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Lepton Electric Dipole Moments in Non-Degenerate Supersymmetric Seesaw Models

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Abstract

In the context of supersymmetric seesaw models of neutrino masses with non-degenerate heavy neutrinos, we show that Dirac Yukawa interactions $N_i^c(Y_\nu)_{ij}L_jH_2$ induce large threshold corrections to the slepton soft masses via renormalization. While still yielding rates for lepton-flavour-violating processes below the experimental bounds, these contributions may increase the muon and electron electric dipole moments d_μ and d_e by several orders of magnitude. In the leading logarithmic approximation, this is due to three additional physical phases in Y_ν, one of which also contributes to leptogenesis. The naive relation $d_\mu/d_e \approx -m_\mu/m_e$ is violated strongly in the case of successful phenomenological textures for Y_ν, and the values of d_μ and/or d_e may be within the range of interest for the future experiments.

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Understanding particle masses and mixings, is currently one of the most important issues in high-energy physics, together with the associated phenomenology. On the experimental side, this is manifested by many ongoing and planned neutrino oscillation experiments and searches for lepton flavour violation (LFV), as well as by the continuous improvement in the measurements of mixing parameters in the quark sector, most recently of CP violation at the B factories. On the theory side, the importance is manifested by enormous efforts in the interpretation of these measurements using various ideas beyond the Standard Model. The smallness of neutrino masses is commonly explained by the seesaw mechanism [1], which introduces heavy singlet (right-handed) neutrinos N_i with masses somewhat below the unification scale. Requiring the corresponding hierarchy of mass scales to be natural motivates supersymmetrizing the theory to cancel the quadratic divergences in the Higgs boson masses. The absence of large supersymmetric contributions to flavour- and CP-violating processes [2] suggests that the soft supersymmetry-breaking terms are real and universal at the unification scale. In this case, the flavour and CP violation in the slepton sector are entirely induced by the renormalization effects of neutrino coupling parameters [3].

For simplicity, in previous studies of renormalization-induced LFV and CP violation in the lepton sector, the heavy singlet neutrino masses have generally not been distinguished, and hence they have been integrated out at the same scale. In this case, observable rates for LFV processes such as $\mu \to e\gamma$, μ-e conversion in nuclei, $\mu \to eee$ and $\tau \to 3\ell$ [4, 5, 6] can be generated in the minimal supersymmetric seesaw model, but the electric dipole moments (EDMs) of charged leptons are many orders of magnitude below the sensitivities of future experiments [7]. The latter suppression is due to the fact that, when the heavy singlet neutrinos are degenerate, only one of the physical CP phases in the the minimal supersymmetric seesaw model contributes to EDMs, and the CP violation in the lepton sector is entirely connected to the flavour violation, just as in the quark sector in the Standard Model.

However, there is no reason to believe that the heavy neutrino masses M_{N_i} are exactly degenerate. Indeed, the known hierarchies in the quark and lepton masses suggest the opposite. Moreover, models of Yukawa textures [8] that successfully explain the quark, charged lepton and light neutrino parameters in fact predict very hierarchical right-handed neutrino masses [9, 10, 11]. In addition, leptogenesis [12], the only known manifestation of leptonic CP violation so far, requires non-degenerate singlet heavy neutrino masses, and very hierarchical M_{N_i} are preferred in specific models such as a supersymmetric $SO(10)$ grand unified theory [13].

In this letter we investigate the effects of non-degeneracy of the right-handed neutrinos on the LFV and CP-violating observables in the minimal supersymmetric seesaw model. We show that, if the heavy neutrinos are integrated out at different scales, the threshold corrections to the soft slepton mass terms proportional to $\ln(M_{N_i}/M_N)$, where M_N is a geometric mean of M_{N_i}, may strongly affect charged-lepton EDMs, $Br(\mu \to e\gamma)$ and to a lesser extent also $Br(\tau \to \mu\gamma)$. In the leading logarithmic approximation, these non-degeneracy contributions introduce into the EDMs dependences on three additional phases, one of which contributes to leptogenesis, that do not renormalize soft slepton mass terms if the heavy singlet neutrinos are degenerate. The charged-lepton EDMs may be enhanced by *several orders of magnitude* over the degenerate neutrino case, bringing them potentially within the range of interest for foreseeable future experiments [14, 15, 16, 17].

We illustrate our general arguments with specific numerical examples using phenomenologically successful symmetric neutrino mass textures [10, 11]. Taking into account the present bounds on $Br(\mu \to e\gamma)$ and $Br(\tau \to \mu\gamma)$, as well as the recent measurement of $(g_\mu - 2)$ and direct bounds on sparticle masses, the textures considered may yield values of the muon EDM d_μ exceeding $\sim 10^{-26}$ e cm, and the electron EDM d_e may approach the level of $\sim 10^{-31}$ e cm. The naive relation $d_\mu/d_e \approx -m_\mu/m_e$, which holds well in the degenerate heavy-neutrino case, is badly violated for non-degenerate heavy neutrinos. Since the PRISM experiment [15] and neutrino factory stopped-muon experiments [16] aim at a sensitivity $d_\mu \sim 10^{-26}$ e cm, and a newly proposed technology might allow a sensitivity to $d_e \sim 10^{-33}$ e cm [17], our results suggest that the minimal supersymmetric seesaw model may be testable in these experiments. Combining the lepton EDM measurements with the CP- and T-violating observables [18] in neutrino oscillations using superbeams or in neutrino factory long-baseline experiments, with the possible measurement of T-odd asymmetries in $\mu \to eee$ [19], and with CP violation in slepton oscillations at colliders [20] and in $\beta\beta_{0\nu}$ decays, one may be able to test the leptogenesis mechanism using a programme of low-energy experiments. In addition, if d_μ is large, $Br(\tau \to \mu\gamma)$ must be close to the present experimental bound, making possible its detection in B-factory or LHC experiments.

We start our studies by considering the following leptonic superpotential, which implements the seesaw mechanism in a minimal way:

$$W = N_i^c (Y_\nu)_{ij} L_j H_2 - E_i^c (Y_e)_{ij} L_j H_1 + \frac{1}{2} N^c{}_i \mathcal{M}_{ij} N_j^c + \mu H_2 H_1 \,, \tag{1}$$

where the indices i, j run over three generations and $\mathcal{M}_{ij}$ is the heavy singlet-neutrino mass matrix. One can always work in a basis where the charged leptons and the heavy neutrinos both have real and diagonal mass matrices:

$$(Y_e)_{ij} = Y_{e_i}^D \delta_{ij} \,,\; \mathcal{M}_{ij} = \mathcal{M}_i^D \delta_{ij} = \mathrm{diag}(M_{N_1}, M_{N_2}, M_{N_3}) \,. \tag{2}$$

The matrix Y_ν contains six physical phases [21] and can be parametrised as [19]

$$(Y_\nu)_{ij} = Z^\star_{ik} Y^D_{\nu_k} X^\dagger_{kj}, \tag{3}$$

where X is the analogue of the quark CKM matrix in the lepton sector and has only one physical phase, and $Z = P_1 \overline{Z} P_2$, where $\overline{Z}$ is a CKM-type matrix with three real mixing angles and one physical phase, and $P_{1,2} = \mathrm{diag}(e^{i\theta_{1,3}}, e^{i\theta_{2,4}}, 1)$.

The soft supersymmetry-breaking terms in the leptonic sector are

$$\begin{aligned} -\mathcal{L}_{\rm soft} \;\; = \;\; & \tilde{L}^\dagger_i (m^2_{\tilde{L}})_{ij} \tilde{L}_j + \tilde{E}^c_i (m^2_{\tilde{E}})_{ij} \tilde{E}^{c*}_j + \tilde{N}^c_i (m^2_{\tilde{N}})_{ij} \tilde{N}^{c*}_j \\ & + \Big(\tilde{N}^c_i (A_N)_{ij} \tilde{L}_j H_2 - \tilde{E}^c_i (A_e)_{ij} \tilde{L}_j H_1 + \frac{1}{2} \tilde{N}^c_i (B_N)_{ij} \tilde{N}^{c*}_j \\ & + \frac{1}{2} M_1 \tilde{B}\tilde{B} + \frac{1}{2} M_2 \tilde{W}^a \tilde{W}^a + \frac{1}{2} M_3 \tilde{g}^a \tilde{g}^a + h.c. \Big) . \end{aligned} \tag{4}$$

Motivated by experimental upper limits on supersymmetric contributions to LFV and CP-violating effects, we assume universal boundary conditions at the GUT scale $M_{GUT} \sim 2 \times 10^{16}$ GeV:

$$\begin{gathered} (m^2_{\tilde{E}})_{ij} = (m^2_{\tilde{L}})_{ij} = (m^2_{\tilde{N}})_{ij} = m_0^2 \mathbf{1}, \\ m^2_{H_1} = m^2_{H_2} = m_0 \, , \\ (A_e)_{ij} = A_0 (Y_e)_{ij} \, , (A_\nu)_{ij} = A_0 (Y_\nu)_{ij} \, , \\ M_1 = M_2 = M_3 = m_{1/2} \, . \end{gathered} \tag{5}$$

In this case, renormalization induces sensitivity to the neutrino Yukawa couplings Y_ν in the soft supersymmetry-breaking parameters. These may induce measurable LFV decays and CP observables, such as EDMs, as we demonstrate by studying approximate analytical solutions to the renormalization-group equations (RGE).

If the heavy neutrinos are exactly degenerate with a common mass M_N, the flavour-dependent parts of the induced soft supersymmetry-breaking terms are given in the leading-logarithmic approximation by

$$\begin{aligned} \left(\delta m^2_{\tilde{L}}\right)_{ij} \;\; & \approx \;\; -\frac{1}{8\pi^2} (3 m_0^2 + A_0^2)(Y_\nu^\dagger Y_\nu + Y_e^\dagger Y_e)_{ij} \log \frac{M_{GUT}}{M_N} \; , \\ \left(\delta m^2_{\tilde{E}}\right)_{ij} \;\; & \approx \;\; -\frac{1}{4\pi^2} (3 m_0^2 + A_0^2)(Y_e Y_e^\dagger)_{ji} \log \frac{M_{GUT}}{M_N} \; , \\ (\delta A_e)_{ij} \;\; & \approx \;\; -\frac{1}{8\pi^2} A_0 Y_{e_i} (3 Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu)_{ij} \log \frac{M_{GUT}}{M_N} . \end{aligned} \tag{6}$$

Here, the Yukawa coupling constants are given at M_N, and then Y_e is diagonal. This means that $m^2_{\tilde{E}}$ remains diagonal in this approximation. Below M_N, the heavy neutrinos decouple, and the renormalization-group running is given entirely in terms of the MSSM particles and couplings, and is independent of Y_ν. It is important to notice that the only combination of neutrino Yukawa couplings entering (6) is $Y_\nu^\dagger Y_\nu$. It is straightforward to see from (3) that

$$Y_\nu^\dagger Y_\nu = X({Y_\nu}^D)^2 X^\dagger, \tag{7}$$

and CP violation in the charged LFV processes arises only from the single physical phase in the diagonalizing matrix X. This implies that CP-violating phases are induced only in the off-diagonal elements of $(m^2_{\tilde{L}})_{ij}$ and $(A_e)_{ij}$, and further indicates that lepton-flavour-conserving but CP-violating observables such as the electric dipole moments of charged leptons are naturally suppressed [7, 19, 22] [1]. Exactly as in the Standard Model for quarks, three generations are necessary for physical CP violation.

On the other hand, if the heavy neutrinos are non-degenerate: $M_{N_i} \neq M_{N_j}$, one obtains additional corrections from the RGEs:

$$(\delta m^2_{\tilde{L}})_{ij} \to (\delta m^2_{\tilde{L}})_{ij} + \left(\tilde{\delta} m^2_{\tilde{L}}\right)_{ij}, \tag{8}$$

where

$$\left(\tilde{\delta} m^2_{\tilde{L}}\right)_{ij} \approx -\frac{1}{8\pi^2}(3m_0^2 + A_0^2)(Y_\nu^\dagger L Y_\nu)_{ij} \quad : \quad L \equiv \log\frac{M_N}{M_{N_i}}\delta_{ij} \ . \tag{9}$$

Here M_N should now be interpreted as the geometric mean of the heavy neutrino mass eigenvalues M_{N_i}. The potentially large logarithm $\log\frac{M_N}{M_{N_i}}$ arises from the distinct thresholds of the heavy neutrinos. Its most important effect concerns CP violation. Namely, according to (3), the first term in (9) contains the matrix factor

$$Y^\dagger L Y = X Y^D P_2 \overline{Z}^T L \overline{Z}^* P_2^* Y^D X^\dagger, \tag{10}$$

which induces dependences on the phases in $\overline{Z}$ and P_2. In the three-generation case, there are two independent entries in the traceless diagonal matrix L, so the renormalization induces in principle dependences on two new combinations of these phases, as well as the single phase in $Y_\nu^\dagger Y_\nu$ [2].

What is the physical interpretation of these new phases? At present, our only experimental knowledge of CP violation in the lepton sector comes from the baryon asymmetry of

[1] Similar arguments hold also for the renormalization-induced neutron EDM in supersymmetric models [23].

[2] Moreover, CP violation could in principle now be observable even in a two-generation model.

the Universe, assuming that this originates from leptogenesis. The L asymmetry ε^i in the decay of an individual heavy-neutrino species N_i^c is given in the supersymmetric case [24] by

$$\begin{aligned}\varepsilon^i &= -\frac{1}{8\pi}\sum_l \frac{\mathrm{Im}\Big[\left(Y_\nu Y_\nu{}^\dagger\right)^{li}\left(Y_\nu Y_\nu{}^\dagger\right)^{li}\Big]}{\sum_j |Y_\nu{}^{ij}|^2} \\ &\quad \sqrt{x_l}\Big[\mathrm{Log}(1+1/x_l)+\frac{2}{(x_l-1)}\Big], \end{aligned} \tag{11}$$

where $x_l \equiv (M_{N_l}/M_{N_i})^2$, and both vertex and self-energy type loop diagrams are taken into account. This L asymmetry is converted into the observed baryon asymmetry by sphalerons acting before the electroweak phase transition. It is clear from (11) that the generated asymmetry depends only on the three phases in

$$Y_\nu Y_\nu^\dagger = P_1^\star \overline{Z}^\star (Y_\nu^D)^2 \overline{Z}^T P_1\,, \tag{12}$$

namely the single phase in $\overline{Z}$ and the two phases $\theta_{1,2}$ in P_1. We note that, according to (10), the CP-violating observables at low energies depend on the leptogenesis phase in $\overline{Z}$ and on the two phases in P_2. We have explicitly checked with our numerical programs that the dependence of the EDMs on the phases in P_1 is negligible.

Our main objective in this letter is to show that the muon and the electron EDMs are enhanced if the heavy neutrinos are non-degenerate and depend strongly on the leptogenesis phase as discussed above. We now illustrate this with approximate analytical expressions in a mass-insertion approximation.

Since the EDMs are flavour-diagonal observables and induced by the radiative corrections to the soft supersymmetry-breaking terms, the leading contributions are proportional to $\mathcal{O}(\log^3 \frac{M_{GUT}}{M_N})$ and to $\mathcal{O}(\log^2 \frac{M_{GUT}}{M_N} \log \frac{M_N}{M_{N_i}})$. The former contributions are present even if the heavy neutrinos are degenerate, whilst the latter require non-degenerate heavy neutrinos. In order to evaluate the contribution to the EDMs at $\mathcal{O}(\log^3 \frac{M_{GUT}}{M_N})$, we need the corrections to the soft supersymmetry-breaking terms at $\mathcal{O}(\log^2 \frac{M_{GUT}}{M_N})$, which are

$$\begin{aligned}\left(\delta^{(2)} m_{\tilde L}^2\right)_{ij} &\approx \frac{4}{(4\pi)^2} A_0^2 (3Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu + 3Y_e^\dagger Y_e Y_e^\dagger Y_e + \{Y_e^\dagger Y_e,\ Y_\nu^\dagger Y_\nu\})_{ij} \log^2 \frac{M_{GUT}}{M_N}\ , \\ \left(\delta^{(2)} m_{\tilde E}^2\right)_{ij} &\approx \frac{8}{(4\pi)^2} A_0^2 (3Y_e Y_e^\dagger Y_e Y_e^\dagger + Y_e Y_\nu^\dagger Y_\nu Y_e^\dagger)_{ji} \log^2 \frac{M_{GUT}}{M_N}\ , \\ (\delta^{(2)} A_e)_{ij} &\approx 0. \end{aligned} \tag{13}$$

From these equations and from (6), non-vanishing contributions to EDMs arise from the combinations $\mathrm{Im}[\delta A_e \delta A_e^\dagger \delta A_e]_{ii}$, $\mathrm{Im}[\delta A_e Y_e^\dagger Y_e \delta^{(2)} m_{\tilde L}^2]_{ii}$ of order $\mathcal{O}(\log^3 \frac{M_{GUT}}{M_N})$. They are propotional to Jarlskog invariants

$$J_{\nu_i} = \mathrm{Im}[Y_e Y_\nu^\dagger Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu]_{ii}, \tag{14}$$

which consist of only the Yukawa couplings Y_ν and Y_e, and depend on only one phase in X. On the other hand, if the heavy neutrinos are not degenerate in mass, there are corrections of $\mathcal{O}(\log \frac{M_{GUT}}{M_N} \log \frac{M_N}{M_{N_i}})$ of the form

$$
\begin{aligned}
\left(\tilde{\delta}^{(2)} m_{\tilde{L}}^2\right)_{ij} &\approx \frac{18}{(4\pi)^4}(m_0^2 + A_0^2)\{Y_\nu^\dagger L Y_\nu, Y_\nu^\dagger Y_\nu\} \log \frac{M_{GUT}}{M_N} \ , \\
\left(\tilde{\delta}^{(2)} m_{\tilde{E}}^2\right)_{ij} &\approx 0, \\
\left(\tilde{\delta}^{(2)} A_e\right)_{ij} &\approx \frac{1}{(4\pi)^4} A_0 Y_e (11\{Y_\nu^\dagger L Y_\nu, Y_\nu^\dagger Y_\nu\} + 7[Y_\nu^\dagger L Y_\nu, Y_\nu^\dagger Y_\nu])_{ij} \log \frac{M_{GUT}}{M_N} \ , \qquad (15)
\end{aligned}
$$

where we have neglected terms with the $Y_e^\dagger Y_e$ factors. The crucial point is that the second term in $\tilde{\delta}^{(2)} A_e$ can have imaginary parts in its diagonal components, and thus can contribute to the electric dipole moment [3]. Therefore, the explicit comparison of (13) and (15) leads us to the following conclusions:

(i) While the induced lepton EDMs depend on the single physical phase in $Y_\nu^\dagger Y_\nu$ if the heavy neutrinos are degenerate, for the non-degenerate case the EDMs depend on three combinations of phases in $Y_\nu^\dagger Y_\nu$ and $\overline{Z} P_2$, including the leptogenesis phase. In the latter case CP violation can occur even if there are two generations of particles.

(ii) The EDMs depend very strongly on the non-degeneracy of heavy neutrinos. In particular, a step-function-like enhancement of (some) EDMs is expected when going from the degenerate to the non-degenerate case. This is associated with the disappearance of the suppression of CP invariants by the light generations.

(iii) The EDMs depend strongly on A_0.

With these observations in mind, we proceed to exact numerical calculations.

In SUSY models the LFV decays $l_i \to l_j \gamma$ and the lepton EDMs are both generated by one-loop Feynman diagrams with neutralinos/charged sleptons and charginos/sneutrinos running in the loop. For the LFV decays, the effective Lagrangian reads

$$
\mathcal{L} \ = \ -\frac{e}{2}\left\{ m_i A_L \bar{l}_{jR} \sigma^{\mu\nu} l_{iL} F_{\mu\nu} + m_i A_R \bar{l}_{jL} \sigma^{\mu\nu} l_{iR} F_{\mu\nu} \right\}, \qquad (16)
$$

and the decay rates are given by

$$
\Gamma(l_i \to l_j \gamma) = \frac{e^2}{16\pi} m_i^5 \left(|A_L|^2 + |A_R|^2\right) . \qquad (17)
$$

[3] Whilst the combination $(\tilde{\delta} A_e \delta m_{\tilde{L}}^2)_{ii}$ has an imaginary part, it does not contribute to the electric dipole moments, since $\mathrm{Im}[(\delta A_e + \tilde{\delta} A_e)(\delta m_{\tilde{L}}^2 + \tilde{\delta} m_{\tilde{L}}^2)]_{ii} = 0$.

The explicit expressions for $A_{L,R}$ in terms of the supersymmetric charged- and neutral-current parameters can be found in [5, 6], and we do not present them here. Similarly, the electric dipole moment of a lepton l is defined as the coefficient d_l of the effective interaction

$$\mathcal{L} = -\frac{i}{2} d_l \, \bar{l} \, \sigma_{\mu\nu} \gamma_5 \, l \, F^{\mu\nu}, \tag{18}$$

and can be expressed as

$$d_l = d_l^{\chi^+} + d_l^{\chi^0} \,, \tag{19}$$

where [25, 22]

$$d_l^{\chi^+} = -\frac{e}{(4\pi)^2} \sum_{A=1}^{2} \sum_{X=1}^{3} \mathrm{Im}(C^L_{lAX} C^{R*}_{lAX}) \, \frac{m_{\chi_A^+}}{m^2_{\tilde{\nu}_X}} \mathrm{A}\Big(\frac{m^2_{\chi_A^+}}{m^2_{\tilde{\nu}_X}}\Big) \,, \tag{20}$$

$$d_l^{\chi^0} = -\frac{e}{(4\pi)^2} \sum_{A=1}^{4} \sum_{X=1}^{6} \mathrm{Im}(N^L_{lAX} N^{R*}_{lAX}) \frac{m_{\chi_A^0}}{M^2_{\tilde{l}_X}} \, \mathrm{B}\Big(\frac{m^2_{\chi_A^0}}{M^2_{\tilde{l}_X}}\Big) \,, \tag{21}$$

and the loop functions are given by

$$A(x) = \frac{1}{2(1-x)^2}\Big(3 - x + \frac{2\log x}{1-x}\Big) \,,$$
$$B(x) = \frac{1}{2(1-x)^2}\Big(1 + x + \frac{2r\log x}{1-x}\Big) \,.$$

Here the relevant chargino and neutralino couplings $C^{L,R}$ and $N^{L,R}$ can be found in [5, 6].

However, in the present scenario where the CP-violating phases are generated only in the off-diagonal elements of the slepton mass matrix, it may turn out that (20),(21) are not useful in numerical calculations. Because of finite computer accuracy, we find that orders of magnitude numerical errors in evaluating the EDMs may occur if the off-diagonal CP phases are small. Therefore we present here the expressions for EDMs in the mass-insertion approximation. These allow precise and reliable calculation of EDMs even for very small CP-violating effects.

When the CP-violating phases are attributed to the off-diagonal components in the slepton mass matrix, the Bino-like neutralino diagrams are the dominant ones. Mass insertions to order n then yield the formula:

$$\begin{aligned}\frac{d_l}{e} = \; & \frac{g_Y^2}{(4\pi)^2} \sum_A O_{A1} M_{\tilde{\chi}_A} (O_{A1} + O_{A1}/\tan\theta_W) \\ & \sum_n \sum_{l_1, \cdots l_{n-1}} \mathrm{Im}\left[\Delta(m^2_{\tilde{l}})_{l',l_1} \Delta(m^2_{\tilde{l}})_{l_1 l_2} \cdots \Delta(m^2_{\tilde{l}})_{l_{n-1} l}\right] \\ & f_n((m^2_{\tilde{l}})_{l'l'}, (m^2_{\tilde{l}})_{l_1 l_1}, \cdots, (m^2_{\tilde{l}})_{l_{n-1} l_{n-1}}, (m^2_{\tilde{l}})_{ll}; |M_{\tilde{\chi}_A}|^2) \,,\end{aligned} \tag{22}$$

where O is the neutralino mixing matrix, $l' = l + 3$, $l_1, ..., l_{n-1} = 1, ..., 6$, $A = 1, ..., 4$ and $\Delta(m^2_{\tilde{l}})_{l_1,l_2} = (m^2_{\tilde{l}})_{l_1,l_2}(i \neq j)$. The mass functions f_n are given by the finite differences

$$f_n(m_1^2, \cdots, m_{n+1}^2; |M|^2) = \frac{1}{m_1^2 - m_2^2} \Big(f_{n-1}(m_1^2, m_3^2, \cdots, m_{n+1}^2; |M|^2) - f_{n-1}(m_2^2, m_3^2, \cdots, m_{n+1}^2; |M|^2) \Big) , \quad (23)$$

where the zeroth function is $f_0(m^2; |M|^2) = 1/|M|^2 f(m^2/|M|^2)$ and

$$f(x) = \frac{1}{2(1-x)^3} \Big(1 - x^2 + 2x \log x \Big) . \quad (24)$$

For a precise evaluation of EDMs even in a case of degenerate hearvy neutrinos, sixth-order terms are needed, since they yield $\mathrm{Im}[A_e m^2_{\tilde{L}} A_e^\dagger A_e m^2_{\tilde{L}} m^2_{\tilde{L}}]$. Comparison with the exact calculation also shows that this order is also sufficient for an accurate result.

The current experimental bounds on the LFV decays are $Br(\mu \to e\gamma) \lesssim 1.2 \times 10^{-11}$ [26] and $Br(\tau \to \mu\gamma) \lesssim 1.1 \times 10^{-6}$ [27]. An experiment with the sensitivity $Br(\mu \to e\gamma) \sim 10^{-14}$ is proposed at PSI [28] and the stopped-muon experiments at neutrino factories will reach $Br(\mu \to e\gamma) \sim 10^{-15}$ [16]. The bound on $Br(\tau \to \mu\gamma)$ will be improved at LHC and B-factories by an order of magnitude.

For EDMs the bounds are $d_e < 4.3 \times 10^{-27}$ e cm for the electron [29], $d_\mu = (3.7 \pm 3.4) \times 10^{-19}$ e cm for the muon [30], and $|d_\tau| < 3.1 \times 10^{-16}$ e cm for the τ [31]. An experiment has been proposed at BNL that could improve the sensitivity to d_μ down to $d_\mu \sim 10^{-24}$ e cm [14], and PRISM [15] and neutrino factory [16] experiments aim at sensitivities $d_\mu \sim 5 \times 10^{-26}$ e cm. Recently it has been proposed [17] that using new technology one could improve the upper bound on the electron EDM by six orders of magnitude and reach $d_e \sim 10^{-33}$ e cm. We shall show that these expected sensitivities will allow one to test the minimal supersymmetric seesaw model.

Our calculational procedure is to fix the gauge and the quark and charged-lepton Yukawa couplings at M_Z and run them to the scale M_{N_1} using the one-loop MSSM RGEs. Above M_{N_1} we use the RGE-s for the SUSY seesaw model [4] and integrate–in the heavy neutrinos each at its own mass scale. At the GUT scale we choose universal boundary conditions for the soft masses and run all the parameters down again, integrating each N_i out at its own mass scale. In this way we get at the M_Z scale the 3×3 soft mass matrices which include the heavy neutrino threshold effects. These are used to calculate the LFV processes and the EDMs.

[4] Notice that in addition to notational differences in the superpotential and soft mass terms, also the sign of the Yukawa Lagrangian varies in the literature. This affects also the RGEs, as may be seen by comparing [5] with [32]. The plots here are made in the convention of [32].

In order to implement this procedure, we have to specify the neutrino Yukawa matrix Y_ν which is the only source of LFV and CP violation in our model. We follow recent works on the Yukawa textures [8, 10, 11], and choose a generic form of Y_ν of the form

$$Y_\nu = Y_0 \begin{pmatrix} 0 & c\,\varepsilon_\nu^3 & d\,\varepsilon_\nu^3 \\ c\,\varepsilon_\nu^3 & a\,\varepsilon_\nu^2 & b\,\varepsilon_\nu^2 \\ d\,\varepsilon_\nu^3 & b\,\varepsilon_\nu^2 & e^{i\psi} \end{pmatrix} . \tag{25}$$

Here ε_ν is a hierarchy parameter which determines the flavour mixings, a, b, c and d are complex numbers, and Y_0 is an overall scale factor. These textures are constructed to satisfy the measured neutrino mass and mixing parameters via the seesaw mechanism, and in different models the parameters in (25) may have different values. For example, in the SO(10) GUT-motivated model [11] $d = 0$ has been taken for simplicity, while the model [10] with SU(3) family symmetry predicts $a = b$ and $d = c$. The parameter ε_ν is in principle a free parameter to be fixed from neutrino phenomenology. These models predict also very hierarchical heavy neutrinos. Following [11], we fix their hierarchy to be

$$M_{N_1} \ : \ M_{N_2} \ : \ M_{N_3} \ = \ \varepsilon_N^6 \ : \ \varepsilon_N^4 \ : \ 1 \, , \tag{26}$$

but we treat ε_N as an independent phenomenological parameter. This allows us roughly to cover the phenomenology of different models, as we discuss below.

We emphasise that the details of the effective light-neutrino parameters (such as neutrino mixing angles) depend in general strongly on the details of the texture (25), as well as on the form of the right-handed neutrino mass matrix. However, the renormalization-induced observables are insensitive to these details, being smooth functions of the parameters in (25) and the heavy neutrino mass eigenvalues only. This is because the seesaw mechanism $m_\nu \sim Y_\nu^T \mathcal{M}^{-1} Y_\nu$ and the equations (13), (15) depend on the neutrino parameters in completely different ways. Therefore our phenomenological analyses of LFV and EDMs is only weakly dependent on the exact effective neutrino mixing parameters and is quite general for this type of texture models.

We also note that, because the textures are very hierarchical, the renormalization effects do not spoil their generic structure. However, if fine tunings of the parameters in (25) are required in order to achieve the correct light neutrino mixing pattern at low energies, these fine tunings may not survive the renormalization. Again, our results on the LFV and EDMs are independent of these fine tunings and are therefore general.

We first choose $Y_0 = 1$, $a = 1e^{-i\pi/3}$, $b = 1e^{i0.3\pi}$, $c = d = 1e^{i\pi/2}$ and $\psi = \pi/2$ in (25). Since (25) predicts hierarchical neutrinos, we must have [5] $\varepsilon_\nu \sim \sqrt{\Delta m^2_{sol}/\Delta m^2_{atm}}$. For the

[5] Strictly speaking, this is true if ε_ν determines the hierarchy in the effective low-energy neutrino masses,

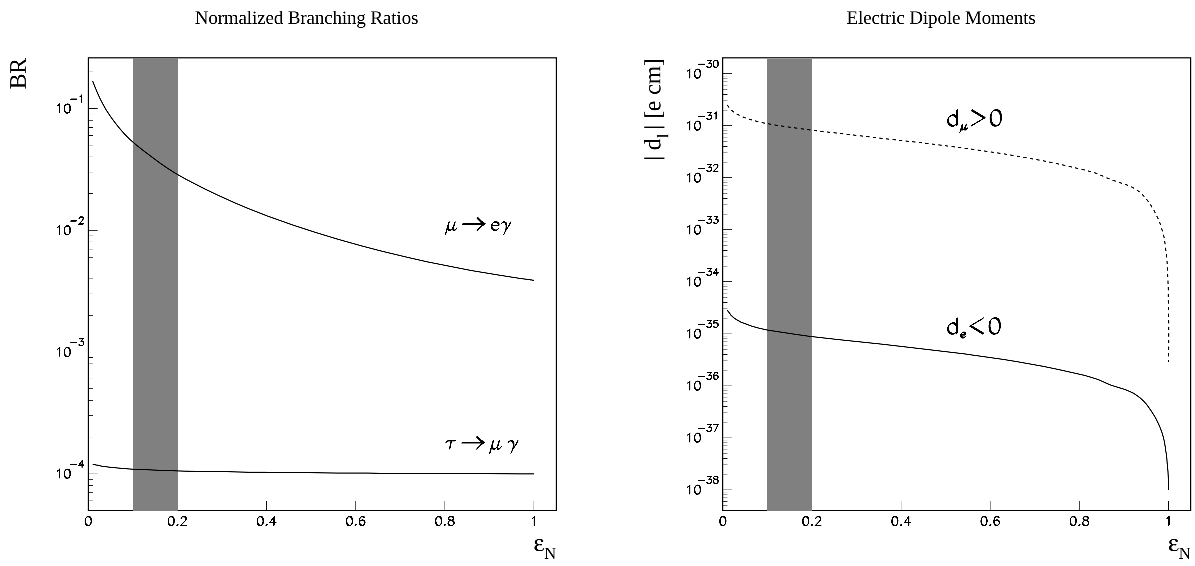

Figure 1: *Branching ratios for $\mu \to e\gamma$ and $\tau \to \mu\gamma$ normalized to their present experimental bounds, and the electron and muon electric dipole moments as functions of the heavy neutrino non-degeneracy parameter ε_N. Texture parameter values are given in the text, and the favoured range of ε_N is indicated by a vertical shaded band.*

currently favoured large-mixing-angle solution to the solar-neutrino problem, the present oscillation data gives at 95% C.L. the range $\Delta m^2_{sol} = (2 \times 10^{-5} - 5 \times 10^{-4})$ eV^2, while for the atmospheric neutrinos one has $\Delta m^2_{atm} = (1.4 \times 10^{-3} - 6 \times 10^{-3})$ eV^2, implying phenomenologically $0.6 \gtrsim \varepsilon_\nu \gtrsim 0.06$. In our numerical examples we fix $\varepsilon_\nu = 0.2$ as in [9]. The seesaw fixes heavy neutrino masses in terms of $M_{N_3} = 5 \times 10^{14}$ GeV. Taking $\tan\beta = 10$, $m_{1/2} = 300$ GeV, $m_0 = 100$ GeV, $A_0 = 0$ and $sign(\mu) = +1$, we plot in Fig. 1 the branching rations of $\mu \to e\gamma$ and $\tau \to \mu\gamma$ normalized to the present experimental bounds, and the electron and muon EDMs as functions of the heavy neutrino non-degeneracy parameter ε_N. Whilst $Br(\tau \to \mu\gamma)$ depends weakly on ε_N, $Br(\mu \to e\gamma)$ may be increased by more than an order of magnitude for hierarchical heavy neutrinos. The dependence of $Br(\mu \to e\gamma)$ on ε_N is smooth. We note that for this choice of parameters $(g_\mu - 2) = 4.4 \times 10^{-9}$ in a good agreement with the recent measurement [30].

However, the EDMs show very strong dependences on ε_N. For $\varepsilon_N = 1$ we always find that $d_\mu/d_e \approx -m_\mu/m_e$ is satisfied with good accuracy. But, already for very small deviations from unity, the EDMs show very sharp step-function-like increases. These are typically followed

as happens in the models considered.

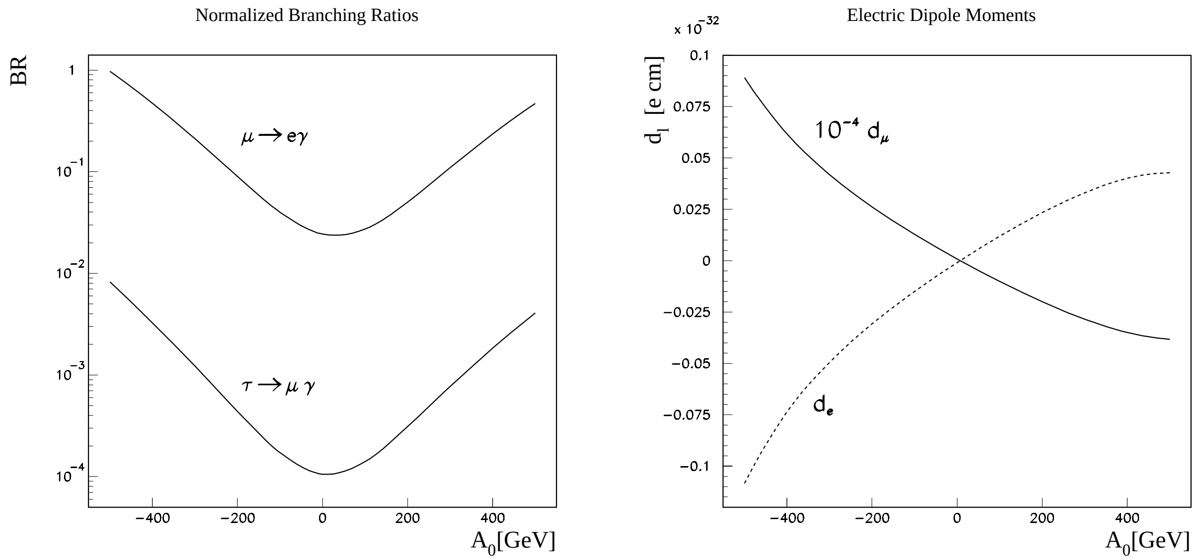

Figure 2: *Normalized branching ratios and EDMs as functions of the trilinear soft mass A_0 for $\varepsilon_N = 0.2$. The rest of the parameters are the same as in Fig. 1.*

by a steady increase as ε_N decreases. We note that the signs of d_e, d_μ and d_μ/d_e are not fixed, and vary depending on the phases in (25). For the given example, d_μ may be enhanced by five and d_e by four orders of magnitude compared to the degenerate heavy neutrino case [6]. Moreover, as noted by the shaded vertical bands in the Fig. 1, the range of ε_N suggested by neutrino mixing phenomenology favours relatively large values of the EDMs.

We emphasize that, in the texture models considered, values of $\varepsilon_N \sim 1$ do not give correct neutrino masses and mixings. Therefore, our discussion of that region just exemplifies the importance of the heavy neutrino non-degeneracy effects on EDMs. The physically meaningful region in Fig. 1 is at small values of ε_N where the EDMs are maximised. In the model [11] the natural value is $\varepsilon_\nu \approx \varepsilon_N \sim 0.2$. However, the model [10] predicts an even stronger heavy neutrino hierarchy: $\varepsilon_N^4 : \varepsilon_N^3 : 1$, with $\varepsilon_N \sim \varepsilon_\nu^2$, whilst the texture for the neutrino Dirac Yukawa matrix is the same. Because the hierarchy between N_3 and N_2 dominates in the EDMs, the appropriate range is $\varepsilon_N \sim 0.1$. The vertical band of shading in Fig. 1 indicates the range $0.1 \lesssim \varepsilon_N \lesssim 0.2$ favoured in our sampling of models.

The dependence of the LFV processes and EDMs on the SUSY soft masses $m_{1/2}$ and m_0 enters mainly through sparticle propagators and is not dramatic. However, as we have shown

[6]If one takes $d = 0$ in (25), the electron EDM is reduced by less than an order of magnitude, and the effect on the other observables is completely negligible.

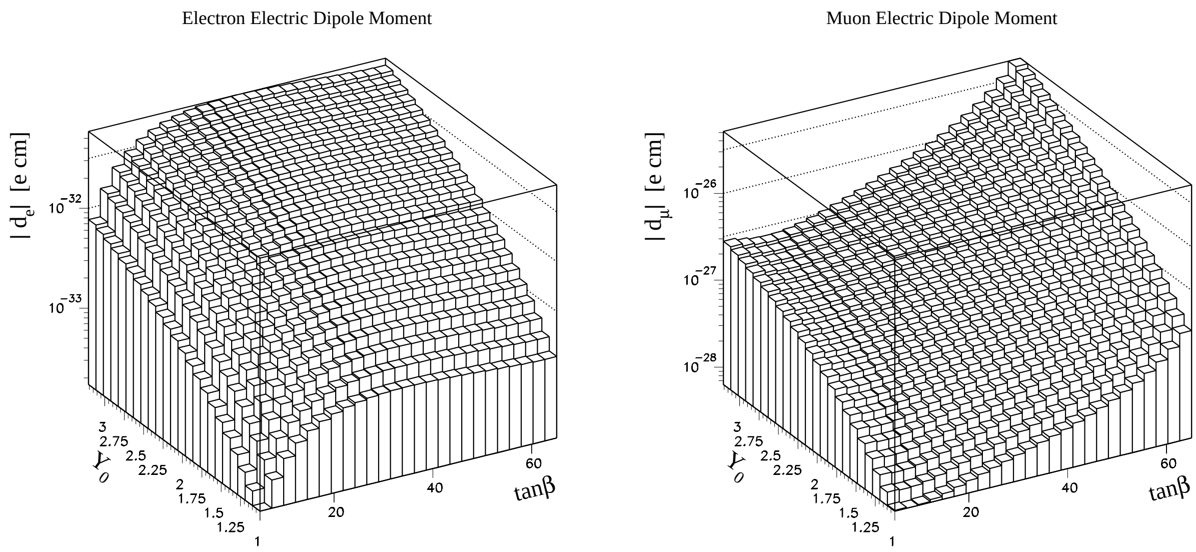

Figure 3: *Absolute values of EDMs as functions of* $\tan\beta$ *and the overall Yukawa scale factor* Y_0. *Other parameter values are given in the text.*

before, the dependence of EDMs on A_0 is non-trivial. This is shown in Fig. 2 where we plot the normalized branching ratios and EDMs as functions of A_0. Notice that d_μ is scaled by 10^{-4} to fit the figure. We have fixed $\varepsilon_N = 0.2$ and chosen all the other parameters as in Fig. 1. Comparison with Fig. 1 shows that, for $A_0 \neq 0$, an additional two orders of magnitude enhancement of the EDMs is possible compared to the $A_0 = 0$ case. Also the signs of the EDMs depend on the sign of A_0.

How large EDMs can one obtain in those texture models? We take $a = 3e^{i0.73\pi}$, $b = 3e^{i\pi/5}$, $c = d = 0.05e^{i\pi/2}$, $\psi = \pi/2$ and plot in Fig. 3 the absolute values of the electron and muon EDMs as functions of $\tan\beta$ and the overall Yukawa scale factor Y_0. For the given choice of phases both d_e and d_μ are negative. The branching ratios of $\mu \to e\gamma$ and $\tau \to \mu\gamma$ are below the present experimental bounds over the whole considered parameter space. The heavy neutrino mass M_{N_3} is calculated from the seesaw mechanism taking the heaviest light neutrino mass to be $m_{\nu_3} = 0.06$ eV, and the neutrino mass hierarchy is fixed by $\varepsilon_N = 0.1$. While increasing $\tan\beta$ from 5 to 65, we also increase the soft mass term m_0 linearly from 100 GeV to 700 GeV, and decrease A_0 from -500 GeV to -700 GeV. The gaugino mass is fixed to be $m_{1/2} = 400$ GeV. Note that, for $sign(\mu) = +1$, the supersymmetric radiative corrections reduce the b-quark Yukawa coupling by $\sim 50\%$ and its running is under control even for such a large $\tan\beta$ as 65.

We see in Fig. 3 that $|d_e|$ and $|d_\mu|$ depend quite strongly on the size of the Yukawa couplings as well as on $\tan\beta$. While the former dependence is trivially expected, the latter is particular to the considered texture (25). Because this texture is hierarchical with small off-diagonal elements, the renormalization-induced EDMs depend on the size of the charged-lepton Yukawa matrix Y_e which is maximized for large $\tan\beta$. For different textures the $\tan\beta$ behaviour might be different.

We point out that for $Y_0 \sim 3$ the largest Yukawa coupling $(Y_\nu)_{33}$ is almost at the fixed point. It is interesting that the Super-Kamiokande measurement of light neutrino masses, the seesaw mechanism and the fixed-point idea for up-type Yukawas are perfectly consistent with each other. Starting with some large value for the Yukawa coupling at M_{GUT}, at the scale $M_{N_3} \sim 5 \times 10^{15}$ GeV where the heaviest singlet neutrino decouples, one always gets $(Y_\nu)_{33} \sim 3$, which implies just the correct effective neutrino mass for Super-Kamiokande. Thus the fixed point idea would suggest large values for EDMs. Therefore, $|d_\mu|$ may exceed the 10^{-26} e cm level, and $|d_e|$ exceeds 10^{-33} e cm for almost the entire parameter space considered. These values for $|d_\mu|$ and $|d_e|$ are in the range of interest for future experiments [15, 16, 17].

We stress that our analyses of EDMs using textures is not phenomenologically general nor exhaustive. It just illustrates what can happen in realistic models. In more general cases (choosing purely phenomenological Y_ν, relaxing the exact universality conditions on the soft supersymmetry-breaking terms, etc.) somewhat different results can be expected. In particular, large EDMs may appear even when $\tan\beta$ is not large. For example, working with the Yukawa texture predicted by the $U(2)$ flavour model [33], large EDMs can be achieved for small values of $\tan\beta$ if some hierarchy between M_{N_3} and M_{N_2} is allowed. This is because this texture is asymmetric with large (33) and (23) elements.

In conclusion: we have shown that, in the minimal supersymmetric seesaw model with non-degenerate heavy neutrino masses, the charged-lepton EDMs may be enhanced by several orders of magnitude compared to the degenerate heavy-neutrino case. This is because, in the leading logarithmic approximation, three additional physical phases in Y_ν renormalize the soft supersymmetry-breaking mass terms. One of these phases contributes also to leptogenesis. We show that, within a plausible class of Yukawa texture models, the charged-lepton EDMs may reach values testable in proposed experiments, in agreement with general analyses [34]. In more general cases, we expect the allowed values of EDMs to increase further. Combining the measurements of EDMs with all possible neutrino and charged-lepton-flavour- and CP-violating measurements would thus allow us to obtain information on the leptogenesis mechanism from low-energy experiments.

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References

[1] M. Gell-Mann, P. Ramond and R. Slansky, Proceedings of the Supergravity Stony Brook Workshop, New York, 1979, eds. P. Van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam); T. Yanagida, Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, Japan 1979 (edited by A. Sawada and A. Sugamoto, KEK Report No. 79-18, Tsukuba); R. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44** (1980) 912.

[2] For a general analyses, see, *e.g.* , F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B **477** (1996) 321.

[3] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57** (1986) 961.

[4] For a review and references see, *e.g.* , Y. Kuno and Y. Okada, Rev. Mod. Phys. **73** (2001) 151.

[5] J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D **53** (1996) 2442; J. Hisano and D. Nomura, Phys. Rev. D **59** (1999) 116005.

[6] Y. Okada, K. Okumura and Y. Shimizu, Phys. Rev. D **58** (1998) 051901; Phys. Rev. D **61** (2000) 094001.

[7] A. Romanino and A. Strumia, hep-ph/0108275; and references therein.

[8] R. G. Roberts, A. Romanino, G. G. Ross and L. Velasco-Sevilla, hep-ph/0104088; and references therein.

[9] S. Lavignac, I. Masina and C. A. Savoy, hep-ph/0106245.

[10] S. F. King and G. G. Ross, hep-ph/0108112.

[11] W. Buchmüller and D. Wyler, hep-ph/0108216.

[12] M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45.

[13] W. Buchmüller and M. Plümacher, Phys. Lett. B **389** (1996) 73; M. Plümacher, Phys. Lett. **530** (1998) 207; W. Buchmüller and T. Yanagida, Phys. Lett. B **445** (1999) 399.

[14] Y. K. Semertzidis *et al.*, hep-ph/0012087.

[15] M. Furusaka *et al.*, JAERI/KEK Joint Project Proposal *The Joint Project for High-Intensity Proton Accelerators*, KEK-REPORT-99-4, JAERI-TECH-99-056.

[16] J. Äystö *et al.*, *Physics with Low-Energy Muons at a Neutrino Factory Complex*, CERN-TH/2001-231, hep-ph/0109217.

[17] S. K. Lamoreaux, nucl-ex/0109014.

[18] For a reference list see, *e.g.* , T. Ota, J. Sato and Y. Kuno, hep-ph/0107007.

[19] J. Ellis, J. Hisano, S. Lola and M. Raidal, hep-ph/0109125.

[20] N. Arkani-Hamed, J. L. Feng, L. J. Hall and H. Cheng, Nucl. Phys. B **505** (1997) 3.

[21] For an early analysis of phase counting, see J. Schechter and J. W. Valle, Phys. Rev. D **22** (1980) 2227; Phys. Rev. D **23** (1981) 1666.

[22] S. Abel, S. Khalil and O. Lebedev, Nucl. Phys. B **606** (2001) 151.

[23] S. Bertolini and F. Vissani, Phys. Lett. B **324** (1994) 164; T. Inui, Y. Mimura, N. Sakai and T. Sasaki, Nucl. Phys. B **449** (1995) 49; S. A. Abel, W. N. Cottingham and I. B. Whittingham, Phys. Lett. B **370** (1996) 106; A. Romanino and A. Strumia, Nucl. Phys. B **490** (1997) 3; C. Hamzaoui, M. Pospelov and R. Roiban, Phys. Rev. D **56** (1997) 4295.

[24] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B **384** (1996) 169; W. Buchmüller and M. Plümacher, Phys. Rept. **320** (1999) 329; M. Flanz, E. A. Paschos, U. Sarkar and J. Weiss, Phys. Lett. B **389** (1996) 693; A. Pilaftsis, Phys. Rev. D **56** (1997) 5431, Int. J. Mod. Phys. A **14** (1999) 1811.

[25] T. Ibrahim and P. Nath, Phys. Rev. D **57** (1998) 478 [Erratum - *ibid.* **58** (1998) 019901].

[26] M. L. Brooks *et al.*, [MEGA Collaboration], Phys. Rev. Lett. **83** (1999) 1521.

[27] S. Ahmed *et al.*, [CLEO Collaboration], Phys. Rev. D **61** (2000) 071101.

[28] L.M. Barkov *et al.*, Research Proposal for experiment at PSI (1999).

[29] E. D. Commins, S. B. Ross, D. DeMille and B. C. Regan, Phys. Rev. A **50** (1994) 2960.

[30] H. N. Brown *et al.* [Muon g-2 Collaboration], Phys. Rev. Lett. **86** (2001) 2227.

[31] M. Acciarri *et al.* [L3 Collaboration], Phys. Lett. B **434** (1998) 169.

[32] S. P. Martin and M. T. Vaughn, Phys. Rev. D **50** (1994) 2282.

[33] R. Barbieri, P. Creminelli and A. Romanino, Nucl. Phys. B **559** (1999) 17.

[34] T. Ibrahim and P. Nath, hep-ph/0105025; J. L. Feng, K. T. Matchev and Y. Shadmi, hep-ph/0107182 and hep-ph/0110157;