

Kaluza-Klein Excitations of W and Z at the LHC?

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Abstract

Deviations from standard electroweak physics arise in the framework of a Randall-Sundrum model, with matter and gauge fields in the bulk and the Higgs field localized on the TeV brane. We focus in particular on modifications associated with the weak mixing angle. Comparison with the electroweak precision data yields a rather stringent lower bound of about 10 TeV on the masses of the lowest Kaluza-Klein excitation of the W and Z bosons. With some optimistic assumptions the bound could be lowered to about 7 TeV.

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1 Introduction

The standard model (SM), as formulated for a flat four-dimensional space-time, can be easily generalized to the RS model [1–5]. In the RS model, the four dimensional world around us arises from the compactification of a curved 5D geometry. The fifth dimension is a S_1/\mathbb{Z}_2 orbifold and contains two 4D branes at the orbifold fixed points at $y = 0$ and $y = \pi R$. The 5D geometry is a slice of AdS with the line element $ds^2 = dy^2 + e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu$, where $\sigma(y) = k|y|$, k measures the curvature along the fifth dimension, x represents the familiar four dimensions and y represents the fifth dimension. The brane at $y = 0$ is called the Planck-brane and the brane at $y = \pi R$ is called the TeV-brane because the typical mass scale on each of the branes is of the order of the effective 4D Planck mass and a TeV respectively. This exponential hierarchy of energy scales is generated by the warp factor $\Omega = e^{-\pi k R}$, with $kR \simeq 11$, and provides a new solution to the gauge hierarchy problem.

By compactifying the fifth dimension, a field living in the bulk, i.e. in the full 5D space-time, can be decomposed into an infinite number of 4D fields with different effective 4D masses using a method known as the Kaluza-Klein (KK) decomposition. Depending on the choice of k and R , there is a large gap between the mass of the lightest KK mode of a bulk field and its next excited state. The mass of the first excited state is typically of the order of 10 TeV (since experimental constraints rule out excited states which are significantly lighter), while the mass of the ground state is constrained by experimental data [6, 7], since such fields correspond to the ones we actually observe in accelerators.

Bulk fields allow one to address problems related to non-renormalizable operators within the RS framework, fermion masses and mixings [3, 5], neutrino masses [8, 9] and gauge coupling unification [10, 11]. With bulk gauge fields, the couplings and masses of the weak gauge bosons deviate from their SM values. In ref. [12] the modifications of couplings and masses were treated independently of each other. This is a good approximation if the SM fermions live close to one of the two branes. In this letter we present a combined analysis of the two effects in order to cover the case where the fermions are weakly localized or delocalized in the extra dimension as well. Weakly localized fermions are expected for instance, if the warped geometry also induces the fermion mass hierarchy [5].

We find that the lowest KK excitations of gauge bosons and fermions have to be heavier than about 10 TeV. There is no window for “light” KK states even for delocalized fermions, as was the case for the constraints derived in refs. [3, 4]. Hence, it is questionable if these excitations can be directly studied at the LHC. Indirect evidence for extra dimensions, e.g. rare processes

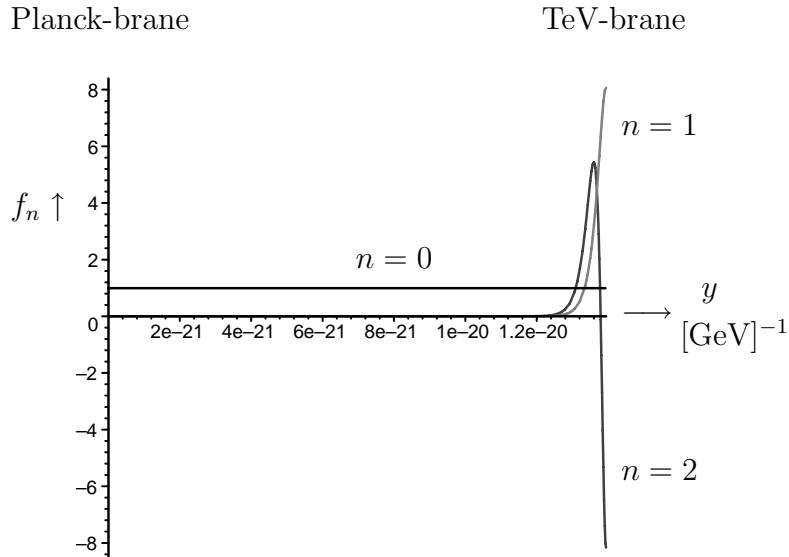


Figure 1: The ground state and the first two excited states of the Z boson when $kR = 10.83$, $k = \overline{M}_{\text{Pl}} = 2.44 \times 10^{18}$ GeV and $a_Z = 0.1849$.

such as $n-\bar{n}$ oscillations or $\mu \rightarrow e + \gamma$ [13, 9], are therefore very important. Nevertheless, one may speculate if the scenario presented here could account for the small deviations of the SM predictions from recent electroweak precision data [14, 15]. This might allow a lowering of the bound to around 7 TeV.

2 The Kaluza-Klein reduction

With some slight modifications, the SM is easily embedded in a warped geometry. All of the fields are assumed to live in the bulk, except the Higgs field, which is confined to the TeV-brane [2, 12]. (This can be considered as an approximation to the case where the Higgs field lives in the bulk but is concentrated exponentially around the TeV-brane. The ground state of a scalar field behaves as $e^{4\sigma}$ and for values of kR around 11, this approximation is accurate up to a few permille [9].)

The masses of the weak gauge bosons are generated by spontaneous symmetry breaking arising from the Higgs mechanism, $M^2(y) = a^2 k \delta(y - \pi R)$, where a is a dimensionless parameter which is determined by how strongly the gauge boson couples to the Higgs field. Since there is no large hierarchy between the weak and KK scales, the 5D mass term M^2 should be included

in the KK reduction of the weak gauge bosons from the very beginning [12]. The masses and 5D wave functions of a gauge field are obtained from its 5D equations of motion, $\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}g^{MN}g^{RS}F_{NS}) - M^2(y)g^{RS}A_S = 0$. Here, the $A_4 = 0$ gauge is imposed and this is only possible because the Higgs field is localized on a brane. If the Higgs were to propagate in the bulk instead, it would not be possible to impose both the unitarity and the $A_4 = 0$ gauge simultaneously and we would have to contend with A_4 excitations as well. Using the method of separation of variables, a gauge field can be decomposed as follows:

$$A_\mu(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} A_\mu^{(n)}(x) f_n(y), \quad (1)$$

where $A_\mu^{(n)}$ satisfies the field equation for a gauge boson of mass m_n . This expansion is only valid in the weak coupling limit for non-Abelian gauge fields. However, if we drop the requirement that the $A_\mu^{(n)}$'s satisfy the 4D equations of motion for a Yang-Mills field, any field configuration can be decomposed according to eq. (1). The wave functions f_n follow from $(\partial_y^2 - 2\sigma'\partial_y - M^2 + e^{2\sigma}m_n^2)f_n = 0$, with $\sigma' = d\sigma/dy$. The f_n are normalized by the condition $\frac{1}{2\pi R} \int_{-\pi R}^{\pi R} f_n(y)^2 dy = 1$. The solution is given by [17]

$$f_n = \frac{e^\sigma (J_1(\frac{m_n}{k}e^\sigma) + b_n Y_1(\frac{m_n}{k}e^\sigma))}{N_n}, \quad (2)$$

where N_n is the normalization constant, and b_n and m_n are obtained by solving the following system of equations [12]:

$$b(m, a^2) = \frac{-\frac{a^2}{2}J_1(\frac{m}{k}) + \frac{m}{k}J_0(\frac{m}{k})}{-\frac{a^2}{2}Y_1(\frac{m}{k}) + \frac{m}{k}Y_0(\frac{m}{k})}, \quad (3)$$

$$b_n(e^{-\pi k R} k x_n, 0) = b(k x_n, -a^2),$$

where $x_n = e^{\pi k R} m_n/k$. In fig. 1 we present the wave functions of the ground state and the first two excited states of the Z boson.

Bulk fermions are described by the 5D equation of motion $(g^{MN}\gamma_M(\partial_M + \Gamma_M) + m_\Psi)\Psi = 0$, where $m_\Psi = c\sigma'$ and Γ_M is the spin connection in the tetrad formulation. The Dirac mass term m_Ψ takes on the functional form it has because of the parity restriction $\Psi(-y) = \pm\gamma_5\Psi(y)$. For the same reason, the KK ground state of a fermion can only be either left-handed or right-handed. These ground states are identified with the fermion fields actually observed by experiments. Again, the 5D field is decomposed into a KK tower [8, 3],

$$\Psi(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \Psi^{(n)}(x) f_n^c(y), \quad (4)$$

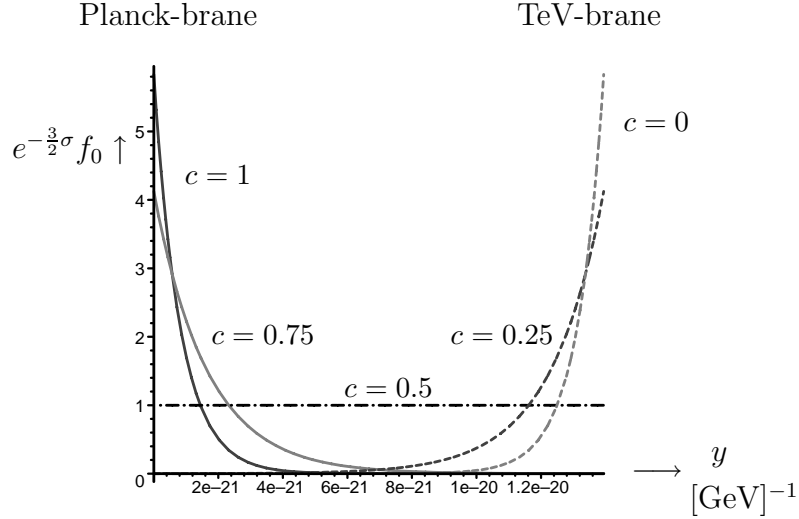


Figure 2: The wave functions for the ground state fermions for different values of c .

where $\Psi^{(n)}(x)$ obeys the equation of motion for a fermion of mass m_n ,

$$f_0^c(y) = \frac{e^{(2-c)\sigma}}{N_0}, \quad (5)$$

and

$$f_n^c(y) = \frac{e^{\frac{5}{2}\sigma} (J_\alpha(\frac{m_n}{k} e^\sigma) + b_n Y_\alpha(\frac{m_n}{k} e^\sigma))}{N_n}, \quad (6)$$

for $n > 0$. The wave functions f_n^c are normalized by

$$\int_{-\pi R}^{\pi R} \frac{e^{-3\sigma} f_n^c(y)^2}{2\pi k R} dy = 1. \quad (7)$$

Fig. 2 shows the wave function for the ground state fermions for different values of c . For $c > 1/2$ ($< 1/2$) the zero mode is localized towards the Planck-brane (TeV-brane). For c close to $1/2$ the zero mode is only weakly localized or even delocalized ($c = 1/2$).

3 Measuring the Weak mixing angle

Experimentally, there are several ways to measure the weak mixing angle, θ_W . Although the equations of the SM remain unchanged in 5D, the effective 4D behavior of the fields is slightly modified. In particular, measurements of θ_W made on the assumption of a flat 4D space-time would result in a value

different from the actual 5D weak mixing angle, θ_5 . The weak mixing angle can be calculated from the ratio of the masses of the W and Z bosons,

$$\cos^2 \theta_1 = \frac{m_W^2}{m_Z^2}, \quad (8)$$

where the subscript 1 indicates that this is the first possible definition. Alternatively, θ_W can also be obtained from the corresponding ratio of the gauge coupling strengths. In the SM, both definitions are equivalent (at the tree level).

In the framework of warped models, deviations from standard 4D electroweak physics arise. The relationship between the (5D) gauge coupling strengths and the physical gauge boson mass is no longer linear because of eq. 3 [12]. As a result, the W mass gets shifted upward compared to the SM.⁴ Furthermore, the couplings of the weak gauge bosons to fermions are modified. In particular, they become dependent on the localization of the fermion within the extra dimension [12]. The effective 4D coupling between the ground state of a fermion and the n^{th} excited state of a gauge boson is given by [3]

$$g^{(n)} = \frac{g^{(5)}}{(2\pi R)^{3/2}} \int_{-\pi R}^{\pi R} e^{\sigma} f_0^c(y)^2 f_n(y) dy, \quad (9)$$

where $g^{(5)}$ is the 5D gauge coupling.

In the approach presented above electroweak symmetry breaking is treated in the 5D framework. Corrections to weak gauge boson masses and gauge coupling strengths arise automatically in the KK reduction. One can interpret these results also as a mixing effect between a massless (y independent) zero mode of a gauge field and its (y dependent) KK states. The mixing between the weak gauge bosons and their KK excitations is on the order of m_W^2/M_{KK}^2 . This explains why all deviations from SM physics are proportional $1/M_{KK}^2$. Note that somewhat similar results are obtained for the case of a flat extra dimension [16].

In the following, we perform a *tree level* comparison between the warped model with bulk fields and the ordinary SM. This procedure is justified because SM-like quantum corrections are essentially the same in both models. The tiny modifications (of order 10^{-3}) of the tree level physics will only cause sub-leading corrections in the loops. In the warped model, there are additional radiative corrections coming from loops involving KK states. However, their influence is small because of the large masses (about 10 TeV) of the excited states.

⁴Curiously, the new experimental value for m_W is indeed slightly above the SM prediction [14].

After electroweak symmetry breaking, the neutral gauge bosons in the bulk mix via the 5D weak mixing angle $\tan \theta_5 = g_1^{(5)}/g_2^{(5)}$, where $g_2^{(5)}$ and $g_1^{(5)}$ denote the $SU(2)$ and $U(1)$ gauge coupling strengths in five dimensions respectively. The gauge coupling strength of the Z boson in 5D is then given by $g_Z^{(5)} = g_2^{(5)}/\cos \theta_5$. Analogous relations hold for the W boson and the photon. The effective 4D gauge couplings to fermions, g_W , g_Z and g_γ are obtained from the $n = 0$ case of eq. 9, using the relevant 5D gauge coupling strengths. From these quantities, the weak mixing angle can be defined as

$$\cos^2 \theta_2 = \frac{g_W^2}{g_Z^2}, \quad (10)$$

or

$$\cos^2 \theta_3 = 1 - \frac{g_\gamma^2}{g_W^2}. \quad (11)$$

Experimentally, these definitions can be regarded as comparisons of the strengths of the charged and neutral current, and of the electric and charged current respectively.

In the 4D SM, the values of the weak mixing angle inferred from eqs. 8, 10 and 11 agree, of course. Since in the warped model, the gauge coupling strengths and gauge bosons masses are obtained from eqs. 3 and 9, this is no longer the case. The differences between these angles, as measured by

$$\Delta_{ij} = \cos^2 \theta_i - \cos^2 \theta_j, \quad (12)$$

is too small to be measured, given the current range of uncertainty in the experimental data. However, upper bounds can be set on the Δ 's based on the range of uncertainty. This in turn provides important constraints on the parameters of the warped SM model. Rather than listing the permissible range of values for the internal parameters of the model, a more transparent way of expressing these constraints from an experimental perspective is to provide a lower bound for the masses of the first KK excitation of the weak gauge bosons.

4 Numerical analysis

The graphs in this section show the constraints on the mass of the first KK excitation for different values of k/M_5 , where M_5 (the Planck mass in 5D) has been chosen to fit the observed value for the Planck mass in 4D, $M_{\text{Pl}}^2 \approx M_5^3/k$ [1]. In each of the graphs, the lower bounds for the mass of the excited Z boson, $M_{KK} \equiv M_Z^{(1)}$ (the difference in mass between the excited Z

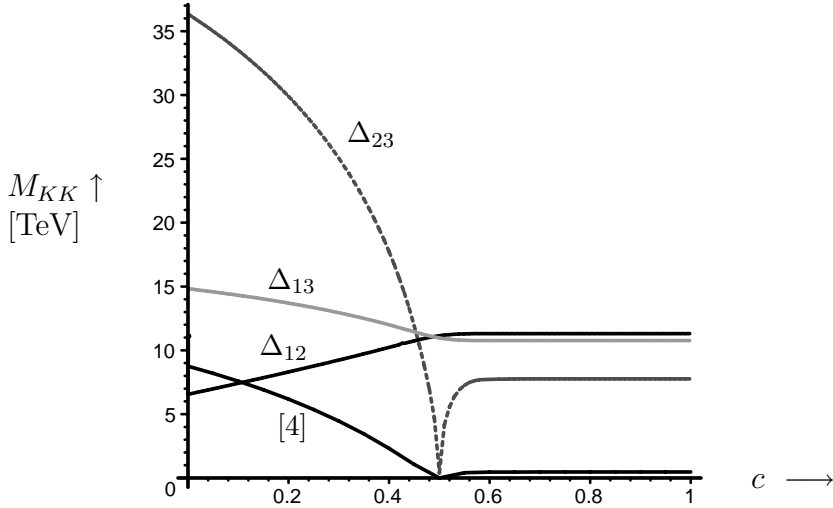


Figure 3: The three lower bound constraints and the limit of ref. [4] for $k = M_5$ as a function of c .

and W bosons is insignificant) arising from each of the three constraints are plotted. The mass of the first KK state is adjusted in our model by varying the radius of the orbifold. In all the following graphs, we use the experimental constraints $\Delta_{12} \leq 1.2 \times 10^{-3}$, $\Delta_{13} \leq 1.2 \times 10^{-3}$ and $\Delta_{23} \leq 1.6 \times 10^{-4}$, based on data from the Particle Data Group [6]. The bound on $\Delta_{12,13}$ is dominated by the experimental error of the W mass, $M_W = 80.419 \pm 0.056$ GeV, which results in an uncertainty in $\cos^2 \theta_1$ of 1.1×10^{-3} . For the weak mixing angle, we take $\sin^2 \theta_W = 0.23117 \pm 0.00016$ with the error providing the constraint on Δ_{23} .

The numerical analysis is done in the following manner. We fix the values of kR , k/M_5 and c . From the equation for the KK gauge boson spectrum (3), we determine the gauge boson mass parameters, $a^2(W)$ and $a^2(Z)$ by fitting the ground state mass to the experimental W and Z masses. The masses of the KK excitations are then also fixed, as is the value of $\cos^2 \theta_1$. The 5D weak mixing angle follows from $\cos^2 \theta_5 = a^2(W)/a^2(Z)$. With this information, we can calculate the ratios of the effective gauge coupling strengths (9) which enter the definitions of $\cos^2 \theta_{2,3}$. The deviations from SM physics are then easily obtained from eq. (12). We stress again that for different values of the KK mass, the deviations scale as $\Delta_{ij} \sim 1/M_{KK}^2$.

In fig. 3 we display the three different constraints on M_{KK} as a function of the 5D fermion mass for $k/M_5 = 1$. For fermions located close to the Planck-brane, $c > 1/2$, the strongest bound, $M_{KK} \gtrsim 11.1$ TeV, arises from

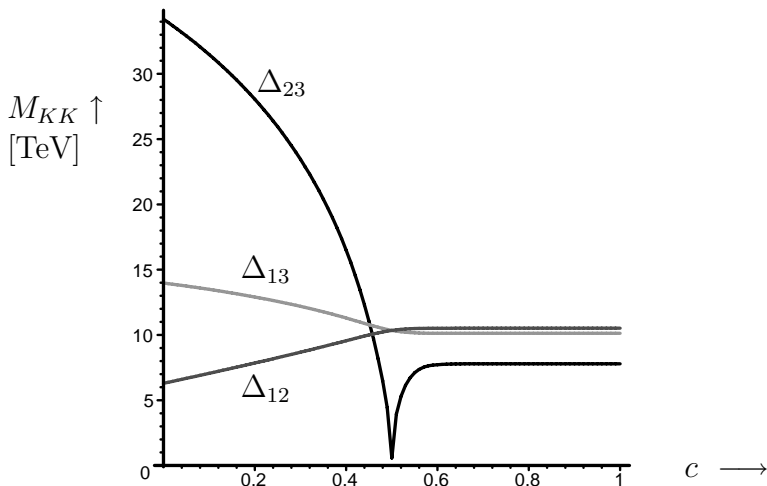


Figure 4: The three lower bound constraints for $k = M_5/100$ as a function of c .

Δ_{12} , i.e. from the modification of the ratio of the weak gauge boson masses. The constraint from Δ_{23} , which only involves the effective gauge coupling strengths, gives a somewhat lower bound, $M_{KK} \gtrsim 7.8$ TeV. This is because near the Planck-brane, the gauge boson wave function is almost flat and independent of the gauge boson mass [12]. Therefore, Δ_{12} and Δ_{13} lead to almost identical results. For $c \lesssim 1/2$, the fermions become sensitive to the dip in the gauge boson wave function at the TeV-brane [12] and the gauge couplings deviate from their SM values. The constraint from Δ_{23} takes over and pushes the value of M_{KK} far above 10 TeV. For fermions strictly confined to the TeV-brane ($c \rightarrow -\infty$), we find $M_{KK} \gtrsim 62$ TeV. For comparison, we also include the lower bound on M_{KK} from ref. [4] in fig. 3. It arises from the contribution of the excited gauge bosons to electroweak observables, like for example, effective 4-fermion interactions. Our bounds turn out to be much stronger for all values of c .⁵ In particular, $M_{KK} \sim 1$ TeV is excluded even for fermions close to the Planck-brane. This is the main result of the present paper.

In fig. 4, we give the corresponding results for $k/M_5 = 0.01$. Qualitatively, the picture is unchanged, but the constraints on M_{KK} are slightly looser by about 7 percent. For fermions localized close to the Planck-brane, the bound from Δ_{12} on M_{KK} is relaxed to 10.3 TeV. In general, the bound on M_{KK} depends logarithmically on k/M_5 , with the relative change given

⁵This justifies our neglect of loops containing KK excitations in the comparison with the SM.

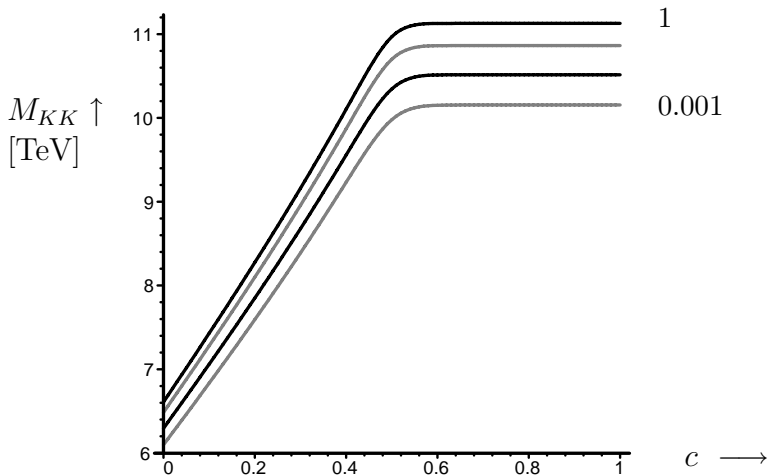


Figure 5: The constraints obtained from Δ_{12} for different values of $k/M_5 = 1, 0.1, 0.01, 0.001$ (from top to bottom).

approximately by $0.015 \ln(k/M_5)$. In fig. 5, this behavior is demonstrated for Δ_{12} .

So far we have only included a single 5D mass parameter c . If all SM fermions have a common location in the extra dimension, the constraint $c \lesssim 0.3$ applies. Otherwise the overlap of the fermion wave functions and the Higgs is insufficient to generate the observed top quark mass. Then the KK scale has to be larger than about 25 TeV, making this scenario less attractive. However, in the SM there is no symmetry that requires the c parameters of different fermion species to be degenerate. The c parameters can be chosen in such a way as to generate the fermion masses and mixings without introducing hierarchies in the 5D Yukawa couplings [5]. The leptons and light quarks then turn out to have c parameters larger than $1/2$. Since these particles are the ones used in experiments, the 11 TeV bound on M_{KK} applies also for this case.

5 Discussion and summary

According to ref. [4], the LHC can probe M_{KK} up to at most 7 TeV. From our results so far, it seems unlikely that KK excitations of gauge bosons can be produced directly at the LHC. Still, there may be interesting experimental signatures of the presented scenario, like for instance, exotic processes such as lepton flavor violation or $n - \bar{n}$ oscillations [9].

One should keep in mind however, that the crucial input in deriving the 10

TeV constraint is the tolerated deviation in the W mass. We took $\Delta M_W = 56$ MeV [6]. The current experimental accuracy on the W mass is about 34 MeV [7]. If we were to take this value to be ΔM_W , the quantities $\Delta_{12,13}$ would have to be smaller than 0.86×10^{-3} . The lower bound on M_{KK} would increase to 13.1 TeV (for $k = M_5$). Most interestingly, however, recent data seem to favor a W mass which is somewhat above the SM prediction. At the 1σ level, the SM prediction and the experimental result have no overlap [14]. The central values differ by about 76 MeV for a Higgs mass of 115 GeV. Taking this value for ΔM_W results in $M_{KK} \gtrsim 9.4$ TeV (for $k = M_5$). For larger Higgs masses, the SM deviates even more from experiment. For a Higgs mass of 250 GeV, the SM prediction is about 130 MeV lower than the value found experimentally. This could be accounted for by KK gauge bosons with a mass of 7 TeV. Thus, being very optimistic, KK gauge boson excitations may be at the verge of being discovered at the LHC.

For $M_{KK} \sim 7$ TeV, the modifications to the effective gauge coupling strengths push Δ_{23} close to its allowed value, as can be observed in fig. 3. Since Δ_{23} is rather sensitive to the location of fermions, experiments with different quarks and/or leptons may actually result in different values for $\sin^2 \theta_W$. This may be of some relevance to the 3.6σ discrepancy in measuring $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ using leptonic and hadronic asymmetries [15].

In conclusion, our calculations suggest that the KK excitations of W and Z within the RS model are expected to be rather heavy, of the order of 10 TeV or so. With some caveats, this may be brought down to about 7 TeV, which would make their discovery at the LHC much easier.

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References

- [1] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83** (1999) 3370.
- [2] S. Chang, J. Hisano, H. Nakano, N. Okada and Yamaguchi, *Phys. Rev.* **D62** (2000) 084025 [hep-ph/9912498].

- [3] T. Gherghetta and A. Pomarol, *Nucl. Phys.* **B586** (2000) 141 [hep-ph/0003129].
- [4] H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Phys. Rev.* **D63** (2001) 075004 [hep-ph/0006041].
- [5] S.J. Huber and Q. Shafi, *Phys. Lett.* **B498** (2001) 256 [hep-ph/0010196].
- [6] D.E. Groom *et al.*, Particle Data Group, *Eur. Phys. J.* **C15** (2000) 1.
- [7] N.K. Watson, hep-ph/0105230.
- [8] Y. Grossman and M. Neubert, *Phys. Lett.* **B474** (2000) 361 [hep-ph/9912408].
- [9] S.J. Huber and Q. Shafi, *Phys. Lett.* **B512** (2001) 365 [hep-ph/0104293].
- [10] A. Pomarol, *Phys. Rev. Lett.* **85** (2000) 4004 [hep-ph/0005293].
- [11] L.J. Randall and M.D. Schwartz, hep-th/0108114, hep-th/0108115.
- [12] S.J. Huber and Q. Shafi, *Phys. Rev.* **D63** (2001) 045010 [hep-ph/0005286].
- [13] R. Kitano, *Phys. Lett.* **B481** (2000) 39 [hep-ph/0002279].
- [14] G. Weiglein, hep-ph/0108063.
- [15] The LEP Collaborations ALEPH, DELPHI, L3, OPAL, the LEP Electroweak Working Group and the SLD Heavy Flavour and Electroweak Groups, LEPEWWG/2001-01.
- [16] A. Delgado, A. Pomarol and M. Quiros, *JHEP* **0001** (2000) 030 [hep-ph/9911252].
- [17] H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Phys. Lett.* **B473** (2000) 43; A. Pomarol, *Phys. Lett.* **B486** (2000) 153.
- [18] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, *Phys. Lett.* **B479** (2000) 43.