

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH
CERN – EST

CERN EST/2001-004 (SM)

**REFLECTION OF INFRARED RADIATION
FROM THIN ALUMINIUM LAYERS**

S. Calatroni

Abstract

The thermal shielding of the LHC magnets cryostats will make use of Multi-Layer Insulation. This is a sandwich of several Mylar (polyester) foils 6 μm thick coated with a thin film of aluminium, having a thickness of some 30 nm. The thickness of the aluminium film must be kept at a minimum to minimise lateral thermal conduction. The outer layer of this sandwich stays at a temperature of 20 K or below, and receives IR radiation from surfaces at 77 K (wavelength of 37.6 μm at the peak of blackbody radiation), which should be reflected with the highest efficiency.

The minimum thickness for the aluminium layer to avoid transmission of the radiation can be calculated by making use of the skin effect theory, taking into account the changes in electrical properties that are due to the extremely low thickness of the film.

Geneva, Switzerland
September 2001

1. THEORETICAL BACKGROUND

1.1 Influence of thickness on the resistivity of thin films

The electrical resistivity of metals is usually described by the formula $\rho = m/ne^2\tau$ where m is the effective mass of the electrons in the metal, n their density, e the electric charge and τ is the relaxation time of the electrons. In terms of the mean free path ℓ it can be written as $\rho = mv_F/ne^2\ell$ ⁱ⁾. Phonons are the major source of scattering at room temperature, but by cooling down the material their density is reduced and other sources of scattering can dominate. This is often described by the Matthiessen rule which states that $\rho(T) = \rho_{ph}(T) + \rho_{def}$, assuming implicitly that the scattering by phonons and by all the different types of defects have the same probability. In the case of aluminium $\rho_{ph} = 2.66 \mu\Omega\text{cm}$ at 293 K, and practically vanishes below 20 K, while ρ_{def} can range from above $1 \mu\Omega\text{cm}$ to lower than $1 \text{n}\Omega\text{cm}$ for high quality specimens. The mean free path is $0.045 \mu\text{m}$ at 293 K, and scales inversely proportional to the resistivity.

It is obvious that the surfaces of the specimen are also a source of scattering. In the case of films having a thickness d of the order or lower than $0.045 \mu\text{m}$ this can be a dominant process even at room temperature. A rough approximation of this effect can be done by simply applying the Matthiessen formula $\rho_{film} = \rho_{bulk} + mv_F/ne^2d$, replacing the mean free path ℓ with the thickness d . The quantity mv_F/ne^2 which depends only on intrinsic quantities of the metal is often quoted as $(\rho\ell)_{bulk}$ and in the case of aluminium is approximately equal to $1.2 \times 10^{-15} \Omega\text{m}^2$ [1]. More detailed calculations [2, 3] show that the Matthiessen rule overestimates the resistivity, as illustrated in Figure 1 where the result of the numerical integration of the Fuchs formulae [2] is compared with the Matthiessen formula. It is worth mentioning that the Fuchs theory includes the possibility that the scattering of electrons at the surfaces is either specular (elastic scattering implying only a change of direction) or diffuse (resulting in a redefinition of the wave function). However even the finest experiments have never identified a fraction of specular electrons larger than a few percent. For our purpose we assume that this fraction is equal to zero.

1.2 Surface resistance and reflection coefficient

Electromagnetic waves penetrate into metals, where some energy is dissipated by the currents induced by the high-frequency fields. In the well-known case of the normal skin effect the penetration depth is equal to $\delta = (2\rho/\omega\mu_0)^{1/2}$, where ω is the angular frequency of the RF wave. The power dissipated is related to the surface resistance $R_s = \rho/\delta$ (which by definition is the ratio of the fields E/H at the surface of the metal) and equal to $1/2R_sH^2$. The mechanism of power dissipation is the classic Joule effect, assuming that the induced currents are locally proportional to the electric field, $\mathbf{J} = \mathbf{E}/\rho$.

This formulation is no longer valid when $\delta < \ell$, regime described by the theory of the anomalous skin effect. A commonly accepted picture is that, when the mean free path is larger than the penetration depth, the conduction electrons “escape” from the region where the field is penetrating, and thus are no longer effective in screening the fields. As a consequence, an increase in mean free path does not further reduce the surface resistance. This happens either at high frequency, or at low temperature where the electron mean free path ℓ increases due to the reduced scattering by phonons. In the case of aluminium at room temperature $\delta = \ell$ at about 3 THz. It is beyond the scope of this short note to recall all the details of the anomalous skin effect. It is interesting to mention that in this regime the relation between field and current is no longer local [4 - 7].

ⁱ⁾ The relaxation time τ is the average interval between two successive scatterings of the conduction electrons in the metal, and is related to the mean free path through the obvious relation $\tau = \ell/v_F$, v_F being the Fermi velocity. These scatterings are the origin of the electrical resistivity and are due to phonons, impurities, lattice defects, grain boundaries and also the surfaces of the metal.

When further increasing the frequency (or further increasing the mean free path, which in practice requires specimens of extremely high purity) another regime sets in when $\omega\tau \gg 1$, where τ is the relaxation time. In this regime the period of the electromagnetic wave is typically much smaller than the relaxation time and the electrons cannot escape during a full RF period, and thus the surface resistance can again be described by the normal skin effect formulas. It is worth mentioning that the opposite condition $\omega\tau \ll 1$ is implicit in the condition $\delta > \ell$ for the normal skin effect regime.

The fraction of electromagnetic energy reflected by a metallic surface is characterised by the reflection coefficient r , assuming the incident energy i equal to unity. This satisfies the obvious relation $i = 1 = r + d + t$, d being the fraction of energy dissipated in the metal (calculable with the surface resistance) and t the fraction which is transmitted (usually $t \neq 0$ only for very thin layers). The reflection coefficient may be approximated by $r \approx (1 - 4R_s/R_{vacuum})$, whenever R_s is much smaller than the vacuum impedance $R_{vacuum} = 376.6 \Omega$. The result is that in almost all practical cases, r is very close to unit except in some special conditions, and it is often more convenient to work with the quantity $(1 - r)$ ⁱⁱ⁾ Figure 2 [4] illustrates this fact for a thick silver slab, showing also the difference between the predictions of the classical model and the full theory, which takes into account the anomalous effect. As discussed above, the classical theory gives correct results at very high frequencies however.

2. REFLECTIVITY OF THIN ALUMINIUM LAYERS

The resistivity of very thin aluminium films as a function of temperature for different bulk RRR values (defined here as the ratio of the resistivity at 293 K and 20 K) and thickness is illustrated in Figure 3. This has been calculated using the pessimistic Matthiessen formulation, which overestimates the thickness effect as already illustrated in Figure 1. The resistivity of thin films is clearly dominated by the size effect, while the RRR plays only a secondary role. For example, a 30 nm film has a room temperature resistivity which is almost three times larger than the corresponding bulk value (two times if the calculation is done with the full Fuchs theory), and its RRR is about 1.6 whatever the purity of the bulk might be. This modification of the resistivity must of course be taken into account for the calculation of the reflectivity.

The value of $\omega\tau$ at 20 K in the case of an incident IR radiation of 37.6 μm wavelength is approximately 1.5, 3 and 6 for the films of 30 nm, 60 nm and 120 nm respectively, almost independently of the bulk RRR value. Although the strong inequality $\omega\tau \gg 1$ is not respected, the calculation of the reflectivity can be performed with the formulae of the normal skin effect without introducing serious deviations [8, 9]. It is however necessary to use a set of equations, which allow for the transmission of the radiation through the film in case its thickness is too small [10]. The reflection coefficient as a function of thickness is thus illustrated in Figure 4, for films having the same bulk RRR as in Figure 3. To obtain a reflectivity better than 99%, the thickness of the aluminium layer must be larger than 40 nm. The result depends only negligibly on the bulk RRR for films less than one μm thick, since the electrical resistivity is dominated by the thickness effect. For very thick films (solid foils for example) the reflectivity depends only on the purity however. At very small thickness the reflectivity decreases progressively, essentially because of the transmission of the wave through the film. The value of zero is however not reached even in the absence of a film, because the Mylar has a refractive index of 1.64 at this wavelength, causing a reflection at the vacuum/Mylar interface.

ⁱⁱ⁾ As a classical example, the surface resistance of an infinitely thick slab of aluminium at room temperature in the classic skin effect regime where $\delta > \ell$ ($\omega/2\pi < 3$ THz) is always lower than 0.56Ω , resulting in a reflectivity $r > 0.994$.

3. CONCLUSIONS

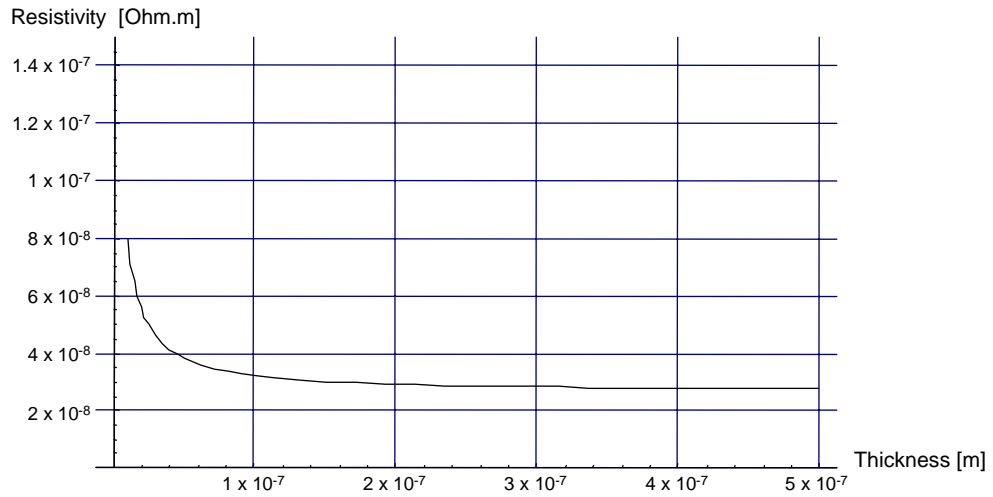
To achieve reflection $> 99\%$ of IR radiation produced by surfaces at 77 K, an aluminium film at 20 K should be at least 40 nm thick. This has been calculated using the appropriate formulae for the reflection of an electromagnetic wave, taking into account the increase in resistivity due to the small thickness of the film. Possible non-uniformity of the coating may suggest increasing the thickness above this value to have a sufficient margin for tolerances. The presence of some oxide on the surface of the metal might nevertheless reduce the reflectivity below the theoretical values.

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Figure 1: Resistivity as a function of thickness for an aluminium film at room temperature. For large thickness both plots converge to the bulk value of $2.66 \mu\Omega\text{cm}$.

(a) Numerical integration of the full Fuchs theory, assuming totally diffuse scattering at the surface



(b) Simplified formulation based on the Matthiessen rule.

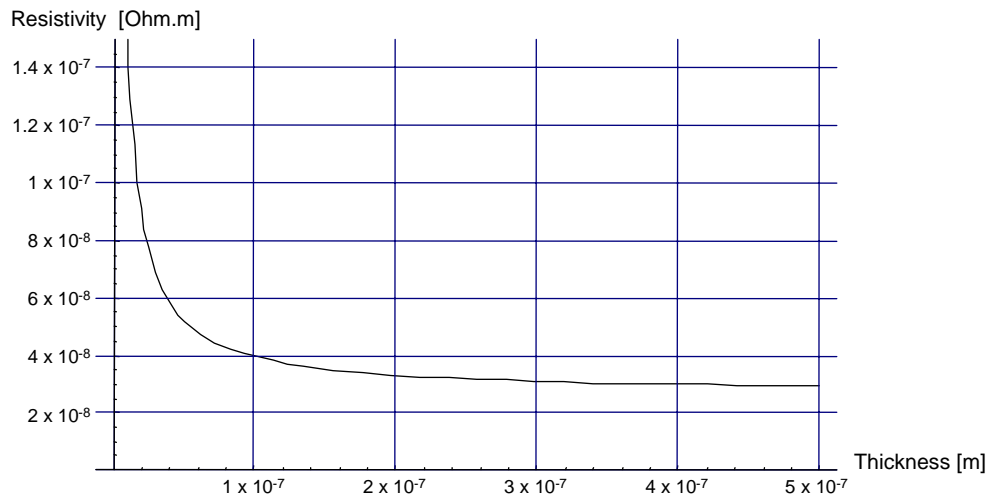


Figure 2: Reflection coefficient, plotted as $\text{Log}_{10}(1-r)$, for an $\text{RRR} = 480$ thick silver slab at liquid helium temperature as a function of $\text{Log}_{10}(\text{wavelength in meters})$ [4].

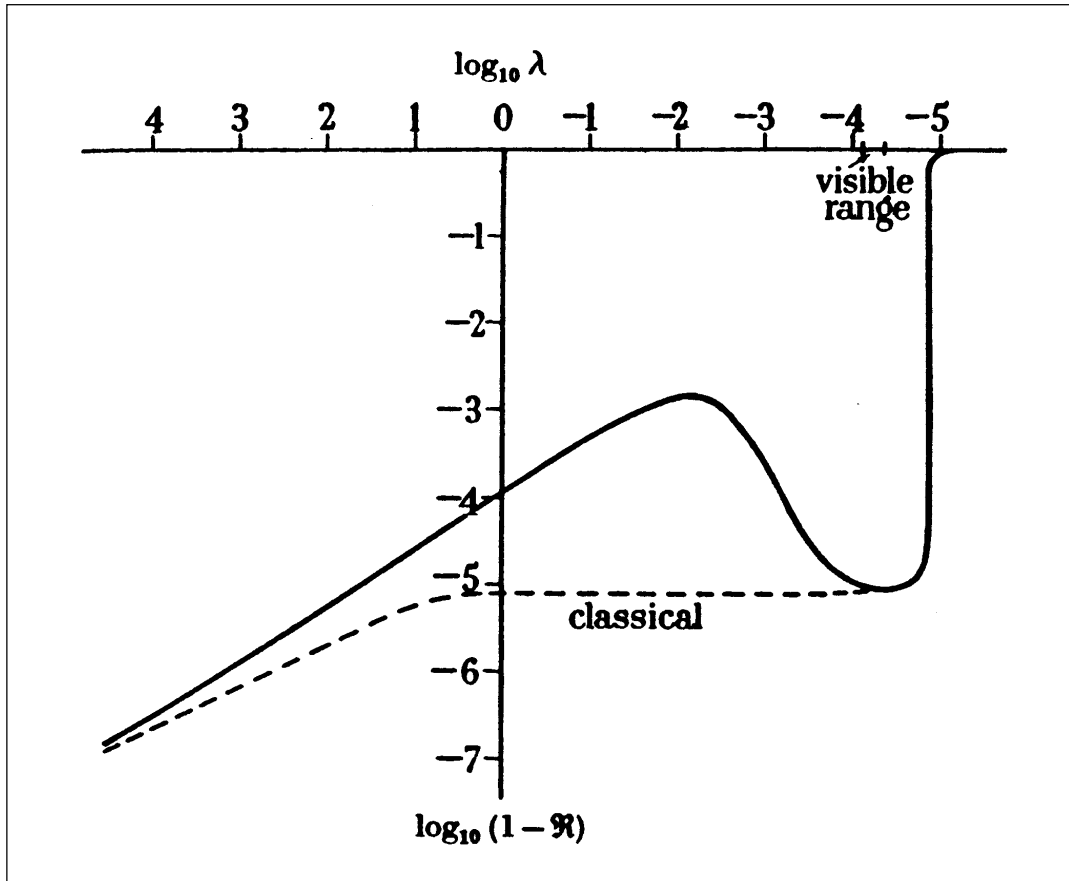
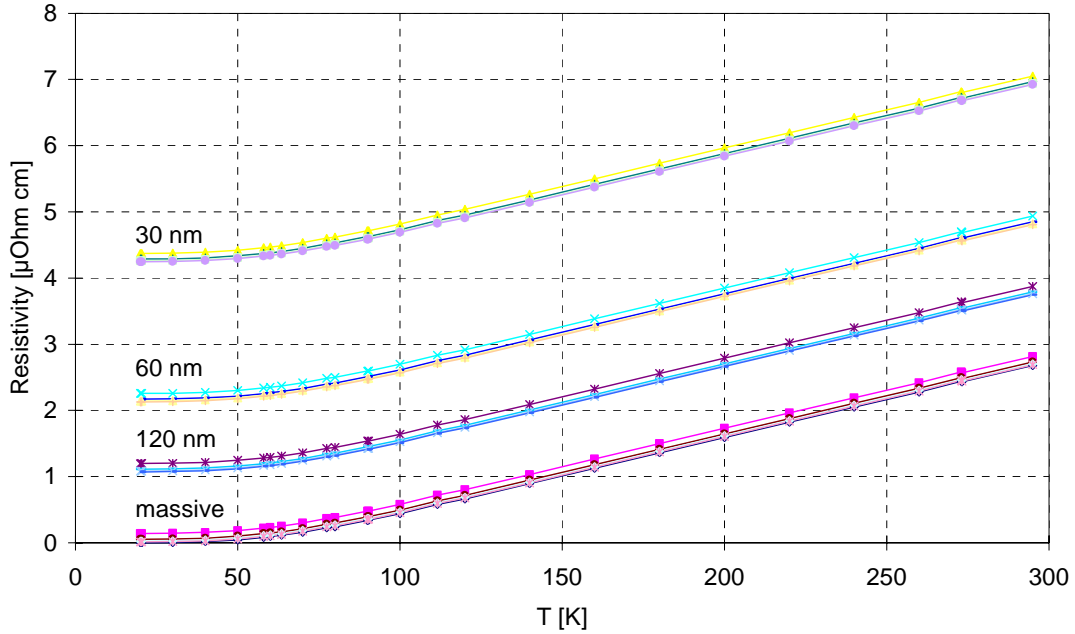


Figure 3: Resistivity as a function of temperature for massive aluminium and for films of 30 nm, 60 nm and 120 nm, calculated assuming a bulk RRR of 20, 50, 100 and 200. Data for a massive reference sample of RRR 3600 are included.

(a) Plot in linear scale. For a given thickness, the data points of films with different RRR values are almost superposed.



(b) The same data plotted in logarithmic scale illustrating the change with RRR of the low-temperature resistivity.

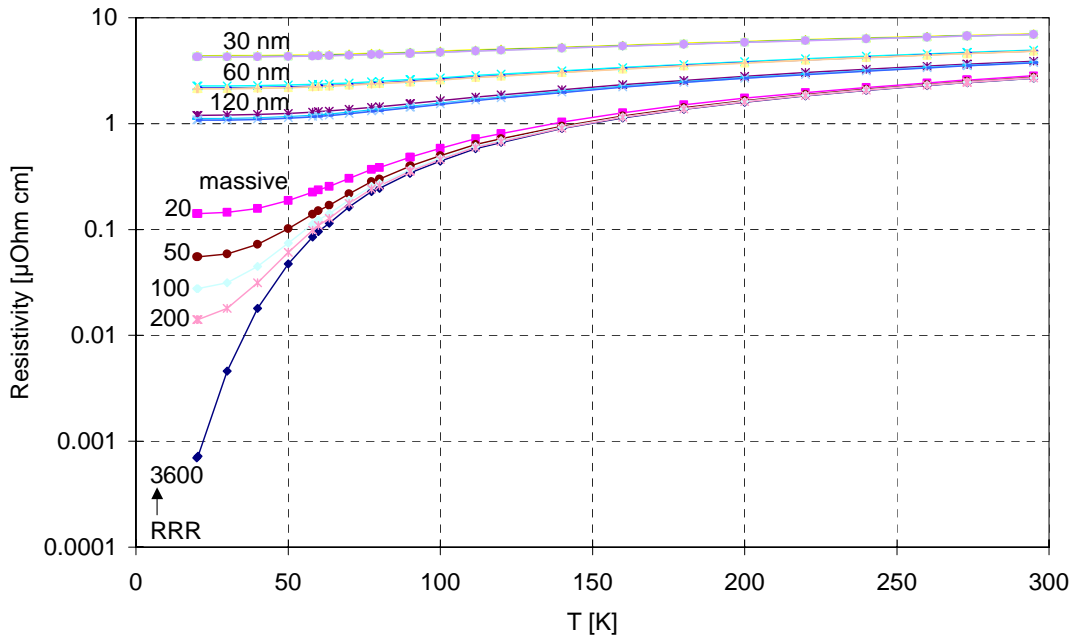
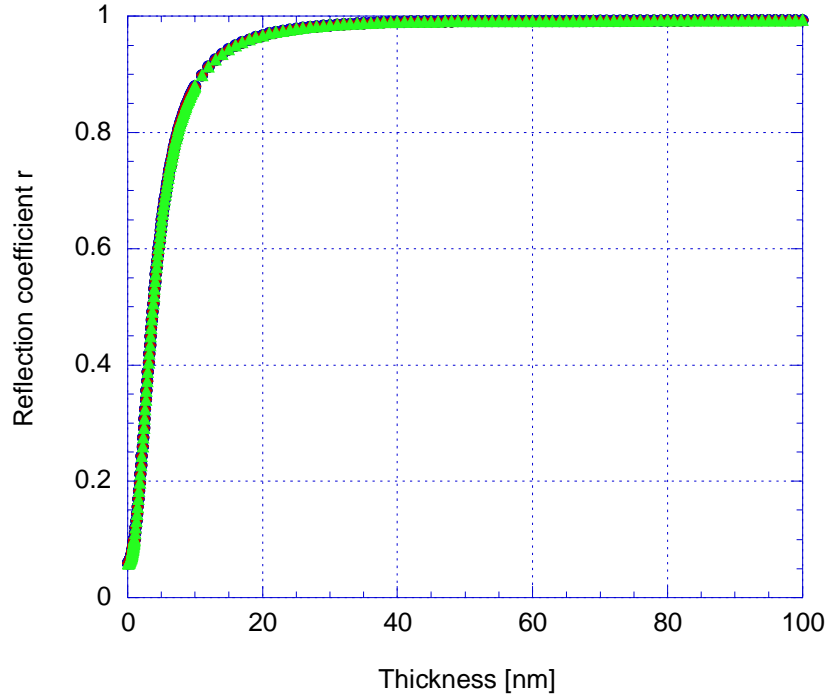


Figure 4:

(a) Reflection coefficient r as a function of the thickness of an aluminium film at 20 K, for an incident IR wavelength corresponding to 77 K. The calculation is performed for four bulk RRR values as in Figure 3, although the curves look coincident.



(b) The quantity $(1 - r)$ calculated from the above plot. When $(1 - r)$ decreases from 0.02 to 0.01 the reflected electromagnetic energy doubles. It should be noted that the thickness effect is no longer dominant for thickness larger than about 1 μm , where the purity starts to play a role.

