde{\ell}_1}^2) \\
&\quad + (f_{RR}^i f_{RL}^j + f_{LL}^i f_{LR}^j) m_t m_{\tilde{\chi}_i^0} (t + m_\ell^2 - m_{\tilde{\ell}_1}^2) \\
&\quad - (f_{RL}^i f_{LR}^j + f_{LR}^i f_{RL}^j) m_t m_\ell 2t - (f_{RR}^i f_{LL}^j + f_{LL}^i f_{RR}^j) 2m_t m_\ell m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0}) \\
&\quad / ((t - m_{\tilde{\chi}_i^0}^2)(t - m_{\tilde{\chi}_j^0}^2)) \\
|\mathcal{T}|^2 &= \sum_{i,j=1}^4 \mathcal{T}_1 \times \mathcal{T}_1 \quad (\text{A22})
\end{aligned}$$

$$\tilde{t}_1 \tilde{\ell} \longrightarrow b \nu$$

I. t-channel $\tilde{\chi}_{(1,2)}^+$ exchange

$$\begin{aligned}
f_{LL}^i &= C_{\tilde{t}_1-b-\tilde{\chi}_i^+}^L C_{\tilde{\ell}_1-\nu-\tilde{\chi}_i^+}^L \\
f_{LR}^i &= C_{\tilde{t}_1-b-\tilde{\chi}_i^+}^L C_{\tilde{\ell}_1-\nu-\tilde{\chi}_i^+}^R \\
f_{RL}^i &= C_{\tilde{t}_1-b-\tilde{\chi}_i^+}^R C_{\tilde{\ell}_1-\nu-\tilde{\chi}_i^+}^L \\
f_{RR}^i &= C_{\tilde{t}_1-b-\tilde{\chi}_i^+}^R C_{\tilde{\ell}_1-\nu-\tilde{\chi}_i^+}^R
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}_1 \times \mathcal{T}_1 &= ((f_{LR}^i f_{RL}^j + f_{RL}^i f_{LR}^j)((m_{\tilde{\ell}_1}^2 - m_b^2 - t)(t - m_{\tilde{\ell}_1}^2) - t(s - m_b^2)) \\
&\quad + (f_{LL}^i f_{LL}^j + f_{RR}^i f_{RR}^j) m_{\tilde{\chi}_i^+} m_{\tilde{\chi}_j^+} (s - m_b^2) \\
&\quad + (f_{RL}^i f_{LL}^j + f_{LR}^i f_{RR}^j) m_b m_{\tilde{\chi}_j^+} (t - m_{\tilde{\ell}_1}^2) \\
&\quad + (f_{RR}^i f_{RL}^j + f_{LL}^i f_{LR}^j) m_b m_{\tilde{\chi}_i^+} (t - m_{\tilde{\ell}_1}^2)) \\
&\quad / ((t - m_{\tilde{\chi}_i^+}^2)(t - m_{\tilde{\chi}_j^+}^2)) \\
|\mathcal{T}|^2 &= \sum_{i,j=1}^2 \mathcal{T}_1 \times \mathcal{T}_1 \tag{A23}
\end{aligned}$$

$$\tilde{t}_1 \tilde{\ell}^* \longrightarrow t \bar{\ell}$$

I. t-channel $\tilde{\chi}_{(1,2,3,4)}^0$ exchange

$$\begin{aligned}
f_{LL}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^L C_{\tilde{\ell}_1-\ell-\tilde{\chi}_i^0}^L \\
f_{LR}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^L C_{\tilde{\ell}_1-\ell-\tilde{\chi}_i^0}^R \\
f_{RL}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^R C_{\tilde{\ell}_1-\ell-\tilde{\chi}_i^0}^L \\
f_{RR}^i &= C_{\tilde{t}_1-t-\tilde{\chi}_i^0}^R C_{\tilde{\ell}_1-\ell-\tilde{\chi}_i^0}^R \\
\mathcal{T}_1 \times \mathcal{T}_1 &= ((f_{LL}^i f_{LL}^j + f_{RR}^i f_{RR}^j)((m_{\tilde{t}_1}^2 - m_t^2 - t)(t + m_{\tilde{\ell}}^2 - m_{\tilde{\ell}_1}^2) - t(s - m_t^2 - m_{\tilde{\ell}}^2)) \\
&\quad + (f_{LR}^i f_{RL}^j + f_{RL}^i f_{LR}^j) m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} (s - m_t^2 - m_{\tilde{\ell}}^2) \\
&\quad - (f_{LL}^i f_{RL}^j + f_{RR}^i f_{LR}^j) m_{\tilde{\ell}} m_{\tilde{\chi}_j^0} (m_{\tilde{t}_1}^2 - m_t^2 - t) \\
&\quad - (f_{LR}^i f_{LL}^j + f_{RL}^i f_{RR}^j) m_{\tilde{\ell}} m_{\tilde{\chi}_i^0} (m_{\tilde{t}_1}^2 - m_t^2 - t) \\
&\quad + (f_{RR}^i f_{RL}^j + f_{LL}^i f_{LR}^j) m_t m_{\tilde{\chi}_j^0} (t + m_{\tilde{\ell}}^2 - m_{\tilde{\ell}_1}^2) \\
&\quad + (f_{RL}^i f_{LL}^j + f_{LR}^i f_{RR}^j) m_t m_{\tilde{\chi}_i^0} (t + m_{\tilde{\ell}}^2 - m_{\tilde{\ell}_1}^2) \\
&\quad - (f_{RR}^i f_{LL}^j + f_{LL}^i f_{RR}^j) m_t m_{\tilde{\ell}} 2t - (f_{RL}^i f_{RL}^j + f_{LR}^i f_{LR}^j) 2 m_t m_{\tilde{\ell}} m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0}) \\
&\quad / ((t - m_{\tilde{\chi}_i^0}^2)(t - m_{\tilde{\chi}_j^0}^2)) \\
|\mathcal{T}|^2 &= \sum_{i,j=1}^4 \mathcal{T}_1 \times \mathcal{T}_1 \tag{A24}
\end{aligned}$$

References

- [1] J. Ellis, J.S. Hagelin, D.V. Nanopoulos, K.A. Olive and M. Srednicki, Nucl. Phys. B **238**, 453 (1984); see also H. Goldberg, Phys. Rev. Lett. **50**, 1419 (1983).
- [2] C. Boehm, A. Djouadi and M. Drees, Phys. Rev. D **62**, 035012 (2000) [arXiv:hep-ph/9911496].
- [3] N. Bahcall, J. P. Ostriker, S. Perlmutter and P. J. Steinhardt, Science **284**, 1481 (1999).
- [4] K. Griest and D. Seckel, Phys. Rev. D **43**, 3191 (1991).
- [5] S. Mizuta and M. Yamaguchi, Phys. Lett. B **298**, 120 (1993) [arXiv:hep-ph/9208251].
- [6] J. Edsjo and P. Gondolo, Phys. Rev. D **56**, 1879 (1997) [arXiv:hep-ph/9704361].
- [7] J. Ellis, T. Falk and K.A. Olive, Phys. Lett. B **444**, 367 (1998).
- [8] J. Ellis, T. Falk, K.A. Olive and M. Srednicki, Astropart. Phys. **13** 181 (2000).
- [9] M. E. Gómez, G. Lazarides and C. Pallis, Phys. Rev. D **61**, 123512 (2000) [arXiv:hep-ph/9907261] and Phys. Lett. B **487**, 313 (2000) [arXiv:hep-ph/0004028]; see also: A. B. Lahanas, D. V. Nanopoulos and V. C. Spanos, Phys. Rev. D **62**, 023515 (2000), [arXiv:hep-ph/9909497].
- [10] R. Arnowitt, B. Dutta and Y. Santoso, Nucl. Phys. B **606**, 59 (2001) [arXiv:hep-ph/0102181].
- [11] J. L. Feng, K. T. Matchev and T. Moroi, Phys. Rev. Lett. **84**, 2322 (2000) [arXiv:hep-ph/9908309]; J. L. Feng, K. T. Matchev and T. Moroi, Phys. Rev. D **61**, 075005 (2000) [arXiv:hep-ph/9909334]; J. L. Feng, K. T. Matchev and F. Wilczek, Phys. Lett. B **482**, 388 (2000) [arXiv:hep-ph/0004043].
- [12] J. R. Ellis, T. Falk, G. Ganis, K. A. Olive and M. Srednicki, Phys. Lett. B **510**, 236 (2001) [arXiv:hep-ph/0102098].
- [13] M. Drees and M. M. Nojiri, Phys. Rev. D **47**, 376 (1993); H. Baer and M. Brhlik, Phys. Rev. D **53**, 597 (1996) and Phys. Rev. D **57**, 567 (1998); H. Baer, M. Brhlik, M. A. Diaz,

- J. Ferrandis, P. Mercadante, P. Quintana and X. Tata, Phys. Rev. D **63**, 015007 (2001);
A. B. Lahanas and V. C. Spanos, hep-ph/0106345.
- [14] M. Srednicki, R. Watkins and K. A. Olive, Nucl. Phys. B **310**, 693 (1988).
- [15] P. Gondolo and G. Gelmini, Nucl. Phys. B **360**, 145 (1991).
- [16] T. Falk, K. A. Olive and M. Srednicki, Phys. Lett. B **339**, 248 (1994) [arXiv:hep-ph/9409270].
- [17] H. N. Brown *et al.* [Muon g-2 Collaboration], Phys. Rev. Lett. **86**, 2227 (2001) [hep-ex/0102017].
- [18] S. Heinemeyer, W. Hollik and G. Weiglein, Comput. Phys. Commun. **124**, 76 (2000) [arXiv:hep-ph/9812320]; S. Heinemeyer, W. Hollik and G. Weiglein, Eur. Phys. J. C **9**, 343 (1999) [arXiv:hep-ph/9812472].
- [19] H. Baer, M. Brhlik and D. Castaño, Phys. Rev. D **54** (1996) 6944; S. Abel and T. Falk, Phys. Lett. B **444** (1998) 427.