Symmetries of Topological Field Theories in the BV-framework

F. Gieres^{a)}, J.M. Grimstrup^{b)1}, H. Nieder^{c)}, T. Pisar^{b)2}, M. Schweda^{b)3}

 ^{a)} Institut de Physique Nucléaire, Université Claude Bernard 43, boulevard du 11 novembre 1918
 F - 69622 - Villeurbanne (France)

^{b)} Institut für Theoretische Physik, Technische Universität Wien Wiedner Hauptstraße 8-10 A - 1040 Wien (Austria)

^{c)} Theory Division, CERNCH - 1211 - Geneva 23 (Switzerland)

Abstract. Topological field theories of Schwarz-type generally admit symmetries whose algebra does not close off-shell, e.g. the basic symmetries of BF models or vector supersymmetry of the gauge-fixed action for Chern-Simons theory (this symmetry being at the origin of the perturbative finiteness of the theory). We present a detailed discussion of all these symmetries within the algebraic approach to the Batalin-Vilkovisky formalism. Moreover, we discuss the general algebraic construction of topological models of both Schwarz- and Witten-type.

¹Work supported by "The Danish Research Academy".

 $^{^2 \}rm Work$ supported by the "Fonds zur Förderung der Wissenschaftlichen Forschung" under Project Grant Number P11582-PHY.

³email: mschweda@tph.tuwien.ac.at

Contents

1	Intr	roduction	1
2	Ger	neral setting	2
3	Example 1: Chern-Simons theory in \mathbb{R}^3		
	3.1	Symmetries of the classical action	2
		3.1.1 The model and its symmetries	2
		3.1.2 Geometric framework of BRST- and BV-approaches	2
		3.1.3 Generalized fields and derivatives	3
		3.1.4 Derivation of s -variations from a horizontality condition	3
		3.1.5 Construction of the minimal BV-action	4
		3.1.6 Gauge-fixing	5
		3.1.7 BV-interpretation	6
		3.1.8 Elimination of antifields	6
		3.1.9 BV versus BRST	8
	3.2	VSUSY	8
		3.2.1 VSUSY-transformations	8
		$3.2.2$ Ward identity \ldots	10
		3.2.3 On the algebra of symmetries	11
		3.2.4 BV versus BRST	12
4	Exa	ample 2: BF model in \mathbb{R}^4	12
_	4.1	Symmetries of the classical action	12
		4.1.1 The model and its symmetries	12
		4.1.2 Horizontality conditions and <i>s</i> -transformations	12
		4.1.3 Minimal BV-action	13
		4.1.4 Gauge-fixing and elimination of antifields	14
	4.2	VSUSY	16
5	Ger	neral case	17
0	5.1		17
	0.1	5.1.1 Generalized forms and duality	
		5.1.2 Field content and construction of models	18
		5.1.3 Examples	19
		5.1.4 General features	22
		5.1.5 Other models and possible generalizations	 22
		5.1.6 Master equation and gauge-fixing	23
	5.2	VSUSY	23
6	Cor	nclusion	25

1 Introduction

The present paper is devoted to the algebraic construction [1] and to the symmetries of topological field theories of Schwarz-type (see [2] for a review of the latter theories). The classical action of some of these models (e.g. BF models in a space-time of dimension $d \ge 4$ [3, 4]) admits on-shell reducible symmetries and thus leads to a BRST-operator which is only nilpotent on-shell. Moreover, the gauge-fixed action for these models admits a supersymmetry-like invariance, the so-called vector supersymmetry (VSUSY), which also generates an on-shell algebra [5]-[8]. Such on-shell invariances raise problems upon quantization of these theories.

The Lagrangian Batalin-Vilkovisky (BV) -formalism represents a systematic approach to this problematics [9]. (For a short summary, see reference [10].) In fact, in this canonical (symplectic) setting, all fields are supplemented from the beginning on with antifields and these additional variables ensure off-shell closure of the symmetry algebras. The antifields of the BV-formalism correspond to the external sources of the standard BRST-approach and can be expressed in terms of the latter. In this way, one recovers symmetry algebras for the basic fields whose closure is guaranteed by the external sources. The latter transformations coincide with those obtained by the action of the linearized Slavnov-Taylor operator in the standard BRST-approach.

As matter of fact [7, 2], topological field theories of Schwarz-type provide a neat application for the BV-formalism which is often discussed in quite general terms in the literature [11]. The algebraic approach to this formalism was pioneered by H. Ikemori [1] and applied to various models in the sequel [12, 13] (see also [14] for earlier work and [15] for an interesting fieldtheoretic interpretation). We will incorporate VSUSY in this framework and show that this yields a major simplification with respect to the algebraic approach to the BRST-formalism [16], thus simplifying the study of the renormalization and finiteness properties of topological models [17].

In the present work, we restrict our attention to models in flat d-dimensional space-time, but a generalization of VSUSY to arbitrary manifolds can be achieved [18]. In fact, the latter allows to tackle the relationship between BF models and gravity [3, 19] where a VSUSY-like invariance also exists [20].

The fundamental ingredients of the algebraic approach [1, 12] are extended forms (corresponding to the complete ladders of the BV-formalism) : the latter can be used to write down action functionals as well as horizontality conditions or Russian formulae [21, 22] summarizing the basic symmetries of the action. The essential tool for describing VSUSY-transformations is given by the so-called \emptyset -symmetry condition introduced in reference [16].

Our paper is organized as follows. In section 3, we discuss in detail the examples of 3dimensional Chern-Simons theory and of the 4-dimensional BF model [1]. In doing so, we will make contact at all stages with previous studies of these models within the BRST- or BVsetting. In particular, we will elaborate on VSUSY and carry out explicitly the elimination of antifields so as to allow for a comparison with the results obtained from other lines of reasoning. To anticipate our conclusions, we already indicate that the algebraic approach allows to recover various known results (or slight generalizations thereof) almost effortless, in a systematic way and in a quite compact form. As a by-product, we will present a novel interpretation of the VSUSY-algebra in subsection 3.2.3. Our study of concrete examples provides the hindsight for formulating some general principles for the algebraic construction of topological models. This will be the subject of section 5 where we also summarize the different classes of topological models of both Schwarz- and Witten-type which can be obtained along these lines.

2 General setting

The models to be discussed admit a Lie algebra of symmetries and the involved fields are *p*-forms with values in this Lie algebra. In particular, we always have the Yang-Mills connection 1-form A and the associated curvature 2-form $F = dA + \frac{1}{2}[A, A]$. The field strength of any additional field φ is given by its covariant derivative $D\varphi = d\varphi + [A, \varphi]$. We will only be concerned with the classical theory and the fields occurring in the initial invariant action of a model will be referred to as *classical* fields.

3 Example 1: Chern-Simons theory in \mathbb{R}^3

3.1 Symmetries of the classical action

3.1.1 The model and its symmetries

The action

$$S_{\rm inv}[A] = \frac{1}{2} \int_{\mathbf{R}^3} \operatorname{tr} \{ A dA + \frac{2}{3} A A A \}$$
 (1)

is invariant under infinitesimal gauge transformations, $\delta A = Dc$, and it leads to the equation of motion F = 0, i.e. a zero-curvature condition for the connection A. In expression (1) and in the following, the wedge product sign is omitted.

The gauge invariance of the functional (1) represents an off-shell, irreducible symmetry and therefore the BV-description of this invariance leads, up to minor modifications, to the same results as the BRST-approach. In fact, by starting our study with a symmetry of this simple type, we can best recognize the precise correspondence between both formalisms.

3.1.2 Geometric framework of BRST- and BV-approaches

By way of motivation and to fix the notation, we recall a few facts concerning the geometric framework of the *BRST-approach* [8, 22]. In this setting, infinitesimal symmetry parameters are turned into ghost fields. Thus, the geometric sector of the Chern-Simons model involves the classical field A and the ghost field c associated to infinitesimal gauge transformations. Lower and upper indices of a field label its form degree and ghost-number, respectively. For each field, the ghost-number is added to the form degree in order to define a *total degree* and all commutators are assumed to be graded with respect to this degree. The BRST-operator s increases the ghost-number by one unit, but it does not modify the form degree of fields. In view of the definition of Green functions or the formulation of the Slavnov-Taylor identity, one also introduces external sources γ_2^{-1} and σ_3^{-2} which couple to the (non-linear) BRST-transformations of A and c, respectively: this amounts to the addition of a term

$$S_{\text{ext}} = \int_{\mathbf{R}^3} \operatorname{tr} \left\{ \gamma_2^{-1} s A + \sigma_3^{-2} s c \right\}$$

to the action. The latter contribution is s-invariant since the operator s is nilpotent and since the external fields γ and σ are assumed to be s-inert.

In the *BV*-approach that we will consider here, one starts with the fields $(\Phi^a) = (A, c)$ together with the corresponding antifields $(\Phi_a^*) = (A^*, c^*) \equiv (A_2^{-1}, A_3^{-2})$ which have the same index structure as the sources $(\gamma_2^{-1}, \sigma_3^{-2})$. All of these fields then define the geometric or minimal sector of the theory. In the sequel (section 3.1.4), s-variations are defined for all of these variables, the transformations of A^* and c^* being non-trivial. Direct contact with the BRST-approach is established at a later stage (after carrying out the gauge-fixing procedure) by eliminating the antifields (A^*, c^*) in terms of external sources (γ, σ) - see sections 3.1.8 and 3.1.9.

3.1.3 Generalized fields and derivatives

In the BRST-approach, the ghost field c is added to the connection A in order to define the generalized field $\tilde{A} = A + c$. In the BV-approach, the corresponding antifields are also included [1] so as to obtain the generalized field (or extended form)

$$\tilde{A} = A_3^{-2} + A_2^{-1} + A + c$$

= $c^* + A^* + A + c$. (2)

The latter contains fields of *all* form degrees which are allowed by three-dimensional space-time (and therefore it also involves fields with *negative* ghost-number). It is referred to as a *complete ladder* and it can be viewed as a "self-dual" quantity since it involves the basic fields A and c together with their antifields [1, 12], see section 5 below.

The s-differential is added to the exterior derivative d so as to define the generalized derivative $\tilde{d} = d + s$. In the same vein, one introduces the generalized field strengths of \tilde{A} and φ :

$$\tilde{F} = \tilde{d}\tilde{A} + \frac{1}{2}[\tilde{A}, \tilde{A}] \qquad , \qquad \tilde{D}\varphi = \tilde{d}\varphi + [\tilde{A}, \varphi] \,. \tag{3}$$

Actually, it also proves useful [12] to define the generalized fields

$$F^{\tilde{A}} = d\tilde{A} + \frac{1}{2} [\tilde{A}, \tilde{A}] \qquad , \qquad D^{\tilde{A}} \varphi = d\varphi + [\tilde{A}, \varphi] \,, \tag{4}$$

which satisfy the Bianchi identities

$$D^{\tilde{A}}F^{\tilde{A}} = 0 \qquad , \qquad D^{\tilde{A}}D^{\tilde{A}}\varphi = [F^{\tilde{A}},\varphi] \,. \tag{5}$$

3.1.4 Derivation of *s*-variations from a horizontality condition

Just like the standard BRST-transformations, the *s*-variations in the BV-framework can be obtained from a *horizontality condition*. For the topological models under consideration, the latter is obtained by replacing ordinary fields by generalized fields in the equations of motion of the model [1]. Thus, for Chern-Simons theory, one imposes the generalized zero-curvature condition

$$\tilde{F} = 0 . (6)$$

By virtue of the definition of \tilde{F} , this relation is equivalent to

$$s\tilde{A} = -F^{\tilde{A}},\tag{7}$$

i.e. condition (6) determines the s-variations of all component fields of \tilde{A} [1, 12]. The resulting transformations read

$$sc^{*} = -DA^{*} - [c, c^{*}] , \qquad sA = -Dc$$

$$sA^{*} = -F - [c, A^{*}] , \qquad sc = -\frac{1}{2}[c, c]$$
(8)

and they are nilpotent by virtue of the Bianchi identities (5), see reference [12, 16].

In equations (8), we recognize the standard BRST-transformations of the basic fields A and c. If we were to eliminate the antifields by setting them to zero together with their *s*-variations, relations (8) would lead to the classical equations of motion of the model. Accordingly, this method of elimination is inappropriate and another procedure will be considered after performing the gauge-fixing.

3.1.5 Construction of the minimal BV-action

Our goal is to extend the invariant action (1) to a s-invariant functional for the fields $(\Phi^a; \Phi_a^*) = (A, c; A^*, c^*)$ of the minimal sector. For its construction, we follow a purely algebraic reasoning, i.e. we ignore the space-time dimension. The Chern-Simons Lagrangian satisfies

$$d\operatorname{tr}\left\{AdA + \frac{2}{3}A^{3}\right\} = \operatorname{tr}\left(FF\right),$$

henceforth a relation of the same form holds for the 'tilde'-variables :

$$\tilde{d} \operatorname{tr} \left\{ \tilde{A} \tilde{d} \tilde{A} + \frac{2}{3} \tilde{A}^3 \right\} = \operatorname{tr} \left(\tilde{F} \tilde{F} \right).$$

By virtue of the horizontality condition $\tilde{F} = 0$, the right-hand-side vanishes and, as a consequence of $\tilde{d} = d + s$, we have

$$s \operatorname{tr} \left\{ \tilde{A}\tilde{d}\tilde{A} + \frac{2}{3}\tilde{A}^{3} \right\} = -d \operatorname{tr} \left\{ \tilde{A}\tilde{d}\tilde{A} + \frac{2}{3}\tilde{A}^{3} \right\}.$$

$$\tag{9}$$

The polynomial tr $\{...\}$ involves $\tilde{d}\tilde{A}$ and therefore it *explicitly* contains *s*-variations of fields. Since we do not want such terms to appear in our action, we will eliminate them in terms of others. By virtue of $\tilde{d} = d + s$, the left-hand-side of equation (9) reads

LHS =
$$s \operatorname{tr} \{ \tilde{A} d\tilde{A} + \frac{2}{3} \tilde{A}^3 \} + I$$

with

$$I \equiv s \operatorname{tr} \{ \tilde{A}s\tilde{A} \} = \operatorname{tr} \{ s\tilde{A}s\tilde{A} \} = \operatorname{tr} \{ F^{\tilde{A}}F^{\tilde{A}} \} = d \operatorname{tr} \{ \tilde{A}d\tilde{A} + \frac{2}{3}\tilde{A}^{3} \}.$$

Using $0 = \tilde{F} = \tilde{d}\tilde{A} + \tilde{A}^2$, the right-hand-side of equation (9) can be rewritten as

$$\operatorname{RHS} = d \operatorname{tr} \left\{ \frac{1}{3} \tilde{A}^3 \right\}.$$

Thus, as a final result, we find the *cocycle condition*

$$s \operatorname{tr} \left\{ \tilde{A}d\tilde{A} + \frac{2}{3}\tilde{A}^{3} \right\} = -d \operatorname{tr} \left\{ \tilde{A}d\tilde{A} + \frac{1}{3}\tilde{A}^{3} \right\}.$$

$$\tag{10}$$

The polynomial tr $\{...\}$ on the LHS involves contributions of different form degrees. After integrating its 3-form part over space-time [1], we obtain the so-called *minimal BV-action* which is s-invariant by virtue of relation (10):

$$S_{\min}[\Phi^{a}; \Phi_{a}^{*}] \equiv \frac{1}{2} \int_{\mathbf{R}^{3}} \operatorname{tr} \left\{ \tilde{A}d\tilde{A} + \frac{2}{3}\tilde{A}\tilde{A}\tilde{A} \right\} \Big|_{3}^{0}$$

$$= S_{inv}[A] + \int_{\mathbf{R}^{3}} \operatorname{tr} \left\{ A^{*}Dc + c^{*}cc \right\}$$

$$= S_{inv}[A] - \int_{\mathbf{R}^{3}} \operatorname{tr} \left\{ \Phi_{a}^{*}s\Phi^{a} \right\}.$$
 (11)

The s-invariance of the second term is non-trivial, since the antifields A^* and c^* transform non-trivially under the s-operation. The simple structure of this term reflects the fact that the basic fields $(\Phi^a) = (A, c)$ transform among themselves: if they were to mix with the antifields, additional terms would appear in (11), see the BF model below.

3.1.6 Gauge-fixing

We fix the gauge by imposing the Lorentz condition $d \star A = 0$ where $\star A$ denotes the Hodge-dual of the 1-form A. This condition is implemented in the action by adding the s-exact functional $S_{\rm gf} = s \Psi_{\rm gf}$ where the gauge-fixing fermion $\Psi_{\rm gf}$ is given (in the Landau gauge) by

$$\Psi_{\rm gf} = \int_{\mathbf{R}^3} \operatorname{tr} \left\{ \bar{c} \left(d \star A \right) \right\} \,. \tag{12}$$

The antighost \bar{c} appearing in this expression forms a BRST-doublet with an auxiliary field b, i.e.

$$s\bar{c} = b$$
 , $sb = 0$. (13)

As for the geometric sector, one also introduces the corresponding antifields (\bar{c}^*, b^*) and couples them to the *s*-variations of \bar{c} and *b*, respectively:

$$S_{\text{aux}}[\bar{c}, b; \bar{c}^*, b^*] \equiv -\int_{\mathbf{R}^3} \text{tr} \{ \bar{c}^* s \bar{c} + b^* s b \}$$

$$= -\int_{\mathbf{R}^3} \text{tr} \{ \bar{c}^* b \}.$$
(14)

The antifields (\bar{c}^*, b^*) are again assumed to form a BRST-doublet, but one transforming "the other way round",

$$s\bar{c}^* = 0$$
 , $sb^* = \bar{c}^*$. (15)

This guarantees the s-invariance of S_{aux} .

By adding the functional (14) to the minimal action (11), one obtains the so-called *non-minimal BV-action* which depends on the fields $(\Phi^A) = (A, c, \bar{c}, b)$ and the associated antifields

 (Φ_{A}^{*}) :

$$S_{\rm nm}[\Phi^A; \Phi^*_A] \equiv S_{\rm min} + S_{\rm aux}$$

= $S_{\rm inv} + \int_{\mathbf{R}^3} \operatorname{tr} \{A^*Dc + c^*cc - \bar{c}^*b\}$
= $S_{\rm inv} - \int_{\mathbf{R}^3} \operatorname{tr} \{\Phi^*_A s \Phi^A\}.$ (16)

Note that this action is s-invariant and that it does not include the gauge-fixing functional $S_{\rm gf} = s \Psi_{\rm gf}$.

3.1.7 BV-interpretation

Let $(\Phi^A) = (\Phi^a, \bar{C}^\alpha, \Pi^\alpha)$ collectively denote all fields, i.e. the classical and ghost fields (Φ^a) defining the minimal sector, the antighosts \bar{C}^α and the multiplier fields Π^α . Accordingly, let $(\Phi^*_A) = (\Phi^*_a, \bar{C}^*_\alpha, \Pi^*_\alpha)$ denote the associated antifields and let $(\Theta^A) = (\Phi^A; \Phi^*_A)$. Quite generally, if Φ^A has index structure $(\Phi^A)^g_p$, then the corresponding antifield has index structure $(\Phi^*_A)^{-g-1}_{d-p}$ where d denotes the space-time dimension. Accordingly, for a space-time \mathcal{M}_d of odd (even) dimension, the fields and their antifields have the same (opposite) parity.

For any two functionals $X[\Theta^A]$ and $Y[\Theta^A]$ of the fields (Θ^A) , the *BV*-bracket is the graded bracket defined by

$$\{X,Y\} = \int_{\mathcal{M}_d} \operatorname{tr} \left\{ \frac{\delta X}{\delta \Phi^A} \frac{\delta Y}{\delta \Phi_A^*} \pm \frac{\delta Y}{\delta \Phi^A} \frac{\delta X}{\delta \Phi_A^*} \right\},\tag{17}$$

where the sign depends on the Grassmann parity of X and Y. (Our convention to use *left* functional derivatives differs from the one which is frequently used in the literature [11].)

Let $\Gamma[\Phi^A, \Phi^*_A]$ be the non-minimal BV-action (16) as defined on $\mathcal{M}_d = \mathbb{R}^3$. Then, the latter is a solution of the *classical BV master equation*

$$\{\Gamma, \Gamma\} = 0$$
 i.e. $\int_{\mathcal{M}_d} \operatorname{tr} \left\{ \frac{\delta\Gamma}{\delta\Phi^A} \frac{\delta\Gamma}{\delta\Phi_A^*} \right\} = 0$ (18)

and the s-variations of fields and antifields are given by

$$s\Theta^A = \{\Gamma, \Theta^A\},\tag{19}$$

i.e.

$$s\Phi^A = -\frac{\delta\Gamma}{\delta\Phi_A^*}$$
 , $s\Phi_A^* = -\frac{\delta\Gamma}{\delta\Phi^A}$ (20)

for $(\Phi^A) = (A, c, \bar{c}, b)$. Indeed, the explicit expressions following from (20) with $\Gamma = S_{nm}$ coincide with the transformation laws (8), (13) and (15). Since $s\Gamma = \{\Gamma, \Gamma\}$, the master equation (18) expresses the *s*-invariance of Γ . The off-shell nilpotency of the *s*-operator can be viewed as a consequence of the graded Jacobi identity for the BV-bracket.

As a matter of fact, the functional S_{\min} already represents a solution of the master equation which only depends on the variables $(\Phi^a; \Phi^*_a) = (A, c; A^*, c^*)$, i.e.

$$s\Phi^{a} = -\frac{\delta S_{\min}}{\delta\Phi_{a}^{*}} \quad , \qquad s\Phi_{a}^{*} = -\frac{\delta S_{\min}}{\delta\Phi^{a}} \,. \tag{21}$$

The latter result confirms the identification between antifields and forms of negative ghostnumber considered in equation (2). It also explains why the choice $\Phi_a^* = 0 = s\Phi_a^*$ implies the classical equations of motion [12].

3.1.8 Elimination of antifields

Since antifields have been associated to all fields, *external sources* are also introduced for each field (and not only for those transforming non-linearly under the *s*-operation, as is usually done in the BRST-approach). The sources associated to $(\Phi^A) = (A, c, \bar{c}, b)$ are denoted by $(\rho_A) \equiv (\gamma, \sigma, \bar{\sigma}, \lambda)$. Then, the antifields $(\Phi^*_A) = (A^*, c^*, \bar{c}^*, b^*)$ are eliminated in terms of these sources by virtue of the general prescription

$$\Phi_A^* = -\hat{\rho}_A \equiv -\rho_A + (-1)^{(d+1)|\Phi^A|+d} \frac{\delta \Psi_{\rm gf}}{\delta \Phi^A}, \qquad (22)$$

where $|\Phi^A|$ denotes the total degree of the field Φ^A and d the space-time dimension (i.e. d = 3 in our example). Thus, we get

$$A^* = -(\gamma + \star d\bar{c}) \equiv -\hat{\gamma} , \quad c^* = -\sigma \bar{c}^* = -(\bar{\sigma} + d \star A) \equiv -\hat{\bar{\sigma}} , \quad b^* = -\lambda ,$$

$$(23)$$

i.e. the antifields become the "hatted" sources $(\hat{\rho}_A) \equiv (\hat{\gamma}, \sigma, \hat{\sigma}, \lambda)$ which amount to a reparametrization of certain sources.

After substituting these expressions into the non-minimal BV-action (16), one obtains the following functional which only depends on the "hatted" sources:

$$\Sigma = S_{\rm inv} + \int_{\mathbf{R}^3} \operatorname{tr} \left\{ \hat{\rho}_A s \Phi^A \right\}$$

= $S_{\rm inv} + \int_{\mathbf{R}^3} \operatorname{tr} \left\{ s \Phi_A \frac{\delta \Psi_{\rm gf}}{\delta \Phi^A} \right\} + \int_{\mathbf{R}^3} \operatorname{tr} \left\{ \rho_A s \Phi^A \right\}$
= $S_{\rm inv} + \int_{\mathbf{R}^3} \operatorname{tr} \left\{ bd \star A + \bar{c}d \star Dc \right\} + \int_{\mathbf{R}^3} \operatorname{tr} \left\{ \gamma s A + \sigma s c + \bar{\sigma} s \bar{c} + \lambda s b \right\}.$ (24)

Actually, the transformation law sb = 0 implies that the last term vanishes so that Σ does not depend on the source λ . Thus, we have

$$\Sigma[\Phi^A, \rho_A] \equiv S_{\rm nm}|_{\Phi^*_A} = S_{\rm inv} + S_{\rm gf} + S_{\rm ext} , \qquad (25)$$

where S_{inv} represents the classical, gauge invariant action, $S_{gf} = s\Psi_{gf}$ the associated gaugefixing functional and S_{ext} the linear coupling of external sources ρ_A to the *s*-variations of the fields Φ^A . The action Σ represents the *s*-invariant vertex functional (at the classical level) and coincides with the result obtained from the usual BRST procedure (e.g. see [8]), except for the presence of the external sources $(\bar{\sigma}, \lambda)$ coupling to the *s*-variations of the BRST-doublet (\bar{c}, b) .

By substituting expressions (23) into the s-variations of the antifields, we obtain those of the sources: the variables $(\lambda, \hat{\sigma})$ transform like a BRST-doublet,

$$s\lambda = \hat{\bar{\sigma}} \quad , \qquad s\hat{\bar{\sigma}} = 0 \,,$$
 (26)

and thereby the s-variations of the sources take the explicit form

$$s\gamma = F - [c, \hat{\gamma}] - \star db \quad , \qquad s\bar{\sigma} = d \star Dc s\sigma = -D\hat{\gamma} - [c, \sigma] \quad , \qquad s\lambda = \bar{\sigma} + d \star A \,.$$

$$(27)$$

The transformation laws of all variables can be summarized by

$$s\Phi^A = \frac{\delta\Sigma}{\delta\rho_A}$$
 , $s\rho_A = \frac{\delta\Sigma}{\delta\Phi^A}$ (28)

with $(\Phi^A) = (A, c, \bar{c}, b)$ and $(\rho_A) = (\gamma, \sigma, \bar{\sigma}, \lambda)$. These relations are the relict of the BV-variations (20) after the elimination of all antifields.

Obviously, the s-variations (28) determine the linearized Slavnov-Taylor operator, i.e.

$$s = S_{\Sigma} \equiv \int_{\mathbf{R}^3} \operatorname{tr} \left\{ \frac{\delta \Sigma}{\delta \Phi^A} \frac{\delta}{\delta \rho_A} + \frac{\delta \Sigma}{\delta \rho_A} \frac{\delta}{\delta \Phi^A} \right\} \,. \tag{29}$$

Thus, the BV master equation (18) becomes the Slavnov-Taylor identity

$$\mathcal{S}(\Sigma) \equiv \int_{\mathbf{R}^3} \operatorname{tr} \left\{ \frac{\delta \Sigma}{\delta \Phi^A} \frac{\delta \Sigma}{\delta \rho_A} \right\} = 0, \qquad (30)$$

which ensures that $s^2 = 0$ off-shell and that $s\Sigma = S_{\Sigma}(\Sigma) = 2S(\Sigma) = 0$. This interpretation of the master equation is a cornerstone of the theory and is further discussed in the literature, both at the classical and quantum level [9, 11]. Here, we only put forward two points. First, we note that the hatted sources associated to antighosts and multipliers form BRST-doublets (see eqs.(26)) which simplifies the cohomogical analysis of the quantum theory. Second, we remark that the Slavnov-Taylor identity (30) of the BV-approach has a more symmetric form than the one of the BRST-approach where one only introduces external sources for those fields which transform non-linearly, i.e. A and c.

3.1.9 BV versus BRST

Let us summarize the conclusions that we can draw from our discussion of the Chern-Simons theory at the classical level (and which are in accordance with the general results [9, 11]). For an off-shell, irreducible symmetry (like YM-invariance), the differences between the BV-approach and the BRST-procedure are two-fold :

- The s-operator of the BV-formalism represents the linearized Slavnov-Taylor operator: unlike the standard BRST-differential, this operator does *not* leave the external sources invariant. (The non-trivial transformation laws of the sources follow from the non-trivial *s*-variations of antifields given by equations (20), after the elimination of antifields in terms of sources by virtue of the gauge fermion $\Psi_{\rm gf}$.)
- In the BV-approach, one introduces sources for all fields, not only for those transforming non-linearly, as one usually does in the BRST-framework. Of course, the latter framework also *allows* for the inclusion of such sources: although they are not particularly useful, they lead to a more symmetric expression for the Slavnov-Taylor identity (and also for the Ward identities, see equation (40) below and comments thereafter).

3.2 VSUSY

3.2.1 VSUSY-transformations

After performing the gauge-fixing for the invariant action (1) in a Landau-type gauge, the gauge-fixed action is invariant under VSUSY-transformations [5]. In this section, we will introduce these transformations as well as their algebra and, in the next section, we will discuss the induced variation of the BV-action.

At the infinitesimal level, VSUSY-transformations are described by an operator δ_{τ} where $\tau \equiv \tau^{\mu} \partial_{\mu}$ represents a constant *s*-invariant vector field of ghost-number zero. The variation δ_{τ} acts as an antiderivation which lowers the ghost-number by one unit and which anticommutes with *d*. The operators *s* and δ_{τ} satisfy a graded algebra of Wess-Zumino type :

$$[s, \delta_{\tau}] = \mathcal{L}_{\tau} + \text{equations of motion} .$$
(31)

Here, $\mathcal{L}_{\tau} = [i_{\tau}, d]$ denotes the Lie derivative with respect to the vector field τ and i_{τ} the interior product with τ .

In the *BRST-approach*, the VSUSY-variations for topological models of Schwarz-type can be derived from the so-called \emptyset -type symmetry condition [16]

$$\delta_{\tau}\tilde{A} = i_{\tau}\tilde{A} . \tag{32}$$

In the *BV-approach*, we start from the same expression, the only difference being that \tilde{A} now involves both fields of positive and negative ghost-number. Substitution of the expansion (2) into (32) yields the VSUSY-variations in the geometric sector,

$$\delta_{\tau}c = i_{\tau}A \quad , \quad \delta_{\tau}A = i_{\tau}A^* \quad , \quad \delta_{\tau}A^* = i_{\tau}c^* \quad , \quad \delta_{\tau}c^* = 0 \,.$$
 (33)

We note that, if two fields are related by VSUSY (e.g. $c \xrightarrow{\delta_{\tau}} A$), then the corresponding antifields are related "the other way round" (i.e. $A^* \xrightarrow{\delta_{\tau}} c^*$). This feature represents a useful *guideline* for dealing with more complex models or field contents.

Using (32) and $s\tilde{A} = -F^A$, it can be explicitly shown that $[s, \delta_{\tau}]\tilde{A} = \mathcal{L}_{\tau}\tilde{A}$, i.e. the VSUSYalgebra is satisfied off-shell for all fields of the geometric sector. (In fact, this result holds by construction [16].)

We now turn to the transformation laws of the remaining fields and antifields. The δ_{τ} -variation of \bar{c} has to vanish for dimensional and ghost-number reasons ("there is nothing it can transform into"). If we require the VSUSY-algebra to be satisfied off-shell for all fields, we readily obtain the variation of b:

$$\delta_{\tau}\bar{c} = 0 \qquad \Rightarrow \qquad \delta_{\tau}b = \delta_{\tau}s\bar{c} = (\mathcal{L}_{\tau} - s\delta_{\tau})\bar{c} = \mathcal{L}_{\tau}\bar{c} . \tag{34}$$

As for the s-variations, the associated doublet of antifields (\bar{c}^*, b^*) is assumed to transform "the other way round" (in accordance with the general guideline indicated above) :

$$\delta_{\tau}b^* = 0 \qquad , \qquad \delta_{\tau}\bar{c}^* = \mathcal{L}_{\tau}b^* \ . \tag{35}$$

After eliminating all antifields in terms of sources by virtue of equations (23), we get the VSUSY-variations

$$\delta_{\tau}c = i_{\tau}A \qquad , \qquad \delta_{\tau}\hat{\gamma} = \delta_{\tau}\gamma = i_{\tau}\sigma \delta_{\tau}A = -i_{\tau}\hat{\gamma} = -i_{\tau}(\gamma + \star d\bar{c}) \qquad , \qquad \delta_{\tau}\sigma = 0$$
(36)

and

$$\begin{aligned} \delta_{\tau}\bar{c} &= 0 & , \quad \delta_{\tau}b = \mathcal{L}_{\tau}\bar{c} \\ \delta_{\tau}\bar{\sigma} &= \mathcal{L}_{\tau}\lambda + d \star i_{\tau}\hat{\gamma} & , \quad \delta_{\tau}\lambda = 0 , \end{aligned} \tag{37}$$

where the variation of $\bar{\sigma}$ is a consequence of $\delta_{\tau} \hat{\bar{\sigma}} = \mathcal{L}_{\tau} \lambda$. It can be checked explicitly that the VSUSY-algebra holds off-shell for all fields and sources.

If we set the sources to zero, we recover the results of the standard BRST-approach for the VSUSY-variations of the fields (A, c, \bar{c}, b) [5]. For these fields, the *s*-variations of the BV- and BRST-approaches coincide so that we also recover the on-shell VSUSY-algebra. If sources are included in the BRST-framework for the discussion of Ward identities, a different argumentation from the one considered here leads to the same VSUSY-variations for (A, c, \bar{c}, b) and (γ, σ) [6, 8]. In fact, external sources (antifields) somehow play the same rôle as *auxiliary fields* in supersymmetric field theories in that they lead to a symmetry algebra which closes off-shell [7, 10].

3.2.2 Ward identity

The transformation law (32) induces the following variation of the minimal BV-action (11):

$$\delta_{\tau} S_{\min} = \frac{1}{2} \int_{\mathbf{R}^3} \operatorname{tr} \left\{ \tilde{A} \mathcal{L}_{\tau} \tilde{A} \right\} \Big|_3^{-1}$$
$$= \int_{\mathbf{R}^3} \operatorname{tr} \left\{ A^* \mathcal{L}_{\tau} A + c^* \mathcal{L}_{\tau} c \right\} \,.$$

The transformations (34),(35) yield a similar expression for the variation of the auxiliary action (14):

$$\delta_{\tau} S_{\text{aux}} = \int_{\mathbf{R}^3} \operatorname{tr} \left\{ b^* \mathcal{L}_{\tau} b + \bar{c}^* \mathcal{L}_{\tau} \bar{c} \right\} \,.$$

Thus, the non-minimal BV-action $S_{nm} = S_{min} + S_{aux}$ satisfies a broken Ward identity:

$$\mathcal{W}_{\tau}S_{\rm nm} \equiv \int_{\mathbf{R}^3} \operatorname{tr} \left\{ \delta_{\tau} \Phi^A \frac{\delta S_{\rm nm}}{\delta \Phi^A} + \delta_{\tau} \Phi^*_A \frac{\delta S_{\rm nm}}{\delta \Phi^*_A} \right\} = \int_{\mathbf{R}^3} \operatorname{tr} \left\{ \Phi^*_A \mathcal{L}_{\tau} \Phi^A \right\} \,. \tag{38}$$

The breaking is linear in the fields (and also in the antifields). Henceforth, it is unproblematic for the quantum theory since 'insertions' that are linear in quantum fields are not renormalized by quantum corrections. The result (38) can also be derived from expression (16) by substituting the variations (33)-(35) and using $[s, \delta_{\tau}] = \mathcal{L}_{\tau}$.

After elimination of the antifields, the Ward identity (38) takes the form

$$\mathcal{W}_{\tau}\Sigma \equiv \int_{\mathbf{R}^{3}} \operatorname{tr} \left\{ \delta_{\tau} \Phi^{A} \frac{\delta\Sigma}{\delta\Phi^{A}} + \delta_{\tau} \rho_{A} \frac{\delta\Sigma}{\delta\rho_{A}} \right\} = \Delta_{\tau}$$
(39)

with

$$\Delta_{\tau} = -\int_{\mathbf{R}^3} \operatorname{tr} \left\{ \rho_A \mathcal{L}_{\tau} \Phi^A \right\} = -\int_{\mathbf{R}^3} \operatorname{tr} \left\{ \gamma \mathcal{L}_{\tau} A + \sigma \mathcal{L}_{\tau} c + \lambda \mathcal{L}_{\tau} b + \bar{\sigma} \mathcal{L}_{\tau} \bar{c} \right\} \,. \tag{40}$$

The Ward identity (39), with $\delta_{\tau} \Phi^A$ and $\delta_{\tau} \rho_A$ given by eqs. (36)(37), has the same form as the one found in the BRST-framework [8, 23] where the sources $\lambda, \bar{\sigma}$ are not considered. Yet, it is their inclusion which leads to the quite symmetric expression (40).

The fact that the breaking is linear in the quantum fields and in the sources only holds in the *Landau gauge* that we have chosen in the gauge-fixing fermion (12): in a different gauge, implemented by the gauge-fermion

$$\Psi_{\rm gf} = \int_{\mathbf{R}^3} \operatorname{tr} \left\{ \bar{c} (d \star A - \frac{\alpha}{2} \star b) \right\} \quad \text{with } \alpha \in \mathbf{R}^* \,,$$

the breaking term Δ_{τ} is non-linear in the quantum fields.

To summarize the two previous sections, we can say that the BV-approach readily leads to a VSUSY-algebra which closes off-shell and to a Ward identity which is broken by a term that is linear in the quantum fields and sources (in the Landau gauge).

3.2.3 On the algebra of symmetries

Before proceeding further, we come back shortly to the algebra of symmetries which may be summarized as follows. The basic operators

$$d \equiv \delta_1^0 \quad , \quad s \equiv \delta_0^1 \quad , \quad i_\tau \equiv \delta_{-1}^0 \quad , \quad \delta_\tau \equiv \delta_0^{-1}$$

modify the form degree p and ghost-number g of a field φ according to

$$\delta_m^n\left(\varphi_p^g\right) = \left(\delta\varphi\right)_{p+m}^{g+n}$$

and satisfy the graded algebra

$$[\delta_m^n, \delta_k^l] = \delta_{m+k,0} \, \delta_{n+l,0} \, \mathcal{L}_\tau \; .$$

It is interesting to compare the action of the operators δ_{τ} and s on the basic fields. For this purpose, we decompose s according to $s = s_0 + s_1$ where s_0 and s_1 represent, respectively, the linear and non-linear parts of the operator. By virtue of $s\tilde{A} = -F^{\tilde{A}}$, the action of s_0 on the generalized field \tilde{A} is given by

$$s_0 \tilde{A} = -d\tilde{A} \,. \tag{41}$$

Comparison with (32) shows that each of the operators s_0 and δ_{τ} acts in the same fashion on all fields occurring in the expansion \tilde{A} . However, the two operators act into opposite directions inside the ladder \tilde{A} : while s_0 increases the ghost-number by one unit, δ_{τ} lowers it by the same amount,

$$\tilde{A} = c^* + \overset{s_0}{\underset{\delta_{\tau}}{\overset{s_0}{\overset{\delta_{\tau}}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}}{\overset{\delta_{\tau}}{\overset{\delta}}}{\overset{\delta_{\tau}}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta_{\tau}}{\overset{\delta}}{\overset{\delta_{\tau}}}{\overset{\delta_{\tau}}}}{\overset{\delta_{\tau}}{\overset{\delta}}{\overset{\delta}}{\overset{\delta}}}$$

both operators being related by

$$[s_0, \delta_\tau] = \mathcal{L}_\tau \ . \tag{43}$$

The linear part s_0 of the s-operator (which already determines the non-abelian structure of the theory to a large extent [24]) also allows for a unified formulation of all symmetries. To present this geometric description, we define (in analogy to $\tilde{d} = d + s$)

$$\widetilde{d}_0 \equiv d + s_0
\widetilde{i}_\tau \equiv i_\tau - \delta_\tau .$$
(44)

The horizontality condition (41) defining s_0 and the symmetry condition (32) defining δ_{τ} are then equivalent to

$$\tilde{d}_0 \tilde{A} = 0 \qquad , \qquad \tilde{i}_\tau \tilde{A} = 0 \tag{45}$$

and the compatibility condition for these two equations,

$$0 = [\tilde{d}_0, \tilde{i}_\tau] = [d, i_\tau] - [s_0, \delta_\tau]$$

is the VSUSY-algebra relation (43).

3.2.4 BV versus BRST

Quite generally, we can say the following. Once external sources (associated to non-linear field variations) are introduced in the *BRST-framework* for discussing Ward identities, one recovers the same results for VSUSY-transformations as in the BV-approach and also the same type of expression for the breaking of VSUSY. Yet, in the *BV-framework* where sources are introduced for all fields under the disguise of antifields, the VSUSY-breaking term has a more symmetric form.

Concerning the derivation of VSUSY-variations (for topological models of Schwarz-type) by virtue of the symmetry condition (32), the conclusion is as follows. This symmetry condition can be taken as a starting point in the standard BRST-formalism [16], but the derivation of symmetry transformations is already involved for a simple model like Chern-Simons theory due to the fact that one has to refer to its equations of motion. By contrast, the *BV-formalism* allows for a quite simple and straightforward study of VSUSY.

4 Example 2: BF model in \mathbb{R}^4

The approach to the BF model closely follows the lines of the Chern-Simons theory, henceforth we will only emphasize the new features that it exhibits.

4.1 Symmetries of the classical action

4.1.1 The model and its symmetries

The BF model in \mathbb{R}^4 involves two gauge potentials: the YM 1-form A and the 2-form potential $B \equiv B_2^0$, i.e. a Lie algebra-valued 2-form transforming under the adjoint representation of the gauge group. The model is characterized by the action

$$S_{\rm inv}[A,B] = \int_{\mathbf{R}^4} \operatorname{tr} \left\{ BF \right\},\tag{46}$$

which leads to the equations of motion

$$F = 0 \qquad \text{and} \qquad DB = 0. \tag{47}$$

The functional (46) is not only invariant under ordinary gauge transformations, but also under the local symmetry

$$\delta B = DB_1. \tag{48}$$

By virtue of the second Bianchi identity $D(DB_0) = [F, B_0]$ and the equation of motion F = 0, the right-hand-side of (48) is on-shell invariant under the transformation $\delta B_1 = DB_0$. Thus, the symmetry (48) is one-stage reducible on-shell.

4.1.2 Horizontality conditions and s-transformations

Apart from the ghost c parametrizing ordinary gauge transformations, we now have ghosts $B_1^1 \equiv B_1$ and $B_0^2 \equiv B_0$ parametrizing the reducible symmetry (48). Thus, one introduces generalized forms [1]

$$\tilde{A} = A_4^{-3} + A_3^{-2} + A_2^{-1} + A + c , \qquad \tilde{B} = B_4^{-2} + B_3^{-1} + B + B_1^{1} + B_0^{2}
= B_0^* + B_1^* + B^* + A + c , \qquad \tilde{B} = c^* + A^* + B + B_1 + B_0,$$
(49)

where $B_0^* \equiv (B_0)^*$, $B_1^* \equiv (B_1)^*$ and where the identification of *antifields* has been performed as for the Chern-Simons theory, i.e. by considering the index structure of all fields (see section 3.1.2). The gauge potentials A and B and, more generally, the generalized fields \tilde{A} and \tilde{B} can be viewed as dual to each other (see references [12, 1] and section 5 below).

In view of the equations of motion (47), one postulates the *horizontality conditions* [1]

$$\tilde{F} = 0$$
 and $\tilde{D}\tilde{B} = 0$. (50)

These relations are equivalent to

$$s\tilde{A} = -F^{\tilde{A}}$$
 and $s\tilde{B} = -D^{\tilde{A}}\tilde{B}$ (51)

and thereby determine all s-variations : by substitution of expressions (49), one obtains

$$sc = -\frac{1}{2}[c,c] , \qquad sB^* = -F - [c,B^*]$$

$$sA = -Dc , \qquad sB_1^* = -DB^* - [c,B_1^*]$$

$$sB_0^* = -DB_1^* - [c,B_0^*] - \frac{1}{2}[B^*,B^*] \qquad (52)$$

and

$$sB_{0} = -[c, B_{0}]$$

$$sB_{1} = -DB_{0} - [c, B_{1}]$$

$$sB = -DB_{1} - [c, B] - [B^{*}, B_{0}]$$

$$sA^{*} = -DB - [c, A^{*}] - [B^{*}, B_{1}] - [B^{*}_{1}, B_{0}]$$

$$sc^{*} = -DA^{*} - [c, c^{*}] - [B^{*}, B] - [B^{*}_{1}, B_{1}] - [B^{*}_{0}, B_{0}].$$
(53)

The fields and antifields of the minimal sector can be collected in $(\Phi^a) = (A, c, B, B_1, B_0)$ and $(\Phi_a^*) = (A^*, c^*, B^*, B_1^*, B_0^*)$. By construction, the *s*-variations of these variables as given by (52) and (53) are nilpotent off-shell. The fact that the transformation law of the classical field *B* involves the antifield B^* reflects the fact that the symmetry algebra generated by (48) closes only on-shell. If all antifields are set to zero, one recovers the standard BRST-transformations of (A, c, B, B_1, B_0) which are only nilpotent on the mass-shell.

4.1.3 Minimal BV-action

Proceeding along the lines of section 3.1.5, we can extend the classical action (46). From the horizontality conditions (51), we obtain the *cocycle condition* [12]

$$s \operatorname{tr} \{ \tilde{B} F^{\tilde{A}} \} = -d \operatorname{tr} \{ \tilde{B} F^{\tilde{A}} \},$$

which yields the s-invariant minimal BV-action [1, 12]

$$S_{\min}[\Phi^a; \Phi_a^*] \equiv \int_{\mathbf{R}^4} \operatorname{tr} \left\{ \tilde{B} F^{\tilde{A}} \right\} \Big|_4^0 \,. \tag{54}$$

Substitution of the expansions (49) leads to the explicit expression

$$S_{\min} = S_{inv} - \int_{\mathbf{R}^4} \operatorname{tr} \left\{ \Phi_a^* \, s \Phi^a \right\} - \frac{1}{2} \int_{\mathbf{R}^4} \operatorname{tr} \left\{ B^*[B^*, B_0] \right\},\tag{55}$$

where the last term reflects the antifield dependence of the transformation law of B. We note that all of the s-variations (52) and (53) have the form BV-form (21) which confirms the identification of antifields made in (49).

4.1.4 Gauge-fixing and elimination of antifields

Gauge fermion and auxiliary fields Gauge-fixing of all symmetries, i.e. of YM-invariance and of the reducible symmetry of the 2-form potential B, requires a gauge fermion of the form

$$\Psi_{\rm gf} = \int_{\mathbf{R}^4} \operatorname{tr} \left\{ \bar{c} \, d \star A + \bar{c}_1^{-1} d \star B + \bar{c}^{-2} d \star B_1 + \bar{c}^0 \left(d \star \bar{c}_1^{-1} + \alpha \star \pi^{-1} \right) \right\} \quad (\alpha \in \mathbf{R}) \,. \tag{56}$$

The involved antighosts $(\bar{C}^{\alpha}) \equiv (\bar{c}, \bar{c}_1^{-1}, \bar{c}^{-2}, \bar{c}^0)$ are supplemented with auxiliary fields $(\Pi^{\alpha}) \equiv (b, \pi_1, \pi^{-1}, \pi^1)$ so as to define BRST-doublets :

$$s\bar{c} = b , \quad s\bar{c}_1^{-1} = \pi_1 , \quad s\bar{c}^{-2} = \pi^{-1} , \quad s\bar{c}^0 = \pi^1 sb = 0 , \quad s\pi_1 = 0 , \quad s\pi^{-1} = 0 , \quad s\pi^1 = 0.$$
(57)

The corresponding antifields transform in a dual way,

$$s\bar{c}^{*} = 0 , \quad s(\bar{c}_{1}^{-1})^{*} = 0 , \quad s(\bar{c}^{-2})^{*} = 0 , \quad s(\bar{c}^{0})^{*} = 0$$

$$sb^{*} = \bar{c}^{*} , \quad s(\pi_{1})^{*} = -(\bar{c}_{1}^{-1})^{*} , \quad s(\pi^{-1})^{*} = -(\bar{c}^{-2})^{*} , \quad s(\pi^{1})^{*} = -(\bar{c}^{0})^{*}$$
(58)

and thereby ensure the s-invariance of the functional

$$S_{\text{aux}} = -\int_{\mathbf{R}^4} \operatorname{tr}\left\{ (\bar{C}^{\alpha})^* \Pi_{\alpha} \right\},\tag{59}$$

which gives rise to the non-minimal action $S_{nm} = S_{min} + S_{aux}$.

Elimination of antifields Altogether, we have the fields $(\Phi^A) = (\Phi^a, \overline{C}^\alpha, \Pi^\alpha)$ with

$$(\Phi^a) = (A, c, B, B_1, B_0) \quad , \quad (\bar{C}^{\alpha}) = \left(\bar{c}, \bar{c}_1^{-1}, \bar{c}^{-2}, \bar{c}^0\right) \quad , \quad (\Pi^{\alpha}) = \left(b, \pi_1, \pi^{-1}, \pi^1\right) \tag{60}$$

and the associated external sources (ρ_A) are to be denoted as follows :

$$(\gamma, \sigma, \rho_2^{-1}, \rho_3^{-2}, \rho_4^{-3})$$
, $(\bar{\sigma}, \bar{\sigma}_3^0, \bar{\sigma}_4^1, \bar{\sigma}_4^{-1})$, $(\lambda, \lambda_3^{-1}, \lambda_4^0, \lambda_4^{-2})$. (61)

The antifields (Φ_A^*) will now be expressed in terms of these sources by virtue of the prescription (22) with d = 4. For the antifields of the minimal sector, this entails

$$\begin{aligned}
A^* &= -(\gamma - \star d\bar{c}) \equiv -\hat{\gamma} , & B^* &= -(\rho_2^{-1} + \star d\bar{c}_1^{-1}) \equiv -\hat{\rho}_2^{-1} \\
c^* &= -\sigma , & B_1^* = -(\rho_3^{-2} - \star d\bar{c}^{-2}) \equiv -\hat{\rho}_3^{-2} \\
B_0^* &= -\rho_4^{-3},
\end{aligned}$$
(62)

whereas the antifields associated to antighost and multiplier fields are given by

$$\bar{c}^{*} = -(\bar{\sigma} + d \star A) \equiv -\hat{\bar{\sigma}} , \qquad b^{*} = -\lambda
(\bar{c}_{1}^{-1})^{*} = -(\bar{\sigma}_{3}^{0} - d \star B - \star d\bar{c}^{0}) \equiv -\hat{\bar{\sigma}}_{3}^{0} , \qquad (\pi_{1})^{*} = -\lambda_{3}^{-1}
(\bar{c}^{-2})^{*} = -(\bar{\sigma}_{4}^{1} - d \star B_{1}) \equiv -\hat{\sigma}_{4}^{1} , \qquad (\pi^{-1})^{*} = -(\lambda_{4}^{0} + \alpha \star \bar{c}^{0}) \equiv -\hat{\lambda}_{4}^{0}
(\bar{c}^{0})^{*} = -(\bar{\sigma}_{4}^{-1} - \alpha \star \pi^{-1} - d \star \bar{c}_{1}^{-1}) \equiv -\hat{\bar{\sigma}}_{4}^{-1} , \qquad (\pi^{1})^{*} = -\lambda_{4}^{-2} .$$
(63)

Vertex functional The gauge-fixed action including external sources is obtained from $S_{\rm nm} = S_{\rm min} + S_{\rm aux}$ by eliminating antifields according to relations (62)(63). This leads to

$$\Sigma = S_{\text{inv}} + \int_{\mathbf{R}^4} \operatorname{tr} \left\{ \hat{\rho}_A s \Phi^A \right\} + S_{\text{mod}}$$

= $S_{\text{inv}} + s \Psi_{\text{gf}} + \int_{\mathbf{R}^4} \operatorname{tr} \left\{ \rho_A s \Phi^A \right\} + S_{\text{mod}}$
= $S_{\text{inv}} + S_{\text{gf}} + S_{\text{ext}} + S_{\text{mod}}$, (64)

where

$$S_{\text{mod}} = -\frac{1}{2} \int_{\mathbf{R}^4} \operatorname{tr} \left\{ B_0[\hat{\rho}_2^{-1}, \hat{\rho}_2^{-1}] \right\}$$
(65)

is related to the fact that the s-variation of B exhibits an antifield dependence, see equation (53). For $\rho_A = 0$, expression (64) coincides with the one of reference [7] in which external sources are introduced at a different stage.

s-variations After elimination of all antifields, the *s*-variations of the basic fields (A, c, B, B_1, B_0) are exactly the same as before except for the fact that sB now depends on a (hatted) source rather than an antifield:

$$sB = -DB_1 - [c, B] + [\hat{\rho}_2^{-1}, B_0].$$
(66)

The sources associated to the basic fields transform as

$$s\gamma = DB - [\hat{\rho}_{3}^{-2}, B_{0}] - [\hat{\rho}_{2}^{-1}, B_{1}] - [c, \hat{\gamma}] - \star db$$

$$s\sigma = -D\hat{\gamma} - [\rho_{4}^{-3}, B_{0}] - [\hat{\rho}_{3}^{-2}, B_{1}] - [\hat{\rho}_{2}^{-1}, B] - [c, \sigma]$$

$$s\rho_{2}^{-1} = F - [c, \hat{\rho}_{2}^{-1}] + \star d\pi_{1}$$

$$s\rho_{3}^{-2} = -D\hat{\rho}_{2}^{-1} - [c, \hat{\rho}_{3}^{-2}] - \star d\pi^{-1}$$

$$s\rho_{4}^{-3} = -D\hat{\rho}_{3}^{-2} - [c, \rho_{4}^{-3}] + \frac{1}{2}[\hat{\rho}_{2}^{-1}, \hat{\rho}_{2}^{-1}]$$
(67)

and those associated to the antighosts and multipliers transform as

$$s\bar{\sigma} = -d \star Dc \qquad , \qquad s\lambda = \bar{\sigma} + d \star A s\bar{\sigma}_{3}^{0} = d \star (DB_{1} + [c, B] - [\hat{\rho}_{2}^{-1}, B_{0}]) - \star d\pi^{1} \qquad , \qquad s\lambda_{3}^{-1} = -\bar{\sigma}_{3}^{0} + d \star B + \star d\bar{c}^{0} s\bar{\sigma}_{4}^{1} = d \star (DB_{0} + [c, B_{1}^{1}]) \qquad , \qquad s\lambda_{4}^{0} = -\bar{\sigma}_{4}^{1} + d \star B_{1}^{1} - \alpha \star \pi^{1} , \qquad s\lambda_{4}^{-2} = -\bar{\sigma}_{4}^{-1} + \alpha \star \pi^{-1} + d \star \bar{c}_{1}^{-1}.$$
(68)

The s-variations of fields and sources all have the form (28), henceforth the s-operator may again be identified with the linearized Slavnov-Taylor operator S_{Σ} .

BV versus BRST The general conclusions drawn from the Chern-Simons theory also hold in the present case. An extra feature of the BV-formulation for the BF model (which involves a reducible symmetry) is that the *s*-variation of the classical field *B* depends on sources (thereby ensuring the *off-shell nilpotency* of the *s*-operator). Another facet of this issue is the presence of the functional S_{mod} in the vertex functional. While the BV-approach automatically produces such contributions which are non-linear in the external sources, they have to be added "by hand" in the standard BRST-framework, e.g. see [5, 7, 25].

It should be noted that an off-shell formulation for the basic s-variations can eventually be given within the BRST-framework by mimicking the BV-approach, see reference [8].

4.2 VSUSY

VSUSY-transformations of fields As in the standard BRST-approach [16], we start from the Ø-symmetry conditions

$$\delta_{\tau}\tilde{A} = i_{\tau}\tilde{A} \qquad , \qquad \delta_{\tau}\tilde{B} = i_{\tau}\tilde{B} . \tag{69}$$

After spelling out these relations in terms of component fields and eliminating the antifields in terms of sources, we obtain the VSUSY-variations of the basic fields,

$$\begin{aligned}
\delta_{\tau}c &= i_{\tau}A &, \quad \delta_{\tau}B_0 = i_{\tau}B_1 \\
\delta_{\tau}A &= -i_{\tau}\hat{\rho}_2^{-1} &, \quad \delta_{\tau}B_1 = i_{\tau}B \\
&\delta_{\tau}B &= -i_{\tau}\hat{\gamma} .
\end{aligned}$$
(70)

and the variations of the associated sources :

$$\begin{aligned}
\delta_{\tau} \hat{\gamma} &= i_{\tau} \sigma , & \delta_{\tau} \sigma = 0 \\
\delta_{\tau} \hat{\rho}_{2}^{-1} &= i_{\tau} \hat{\rho}_{3}^{-2} , & \delta_{\tau} \hat{\rho}_{3}^{-2} &= i_{\tau} \rho_{4}^{-3} \\
& \delta_{\tau} \rho_{4}^{-3} &= 0.
\end{aligned}$$
(71)

Taking the results of Chern-Simons theory as a guideline (see eq.(36)), we now define the VSUSY-variations of the antighosts $(\bar{C}^{\alpha}) = (\bar{c}, \bar{c}_1^{-1}, \bar{c}^{-2}, \bar{c}^0)$ in such a way that relations (71) hold for the unhatted sources, i.e. such that we have

$$\begin{aligned}
\delta_{\tau} \gamma &= i_{\tau} \sigma , & \delta_{\tau} \sigma = 0 \\
\delta_{\tau} \rho_2^{-1} &= i_{\tau} \rho_3^{-2} , & \delta_{\tau} \rho_3^{-2} &= i_{\tau} \rho_4^{-3} \\
& \delta_{\tau} \rho_4^{-3} &= 0.
\end{aligned}$$
(72)

The transformation laws of the multipliers (Π^{α}) are then determined by the requirement that the VSUSY-algebra is satisfied, see equation (34). Altogether, we find the following variations of antighosts and multipliers :

$$\begin{aligned}
\delta_{\tau}\bar{c} &= 0 &, \qquad \delta_{\tau}b = \mathcal{L}_{\tau}\bar{c} \\
\delta_{\tau}\bar{c}_{1}^{-1} &= g(\tau)\bar{c}^{-2} &, \qquad \delta_{\tau}\pi_{1} = \mathcal{L}_{\tau}\bar{c}_{1}^{-1} + g(\tau)\pi^{-1} \\
\delta_{\tau}\bar{c}^{-2} &= 0 &, \qquad \delta_{\tau}\pi^{-1} = \mathcal{L}_{\tau}\bar{c}^{-2} \\
\delta_{\tau}\bar{c}^{0} &= 0 &, \qquad \delta_{\tau}\pi^{1} = \mathcal{L}_{\tau}\bar{c}^{0} .
\end{aligned}$$
(73)

Here, $g(\tau) = \tau^{\mu} g_{\mu\nu} dx^{\nu}$ denotes the 1-form associated to the vector field τ by virtue of a spacetime metric $(g_{\mu\nu})$ [8, 16].

If we set all sources to zero, we recover the transformation laws and on-shell VSUSY-algebra of the standard BRST-approach [16]. If sources are included in the latter framework for the discussion of Ward identities, considerations different from ours lead to the introduction of the hatted sources (62) and to the variations (70)-(73) [7].

To conclude, we come to the δ_{τ} -variations of the sources (61) associated to the doublet fields $(\bar{C}^{\alpha}, \Pi^{\alpha})$. According to the general guideline indicated after equation (33), these sources (antifields) are assumed to transform "the other way round", in the opposite direction as the fields, see eq.(35):

$$\begin{aligned}
\delta_{\tau}\bar{\sigma} &= \mathcal{L}_{\tau}\lambda - d \star i_{\tau}\hat{\rho}_{2}^{-1} &, & \delta_{\tau}\lambda = 0 \\
\delta_{\tau}\bar{\sigma}_{3}^{0} &= -\mathcal{L}_{\tau}\lambda_{3}^{-1} + d \star i_{\tau}\hat{\gamma} &, & \delta_{\tau}\lambda_{3}^{-1} = 0 \\
\delta_{\tau}\bar{\sigma}_{4}^{1} &= -\mathcal{L}_{\tau}\lambda_{4}^{0} - g(\tau)\bar{\sigma}_{3}^{0} + (1-\alpha) \star \mathcal{L}_{\tau}\bar{c}^{0} &, & \delta_{\tau}\lambda_{4}^{0} = -g(\tau)\lambda_{3}^{-1} \\
\delta_{\tau}\bar{\sigma}_{4}^{-1} &= -\mathcal{L}_{\tau}\lambda_{4}^{-2} - d \star g(\tau)\bar{c}^{-2} + \alpha \star \mathcal{L}_{\tau}\bar{c}^{-2} &, & \delta_{\tau}\lambda_{4}^{-2} = 0 .
\end{aligned}$$
(74)

By construction, the VSUSY-algebra is satisfied off-shell for all fields and sources.

Ward identity The δ_{τ} -variations of fields and antifields induce the broken Ward identity

$$\mathcal{W}_{\tau}S_{\mathrm{nm}} = \int_{\mathbf{R}^4} \mathrm{tr}\left\{\pm \Phi_A^* \mathcal{L}_{\tau} \Phi^A\right\},\tag{75}$$

which takes the following form after elimination of antifields:

$$\mathcal{W}_{\tau}\Sigma = \int_{\mathbf{R}^4} \operatorname{tr}\left\{ (-1)^{|\rho_A|} \rho_A \mathcal{L}_{\tau} \Phi^A \right\}.$$
(76)

Thus, the final result has the same form as for Chern-Simons theory, i.e. we have a breaking which is linear in the sources and in the quantum fields [7, 23]. It is worthwhile to note that this result has been obtained for an arbitrary value of the gauge parameter α . This is in contrast to the usual formulation where VSUSY puts some restrictions on the gauge parameter [7, 8, 23].

5 General case

The topological field theories studied in the previous sections allowed us to make a detailed comparison between the BRST- and BV-approaches to the different types of symmetries that are essential for discussing perturbative renormalization. They also provide concrete illustrations for an algebraic construction of topological models. The goal of the present section is three-fold. First of all, to present some general principles summarizing the algebraic formalism considered so far (section 5.1.2). Second, to investigate which other models can be constructed using this approach and to determine their characteristic features (sections 5.1.3 and 5.1.4). Finally, we wish to provide some general expressions and strategies applicable to all models under consideration (sections 5.1.6 and 5.2).

5.1 Symmetries of the classical action

5.1.1 Generalized forms and duality

Since the algebraic construction involves a gauge field and, more generally, *p*-form potentials, we first recall the general framework for *d*-dimensional space-time \mathcal{M}_d presented in reference [12].

Let us consider $p \in \{0, 1, ..., d\}$. A *p*-form gauge potential $X_p \equiv X_p^0$ with values in a Lie algebra gives rise to a *generalized form*

$$\tilde{X}_p = \sum_{q=0}^d X_{d-q}^{p-d+q} = X_d^{p-d} + X_{d-1}^{p-d+1} + \ldots + X_p + \ldots + X_0^p,$$
(77)

which involves all ghosts and "ghosts for ghosts" as well as some fields with negative ghostnumber. However, in general, the index structure of the latter fields does not allow us to identify them with the antifields associated to the fields appearing in \tilde{X}_p . Rather one has to introduce a so-called *dual form* \tilde{Y}_{d-p-1} with an analogous expansion,

$$\tilde{Y}_{d-p-1} = \sum_{q=0}^{d} Y_{d-q}^{-p-1+q} = Y_d^{-p-1} + \ldots + Y_{d-p-1} + \ldots + Y_0^{d-p-1}.$$
(78)

The generalized forms (77) and (78) are dual to each other in the sense that the fields with negative ghost-number contained in the first one are the antifields associated to the fields with positive ghost-number contained in the second one and vice versa, i.e.

$$\left(X_{p-q}^{q}\right)^{*} = Y_{d-(p-q)}^{-q-1} \qquad \text{for} \quad q = 0, \dots, p$$

$$\left(Y_{d-p-1-q}^{q}\right)^{*} = X_{p+1+q}^{-q-1} \qquad \text{for} \quad q = 0, \dots, d-p-1.$$

$$(79)$$

For instance, for d = 4, the ladder \tilde{A} is dual to the ladder \tilde{B} while \tilde{A} is self-dual for d = 3.

5.1.2 Field content and construction of models

All of the models to be considered involve a gauge field A, hence a generalized form $\tilde{A} \equiv \tilde{X}_1$ and the dual form $\tilde{B} \equiv \tilde{Y}_{d-2}$. Eventually, additional dual pairs $(\tilde{X}_p, \tilde{Y}_{d-p-1})$ with $0 \le p \le d-1$ can be included and coupled to the generalized gauge field \tilde{A} .

We now summarize the general algebraic procedure for constructing models [1, 12].

1. One imposes horizontality conditions on $\tilde{A}, \tilde{B}, \tilde{X}, \tilde{Y}$, i.e. conditions on their field strengths $\tilde{F}, \tilde{D}\tilde{B}, \tilde{D}\tilde{X}, \tilde{D}\tilde{Y}$ which are compatible with the Bianchi identities

 $\tilde{D}\tilde{F} = 0$ and $\tilde{D}^2\tilde{\Omega} = [\tilde{F},\tilde{\Omega}]$ for $\tilde{\Omega} = \tilde{B},\tilde{X},\tilde{Y}$.

This determines nilpotent s-variations for the components of $\tilde{A}, \tilde{B}, \tilde{X}, \tilde{Y}$. In practice, the horizontality conditions are nothing but the *tilted equations of motion* of the model to be defined in the next step: thus, the horizontality conditions fix both the symmetries and the dynamics.

2. One looks for a generalized Lagrangian density $\tilde{\mathcal{L}}$, i.e. a generalized *p*-form which depends on $\tilde{A}, \tilde{B}, ...$ and their exterior derivatives $d\tilde{A}, d\tilde{B}, ...$ and which satisfies the cocycle condition $s\tilde{\mathcal{L}} = d(..)$ where s denotes the operator defined in the first step. Then

$$S_{\min}[\Phi^a, \Phi_a^*] \equiv \int_{\mathcal{M}_d} \tilde{\mathcal{L}} \Big|_d^0 \tag{80}$$

represents an *s*-invariant action extending the classical action

$$S_{\rm inv}[\Phi^a] \equiv S_{\rm min}[\Phi^a, \Phi^*_a = 0] . \qquad (81)$$

Moreover, the s-variations defined in the first step coincide with those generated by the functional (80) according to relations (21), i.e. S_{\min} solves the BV master equation¹.

Since the s-operator is defined in terms of conditions involving the covariant quantities \tilde{F} , $\tilde{D}\tilde{B}$, ..., the polynomial $\tilde{\mathcal{L}}$ depends on $d\tilde{A}, d\tilde{B}, ...$ by virtue of the field strengths $F^{\tilde{A}}, D^{\tilde{A}}\tilde{B}, ...$ By construction, the classical action (81) is invariant under the standard BRST-transformations $s_0\Phi^a \equiv (s\Phi^a)|_{\Phi^*_{\pm}=0}$. If $s\Phi^a$ involves Φ^*_a , then s_0 is only nilpotent on the mass-shell.

As emphasized in reference [12], the algebraic approach proceeds in the opposite order than the usual *BV-algorithm*. In fact, the latter starts with a classical action $S_{inv}[A, B, ..]$ that is invariant under s_0 -variations (which are, in general, only nilpotent on the mass-shell) and the goal then consists of explicitly determining an action

$$\Gamma[\Phi^{A}; \Phi_{A}^{*}] = S_{\text{inv}}[A, B, ..] + \sum_{q=1}^{n} \Phi_{A_{1}}^{*} \cdots \Phi_{A_{q}}^{*} \Delta_{q}[\Phi^{A}],$$

satisfying the master equation and generating nilpotent s-variations by virtue of the definitions (20).

5.1.3 Examples

Mostly following references [1, 12], we will now present an overview of models which can be constructed by the procedure outlined above. As pointed out by L. Baulieu [12, 13], this construction not only yields topological field theories of Schwarz-type, but also theories of Witten-type. We will not spell out the explicit form of the cocycle condition for each model, since the latter can easily be obtained from the given Lagrangian $\tilde{\mathcal{L}}$ by application of the *s*operator. However, we note that (contrary to the indications in [12]), the cocycle condition does not always have the simple form $(s + d)\tilde{\mathcal{L}} = 0$ as was already illustrated by the Chern-Simons theory, see equation (10). The compatibility of horizontality conditions can readily be verified for each example.

¹The explanation of this fact represents an intriguing question.

1) **BF-model in** $d \ge 2$ This model involves the pair (\tilde{A}, \tilde{B}) , the action being given by

$$S_{\min} = \int_{\mathcal{M}_d} \operatorname{tr} \{ \tilde{B} F^{\tilde{A}} \} \Big|_d^0 = \int_{\mathcal{M}_d} \operatorname{tr} \{ BF \} + \text{ s.t.} , \qquad (82)$$

where 's.t.' stands for source terms. The classical equations of motion are the zero-curvature conditions F = 0 = DB. For d = 2, the field B represents a 0-form and does not have a local gauge symmetry (apart from ordinary gauge transformations) [3, 2, 7].

2) BF-model with cosmological constant For d = 3 and d = 4, a term involving a real dimensionless parameter λ can be added [3] to the BF-action for the pair (\tilde{A}, \tilde{B}) . For $\mathbf{d} = \mathbf{3}$, the minimal action reads [1]

$$S_{\min} = \int_{\mathcal{M}_3} \operatorname{tr} \left\{ \tilde{B}F^{\tilde{A}} + \frac{\lambda}{3}\tilde{B}^3 \right\} \Big|_3^0 = \int_{\mathcal{M}_3} \operatorname{tr} \left\{ BF + \frac{\lambda}{3}B^3 \right\} + \text{ s.t.} , \qquad (83)$$

which leads to the classical equations of motion $F + \lambda B^2 = 0 = DB$.

For d = 4, the action

$$S_{\min} = \int_{\mathcal{M}_4} \operatorname{tr} \left\{ \tilde{B}F^{\tilde{A}} + \frac{\lambda}{2}\tilde{B}^2 \right\} \Big|_4^0 = \int_{\mathcal{M}_4} \operatorname{tr} \left\{ BF + \frac{\lambda}{2}B^2 \right\} + \text{ s.t.}$$
(84)

leads to the complete equations of motion $F^{\tilde{A}} + \lambda \tilde{B} = 0 = \tilde{D}\tilde{B}$. From these, we can deduce, amongst others, that

$$sB^* = -[c, B^*] - (F + \lambda B)$$
 (85)

We now impose the complete equation of motion for the 2-form potential B, i.e.

$$0 = \left(F^{\tilde{A}} + \lambda \tilde{B}\right)\Big|_{2}^{0} = \frac{\delta S_{\min}}{\delta B} = -sB^{*} .$$
(86)

This implies that the field B^* can be set to zero consistently. In addition, we choose $\lambda = -1$ for the sake of simplicity. From equations (86)(85), we then conclude that B = F and, by substitution into (84), we obtain

$$S_{\min}|_{\frac{\delta S_{\min}}{\delta B}=0} = \frac{1}{2} \int_{\mathcal{M}_4} \operatorname{tr} \{FF\} + \sum_{\Phi^a=A,c,B_1,B_0} \int_{\mathcal{M}_4} \operatorname{tr} \{\Phi_a^* s \Phi^a\} .$$
(87)

This expression represents the minimal action associated to the topological invariant $\int_{\mathcal{M}_4} \operatorname{tr} \{FF\}$ whose gauge-fixing gives rise to Witten's topological Yang-Mills theory (TYM) [2]. For the latter theory, both the BRST-algebra [26] and the VSUSY-algebra [27, 16] close off-shell for different classes of gauge-fixings so that the introduction of antifields does not seem useful for studying the quantization of the model. Yet, it is quite interesting that TYM whose gauge-fixing procedure refers to self-*duality* conditions can be obtained from an action involving only a *dual* pair of potentials [13].

3a) BF-XY-model For $d \ge 2$, one can add to the BF-model (82) some dual pairs $(\tilde{X}_p, \tilde{Y}_{d-p-1})$ with $0 \le p \le d-1$ coupling to \tilde{A} according to [12]

$$S_{\min} = \int_{\mathcal{M}_d} \operatorname{tr} \left\{ \tilde{B}F^{\tilde{A}} + \sum_{p=0}^{d-1} \tilde{X}_p D^{\tilde{A}} \tilde{Y}_{d-p-1} \right\} \Big|_d^0 = \int_{\mathcal{M}_d} \operatorname{tr} \left\{ BF + \sum_{p=0}^{d-1} X_p DY_{d-p-1} \right\} + \text{s.t.} \quad (88)$$

This action leads to the classical equations of motion $0 = F = DB - \sum_{p=0}^{d-1} (-1)^p [X_p, Y_{d-p-1}] = DX_p = DY_{d-p-1}$. It represents a first order action that is analogous to three-dimensional Chern-Simons theory.

3b) BF-XY-model with BX-coupling For any $d \ge 2$, the 2-form potential X_2 appearing in the previous model can be coupled directly to B [12] with strength $\alpha \in \mathbf{R}$:

$$S_{\min} = \int_{\mathcal{M}_d} \operatorname{tr} \left\{ \tilde{B}(F^{\tilde{A}} + \alpha \tilde{X}_2) + \tilde{X}_2 D^{\tilde{A}} \tilde{Y}_{d-3} \right\} \Big|_d^0 = \int_{\mathcal{M}_d} \operatorname{tr} \left\{ B(F + \alpha X_2) + X_2 D Y_{d-3} \right\} + \text{s.t.}$$
(89)

The classical equations of motion then take the form $0 = F + \alpha X = DY + \alpha B = DB - [X, Y]$. By elimination of X from the action functional by virtue of its algebraic equation of motion, one gets a classical action of the form $\int_{\mathcal{M}_d} d \operatorname{tr} \{FY_{d-3}\}$ which is analogous to TYM [12]. More specifically, for d = 3, this action, $\int_{\mathcal{M}_3} d \operatorname{tr} \{FY_0\} = \int_{\mathcal{M}_3} \operatorname{tr} \{FDY_0\}$ describes magnetic monopoles and its gauge-fixing via Bogomolny's equations yields a topological model that is closely related to four-dimensional TYM [28].

For $\mathbf{d} = \mathbf{4}$, a "dual" form of the model (89) is obtained by exchanging the generalized fields \tilde{B}_2 and \tilde{X}_2 in the pairs $(\tilde{A}_1, \tilde{B}_2), (\tilde{X}_2, \tilde{Y}_1)$:

$$S_{\min} = \int_{\mathcal{M}_4} \operatorname{tr} \left\{ \tilde{X}_2 (F^{\tilde{A}} + \alpha \tilde{B}_2) + \tilde{B}_2 D^{\tilde{A}} \tilde{Y}_1 \right\} \Big|_4^0 .$$
⁽⁹⁰⁾

To this functional one can add a contribution $\int \operatorname{tr} \mathcal{F}(\tilde{B}_2) \Big|_4^0$ of the form $s \int \Delta_4^{-1} + \int d(...)$. Different gauge-fixings then allow to recover the Lagrangian $\operatorname{tr} \{\tau FF\}$ for TYM and the one of the dual theory defined by the duality transformation $\tau \to 1/\tau$ (see reference [29] for this and the following points). The θ -parameter of the theory can be adjusted by adding the topological invariant

$$S_{\text{top}} \equiv \int_{\mathcal{M}_4} d \operatorname{tr} \left\{ a \left(A d A + \frac{2}{3} A A A \right) + b F Y_1 + c Y_1 D Y_1 \right\} \\ = a \int_{\mathcal{M}_4} \operatorname{tr} \left\{ F F \right\} + b \int_{\mathcal{M}_4} \operatorname{tr} \left\{ F D Y_1 \right\} + c \int_{\mathcal{M}_4} \operatorname{tr} \left\{ D Y_1 D Y_1 + F[Y_1, Y_1] \right\}$$

with appropriately chosen complex parameters a, b, c. The different formulations of TYM in two and eight dimensions can be approached along the same lines.

3c) BF-XY-model with mixed couplings Several sets of pairs $(\tilde{X}_p, \tilde{Y}_{d-p-1}), (\tilde{U}_p, \tilde{V}_{d-p-1}), ...$ with $0 \le p \le d-1$ can be considered and coupled by terms of the form [X, Y], [X, U], B[X, U], ...[13]. For concreteness, we consider d = 6 and independent pairs $(\tilde{A}, \tilde{B}_4), (\tilde{X}_2, \tilde{Y}_3), (\tilde{U}_2, \tilde{V}_3), (\tilde{U}_2, \tilde{V}_3), (\tilde{U}_2, \tilde{V}_3)$ with an action $S_{\min} = S_{inv} + s.t.$ where

$$S_{\rm inv} = \int_{\mathcal{M}_6} \operatorname{tr} \left\{ B(F+X) + X(DY + [U, U^c]) + UDV - U^c DV^c + VV^c \right\} \,. \tag{91}$$

By substituting the algebraic equations of motion $0 = F + X = V + DU^c = V^c + DU$ into the latter functional, we obtain the following six-dimensional topological model of Witten-type [13]:

$$S_{\text{inv}} = \int_{\mathcal{M}_6} \operatorname{tr} \left\{ DU DU^c - F[U, U^c] \right\} - \int_{\mathcal{M}_6} \operatorname{tr} \left\{ FDY \right\}$$

$$= \int_{\mathcal{M}_6} d \operatorname{tr} \left\{ UDU^c \right\} - \int_{\mathcal{M}_6} d \operatorname{tr} \left\{ FY \right\} .$$
(92)

The first part of this action admits BRST- and VSUSY-algebras which close off-shell and which have been studied in references [13, 30].

4) 3d Chern-Simons theory and extensions thereof For d = 3, we can choose $\tilde{X}_1 = \tilde{A} = \tilde{Y}_1$ and consider the *Chern-Simons theory* as we did in section 3. One can also combine this theory with the models considered above [12] or include a term $\int_{\mathcal{M}_3} \operatorname{tr} \{X_1 D X_1\}$ [4]. The generalization of Chern-Simons theory to an arbitrary dimension [4] may be discussed as well using the algebraic approach [31].

5) Supersymmetric extensions of the previous models The algebraic formalism admits a supersymmetric extension [32] which should allow to discuss the supersymmetric versions of the previous models, e.g. super BF models (see [33] and references therein).

5.1.4 General features

One may wonder what kind of field theoretic models can be constructed by the algebraic procedure summarized above and which generic features are shared by all of the models that we listed. Obviously, their *field content* is given by *p-form potentials* and they involve at least the connection 1-form A. Only in three dimensions, the corresponding extended form \tilde{A} contains solely A, its ghost c and the associated antifields A^*, c^* . Thus, 3-manifolds are the only ones for which a model involving solely a Yang-Mills potential can be constructed. (Actually, such models can be obtained indirectly in other dimensions by eliminating some fields, as illustrated by example 2 above.)

Otherwise, a common feature of all models constructed above is that their minimal action (involving the classical fields A, ..., the ghosts c, ... and the associated antifields $A^*, c^*, ...$) can be written directly in terms of generalized fields $\tilde{A}, ...$ obeying some generalized zero-curvature conditions $0 = \tilde{F} = ...$ [23]. This fact is related to the following one. The dynamics of fields is described by a metric-independent, first order action, the kinetic term being of the form AdA, BdA, XdY and the gauge invariant interaction being given by some polynomial of the fields. (General arguments supporting that this is the only class of examples have been put forward in reference [31].) All of these theories are of topological nature.

If some of the classical equations of motion are algebraic (and linear in the basic fields), as it is the case in examples 2, 3b, 3c, then they imply all other equations of motion by application of the covariant derivative. Moreover, elimination of fields by some algebraic equations of motion then reduces the first order actions to second order actions analogous to TYM, i.e. topological models of Witten-type.

5.1.5 Other models and possible generalizations

Some examples related to 2-dimensional gravity have been studied in references [12, 23]. In this case, the components of the space-time metric are viewed as gauge potentials associated to the invariance under general coordinate transformations.

A further extension of the formalism is the following. Consider the case of an abelian gauge group. The pairs of potentials $(\tilde{X}_p, \tilde{Y}_{d-p-1})$ can be generalized to mixed dual pairs $(\tilde{X}_p, \tilde{d}\tilde{Y}_{d-p-2})$ and $(\tilde{d}\tilde{X}_p, \tilde{Y}_{d-p-2})$ each of which involves an abelian potential and an abelian field strength. Such pairs appear in the transgression construction of (d+1)-dimensional topological field theories from d-dimensional topological models [34].

A different generalization of the algebraic formalism consists of introducing incomplete ladders and deformations of the operator $\tilde{d} \equiv d + s$ [35]. This approach allows to discuss cohomological aspects of Yang-Mills-type theories or supersymmetric extensions thereof [35, 32].

Finally, we note that Yang-Mills theories can be formulated in terms of a first order action by deforming a BF model [36]. Thus, the algebraic formalism discussed here should also be useful for describing these (non-topological) field theories.

5.1.6 Master equation and gauge-fixing

In this section, we summarize the general recipe for deriving the vertex functional of the theory on a generic space-time manifold \mathcal{M}_d .

The minimal actions $S_{\min}[\Phi^a; \Phi_a^*]$ presented in section 5.1.3 (which have been obtained from a horizontality condition and cocycle condition) satisfy the BV master equation (18). For each of these models, the gauge degrees of freedom have to be fixed by virtue of some gauge-fixing conditions \mathcal{F}_{α} . The latter are implemented in the action by introducing a gauge-fermion $\Psi_{\rm gf}$ of ghost-number -1 depending on antighost fields \bar{C}^{α} :

$$\Psi_{\rm gf}[\Phi^A] = \int_{\mathcal{M}_d} \operatorname{tr} \left\{ \bar{C}^{\alpha} \mathcal{F}_{\alpha} \right\}.$$
(93)

The s-variation of \bar{C}^{α} yields the multiplier field Π^{α} ,

$$s\bar{C}^{\alpha} = \Pi^{\alpha} \quad , \quad s\Pi^{\alpha} = 0$$

$$\tag{94}$$

and the corresponding antifields are assumed to transform "the other way round":

$$s\Pi_{\alpha}^{*} = (-1)^{(d+1)(|\bar{C}^{\alpha}|+1)}\bar{C}_{\alpha}^{*} \quad , \quad s\bar{C}_{\alpha}^{*} = 0 \,.$$
(95)

These trivial ("contractible") BRST-doublets, which do not contribute to the physical content of the theory, are taken into account by adding a contribution

$$S_{\text{aux}}[\bar{C}^*_{\alpha},\Pi^{\alpha}] = -\int_{\mathcal{M}_d} \text{tr} \left\{ \bar{C}^*_{\alpha}\Pi^{\alpha} \right\}$$
(96)

to the action S_{\min} . The resulting non-minimal action depends on the fields $(\Phi^A) = (\Phi^a, \bar{C}^{\alpha}, \Pi^{\alpha})$ and the corresponding antifields $(\Phi^*_A) = (\Phi^*_a, \bar{C}^*_\alpha, \Pi^*_\alpha)$:

$$S_{\rm nm}[\Phi^A, \Phi^*_A] = S_{\rm min}[\Phi^a, \Phi^*_a] + S_{\rm aux}[\bar{C}^*_{\alpha}, \Pi^{\alpha}].$$
(97)

It still solves the master equation.

After elimination of the antifields according to prescription (22), one obtains the *vertex* functional

$$\Sigma[\Phi^{A}, \rho_{A}] = S_{\rm nm}|_{\Phi^{*}_{A}} = S_{\rm inv} + S_{\rm gf} + S_{\rm ext} + S_{\rm mod} , \qquad (98)$$

where $S_{\rm gf} = s \Psi_{\rm gf}$ denotes the gauge-fixing part for the classical, gauge invariant action $S_{\rm inv}$ and where $S_{\rm ext}$ represents the linear coupling of the external sources ρ_A to the *s*-variations of the fields Φ^A .

5.2 VSUSY

In the following, we sketch the general procedure for obtaining the VSUSY-variations of all fields and antifields on $\mathcal{M}_d = \mathbf{R}^d$.

A *p*-form gauge potential X_p generally admits a hierarchy of ghosts $X_{p-1}^1, X_{p-2}^2, ...$ and the gauge-fixing of the corresponding symmetries leads to analogous hierarchies of antighosts. All of these fields can be organized in a *BV-pyramid* culminating in X_p , see Table 1.

For a ladder Ω , the VSUSY-variations are postulated to be given by

$$\delta_{\tau}\tilde{\Omega} = i_{\tau}\tilde{\Omega} , \qquad (99)$$

i.e. VSUSY climbs the ladder from the highest ghost-number to the lowest one. The variations of the classical fields, the ghosts and the associated antifields follow directly from (99) by choosing $\tilde{\Omega} = \tilde{X}_p$, \tilde{Y}_{d-p-1} . As noted after eq.(33), the antifields transform in the other direction than the fields do.

Next, we consider the antighosts with negative ghost-number, i.e. those located on the left half of the BV-pyramid, i.e. $\bar{c}_{p-1}^{-1}, \bar{c}_{p-2}^{-2}, \bar{c}_{p-3}^{-1}, \ldots$ Their variations follow from the arguments preceding equations (73). Those of the associated antifields are inferred from the general guideline that antifields transform in the other direction than fields do, i.e. $\bar{C}^{\alpha} \xrightarrow{\delta_{\tau}} \bar{C}^{\beta}$ implies $\bar{C}_{\beta}^* \xrightarrow{\delta_{\tau}} \bar{C}_{\alpha}^*$, see eq.(74).

All of the remaining *antighosts* have *positive* (more precisely non-negative) *ghost-number*. Those which have the same total degree can be gathered in ladders which correspond to the diagonals on the right half of the pyramid :

$$\tilde{\bar{c}}_{p-2} = \bar{c}_{p-2} + \bar{c}_{p-3}^1 + \ldots + \bar{c}^{p-2}$$

$$\tilde{\bar{c}}_{p-4} = \bar{c}_{p-4} + \bar{c}_{p-5}^1 + \ldots + \bar{c}^{p-4} , \ldots$$
(100)

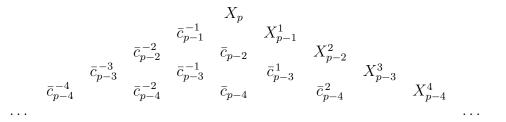


Table 1: X_p -pyramid

These generalized forms are incomplete since they only involve components with positive ghostnumber². Similarly, the *antifields associated to the antighosts* (100) can be collected into ladders containing only components with negative ghost-number :

$$(\tilde{\bar{c}}_{p-2})^* = (\bar{c}^{p-2})^* + \ldots + (\bar{c}_{p-2})^* \quad , \quad \dots$$
 (101)

The transformation law (99) is now postulated for all of these ladders, i.e. for $\tilde{\Omega} = \tilde{\bar{c}}_{p-2n}, (\tilde{\bar{c}}_{p-2n})^*$ with n = 1, 2, ...

The δ_{τ} -variations of the multipliers $\Pi^{\alpha} = s\bar{C}^{\alpha}$ follow from the variations of the \bar{C}^{α} by requiring the VSUSY-algebra $[s, \delta_{\tau}] = \mathcal{L}_{\tau}$ to be satisfied :

$$\delta_{\tau}\Pi^{\alpha} = \delta_{\tau}s\bar{C}^{\alpha} = \mathcal{L}_{\tau}\bar{C}^{\alpha} - s(\delta_{\tau}\bar{C}^{\alpha}) \; .$$

The antifields $(\bar{C}^*_{\alpha}, \Pi^*_{\alpha})$ associated to the BRST-doublets $(\bar{C}^{\alpha}, \Pi^{\alpha})$ again transform "the other way round".

6 Conclusion

As is well-known, the BV-formalism represents a systematic procedure for constructing an *s*-invariant action in the case of a gauge algebra which is reducible and/or only valid on-shell. The *s*-operator of the BV-setting is nothing but the linearized Slavnov-Taylor operator. If the symmetry algebra is only valid on-shell, antifields appear in the *s*-variations and the solution $S_{\rm nm}$ of the Slavnov-Taylor identity involves terms that are quadratic (or of higher oder) in the antifields.

The algebraic framework for the BV-formalism on which we elaborated here, represents an elegant procedure for constructing solutions of the Slavnov-Taylor identity for *topological models of Schwarz-type* as defined in various dimensions. In particular, it allows to obtain quite straightforwardly the VSUSY-transformations which are most useful for dealing with the quantum version of these theories.

As emphasized in section 5, topological models of Witten-type can also be introduced along these lines. However, their BRST- and VSUSY-algebras close off-shell in the standard BRSTapproach and therefore the introduction of antifields is not useful for their description [27, 16, 30].

Our discussion of VSUSY for topological models defined on flat space-time can be generalized to generic manifolds by incorporating VSUSY in the *s*-operation: this leads to an exact rather than a broken Ward identity and it proves to be useful for discussing the relationship between topological models and gravity [18].

To conclude, we note that it would be interesting to gain a deeper geometric understanding [37] of the algebraic construction of topological models summarized in section 5. (Presumably the field theoretic formulation of references [15, 32] provides the appropriate framework for this endeavor.) Such an insight should explain more fully why highly non-trivial solutions of the master equation can be obtained from a simple algebraic procedure.

²Yet, the BV-pyramid does not involve the complete ladder \tilde{X}_p either, but only those components which have positive ghost-number.

Acknowledgments

F.G. wishes to thank M. Schweda and all the members of the Institut für Theoretische Physik of the Technical University of Vienna for the warm hospitality extended to him during part of the present work. He also acknowledges enlightening discussions with O. Piguet, F. Delduc and S. Wolf.

References

 H. Ikemori, "Extended form method of antifield BRST formalism for topological quantum field theories," *Class. Quant. Grav.* 10 (1993) 233, hep-th/9206061.

H. Ikemori, "Extended form method of antifield BRST formalism for BF theories," *Mod.Phys.Lett.* A7 (1992) 3397, hep-th/9205111.

- [2] D. Birmingham, M. Blau, M. Rakowski and G. Thompson, "Topological field theory", *Phys. Rept.* 209 (1991) 129.
- [3] G.T. Horowitz, "Exactly soluble diffeomorphism invariant theories", Commun.Math.Phys. **125** (1989) 417.

M. Blau and G. Thompson, "Topological gauge theories of antisymmetric tensor fields", Ann. Phys. **205** (1991) 130.

A. Karlhede and M. Roček, "Topological quantum field theories in arbitrary dimensions", *Phys.Lett.* **B224** (1989) 58.

- [4] R.C. Myers and V. Periwal, "New symmetries in topological field theories", *Phys.Lett.* B225 (1989) 352.
- [5] D. Birmingham, M. Rakowski and G. Thompson, "Renormalization of topological field theory", Nucl. Phys. B329 (1990) 83.

F. Delduc, F. Gieres and S. P. Sorella, "Supersymmetry of the d = 3 Chern-Simons action in the Landau gauge", *Phys.Lett.* **B225** (1989) 367.

- [6] F. Delduc, C. Lucchesi, O. Piguet and S. P. Sorella, "Exact scale invariance of the Chern-Simons theory in the Landau gauge", Nucl. Phys. B346 (1990) 313.
- [7] E. Guadagnini, N. Maggiore and S.P. Sorella, "Supersymmetric structure of four-dimensional antisymmetric tensor fields", *Phys.Lett.* B255 (1991) 65.

N. Maggiore and S. P. Sorella, "Perturbation theory for antisymmetric tensor fields in four-dimensions", *Int.J.Mod.Phys.* A8 (1993) 929, hep-th/9204044.

C. Lucchesi, O. Piguet and S. P. Sorella, "Renormalization and finiteness of topological BF theories", *Nucl.Phys.* **B395** (1993) 325, hep-th/9208047.

A. Blasi and N. Maggiore, "Infrared and ultraviolet finiteness of topological BF theory in two dimensions", *Class. Quant. Grav.* **10** (1993) 37, hep-th/9207008.

- [8] O. Piguet and S.P. Sorella, Algebraic Renormalization (Springer Verlag, 1995).
 A. Boresch, S. Emery, O. Moritsch, M. Schweda, T. Sommer and H. Zerrouki, Applications of Noncovariant Gauges in the Algebraic Renormalization Procedure (World Scientific, 1998).
- [9] I.A. Batalin and G.A. Vilkovisky, "Gauge algebra and quantization", *Phys.Lett.* 102B (1981) 27.

I.A. Batalin and G.A. Vilkovisky, "Quantization of gauge theories with linearly dependent generators", *Phys.Rev.* **D28** (1983) 2567.

J. Gomis, J. Paris and S. Samuel, "Antibracket, antifields and gauge theory quantization", *Phys.Rept.* **259** (1995) 1, hep-th/9412228.

G. Barnich, F. Brandt and M. Henneaux, "Local BRST cohomology in gauge theories", *Phys.Rept.* **338** (2000) 439, hep-th/0002245.

G. Barnich, "Classical and quantum aspects of the extended antifield formalism", Thèse d'agrégation ULB (June 2000), Proceedings of the Spring School "QFT and Hamiltonian Systems", Calimanesti, (Romania, May 2000), hep-th/0011120.

- [10] L. Baulieu, M. Bellon, S. Ouvry and J. Wallet, "Batalin-Vilkovisky analysis of supersymmetric systems", *Phys.Lett.* B252 (1990) 387.
- [11] S. Weinberg, The Quantum Theory of Fields, Vol.II (Cambridge University Press, 1996).
- [12] L. Baulieu, "B-V quantization and field-anti-field duality for p-form gauge fields, topological field theories and 2-D gravity", Nucl. Phys. B478 (1996) 431.
- [13] L. Baulieu, "On forms with non-Abelian charges and their dualities", *Phys.Lett.* B441 (1998) 250, hep-th/9808055.
- [14] J. C. Wallet, "Algebraic setup for the gauge fixing of BF and super BF systems", *Phys.Lett.* B235 (1990) 71.

L. Baulieu, E. Bergshoeff and E. Sezgin, "Open BRST algebras, ghost unification and string field theory", *Nucl. Phys.* B307 (1988) 348.

M. Abud, J.-P. Ader and L. Cappiello, "A BRST Lagrangian quantization of reducible gauge theories: Non-Abelian *p*-forms and string field theories", *Nuov.Cim.* **105A** (1992) 1507.

- [15] M. Carvalho, L. C. Vilar, C. A. Sasaki and S. P. Sorella, "BRS cohomology of zero curvature systems I. The complete ladder case", *J.Math.Phys.* **37** (1996) 5310, hep-th/9509047.
- [16] F. Gieres, J. Grimstrup, T. Pisar and M. Schweda, "Vector supersymmetry in topological field theories", JHEP 06 (2000) 18, hep-th/0002167.
- T. Pisar, J. Rant and H. Zerrouki, "A generalized p-form model in D = 3", Mod.Phys.Lett. A15 (2000) 1147, hep-th/0002234.
 T. Pisar, "Symmetry content of a generalized p-form model of Schwarz-type in d dimensions", Phys.Lett. B513 (2001) 413, hep-th/0004137.
- [18] C.P. Constantinidis, F. Gieres, O. Piguet and M.S. Sarandy, "On the symmetries of BF models and their relation with gravity", preprint LYCEN 2001-78 (November 2001).
- [19] E. Witten, "2 + 1 dimensional gravity as an exactly soluble system", Nucl. Phys. B311 (1988/89) 46.

 [20] O. Piguet, "Ghost equations and diffeomorphism invariant theories", Class. Quant. Grav. 17 (2000) 3799, hep-th/0005011.

O. Piguet, "Diffeomorphism invariant theories and vector supersymmetry", Talk given at International Conference on Quantization, Gauge Theory and Strings (Conference dedicated to the memory of Professor Efim Fradkin), Moscow, Russia, 5-10 June 2000, hep-th/0010053.

[21] J. Thierry-Mieg, "Geometrical reinterpretation of Faddeev-Popov ghost particles and BRS transformations", J.Math.Phys. 21 (1980) 2834.

L. Bonora and M. Tonin, "Superfield formulation of extended BRS symmetry", *Phys.Lett.* **98B** (1981) 48.

R. Stora, in "New Developments in Quantum Field Theory and Statistical Mechanics", Cargèse 1976, M. Lévy and P. Mitter, eds., NATO ASI Ser. B, Vol. 26 (Plenum Press, 1977).

R. Stora, in "Progress in Gauge Field Theory", Cargèse 1983, G. 't Hooft et al. eds., NATO ASI Ser. B, Vol. 115 (Plenum Press, 1984).

L. Baulieu and J. Thierry-Mieg, "The principle of BRS symmetry: An alternative approach to Yang-Mills theories", *Nucl. Phys.* **B197** (1982) 477.

- [22] R. A. Bertlmann, Anomalies in quantum field theory (Clarendon Press, Oxford 1996).
- [23] L. C. Vilar, O. S. Ventura, C. A. Sasaki and S. P. Sorella, "Algebraic characterization of vector supersymmetry in topological field theories", J.Math.Phys. 39 (1998) 848, hep-th/9706133.
- [24] G. Barnich and M. Henneaux, "Consistent couplings between fields with a gauge freedom and deformations of the master equation", *Phys.Lett.* B311 (1993) 123, hep-th/9304057.

G. Barnich, M. Henneaux and R. Tatar, "Consistent interactions between gauge fields and the local BRST cohomology: The Example of Yang-Mills models", *Int.J.Mod.Phys.* **D3** (1994) 139, hep-th/9307155.

R. Stora, "Local gauge groups in quantum field theory: Perturbative gauge theories" (1997), unpublished.

A. W. Aste and G. Scharf, "Non-abelian gauge theories as a consequence of perturbative quantum gauge invariance", *Int.J.Mod.Phys.* A14 (1999) 3421, hep-th/9803011.

[25] C. Becchi, "An introduction to the renormalization of theories with continuous symmetries, to the chiral models and to their anomalies", Lectures given at the 29th International Universitätswochen für Kernphysik (Schladming, March 1990), Genoa University preprint GEF-TH-10/1990.

C. Becchi, "An introduction to BRS symmetry", Lectures given at the ETH Zürich (May 1996), Genoa University preprint GEF-TH-96-10, hep-th/9607181.

[26] L. Baulieu and I. M. Singer, "Topological Yang-Mills symmetry", Nucl. Phys. B(Proc. Suppl.) 5B (1988) 12. S. Ouvry, R. Stora, and P. van Baal, "On the algebraic characterization of Witten's topological Yang-Mills theory", *Phys.Lett.* **B220** (1989) 159.

- [27] A. Brandhuber, O. Moritsch, M. W. de Oliveira, O. Piguet and M. Schweda, "A renormalized supersymmetry in the topological Yang-Mills field theory", *Nucl. Phys.* B431 (1994) 173, hep-th/9407105.
- [28] L. Baulieu and B. Grossman, "Monopoles and topological field theory", *Phys.Lett.* B214 (1988) 223.
- [29] L. Baulieu and S. L. Shatashvili, "Duality from topological symmetry", JHEP 03 (1999) 11, hep-th/9811198.
- [30] F. Gieres, H. Nieder, T. Pisar, L. Popp and M. Schweda, "Interacting six-dimensional topological field theories", *Mod.Phys.Lett.* A15 (2000) 791, hep-th/0003222.
- [31] O. F. Dayi, "A general solution of the BV master equation and BRST field theories," Mod. Phys.Lett. A8 (1993) 2087, hep-th/9305038.
- [32] L. C. Vilar, S. P. Sorella and C. A. Sasaki, "Zero curvature formalism of the four-dimensional Yang-Mills theory in superspace", J.Math.Phys. 40 (1999) 2735, hep-th/9610220.
- [33] P. Pasanen, "Symmetries and observables for BF theories in superspace", J.Phys. A29 (1996) 8123, hep-th/9609108.
- [34] L. Baulieu and E. Rabinovici, "Self-duality and new TQFTs for forms", JHEP 06 (1998) 6, hep-th/9805122.
- [35] M. Carvalho, L. C. Vilar, C. A. Sasaki and S. P. Sorella, "BRS cohomology of zero curvature systems II. The noncomplete ladder case", J.Math.Phys. 37 (1996) 5325, hep-th/9509048.
- [36] A. S. Cattaneo, P. Cotta-Ramusino, F. Fucito, M. Martellini, M. Rinaldi, A. Tanzini and M. Zeni, "Four-dimensional Yang-Mills theory as a deformation of topological BF theory", *Commun.Math.Phys.* **197** (1998) 571, hep-th/9705123.

A. Accardi, A. Belli, M. Martellini and M. Zeni, "Topological Yang-Mills cohomology in pure Yang-Mills theory", *Phys.Lett.* B431 (1998) 127, hep-th/9803029.

 [37] A. Schwarz, "Geometry of Batalin-Vilkovisky quantization", Commun. Math. Phys. 155 (1993) 249, hep-th/9205088.

A. Schwarz, "Symmetry transformations in Batalin-Vilkovisky formalism", *Lett.Math.Phys.* **31** (1994) 299, hep-th/9310124.