Relating production and masses of the vector and *P*-wave mesons for light and heavy flavours at LEP

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The production rates of primary vector and P-wave mesons in Z^0 hadronic decays are analysed. The mass dependence of production rates for the bottom, charm, strange charm and three families of the light-flavour mesons is found to be very similar, allowing to relate the relative production rates for mesons with different flavours and, possibly, their masses. The strange axial mesons $K_1(1273)$ and $K_1(1402)$ might be assigned to the $1^+(1/2)$ and $1^+(3/2)$ levels degenerate with the $0^+(1/2)$ and $2^+(3/2)$ levels of the $K_0^*(1430)$ and $K_2^*(1430)$, respectively, if the observed $K_0^*(1430)$ mass is replaced by its "bare" $q\bar{q}$ mass corresponding to the K-matrix pole and close to the $K_1(1273)$ mass. Then the $0^+(1/2)$ and $1^+(1/2)$ levels are *below* the $1^+(3/2)$ and $2^+(3/2)$ levels for the strange, charm and bottom mesons.

The LEP experiments accumulated rich information on inclusive production of the light-flavour, charm and bottom mesons in the Z^0 hadronic decays including data on P-wave meson production. In this Letter, we use these data to compare the production of primary vector and P-wave mesons in an attempt to relate the production of these states for light and heavy flavours.

The total production rates of the vector ρ^0, ω, $K^{*0}(892)$ and ϕ, the tensor $f_2(1275)$, $K_2^{*0}(1430)$ and $f_2'(1525)$, and the scalar $f_0(980)$ and $a_0^+(980)$ mesons measured by the LEP experiments [1–7] are presented in Table 1. For the vector and scalar mesons, the measurements from the different LEP experiments agree within errors. Therefore subsequently we used the rates obtained by averaging the results of these experiments, also presented in Table 1. In calculating the errors of averages, the standard procedure suggested by the PDG group [8] was applied. The DELPHI [2] and OPAL [5,6] results on the $f_2(1275)$ and $K_2^{*0}(1430)$ rates are less consistent. The $K_2^{*0}(1430)/f_2(1275)$ ratio from DELPHI, 0.24 ± 0.09, agrees with usually accepted value of the strangeness suppression parameter $\lambda \approx 0.3$. This is also true within large errors for the ratio $f_2'(1525)/K_2^{*0}(1430) = 0.32 \pm 0.20$. The same ratios $K_2^{*0}(1430)/f_2(1275) = 0.56 \pm 0.23$ and $f_2'(1525)/K_2^{*0}(1430) = 0.10 \pm 0.06$ obtained from the $f_2(1275)$ and $K_2^{*0}(1430)$ rates from OPAL and the $f_2'(1525)$ rate from DELPHI differ

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Preprint submitted to Elsevier Preprint *6 December 2001*

Table 1
The total production rates of the vector, tensor and scalar mesons in the light-quark sector measured by the LEP experiments, averaged total rates for the vector and scalar mesons, fractions of primary mesons obtained from the JETSET model and direct rates determined by multiplying the total rates by the fractions of primary mesons. For the $K^*(892)$, $K_2^*(1430)$ and $a_0^+(980)$ antiparticles and charge conjugates are not included into the definition of the rates.

Meson	Total rate	Averaged rate	Fraction	Direct rate
ρ^0	1.45 ± 0.21 [1]	1.23 ± 0.10	0.54	0.664 ± 0.054
	1.19 ± 0.10 [2]			
ω	1.07 ± 0.14 [1]	1.08 ± 0.09	0.57	0.618 ± 0.049
	1.17 ± 0.17 [4]			
	1.04 ± 0.15 [7]			
$K^{*0}(892)$	0.415 ± 0.045 [1]	0.377 ± 0.017	0.60	0.226 ± 0.010
	0.385 ± 0.040 [3]			
	0.379 ± 0.017 [5]			
ϕ	0.122 ± 0.009 [1]	0.0966 ± 0.0073	0.70	0.0676 ± 0.0051
	0.104 ± 0.008 [3]			
	0.091 ± 0.004 [6]			
$f_2(1275)$	0.155 ± 0.021 [2]		0.96	0.149 ± 0.020
	0.214 ± 0.038 [6]			
$K_2^{*0}(1430)$	0.037 ± 0.013 [2]		0.98	0.036 ± 0.013
	0.119 ± 0.044 [5]			
$f_2'(1525)$	0.012 ± 0.006 [2]		0.98	0.012 ± 0.006
$f_0(980)$	0.164 ± 0.021 [2]	0.147 ± 0.011	0.93	0.137 ± 0.010
	0.141 ± 0.013 [6]			
$a_0^+(980)$	0.1350 ± 0.0055 [7]		0.93	0.126 ± 0.051

from the expected value of λ by a factor of 2 and 3, respectively, although consistent with it within 1 and 3 standard deviations. For this reason we subsequently relied on the DELPHI measurements of the $f_2(1275)$ and $K_2^{*0}(1430)$ total rates. The direct production rates were determined by multiplying the total averaged rates by the fractions of primary mesons obtained from the JETSET model [9] given in [10] and reproduced in Table 1. The fractions of the promptly produced vector mesons are quite high. For the tensor mesons, they are close to 1. This facilitates the analysis of the vector and tensor meson direct rates in comparison with the more difficult situation for the pseudoscalar mesons [11,12] [2].

The direct production rates of the vector and tensor mesons per spin projection, $\langle n \rangle/(2J+1)$, are also presented as a function of their mass, M, in Fig. 1. The mass dependences of the ρ^0, ω and $f_2(1275)$, the $K^{*0}(892)$ and $K_2^{*0}(1430)$, the ϕ and $f_2'(1525)$ rates are very similar. The fit of the data to three exponentials $\langle n \rangle/(2J+1) = a\mathrm{e}^{-bM}$, with different normalization parameters a but the *same* slope parameter, yields $b = 4.11 \pm 0.27$ $(\mathrm{GeV}/c^2)^{-1}$.

The probabilities that a charm quark fragments into the P-wave $D_2^{*0}(2460)$,

[2] Although the JETSET model predictions for the ratios of the promptly produced vector and pseudoscalar mesons are also quite compatible with experiment [12].

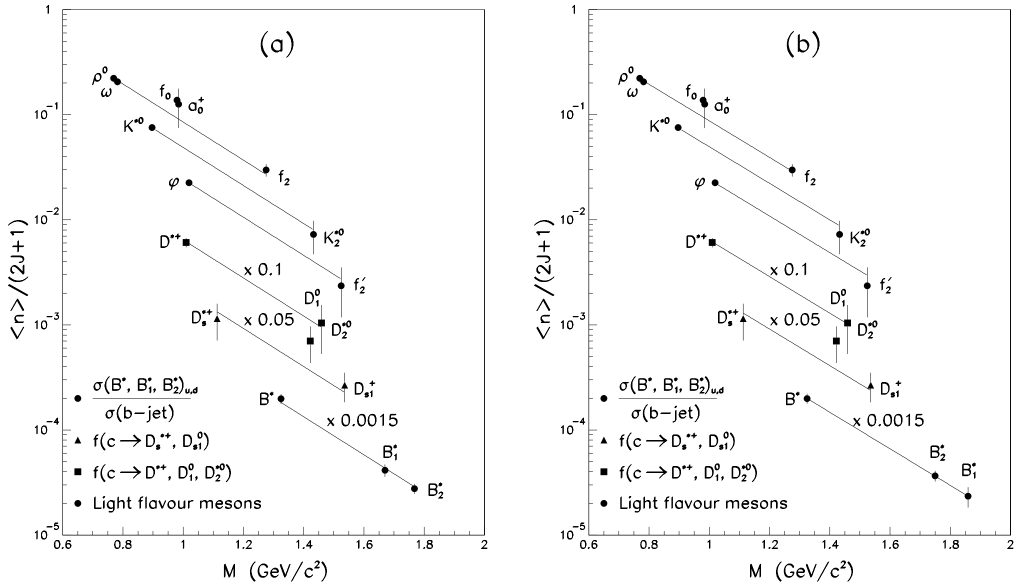

Fig. 1. The mass dependence of direct production rates, $\langle n \rangle$, for light-flavour mesons, fragmentation fractions $f(c \to \mathrm{D}^{*+}, \mathrm{D}_1^0, \mathrm{D}_2^{*0})$ and $f(c \to \mathrm{D}_s^{*+}, \mathrm{D}_{s1}^+)$ for charm mesons, and ratios $\sigma(\mathrm{B}^*, \mathrm{B}_1^*, \mathrm{B}_2^*)_{u,d}/\sigma_{b-\mathrm{jet}}$ for bottom mesons (with their masses and rates from L3 (a) and OPAL (b)), all divided by the spin counting factor $2J+1$. The data points for charm, strange charm and bottom mesons have been scaled by factors of 0.1, 0.05 and 0.0015 respectively and shifted by 1 GeV/c^2 for charm and by 4 GeV/c^2 for bottom mesons for clarity. The solid lines represent the result of the fit of the data for the light-flavour vector and tensor mesons, and the charm and bottom vector and P-wave mesons to six exponentials with the *same* slope.

$\mathrm{D}_1^0(2420)$ and $\mathrm{D}_{s1}^+(2536)$ mesons were measured by OPAL [13]:

$$f(c \to \mathrm{D}_2^{*0}(2460)) = 0.052 \pm 0.026 \quad (1)$$
$$f(c \to \mathrm{D}_1^{0}(2420)) = 0.021 \pm 0.008 \quad (2)$$
$$f(c \to \mathrm{D}_{s1}^{+}(2536)) = 0.016 \pm 0.005. \quad (3)$$

The charm fragmentation fraction into the D^{*+} meson measured by ALEPH [14], DELPHI [15] and OPAL [16] amounted to 0.233 ± 0.015, 0.255 ± 0.017 and 0.222 ± 0.020, respectively. The averaged result is

$$f(c \to \mathrm{D}^{*+}) = 0.238 \pm 0.010. \quad (4)$$

The charm fragmentation fraction into the D_s^{*+} measured by ALEPH [14] is

$$f(c \to \mathrm{D}_s^{*+}) = 0.069 \pm 0.026. \quad (5)$$

Accounting for the $\mathrm{D}_2^*(2460)$, $\mathrm{D}_1(2420)$ and $\mathrm{D}_{s1}^+(2536)$ decays into $\mathrm{D}^{*+}\pi$ and $\mathrm{D}^{*+}\mathrm{K}$, and assuming isospin invariance, the charm fragmentation fraction into the primary D^{*+} meson is

$$f(c \to D^{*+}{}_{\rm prompt}) = 0.183 \pm 0.018. \qquad (6)$$

The D_s^{*+} in Eq. (5) has been considered as promptly produced, since a contribution of possibly seen $D_{s1}^+(2536)$ decay into $D_s^{*+}\gamma$ can be ignored within presently large errors. The values of the charm fragmentation fractions into the primary vector mesons D^{*+} and D_s^{*+} and P-wave mesons $D_1^0(2420)$, $D_2^{*0}(2460)$ and $D_{s1}^+(2536)$, divided by the corresponding spin counting factors $2J+1$, are presented in Fig. 1. As one can see their mass dependence is very similar to the one observed for the light-flavour vector and tensor mesons.

The experimental situation for P-wave meson production in the bottom-quark sector is more complicated. In the quark model one expects for each spectator flavour four different orbitally excited states. For $b\bar{u}$ and $b\bar{d}$ states they are commonly labelled as $B_{u,d}^{**}$. Heavy quark effective symmetry (HQET) [17] groups these four states into two doublets with $j_q = 1/2$ and $j_q = 3/2$ where $\vec{j}_q = \vec{s}_q + \vec{l}$ is the total angular momentum of the light quark. The $j_q = 1/2$ doublet consists of the states B_0 and B_1^* with spins 0 and 1 respectively. The states B_1 and B_2^*, with respective spins 1 and 2, comprise the $j_q = 3/2$ doublet. The splitting between the states in each doublet is expected to be small. The states in the $j_q = 1/2$ doublet are expected to be broad since they can decay through an S-wave transition, whereas the $j_q = 3/2$ states decay through a D-wave transition and are therefore thought to be narrow.

Evidence for the B^{**} states with inclusively reconstructed B mesons has been clearly observed by the LEP experiments [18–21]. The relative rate of all spin states, $\sigma_{B_{u,d}^{**}}/\sigma_{b-\rm jet}$ amounted to 0.214 ± 0.049 [18], 0.270 ± 0.063 [19], 0.320 ± 0.067 [20] and 0.270 ± 0.056 [21]. Besides, ALEPH reported the rate of 0.238 ± 0.085 [22] based on fully reconstructed B mesons. The cited values from ALEPH were obtained using the fraction of $B_{u,d}$ mesons in $Z^0 \to b\bar{b}$ decays, 0.768 ± 0.052, taken from [18]. Averaging these results we obtain

$$\sigma_{B_{u,d}^{**}}/\sigma_{b-\rm jet} = 0.258 \pm 0.027. \qquad (7)$$

A similar value, $0.28 \pm 0.06 \pm 0.03$, was recently measured in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV by CDF [23].

The relative production rates, masses and widths of the four different states contributing to the $B_{u,d}^{**}$ signal are not very well known. In the framework of HQET, attempts have been made by ALEPH [22], L3 [20] and OPAL [24] to determine the masses and widths of at least one of these states. The fitted masses are shown (as bold numbers) in Table 2. The complicated fitting procedures and constraints in these experiments were different, apart from mass splitting between the states belonging to the same j_q doublet. For the narrow states, the constraint $M_{\mathrm{B}_2^*} - M_{\mathrm{B}_1} = 12$ MeV/c^2 was applied by all experiments. For the broad states, ALEPH and L3 applied the same constraint, while OPAL took $M_{\mathrm{B}_1^*} - M_{\mathrm{B}_0} = 20$ MeV/c^2. ALEPH fitted the mass of the B_2^* meson only, with the constraint $M_{\mathrm{B}_2^*} - M_{\mathrm{B}_1^*} = 100$ MeV/c^2. The masses of the states resulting from these constraints are also shown in Table 2.

Table 2
Masses of the P-wave bottom mesons determined by the LEP experiments from the fits to the data (bold numbers) together with masses of their partners used as the constraints in the fits. The branching fractions into $\mathrm{B}^*\pi$ used by ALEPH, L3 and in the present Letter are also shown.

Meson	$J^P_{j_q}$	ALEPH [22]	L3 [20]	OPAL [24]	Br($\mathrm{B}^*\pi$)
B_0	$0^+_{1/2}$	$5627 \pm^{8}_{11} \pm^{6}_{4}$	$5658 \pm 10 \pm 13$	$\mathbf{5839 \pm^{13}_{14} \pm^{34}_{42}}$	0.0
B_1^*	$1^+_{1/2}$	$5639 \pm^{8}_{11} \pm^{6}_{4}$	$\mathbf{5670 \pm 10 \pm 13}$	$5859 \pm^{13}_{14} \pm^{34}_{42}$	1.0
B_1	$1^+_{3/2}$	$5727 \pm^{8}_{11} \pm^{6}_{4}$	$5756 \pm 5 \pm 6$	$\mathbf{5738 \pm^{5}_{6} \pm 7}$	1.0
B_2^*	$2^+_{3/2}$	$\mathbf{5739 \pm^{8}_{11} \pm^{6}_{4}}$	$\mathbf{5768 \pm 5 \pm 6}$	$5750 \pm^{5}_{6} \pm 7$	0.5

The results on the masses of the narrow B_1 and B_2^* are quite consistent bearing in mind the difference in other assumptions in the corresponding fits. The B_1 mass, 5710 ± 20 MeV/c^2, extracted by CDF [23], with the error not including the theoretical uncertainty on the shape of the B^{**} peak, is also consistent with the LEP results. On the other hand, the masses of the broad B_1^* and B_0 states obtained in the L3 fit and constrained by ALEPH are smaller by ≈ 200 MeV/c^2 than the masses determined by OPAL, even if OPAL stressed that the B_0 mass could not be considered as a robust fit result. Theoretical predictions (see [25–30] and references therein) for the masses of the four spin states in the charm and bottom sectors are also different. Some models [27,28] predict that the broad $j_q = 1/2$ states have smaller masses than the narrow $j_q = 3/2$ states, in agreement with the L3 result and ALEPH constraints. Other models [29,30] proposing spin-orbit inversion are more consistent with the OPAL result. The difference in the experimental results might be, at least partly, explained by different assumptions about the relative production rates of the four states. The corresponding proportions were set by ALEPH, L3 and CDF according to simple total spin counting, B_0:B_1^*:B_1:B_2^*= 1:3:3:5. OPAL fixed the relative production rates of the same states to 2:2:3:3.

Our attempt to determine the relative rates of the four spin states is based on the assumption that the mass dependence of their production rates is the same as that observed for the light-flavour and charm mesons in Fig. 1. For this the production rates of the states with the same or very close masses must be set according to simple total spin counting [3]. For the $j_q = 3/2$ and $j_q = 1/2$ states, with presumably different masses, this simple spin counting is expected to be violated. This violation can be accounted for assuming that the mass dependence of the production rates is described by the exponential

[3] Total spin counting works for the vector and pseudoscalar B^* and B [18,31–33] or D_2^{*0} and D_1^0 [13], although even in this case its small violation due to non negligible mass difference can not be excluded [11].

with the same slope parameter b as given earlier for the light-flavour mesons. Then the coefficient characterising the violation of simple spin counting for the B_1^* and B_2^* is

$$\varepsilon = 5B_1^*/(3B_2^*) = e^{b(M_{B_2^*} - M_{B_1^*})}, \tag{8}$$

and the relative production rates of the four different states contributing to the $B_{u,d}^{**}$ signal are set according to the following "modified" total spin counting procedure[4]:

$$B_0 : B_1^* : B_1 : B_2^* = \varepsilon : 3\varepsilon : 3 : 5. \tag{9}$$

The values of ε for the B_2^* and B_1^* masses determined by L3 and OPAL are given in Table 3. The B_0, B_1^*, B_1 and B_2^* relative production rates in b-quark jets (with their overall rate given in Eq. (7)) following from the modified total spin counting (MTSC) rule proposed here are compared with those obtained using the simple total spin counting (STSC) applied by L3 and the proportion B_0:B_1^*:B_1:B_2^*= 2:2:3:3 used by OPAL.

Table 3
The coefficient ε, relative fractions of the four P-wave states and promptly produced $B^*(u,d)$ in b-quark jets (in %) calculated with the B_2^* and B_1^* masses determined by L3 and OPAL and applying modified total spin counting (MTSC), simple total spin counting (STSC), and the OPAL proportion B_0:B_1^*:B_1:B_2^*= 2:2:3:3.

Experiment	L3 (MTSC)	L3 (STSC)	OPAL (MTSC)	OPAL (2:2:3:3)
ε	1.50 ± 0.12	1	$0.64^{0.10}_{0.12}$	
$B_0(u,d)/\sigma_{b-\text{jet}}$	2.75 ± 0.38	2.15 ± 0.23	1.56 ± 0.34	5.2 ± 0.5
$B_1^*(u,d)/\sigma_{b-\text{jet}}$	8.3 ± 1.1	6.5 ± 0.7	4.7 ± 1.0	5.2 ± 0.5
$B_1(u,d)/\sigma_{b-\text{jet}}$	5.5 ± 0.6	6.5 ± 0.7	7.3 ± 0.8	7.7 ± 0.8
$B_2^*(u,d)/\sigma_{b-\text{jet}}$	9.2 ± 1.0	10.8 ± 1.1	12.2 ± 1.4	7.7 ± 0.8
$B^*(u,d)/\sigma_{b-\text{jet}}$	39.6 ± 4.2	39.7 ± 4.2	39.9 ± 4.3	41.2 ± 4.5

The relative B^* production rate in b-quark jet, $\sigma_{B^*}/\sigma_{b-\text{jet}}$, was measured by the LEP experiments for a mixture of the states B_d^*, B_u^* and B_s^* with the following results: 0.677 ± 0.073 [18], 0.650 ± 0.063 [31], 0.690 ± 0.086 [32] and 0.660 ± 0.085 [33], with the averaged value of 0.667 ± 0.037. Assuming that $0.3B_s^*$ are produced for each B_d^*, we obtain

$$\sigma_{B^*(u,d)}/\sigma_{b-\text{jet}} = 0.580 \pm 0.032. \tag{10}$$

For determining the rate of the promptly produced $B^*(ud)$, the decays of P-wave mesons into $B^*\pi$ have to be taken into account. For this, the branching

[4] With the $D^\pm$ and D^{*+} masses and the same b, Eq. (8) yields $\varepsilon = 3D/D^* = 1.80 \pm 0.16$. This is consistent with $\varepsilon = 2.0 \pm 0.3$ following from the value of $P_V = V/(P+V) = 0.595 \pm 0.045$ for these mesons [14], supporting our assumption that the violation of the simple spin counting rule is closely related to the mass difference.

fractions into $B^*\pi$ shown in Table 2 were used (the same as in [20,22]). With the relative production rates from Table 3 calculated using modified total spin counting, this yields $Br(B_J \to B^*\pi) = 0.714 \pm 0.029 \pm 0.068$ and $0.703 \pm 0.040 \pm 0.074$ for the L3 and OPAL results respectively. The additional systematic errors account to half of the difference between these values and the value $Br(B_J \to B^*\pi(X)) = 0.85 \pm 0.29$ found by OPAL [24]. The resulting relative rates of the promptly produced $B^*(ud)$ are presented in Table 3, together with similarly obtained values based on the L3 results with simple total spin counting and OPAL results with the OPAL proportion of the relative rates.

As one can see from Table 3, the relative rates of the four spin states, and especially the B_0/B_2^* ratio, are quite sensitive to the spin counting rules assumed. For the masses of these states from OPAL, this ratio obtained using the OPAL proportion of the rates is larger by a factor of 5.2 than the same ratio obtained with modified total spin counting. This suggests that significantly different fitted values of the masses might be obtained if modified total spin counting were applied instead of the OPAL proportion. On the other hand, the rate of promptly produced B^* is practically insensitive to the difference in the counting rules. The B_2^* rates obtained with modified spin counting at the masses from the L3 and OPAL are consistent within 1.7 standard deviations (or even less since the B_2^* mass from L3 is larger than from OPAL). This suggests that the production rates of the B_2^* and promptly produced B^* are sufficiently reliable to allow comparison of their mass dependence with other data.

The relative rates of B_1^*, B_2^* and promptly produced B^* in b-quark jets calculated using modified total spin counting and divided by the spin counting factors $2J+1$ are shown at the B_1^* and B_2^* masses from L3 in Fig. 1a and at the B_1^* and B_2^* masses from OPAL in Fig. 1b. The fits of the data to six exponentials with different normalization parameters for the six meson families, but the *same* slope parameter $b = 4.17 \pm 0.21$ (GeV/c^2)$^{-1}$ in Fig. 1a and $b = 4.01 \pm 0.19$ (GeV/c^2)$^{-1}$ in Fig. 1b describe the data well (solid lines in Fig. 1). The slope parameters are very close to the value $b = 4.11 \pm 0.27$ (GeV/c^2)$^{-1}$ obtained for the light-flavour mesons. Thus we see that the mass dependences of the production rates per spin projection are indeed very similar for the light-flavour, charm and bottom mesons [5]. One important lesson from this observation is the existence of a close relationship between the masses of the P-wave states and their production rates. If the masses of the $j_q = 1/2$ states with $J^P = 0^+$ and 1^+ are below (above) the masses of the $j = 3/2$ states with $J^P = 1^+$ and 2^+, their production rates per spin projection are larger (smaller) than for the $j_q = 3/2$ states, as shown for the B_1^* and B_2^* in Fig. 1a (Fig. 1b).

[5] For the bottom mesons, this applies, strictly speaking, only to the B^* and B_2^*, since for the B_1^* and B_2^* rates the same mass dependence as for the light-flavour mesons has been imposed by Eqs. (8) and (9).

Apart from the P-wave mesons discussed above, only the production rates of the scalars $a_0^+(980)$ and $f_0(980)$ were measured at LEP [2,6,7]. They are presented in Table 1 and Fig. 1. The $a_0^+(980)$ rate is consistent within errors with the mass dependence of the ρ^0, ω and $f_2(1275)$ rates. The $f_0(980)$ rate is consistent with the $a_0^+(980)$ rate as expected, but appears to be slightly higher than follows from the mass dependence of the ρ^0, ω and $f_2(1275)$ rates. However, this might well be due to overestimated fractions of the promptly produced $a_0^+(980)$ and $f_0(980)$, which are difficult to estimate. These presumably must be comparable with those for the vector mesons (with similar masses), but are higher in JETSET (see Table 1). Such an explanation is supported by the mass dependence of the ρ^0, ω, $a_0^+(980)$, $f_0(980)$ and $f_2(1275)$ total rates [34]. If the production rates of other P-wave mesons follow the same mass dependence as observed in Fig. 1, this allows their production rates to be estimated. For example, the corresponding predictions for the $b_1(1235)$ and $f_1(1420)$ total production rates per Z^0 hadronic decay are 0.102 ± 0.031 and 0.0126 ± 0.0045 if the $f_1(1420)$ is a pure $s\bar{s}$ state.

Moreover, provided that the observed mass dependence of production rates is indeed universal for all flavours, it allows not only the production rates of mesons with different flavours to be related, but also their masses. Indeed, from simple mass rescaling in Fig. 1 one obtains the following phenomenological mass formulae:

$$\mathrm{B}_i = \mathrm{B}_2^* - (\mathrm{B}_2^* - \mathrm{B}^*)\frac{T - P_i}{T - V}, \quad \mathrm{D}_i = \mathrm{D}_2^* - (\mathrm{D}_2^* - \mathrm{D}^*)\frac{T - P_i}{T - V} \tag{11}$$

where V, T and P_i are the masses of the vector, tensor and P-wave (with $J^P = 1^+$ or 0^+) light-flavour mesons corresponding to the masses of their respective charm D^*, D_2^* and D_i, and bottom B^*, B_2^* and B_i partners.

From Eq. (11), with the K^{*0}, K_2^{*0}, $\mathrm{K}_1(1402)$, D^*, D_2^* and B^* masses from PDG [8] and the B_2^* mass, 5752 ± 15 MeV/c^2, taken as the average of the masses obtained by ALEPH, L3 and OPAL (Table 2) and with the error equal to half of the difference between the ALEPH and L3 values, one obtains:

$$M_{\mathrm{B}_1} = 5728 \pm 16 \text{ MeV}/c^2, \quad M_{\mathrm{D}_1^0} = 2433 \pm 6 \text{ MeV}/c^2. \tag{12}$$

With the $\mathrm{K}_1(1273)$ instead of the $\mathrm{K}_1(1402)$ in Eq. (11) one has:

$$M_{\mathrm{B}_1^*} = 5625 \pm 16 \text{ MeV}/c^2, \quad M_{\mathrm{D}_1^{*0}} = 2325 \pm 6 \text{ MeV}/c^2. \tag{13}$$

The limited accuracy of the phenomenological formulae (11) results in additional systematic uncertainty, not accounted for in the mass estimates given in Eqs. (12) and (13). It can roughly be estimated from the mass relation

$(\mathrm{B}_2^* - \mathrm{B}_i)/(\mathrm{B}_2^* - \mathrm{B}^*) = (\mathrm{D}_2^* - \mathrm{D}_i)/(\mathrm{D}_2^* - \mathrm{D}^*)$ following from Eq. (11), which imposes practically the same mass splitting between the B_2^* and B_1 as between the D_2^* and D_1. This is not consistent with the smaller B_2^* and B_1 mass difference of 12 MeV/c^2, required in the fits performed by the LEP experiments, in comparison with the measured D_2^{*0} and D_1^0 mass difference 37 ± 3 MeV/c^2 and may result in possible biases of ≈ 25 MeV/c^2 in our mass estimates.

The B_1 mass given in Eq. (12) agrees within errors with the averaged value of this mass 5740 ± 15 MeV/c^2 from ALEPH, L3 and OPAL and $M_{\mathrm{B}_1} = 5710 \pm 20$ MeV/c^2 from CDF. The mass difference $M_{\mathrm{B}_2^*} - M_{\mathrm{B}_1} = 24 \pm 22$ MeV/c^2 is consistent with the constraint of 12 MeV/c^2 imposed by the LEP experiments. The B_1^* mass given in Eq. (13) is consistent within 2 standard deviations with $M_{\mathrm{B}_1^*} = 5670 \pm 16$ MeV/c^2 from the L3 fit, agrees with $M_{\mathrm{B}_1^*} = 5639\pm^{10}_{12}$ MeV/c^2 from the ALEPH fit, but significantly smaller than the value following from the OPAL fit. The obtained D_1^0 mass is in good agreement with the PDG value 2422.2 ± 1.8 MeV/c^2. The D_1^{*0} mass in Eq. (13) represents our prediction for the mass of the broad, not yet established state.

The physical $\mathrm{K}_1(1273)$ and $\mathrm{K}_1(1402)$ are mixtures of the two SU(3) octet states $^1P_1$ and $^3P_1$. The decay patterns of the $\mathrm{K}_1(1273)$ and $\mathrm{K}_1(1402)$ suggest that these singlet and octet states are almost degenerate, with a mixing angle near 45°. Thus, from the decay amplitudes of the $\mathrm{K}_1(1273)$ and $\mathrm{K}_1(1402)$ into $\rho\mathrm{K}$ and $\mathrm{K}^*\pi$, the ACCMOR collaboration found $\theta = 56° \pm 3°$ [35]. Provided that the heavy quark limit is also appropriate for the strange mesons, the two mixed K_1 mass eigenstates of $J^P = 1^+$ can also be described by the total angular momentum j_q of the light quark with $j_q = 1/2$ and $j_q = 3/2$ expected to be degenerate with the $J^P = 0^+$ and $J^P = 2^+$ states, respectively. By a change of basis one can introduce a new mixing angle θ_K which defines the amount of $j_q = 1/2$ and $j_q = 3/2$ in the physical $\mathrm{K}_1(1273)$ and $\mathrm{K}_1(1402)$ states. According to Isgur [29] (see also [26,36,37]), the $\mathrm{K}_1(1273)$ and $\mathrm{K}_1(1402)$ are quite near to being the pure $j_q = 3/2$ and $j_q = 1/2$ states, respectively. The small splitting between $\mathrm{K}_0^*(1430)$ and $\mathrm{K}_1(1402)$, implying that the $0^+(1/2)$ and $1^+(1/2)$ levels are nearly degenerate as expected for all heavy-light systems, is well consistent with such association. However, this is certainly not a case for the $\mathrm{K}_2^*(1430)$ and $\mathrm{K}_1(1273)$ associated with the $2^+(3/2)$ and $1^+(3/2)$ levels, respectively.

On the other hand, our results following from the mass relations (11) suggest that the $\mathrm{K}_1(1402)$ and $\mathrm{K}_1(1273)$ might be assigned in the heavy quark limit to the $j_q = 3/2$ and $j_q = 1/2$ levels, respectively. This may certainly imply either that our assumption about a universal mass dependence of the production rates for all P-wave states (including the $\mathrm{K}_1(1402)$ and $\mathrm{K}_1(1273)$) fails, or that our phenomenological mass formulae (11) resulting from this assumption are not correct. However, if it is not the case, such an assignment implies that the $j_q = 3/2$ levels corresponding to the $\mathrm{K}_1(1402)$ and $\mathrm{K}_2^*(1430)$ are degenerate, whereas the $j_q = 1/2$ levels corresponding to the $K_1(1273)$ and $K_0^*(1430)$ are quite different, just contrary to the situation discussed earlier. We also notice that an attempt to apply Eq. (11) with the $K_0^*(1430)$ mass for the determination of the B_0 and D_0 masses results, due to the small difference between the $K_2^*(1430)$ and $K_0^*(1430)$ masses, in $M_{B_0} \approx M_{B_2^*}$ and $M_{D_0} \approx M_{D_2^*}$. This is not consistent with the values of the B_1^* and D_1^{*0} masses given in Eq. (13), if the mass difference between the B_1^* and B_0, and also the D_1^{*0} and D_0, is small as expected. For the B_0 and D_0 masses equal to the B_1^* and D_1^{*0} masses given in Eq. (13), Eq. (11) by definition gives the K_0^* mass equal to the $K_1(1273)$ mass. A smaller K_0^* mass is also required in the description of the light-flavour P-wave mesons in the nonrelativistic quark model [38]. As noticed in [38], this can be explained if the observable $K_0^*(1430)$ mass is replaced by its "bare" $q\bar{q}$ mass corresponding to the K-matrix pole. In the K-matrix analysis of the 0^{++}-wave [39], the "bare" K_0^* mass, in one of the two possible solutions, is 1220 ± 70 MeV/c^2, consistent with the $K_1(1273)$ mass. Thus, if this conjecture is correct, the $K_1(1402)$ and $K_1(1273)$ assignment in the heavy quark limit to the $j_q = 3/2$ and $j_q = 1/2$ levels is consistent with the expected degeneracy of the $1^+(3/2)$ and $2^+(3/2)$ and, respectively, the $1^+(1/2)$ and $0^+(1/2)$ levels. It also provides a consistent description of the strange, charm and bottom meson production rates and also their masses and lends support to the models suggesting that the $j_q = 1/2$ levels for the strange, charm and bottom mesons are *below* the $j_q = 3/2$ levels. In particular, our results given in Eq. (13) are in excellent agreement with the predictions [27].

In conclusion we have shown that the mass dependences of the production rates for the six families of primary produced mesons in Z^0 hadronic decays obtained from results of the LEP experiments: the vector and tensor light-flavour mesons, the vector and P-wave charm, strange charm and bottom mesons are very similar. This allows not only the production rates of mesons with different flavours to be related, but also their masses, thus showing an interesting connection between hadron production properties and their masses. Our analysis suggests that the $0^+(1/2)$ and $1^+(1/2)$ levels are *below* the $1^+(3/2)$ and $2^+(3/2)$ levels not only for the charm and bottom but also for the strange mesons. Contrary to the conventional picture, the strange axial mesons $K_1(1273)$ and $K_1(1402)$ might be considered as mainly $1^+(1/2)$ and $1^+(3/2)$ levels, respectively, degenerate with the $0^+(1/2)$ and $2^+(3/2)$ levels of the $K_0^*(1430)$ and $K_2^*(1430)$ if the observed $K_0^*(1430)$ mass is replaced by its "bare" $q\bar{q}$ mass corresponding to the K-matrix pole and close to the $K_1(1273)$ mass. Although these results, if verified by future experiments, do not support the spin-orbit inversion suggested by Isgur [29], they amusingly lend strong support to his conclusion about the key role that the strange quark plays as the link between heavy- and light-quark hadrons.

Acknowledgements

I thank S. Godfrey and W. Venus for discussions and helpful comments.

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