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Thermal phase transitions in cosmology

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We review briefly the current status of thermal phase transitions within the Standard Model and its simplest extensions. We start with an update on QCD thermodynamics, then discuss the electroweak phase transition, particularly in supersymmetric extensions of the Standard Model, and end with a few remarks on the cosmological constraints that thermal phase transitions might impose on even higher scale particle physics.

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1 Introduction

As is well known, the microscopic energy scales of quantum mechanics and the macroscopic properties of our present Universe are intimately connected. For instance, the $\mathcal{O}(\text{eV})$ energy scale of atomic physics manifests itself through the existence of the cosmic microwave background radiation, and the $\mathcal{O}(\text{MeV})$ scale of nuclear physics through the primordial origin of light element abundances. The connection might of course extend even much further on: the small fluctuations observed in the microwave background could have been produced during an early period of inflation, which could well be a manifestation of particle physics at, say, the $\mathcal{O}(10^{15} \text{ GeV})$ scale of grand unification.

With this background, it is natural to expect that the main scales of experimentally accessible particle physics should also leave their marks in cosmology. Could not the $\mathcal{O}(\text{GeV})$ scale of QCD, and the $\mathcal{O}(\text{TeV})$ scale of the electroweak (EW) theory, be related to such cosmological remnants as extragalactic magnetic fields, or baryon asymmetry?

Taking a closer look, it turns out that quite non-trivial conditions have to be met for the latter connections to exist. Among the biggest challenges are:

1. Generic QCD and EW interactions are so strong that it is difficult to deviate from thermal equilibrium, which is a necessary requirement for any cosmological remnant to emerge. Denoting by n a particle density, σ a cross section, v an average velocity, $\alpha = g^2/(4\pi)$ the coupling, T the temperature, and $m_{\text{Pl}} \sim 10^{19} \text{ GeV}$ the Planck mass, interactions fall out of equilibrium provided that

$$\tau \equiv \frac{1}{n\sigma v} \sim \frac{1}{\alpha^2 T} \gg t \equiv \frac{m_{\text{Pl}}}{T^2}, \quad (1)$$

which leads to $T > \alpha^2 m_{\text{Pl}} \sim 10^{15} \text{ GeV}$. Thus, they can fall out of equilibrium only at temperatures where anyway the QCD and EW theories merge to a grand unified theory.

There is one major exception to the conclusion just drawn: a theory possessing a *first order phase transition* falls out of equilibrium, even if microscopic interactions are strong enough to be in local thermal equilibrium above and below the transition point.

2. Spatial fluctuations (relevant to remnants such as magnetic fields) are produced only on very small length scales. Indeed, the horizon of the moment when $T \sim 1 \text{ GeV}$ corresponds today to about 1 light year, and that of $T \sim 1 \text{ TeV}$ to about 1 astronomical unit. Fluctuations can effectively only be produced on scales smaller than these, which leads to miniscule numbers with respect to intergalactic distances.

Again, there is one conceivable way of avoiding the problem: magnetohydrodynamic evolution is very non-linear and could potentially transfer energy to large length scales more rapidly than comoving expansion [1].

The subject of this talk is the analysis of the first of the problems mentioned, the existence of first order phase transitions in QCD and in various versions of the EW theory. The actual generation of cosmological remnants has been treated in other talks at this conference [2, 3], and will only be touched upon very briefly here.

2 QCD thermodynamics

The theory of strong interactions, QCD, is expected to undergo a phase transition at a temperature of the order of $\Lambda_{\overline{\text{MS}}}$. The transition is said to be related to deconfinement and chiral symmetry restoration. However, neither of these are rigorous concepts for physical quark masses, and therefore there might just as well be a smooth gradual change of the properties of the system, instead of an actual singularity.

In any case, a smooth change is not what one naively expects. Indeed, counting the “free” degrees of freedom g_* in the “confined” and “deconfined” phases, one finds a considerable change, suggesting perhaps a strong transition:

$$g_*(T < T_c) = (\text{pions}) + \dots = 17.25, \quad (2)$$

$$g_*(T > T_c) = (\text{gluons}) + \frac{7}{8}(\text{light quarks}) + \dots = 51.25. \quad (3)$$

However, in practice, particles are not free but strongly interacting. This leads to the fact that the properties of the QCD phase transition can, on a quantitative level, only be studied systematically with four-dimensional (4d) finite temperature lattice simulations.

It turns out, furthermore, that such lattice simulations are very demanding. The reason is that the precise characteristics of the transition, such as its order, depend strongly on the symmetries of the system, which are in turn determined by the quark masses (for a review, see [4]). But chiral quarks, as light as they are in Nature, are difficult to fit on the lattices available in practice. Thus it is believed that the properties of the system do change at a temperature $T_c \sim 170$ MeV [5], but whether the change is smooth or, in the large volume limit, discontinuous, remains open.

What is known much better is the behaviour of various thermodynamical quantities, such as the pressure, at temperatures above the critical, $T \gtrsim T_c$. Indeed, there the inclusion of quarks [6] does not appear to change the result qualitatively from the idealised case of pure SU(3) [7]. The general pattern, illustrated in Fig. 1, is that the pressure is small at $T \sim T_c$, rises rapidly at $T \sim (1\dots 2)T_c$, and then levels off, approaching the ideal gas limit very slowly, with a deviation of 10...15% for a long while.

This noticeable deviation from ideal gas thermodynamics has some implications. According to the Einstein equations, the temperature in the Universe evolves as

$$\frac{dT}{dt} = -\frac{\sqrt{24\pi}}{m_{\text{Pl}}} \frac{\sqrt{e(T)}}{d \ln s(T)/dT}. \quad (4)$$

Here $s = p'(T)$, $e = T^2(p(T)/T)'$. Using an equation of state as in Fig. 1, the Universe cools slower than a free gas [9], and the sound velocity $v_s^2 = p'(T)/e'(T)$, characterising hydrodynamic fluctuations, dips around the transition point. These facts do result in a non-trivial fluctuation spectrum, even if long-lasting consequences might not remain.

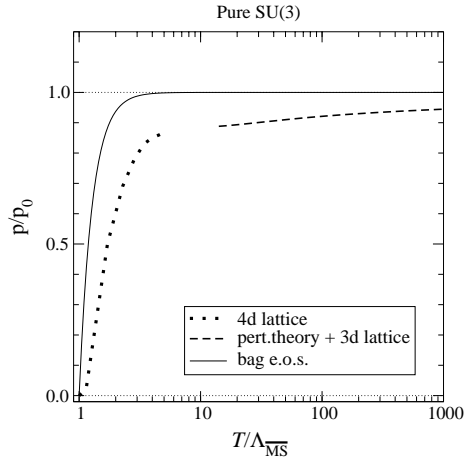


Figure 1: The pressure of pure SU(3) gauge theory, compared with the free Stefan-Boltzmann value $p_0 = (\pi^2/45)(N_c^2 - 1)T^4$, as a function of $T/\Lambda_{\overline{\text{MS}}}$, where $\Lambda_{\overline{\text{MS}}}$ is the scale parameter. The transition takes place at $T \approx \Lambda_{\overline{\text{MS}}}$. The 4d lattice results are from [7], and the curve labelled perturbation theory + 3d lattice, from [8]. The results are compared with the “bag” equation of state, $p_{\text{bag}}(T) \equiv p_0(T) - p_0(T_c)$.

3 EW phase transition

At temperatures of the order of $T_c \sim m_H/g$, where m_H is the Higgs mass and g is the weak coupling constant, the electroweak gauge symmetry gets restored [10]. This may have led to an important consequence, the existence of a matter–antimatter (baryon) asymmetry [11] (for reviews, see [12, 13, 2]). Again, a deviation from equilibrium through a first order phase transition is a necessary requirement. More quantitatively, a 1-loop saddle point computation [14] as well as a non-perturbative evaluation [15] show that the discontinuity in the Higgs expectation value across the phase transition, $\Delta v/T$ (in, say, the Landau gauge) should exceed unity, $\Delta v/T \gtrsim 1.0$.

In contrast to the QCD case, many properties of the EW transition can be addressed in perturbation theory. This is simply because the Higgs mechanism itself is perturbative. Eventually perturbation theory breaks down, though: at finite temperatures the largest loop expansion parameter can be estimated to be [16]

$$\epsilon \sim \frac{g^2 T}{\pi m}, \quad (5)$$

where m is some mass scale. Thus light excitations, $m \lesssim g^2 T$, always present at the phase transition point, lead to an infrared problem [17].

The expansion parameters related to heavy excitations, on the other hand, such as the non-zero Matsubara modes of finite temperature field theory with $m \sim \pi T$, are small. This allows for an essential simplification of the non-perturbative treatment: one can

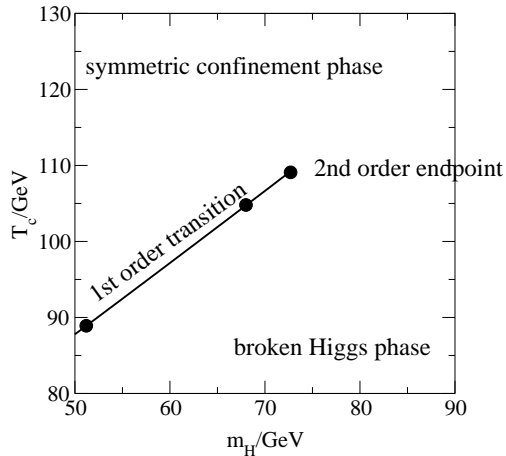


Figure 2: The phase diagram of the physical Standard Model. The blobs indicate simulation points (with error bars) [23, 25], and the solid curve is a fit through them.

integrate out massive modes perturbatively and study only the dynamics of the light modes non-perturbatively [18]. In the first step, the original 4d theory reduces to a three-dimensional (3d) effective one, in a process called dimensional reduction [19]. For studying the EW phase transition in a weakly coupled theory ($m_H \lesssim 250$ GeV), this approach works with a practical accuracy at the per cent level, from the point of view of both dimensional reduction [20]–[22] and numerical simulations [16, 23, 24].

In the case of the Standard Model and many of its extensions [20], the only non-perturbative infrared modes are a Higgs doublet ϕ and the spatial SU(2) and U(1) gauge fields, with field strength tensors F_{ij}, B_{ij} and gauge couplings g_3, g'_3 . The Lagrangian is

$$\mathcal{L}_{3d} = \frac{1}{2} \text{Tr} F_{ij}^2 + \frac{1}{4} B_{ij}^2 + (D_i \phi)^\dagger D_i \phi + m_3^2 \phi^\dagger \phi + \lambda_3 (\phi^\dagger \phi)^2. \quad (6)$$

All knowledge about the physical zero temperature parameters, as well as about the temperature, is encoded in the expressions for the effective couplings $m_3^2, \lambda_3, g_3^2, g_3'^2$.

The properties of the phase transition now depend on the effective couplings in Eq. (6). In particular, since the gauge couplings are fixed and the mass parameter m_3^2 is to be tuned to the phase transition point, such properties are determined by the single dimensionless ratio λ_3/g_3^2 [20]. To get a feeling of this dependence, let us first apply 1-loop perturbation theory. Ignoring the tiny corrections from $g_3'^2/g_3^2$, we find a first order transition with the discontinuity

$$\Delta \frac{v}{gT} = \frac{1}{8\pi} \frac{g_3^2}{\lambda_3} \left[1 + \mathcal{O}\left(\frac{\lambda_3}{g_3^2}\right) \right]. \quad (7)$$

Thus, the transition weakens for large λ_3/g_3^2 . In reality, the line of first order transitions even ends completely, at $\lambda_3/g_3^2 = 0.0983(15)$ [25].

3.1 Standard Model

To be now more specific, let us consider the Standard Model. Then

$$\frac{\lambda_3}{g_3^2} \approx \frac{1}{8} \frac{m_H^2}{m_W^2} + \mathcal{O}\left(\frac{g^2}{(4\pi)^2} \frac{m_{\text{top}}^4}{m_W^4}\right). \quad (8)$$

Applying this (with the actual 1-loop corrections properly included) to the lattice results of [23, 25] leads to Fig. 2. We find that the endpoint location $\lambda_3/g_3^2 = 0.0983(15)$ corresponds in physical units to $m_{H,c} = 72.3(7)$ GeV, $T_c = 109.2(8)$ GeV [25].

As pointed out above, for baryogenesis we would need a first order phase transition, and even a strong one, $\Delta v/T \gtrsim 1$. We have learned that a first order transition only exists for $m_H < 72$ GeV. Furthermore, it has in fact $\Delta v/T \lesssim 1.0$ down to $m_H \sim 10$ GeV [26, 16]! Thus, we observe that experimentally allowed Higgs masses $m_H \gtrsim 115$ GeV [27] are very far from allowing for electroweak baryogenesis.

It appears that primordial magnetic fields present at the time of the EW transition can strengthen the transition quite significantly, but not enough to change the conclusions [28], though they are associated with other intriguing phenomena [29].

3.2 Supersymmetric extensions of the Standard Model

How should the Higgs sector be modified in order to change the pattern above? As the strength of the transition is determined by the scalar self-coupling, cf. Eq. (7), we apparently need some new degree of freedom, which can decrease the effective λ_3 by $\mathcal{O}(100\%)$. It turns out that to get such a large correction, we need a bosonic Matsubara zero mode with an expansion parameter as in Eq. (5). A simple perturbative 1-loop computation shows that at least the sign of such loop corrections is the correct one, negative. But to have an effect of $\mathcal{O}(100\%)$, we need $\epsilon \sim 1$, so that $m \sim g^2 T/\pi$. That is, the degree of freedom should itself be non-perturbative!

In the MSSM, natural candidates for such degrees of freedom are squarks and sleptons. Then there are thermal corrections in their effective mass parameters, appearing as $m_3^2 \sim m_{4d}^2 + g^2 T^2$. Thus, to get a total outcome of order $(g^2 T)^2$, a cancellation must take place, which requires a negative mass parameter m_{4d}^2 . At zero temperature, the physical mass is then roughly $m_{\text{phys}}^2 \lesssim m_{4d}^2 + m_{\text{top}}^2 \lesssim m_{\text{top}}^2$. In order for such a relatively light degree of freedom not to have shown up so far in precision observables, it should be an SU(2) singlet. Since it should also couple strongly to the Higgs, we should choose stops.

Now, the stops come left- and right-handed, \tilde{t}_L, \tilde{t}_R (these states can also mix, but for lack of space we ignore this here). The requirement of having a small violation of the electroweak precision observables means that the weakly interacting left-handed one cannot be light, $m_{\tilde{t}_L} \gg m_{\text{top}}$. This also increases the Higgs mass (see, e.g., [30]),

$$m_H^2 \sim m_Z^2 \cos^2 2\beta + \frac{3g^2}{8\pi^2} \frac{m_{\text{top}}^4}{m_W^2} \ln \frac{m_{\tilde{t}_R} m_{\tilde{t}_L}}{m_{\text{top}}^2}. \quad (9)$$

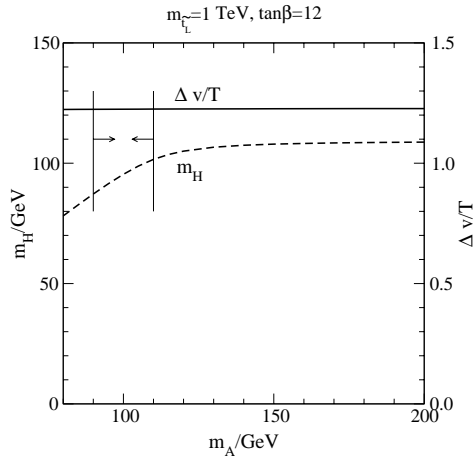


Figure 3: Examples of values (Higgs mass m_H , mass parameter m_A), leading to a large $\Delta v/T$ according to 2-loop perturbation theory, for an optimal choice of $m_{\tilde{t}_R}$ (from [35]). The region indicated with arrows (widening for increasing $m_{\tilde{t}_L}$ [34, 35]) appears to be within experimental constraints, even for the most stringent “no mixing” scenario [27].

On the contrary, the SU(2) singlet stop \tilde{t}_R can be “light”, and serve as the desired new degree of freedom [31]. (Then, of course, we lose half of the correction to m_H in Eq. (9), but this is the price to pay.) It should not be so light that the stop direction gets broken before the phase transition, though, because then one cannot get back to the EW minimum afterwards [32]. Another example of a viable light scalar degree of freedom is the complex gauge singlet of the NMSSM [33].

Under the conditions described above, resummed 2-loop perturbation theory indicates that the EW phase transition can indeed be strong enough for baryogenesis [34]. To check the reliability of perturbation theory, a 3d effective theory (a generalisation of Eq. (6)) has again been constructed and studied with simulations [35], with the encouraging outcome that, in fact, for a light right-handed stop and heavy Higgs, the 2-loop estimates are reliable and even somewhat conservative. This is in strong contrast to the case of the Standard Model at realistic Higgs masses.

Next, we must ask whether this parameter region is indeed in agreement with all experimental data. The most important constraint comes from the lower bound on the Higgs mass [27]. There is a parameter in the MSSM, m_A , which determines whether the Higgs sector resembles that in the Standard Model ($m_A \gtrsim 120$ GeV) or not ($m_A \lesssim 120$ GeV). In the latter case, the experimental lower bound is relaxed [27], and since the transition needs not always get significantly weaker (see Fig. 3), this case is acceptable. If m_A is larger, then the left-handed stop should be quite heavy, $m_{\tilde{t}_L} \gtrsim 2$ TeV, in order to increase the Higgs mass towards the Standard Model value $\gtrsim 115$ GeV (cf. Eq. (9)). Allowing for significant mixing in the stop mass matrix relaxes the Higgs mass bounds quite considerably [27], although it also tends to weaken the transition somewhat [34, 35].

4 Phase transitions at higher scales?

Let us end by considering hypothetical phase transitions at scales even higher than the EW one, possibly related to unification. To discuss this self-consistently, it must be assumed that inflation and unification are not related, but inflation takes place earlier on, maybe at the Planck scale. Then, in clear contrast to the cases considered so far, thermal phase transitions rather generically do tend to produce cosmological remnants, whose non-observation allows to place constraints on possible unification models, or on cosmology. We are here referring to topological defects, such as domain walls, cosmic strings, and monopoles [36]. As an example, let us make some more specific comments on the first ones.

The reason why there is a strong constraint is that the energy density $\delta\rho$ related to domain walls decays much more slowly than radiation. It would lead to $\delta\rho/\rho \gtrsim 10^{-5}$ at photon decoupling, if the domain wall surface energy density is $\sigma \gtrsim (1 \text{ MeV})^3$. This exceeds the fluctuations observed in the cosmic microwave background. Thus *any* new theories leading to thermal domain wall production are excluded [37]!

As timely examples, let us consider models with compact extra dimensions. In case there are gauge fields living in the bulk [38] but no fundamental matter, one generically gets $Z(N_c)$ domain walls [39]. Thus, a class of such models may be constrained by the thermal phase transitions they would undergo in cosmology. It should also be mentioned that the winding of a brane around the extra dimension may lead to various types of cosmic strings and monopoles [40], potentially resulting in other constraints.

5 Conclusions

To summarise, the QCD phase transition is probably very weak, if there at all. However, the system is strongly interacting, and its thermodynamics deviates significantly from that of an ideal gas, which leads to a lengthy period of non-standard expansion.

In EW theories, there is in general no phase transition at all for realistic Higgs masses, unless there is also another light scalar degree of freedom, which plays an essential role in phase transition dynamics. For instance, the transition can be strong enough for baryogenesis in the MSSM if there is one very light ($m_{\tilde{t}_R} \lesssim m_{\text{top}}$) and one rather heavy stop. Either the heavy stop should be really heavy, $m_{\tilde{t}_L} \gtrsim 10 m_{\text{top}}$, or the Higgs sector should contain at least two independent light particles, $m_A \lesssim 120 \text{ GeV}$, in order not to violate experimental constraints. There is much more freedom in the NMSSM.

While little concrete can be said about theories of unification scale physics, one can at least note that the possible overproduction of topological defects in phase transitions can place some constraints on model building and cosmology (on top of many other constraints, of course). A classic example is the monopole problem, but one can also arrive at a domain wall problem in some models with extra dimensions. Such problems can be avoided if the reheating temperature after inflation is below the unification scale.

Finally, let us stress that the topic of this talk has been phase transitions taking place in a system in local thermodynamical equilibrium. In case of low-scale inflation ending at a reheating temperature $T \sim \mathcal{O}(\text{TeV})$, it is also natural to consider non-thermal phase transitions (see, e.g., [41]), which lead to many new physics possibilities.

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