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On the Experimental Effects of the Off-shell Structure in Anomalous Neutral Triple Gauge Vertices

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Abstract

We discuss differences between on-shell and off-shell treatments in the search for anomalous neutral triple gauge couplings in $\mathrm{e^+e^-}$ collisions. We find that the usual on-shell framework represents an optimal starting point, covering all scenarios in which a reasonable experimental sensitivity is expected. We show that off-shell effects lead to negligible deviations at the experimental level, provided that $\mathrm{e^+e^-} \rightarrow \mathrm{f\bar{f}}\gamma$ and $\mathrm{e^+e^-} \rightarrow \mathrm{f\bar{f}f'\bar{f}'}$ analyses are performed in regions where $\mathrm{Z^*} \rightarrow \mathrm{f\bar{f}}, \mathrm{f'\bar{f}'}$ production is dominant. For consistency reasons, we advocate the use of a natural extension of the on-shell definitions, which takes into account the correct off-shell dependences.

1 Introduction

The measurement of triple gauge boson couplings is one of the main items in the physics program of present and future colliders [?]. In this context, anomalous neutral triple gauge couplings (NTGC), which are not present in the Standard Model (SM) at tree level, constitute an interesting possibility for New Physics [?]. Tevatron [?, ?] and LEP collider experiments [?, ?, ?, ?] have carried out systematic searches for ZVV couplings, where V denotes any of the two SM neutral gauge bosons (Z or γ).

Recently it has been claimed [?] that off-shell effects in anomalous couplings can not be ignored, and that the spectrum of possible coupling structures may be larger. LEP analyses on the search for anomalous off-shell couplings have followed [?]. The aim of this paper is to clarify the situation in what respects the different NTGC sets and conventions, and the implications of these choices on present experimental limits.

The study is organized as follows. The first section introduces the usual convention employed in the search for anomalous NTGCs. Next we present a general discussion on NTGCs arising at the lowest order in $\sqrt{s}/\Lambda$, where Λ represents the scale of New Physics. A new convention for the NTGC structures will be suggested at this stage. It will be shown that the new convention should lead to no changes in what respects present experimental results [?,?,?,?,?,?]. A different approach will be used in order to build up the off-shell dependences for the remaining (higher order) NTGCs. The study will be completed with a short discussion on the experimental consequences of imposing $SU(2)_L \times U(1)_Y$ SM symmetry constraints. The conclusions are presented in the last section.

2 The standard convention: on-shell anomalous couplings

The usual definition of anomalous NTGCs is obtained from the vertex structures (see Figure 1):

$$
\begin{aligned}
\Gamma^{\alpha\beta\mu}_{\mathrm{Z}\gamma V} = i\, e \frac{q_V^2 - m_V^2}{m_{\mathrm{Z}}^2} \{\, & h_1^V \, (q_\gamma^\mu g^{\alpha\beta} - q_\gamma^\alpha g^{\beta\mu}) \\
& + \; h_2^V \, \frac{q_V^\alpha}{m_{\mathrm{Z}}^2} \, (q_\gamma q_V \; g^{\beta\mu} - q_\gamma^\mu \; q_V^\beta) \\
& + h_3^V \; \epsilon^{\alpha\beta\mu\rho} \; q_{\gamma\rho} \\
& + \; h_4^V \, \frac{q_V^\alpha}{m_{\mathrm{Z}}^2} \, \epsilon^{\mu\beta\rho\sigma} \; q_{V\rho} \, q_{\gamma\sigma} \, \}
\end{aligned}
\tag{1}
$$

for the $\mathrm{e^+e^-} \to \mathrm{Z}\gamma$ case, and:

$$
\begin{aligned}
\Gamma^{\alpha\beta\mu}_{\mathrm{Z_1 Z_2} V} = i\, e \frac{q_V^2 - m_V^2}{m_{\mathrm{Z}}^2} \{\, & f_4^V \, (q_V^\alpha \; g^{\beta\mu} + q_V^\beta \; g^{\mu\alpha}) \\
& + \; f_5^V \; \epsilon^{\alpha\beta\mu\rho} \, (q_{\mathrm{Z}_1\rho} - q_{\mathrm{Z}_2\rho}) \, \}
\end{aligned}
\tag{2}
$$

for the $\mathrm{e^+e^-} \to \mathrm{ZZ}$ case. The momenta of the particles in the vertex are denoted by q_V (ingoing) and $q_{\mathrm{Z}}, q_\gamma, q_{\mathrm{Z}_1}, q_{\mathrm{Z}_2}$ (outgoing). The electromagnetic coupling, $e = \sqrt{4\pi\alpha}$, and the Z mass, m_{Z}, appear as arbitrary constant factors.

The anomalous ZγV couplings h_1^V, h_2^V ($V = \mathrm{Z}, \gamma$) correspond to CP violating terms, whereas h_3^V, h_4^V are related to CP conserving ones. The anomalous ZZV couplings f_4^V lead to CP violating interactions, whereas f_5^V are associated to a CP conserving structure. All terms violate charge conjugation.

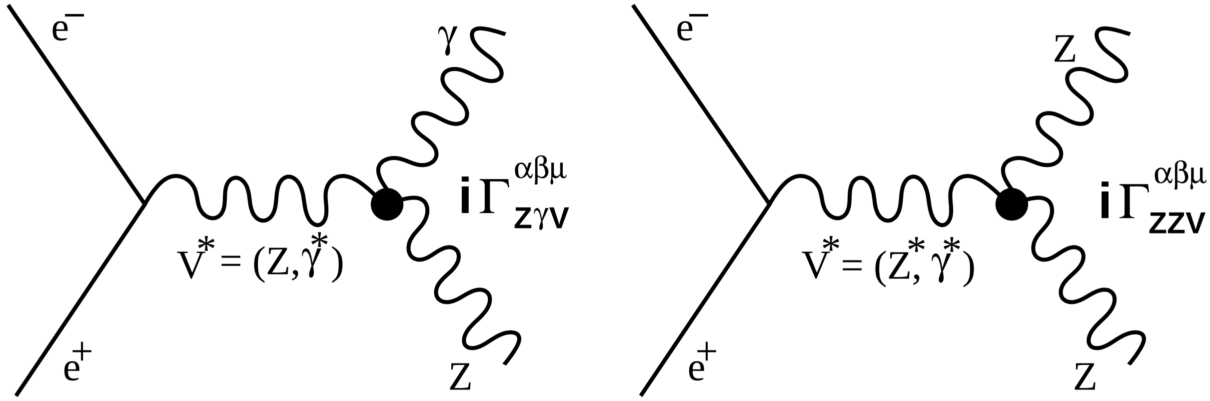

Figure 1: Anomalous vertex structures for $\mathrm{Z}\gamma\mathrm{V}$ (left) and ZZV (right) anomalous couplings.

Both parametrizations were proposed for the first time in Reference **?**. For the $\mathrm{e}^+\mathrm{e}^- \to \mathrm{Z}\gamma$ case, the original proposal had to be modified [**?**] (an extra i factor was included) in order to work with Hermitian Lagrangians for real values of the anomalous couplings.

The previous vertex expressions are the most general ones preserving Lorentz and electromagnetic gauge invariance, assuming that the bosons in the final state are on shell. Let us comment on some features related to the arbitrary factors in the convention:

- The strength of the coupling is assumed to be electromagnetic, but it should be substituted in general by a coupling g, of order one:

$$e \to g\,\sqrt{4\pi}$$

- $h_1^V, h_3^V, f_4^V, f_5^V$ are accompanied by a m_Z^{-2} factor. They correspond to vertices arising from Lagrangians of dimension six or higher. It is convenient to reinterpret them in terms of the new physics scale Λ:

$$\frac{e}{m_\mathrm{Z}^2} \to \frac{g\,\sqrt{4\pi}}{\Lambda^2}$$

- h_2^V, h_4^V are accompanied by a m_Z^{-4} factor, and only appear via Lagrangian terms of dimension eight or higher. Similarly to the previous case, the m_Z^{-4} dimensional factor could be substituted by Λ^{-4}:

$$\frac{e}{m_\mathrm{Z}^4} \to \frac{g\,\sqrt{4\pi}}{\Lambda^4}$$

Since this is just a matter of convention, adopted by all experiments until now, we are not proposing a redefinition in terms of scales of new physics. We just point out that if the sensitivities to h_1^V, h_3^V and h_2^V, h_4^V at center-of-mass energies $\sqrt{s} \gtrsim m_\mathrm{Z}$ turn out to be quite similar this is an artifact of the m_Z^2 factors in the convention. The actual sensitivity to the New Physics scale Λ is reduced in general for the higher dimension terms associated to h_2^V, h_4^V.

3 Off-shell couplings

Reference **?** will be used as the starting point. The paper contains an excellent and comprehensive study of the most general off-shell structures. There it is shown that, at the lowest dimension (six), only the following Lagrangians contain sensible [1)] ZVV vertex information:

$$
\begin{aligned}
\mathcal{O}_1^{\rm ZZZ} &= \tilde{Z}_{\mu\nu}(\partial_\sigma Z^{\sigma\mu})Z^\nu && (3)\\
\mathcal{O}_1^{\rm ZZ\gamma} &= -\tilde{F}_{\mu\nu}Z^\nu(\partial_\sigma Z^{\sigma\mu}) && (4)\\
\mathcal{O}_3^{\rm ZZ\gamma} &= \tilde{Z}^{\mu\nu}Z_\nu(\partial^\sigma F_{\sigma\mu}) && (5)\\
\mathcal{O}_1^{\rm Z\gamma\gamma} &= -\tilde{F}_{\rho\alpha}(\partial_\sigma F^{\sigma\rho})Z^\alpha && (6)\\
\tilde{\mathcal{O}}_1^{\rm ZZZ} &= -Z_\sigma(\partial^\sigma Z^\nu)(\partial_\mu Z^{\mu\nu}) && (7)\\
\tilde{\mathcal{O}}_1^{\rm ZZ\gamma} &= -F^{\mu\beta}Z_\beta(\partial^\sigma Z_{\sigma\mu}) && (8)\\
\tilde{\mathcal{O}}_3^{\rm ZZ\gamma} &= -(\partial_\mu F^{\mu\beta})Z_\alpha(\partial^\alpha Z_\beta) && (9)\\
\tilde{\mathcal{O}}_1^{\rm Z\gamma\gamma} &= -(\partial^\sigma F_{\sigma\mu})Z_\beta F^{\mu\beta} && (10)
\end{aligned}
$$

These terms give rise to anomalous vertices which can be parametrized as follows:

$$
\begin{aligned}
\Gamma^{\alpha\beta\mu}_{\rm ZZZ} &\rightarrow ie\,\frac{f_5^Z}{m_{\rm Z}^2}\left[q_1^2\epsilon^{\alpha\beta\mu\rho}\,(q_{2\rho}-q_{3\rho})+q_2^2\epsilon^{\alpha\beta\mu\rho}\,(q_{3\rho}-q_{1\rho})+q_3^2\epsilon^{\alpha\beta\mu\rho}\,(q_{1\rho}-q_{2\rho})\right] && (11)\\
\Gamma^{\alpha\beta\mu}_{\rm Z\gamma Z} &\rightarrow ie\,\frac{h_3^Z}{m_{\rm Z}^2}\left[(q_3^2-q_1^2)\,\epsilon^{\alpha\beta\mu\rho}\,q_{2\rho}\right] && (12)\\
\Gamma^{\alpha\beta\mu}_{\rm ZZ\gamma} &\rightarrow ie\,\frac{f_5^\gamma}{m_{\rm Z}^2}\left[q_3^2\,\epsilon^{\alpha\beta\mu\rho}\,(q_{1\rho}-q_{2\rho})\right] && (13)\\
\Gamma^{\alpha\beta\mu}_{\rm Z\gamma\gamma} &\rightarrow ie\,\frac{h_3^\gamma}{m_{\rm Z}^2}\left[q_3^2\epsilon^{\alpha\beta\mu\rho}\,q_{2\rho}-q_2^2\epsilon^{\alpha\beta\mu\rho}\,q_{3\rho}\right] && (14)\\
\Gamma^{\alpha\beta\mu}_{\rm ZZZ} &\rightarrow ie\,\frac{f_4^Z}{m_{\rm Z}^2}\left[-q_1^2\,(q_1^\beta g^{\mu\alpha}+q_1^\mu g^{\alpha\beta})-q_2^2\,(q_2^\alpha g^{\beta\mu}+q_2^\mu g^{\alpha\beta})-q_3^2\,(q_3^\alpha g^{\beta\mu}+q_3^\beta g^{\mu\alpha})\right] && (15)\\
\Gamma^{\alpha\beta\mu}_{\rm Z\gamma Z} &\rightarrow ie\,\frac{h_1^Z}{m_{\rm Z}^2}\left[(q_3^2-q_1^2)\,(g^{\alpha\beta}q_2^\mu-g^{\beta\mu}q_2^\alpha)\right] && (16)\\
\Gamma^{\alpha\beta\mu}_{\rm ZZ\gamma} &\rightarrow ie\,\frac{f_4^\gamma}{m_{\rm Z}^2}\left[-q_3^2\,(g^{\beta\mu}q_3^\alpha+g^{\mu\alpha}q_3^\beta)\right] && (17)\\
\Gamma^{\alpha\beta\mu}_{\rm Z\gamma\gamma} &\rightarrow ie\,\frac{h_1^\gamma}{m_{\rm Z}^2}\left[q_2^2\,(q_3^\beta g^{\mu\alpha}-q_3^\alpha g^{\beta\mu})+q_3^2\,(q_2^\mu g^{\alpha\beta}-q_2^\alpha g^{\beta\mu})\right] && (18)
\end{aligned}
$$

where the introduction of the $h_1^V, h_3^V, f_4^V, f_5^V$ parameters will be justified later. The (always outgoing) four-momenta $q_j(j=1,3)$ refer to the particles appearing in the position j of the $V_1V_2V_3$ label. The following index correspondence is assumed: $1\leftrightarrow\alpha$, $2\leftrightarrow\beta$, $3\leftrightarrow\mu$. "Scalar" terms, proportional to q_1^α, q_2^β and q_3^μ, are neglected.

When particles 1 and 2 are assumed to be on-shell bosons, the previous expressions become:

$$
\begin{aligned}
\Gamma^{\alpha\beta\mu}_{\rm ZZZ} &\rightarrow ie\,\frac{f_5^Z}{m_{\rm Z}^2}\left[(q_V^2-m_{\rm Z}^2)\,\epsilon^{\alpha\beta\mu\rho}\,(q_{1\rho}-q_{2\rho})\right] && (19)\\
\Gamma^{\alpha\beta\mu}_{\rm Z\gamma Z} &\rightarrow ie\,\frac{h_3^Z}{m_{\rm Z}^2}\left[(q_V^2-m_{\rm Z}^2)\,\epsilon^{\alpha\beta\mu\rho}\,q_{2\rho}\right] && (20)\\
\Gamma^{\alpha\beta\mu}_{\rm ZZ\gamma} &\rightarrow ie\,\frac{f_5^\gamma}{m_{\rm Z}^2}\left[q_V^2\,\epsilon^{\alpha\beta\mu\rho}\,(q_{1\rho}-q_{2\rho})\right] && (21)
\end{aligned}
$$

[1)] "Scalar" terms are ignored. These terms are only relevant for off-shell decays into very massive fermions, like $\rm Z^*\rightarrow t\bar{t}$.

$$\Gamma^{\alpha\beta\mu}_{Z\gamma\gamma} \rightarrow \quad ie\, \frac{h_3^\gamma}{m_Z^2} \quad \left[q_V^2\, \epsilon^{\alpha\beta\mu\rho}\, q_{2\rho}\right] \tag{22}$$

$$\Gamma^{\alpha\beta\mu}_{ZZZ} \rightarrow \quad ie\, \frac{f_4^Z}{m_Z^2} \quad \left[(q_V^2 - m_Z^2)\,(g^{\beta\mu} q_V^\alpha + g^{\mu\alpha} q_V^\beta)\right] \tag{23}$$

$$\Gamma^{\alpha\beta\mu}_{Z\gamma Z} \rightarrow \quad ie\, \frac{h_1^Z}{m_Z^2} \quad \left[(q_V^2 - m_Z^2)\,(g^{\alpha\beta} q_2^\mu - g^{\beta\mu} q_2^\alpha)\right] \tag{24}$$

$$\Gamma^{\alpha\beta\mu}_{ZZ\gamma} \rightarrow \quad ie\, \frac{f_4^\gamma}{m_Z^2} \quad \left[q_V^2\,(g^{\beta\mu} q_V^\alpha + g^{\mu\alpha} q_V^\beta)\right] \tag{25}$$

$$\Gamma^{\alpha\beta\mu}_{Z\gamma\gamma} \rightarrow \quad ie\, \frac{h_1^\gamma}{m_Z^2} \quad \left[q_V^2\,(g^{\alpha\beta} q_2^\mu - g^{\beta\mu} q_2^\alpha)\right] \tag{26}$$

where $q_V \equiv -q_3$ (ingoing four-momentum).

No new terms are found when the final on-shell particles are assumed to be 1 and 3, or 2 and 3 [2)]. Structures 19-26 exhaust all the on-shell possibilities among neutral gauge bosons. Now the reason for introducing h_1^V, h_3^V, f_4^V and f_5^V becomes evident: all terms lead to the usual convention of equations 1-2 in the on-shell limit. This feature was already noticed in Reference **?**. The correspondence with their notation is the following:

$$l_1^{Z^*Z^*\gamma^*} \equiv \frac{h_3^Z}{m_Z^2}, \qquad l_1^{\gamma^*\gamma^*Z} \equiv \frac{h_3^\gamma}{m_Z^2}, \qquad \tilde{l}_1^{Z^*Z^*\gamma^*} \equiv \frac{h_1^Z}{m_Z^2}, \qquad \tilde{l}_1^{\gamma^*\gamma^*Z} \equiv \frac{h_1^\gamma}{m_Z^2},$$
$$l_1^{Z^*Z^*Z^*} \equiv \frac{f_5^Z}{m_Z^2}, \qquad l_2^{Z^*Z^*\gamma^*} \equiv \frac{f_5^\gamma}{m_Z^2}, \qquad \tilde{l}_1^{Z^*Z^*Z^*} \equiv \frac{f_4^Z}{m_Z^2}, \qquad \tilde{l}_3^{Z^*Z^*\gamma^*} \equiv \frac{f_4^\gamma}{m_Z^2}. \tag{27}$$

As commented before, h_2^V and h_4^V couplings do not appear here because they are associated to Lagrangians of higher dimension. Concerning the most general off-shell vertex structures 11-18, some important comments are necessary:

a) The introduction of the h_j^V and f_j^V couplings in this context implies a redefinition of the convention in present $e^+e^- \rightarrow ZZ$ and $e^+e^- \rightarrow Z\gamma$ analyses. However, the next sections will show that off-shell and on-shell expressions lead to negligible differences at the experimental level.

b) The inclusion of off-shell structures is theoretically well motivated, but it does not imply that experiments should search for anomalous effects in regions with dominant off-shell boson production. The maximal sensitivity is provided by the analysis of $e^+e^- \rightarrow Z^*\gamma \rightarrow f\bar{f}\gamma$ and $e^+e^- \rightarrow Z^*Z^* \rightarrow f\bar{f}f'\bar{f}'$ events in the vicinity of the Z resonances, corresponding to a sensible signal definition of $Z\gamma$ and ZZ final states. There, in addition, "signal" statistics is higher and non-sensitive backgrounds are smaller.

c) The standard $e^+e^- \rightarrow Z\gamma$ and $e^+e^- \rightarrow ZZ$ analyses cover all reasonable types of vertex structures. No additional samples are required in order to complete a search for anomalous effects at the lowest dimension (six). And these terms are guaranteed to be the ones which provide the largest effects from New Physics lying above the center-of-mass energy of the collision: $\Lambda > \sqrt{s}$.

[2)] The surviving terms differ by trivial interchanges of identical bosons indices.

4 On-shell versus off-shell at the experimental level

Comparing equations 11-18 and equations 19-26, the following conclusions are obtained:

- The on-shell and off-shell vertex functions associated to f_5^γ and f_4^γ are identical.
- The on-shell and off-shell vertex functions associated to h_1^γ and h_3^γ coincide in the case of real photon production ($q_2^2 = 0$), i.e. in the relevant case of $\mathrm{e^+e^-} \to \mathrm{Z}^*\gamma$ production.
- The on-shell and off-shell vertex functions associated to h_j^Z, f_j^Z differ by additive terms of order $\dfrac{q_Z^2 - m_Z^2}{q_V^2 - m_Z^2} \approx \dfrac{m_\mathrm{Z}\Gamma_\mathrm{Z}}{s - m_Z^2}$.

Therefore, the only relevant differences between the two set of expressions appear for f_j^Z and h_j^Z. These differences are expected to be small.

In order to quantify the effects of an off-shell treatment on present LEP results [**?**, **?**, **?**, **?**], 100000 $\mathrm{e^+e^-} \to (\mathrm{Z}/\gamma)^*\gamma \to \mathrm{f\bar{f}}\gamma$ and $\mathrm{e^+e^-} \to (\mathrm{Z}/\gamma)^*(\mathrm{Z}/\gamma)^* \to \mathrm{f\bar{f}f'\bar{f}'}$ events at a center-of-mass energy of $\sqrt{s} = 200$ GeV are generated. The values $h_j^Z, f_j^Z = 0.25,\ 0.5,\ 1.0,\ 2.0$ are considered. A more realistic experimental scenario is obtained by selecting events in which the two-fermion invariant masses, $m_\mathrm{f\bar{f}}$, are consistent with the Z mass, $\mid m_\mathrm{f\bar{f}} - m_\mathrm{Z} \mid < 10$ GeV. In addition, a cut on the polar angle of photons, $\mid \cos\theta_\gamma \mid < 0.9$, is applied.

For the h_j^Z case, the study is performed by a reweighting procedure according to the $\mathrm{e^+e^-} \to (\mathrm{Z}/\gamma)^*\gamma \to \mathrm{f\bar{f}}\gamma$ anomalous matrix element, either under off-shell (equations 11-18) or under on-shell [**?**] assumptions.

A first observable sensitive to anomalous couplings is the total cross section. The relative differences between off-shell and on-shell cases are reported in Table 1. These extremely small numbers are somehow expected, since off-shell deviations have similar sizes but different signs above and below the Z mass.

Table 1: Relative difference in the number of expected events, $\Delta N/N$, between off-shell and on-shell analyses at $\sqrt{s} = 200$ GeV. Different values of the h_j^Z anomalous couplings are considered. Cuts on the fermion-pair invariant mass, $\mid m_\mathrm{f\bar{f}} - m_\mathrm{Z} \mid < 10$ GeV, and on the photon polar angle, $\mid \cos\theta_\gamma \mid < 0.9$, are applied.

Coupling value	$\frac{\Delta N}{N}$
$h_1^Z = 0.25$	$(0.9 \pm 0.2)\ 10^{-5}$
$h_1^Z = 0.5$	$(3.1 \pm 0.5)\ 10^{-5}$
$h_1^Z = 1.0$	$(0.8 \pm 0.1)\ 10^{-4}$
$h_1^Z = 2.0$	$(1.4 \pm 0.2)\ 10^{-4}$
$h_3^Z = 0.25$	$(0.3 \pm 0.2)\ 10^{-5}$
$h_3^Z = 0.5$	$(2.1 \pm 0.5)\ 10^{-5}$
$h_3^Z = 1.0$	$(0.7 \pm 0.1)\ 10^{-4}$
$h_3^Z = 2.0$	$(1.3 \pm 0.2)\ 10^{-4}$

Even if the differences in the total rate are negligible, experiments use to combine cross section measurements and shape information in the full phase space. A powerful way to study the effect of the differences in shape is by analyzing the mean values of the optimal observables of the process.

In the general case the differential cross section in the presence of an anomalous coupling h can be expressed as follows:

$$\left.\frac{d^2\sigma}{dO_1\, dO_2}\right|_h = \left.\frac{d^2\sigma}{dO_1\, dO_2}\right|_{h=0} \left(1 + h\, O_1 + h^2\, O_2\right) \tag{28}$$

where the variables O_1 and O_2, also known as *optimal observables*, are functions of the phase space variables of the event, with no explicit dependence on h. The previous equation guarantees that the maximal information on h is obtained by a study of the event density as a function of the variables O_1 and O_2.

For small CP-conserving couplings, $h_3^Z \to 0$, only the O_1 variable contributes. In fact, in the limit of vanishing couplings the maximal sensitivity is obtained by a simultaneous measurement of the total cross section and of the mean value of O_1. For CP-violating couplings like h_1^Z, O_1 is not the relevant variable, since CP-violating and CP-conserving (SM) terms do not interfere [3)]. In this case, O_2 will be considered as the sensitive quantity.

Using the mean values of O_1 and O_2 as inputs, the values for the different couplings are extracted. The difference observed between the measurements of a coupling h using off-shell and on-shell approaches will be denoted by Δh. It quantifies the influence of discrepancies in the shapes of phase space distributions between the two treatments. As observed in Figure 2, the absolute differences at $\sqrt{s} = 200$ GeV never exceed $\mid \Delta h \mid = 0.01$ in the range under study, and are negligible when compared to the present experimental uncertainties [**?**].

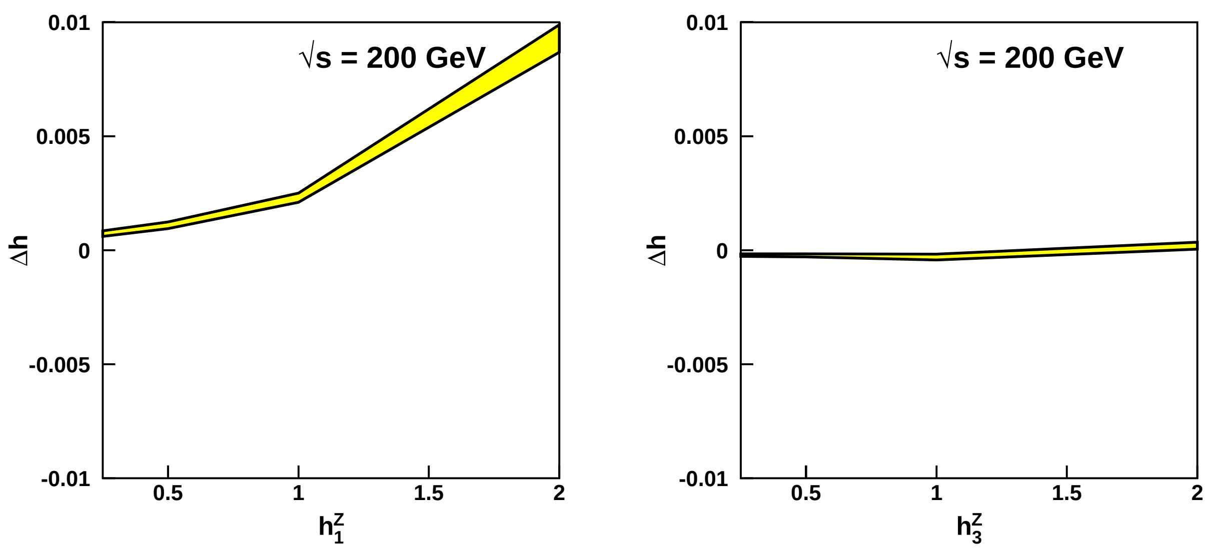

Figure 2: Differences, Δh, between off-shell and on-shell measurements of the anomalous gauge couplings h_1^Z (left) and h_3^Z (right). Measurements are derived from the mean values of the O_2 distribution (for h_1^Z) and of the O_1 distribution (for h_3^Z). The analyzed process is $\mathrm{e^+e^-} \to (\mathrm{Z}/\gamma)^*\gamma \to \mathrm{f\bar{f}}\gamma$ at $\sqrt{s} = 200$ GeV.

[3)] This is strictly true at the same order of perturbative expansion. In practice, some interference remains due to the presence of $im_\mathrm{Z}\Gamma_\mathrm{Z}$ terms in the amplitudes, originating from higher order terms.

For the f_j^Z case, the study is performed by a reweighting procedure according to the $\mathrm{e^+e^-} \rightarrow (\mathrm{Z}/\gamma)^*(\mathrm{Z}/\gamma)^* \rightarrow \mathrm{f\bar{f}f'\bar{f}'}$ anomalous matrix element, either under off-shell (equations 11-18) or on-shell [**?**] assumptions. Again, the relative differences in cross section between the two approaches are extremely small (Table 2). Similarly to the h_j^Z case, the mean values of the optimal observables give access to the values of the f_j^Z couplings. The differences between off-shell and on-shell treatments due to discrepancies in the shape of the phase space distributions are denoted by Δf. Figure 3 shows that the differences never exceed $\mid \Delta f \mid = 0.015$, and are negligible when compared to the present experimental uncertainties [**?**].

Table 2: Relative difference in the number of expected events, $\Delta N/N$, between off-shell and on-shell analyses at $\sqrt{s} = 200$ GeV. Different values of the f_j^Z anomalous couplings are considered. A cut on the relevant fermion-pair invariant masses, $\mid m_{\mathrm{f\bar{f}}} - m_{\mathrm{Z}} \mid < 10$ GeV, is applied.

Coupling value	$\frac{\Delta N}{N}$
$f_4^Z = 0.25$	$(0.6 \pm 0.1)\ 10^{-5}$
$f_4^Z = 0.5$	$(2.1 \pm 0.2)\ 10^{-5}$
$f_4^Z = 1.0$	$(5.8 \pm 0.5)\ 10^{-5}$
$f_4^Z = 2.0$	$(1.1 \pm 0.1)\ 10^{-4}$
$f_5^Z = 0.25$	$(5.4 \pm 0.9)\ 10^{-5}$
$f_5^Z = 0.5$	$(1.7 \pm 0.2)\ 10^{-4}$
$f_5^Z = 1.0$	$(5.3 \pm 0.3)\ 10^{-4}$
$f_5^Z = 2.0$	$(1.5 \pm 0.1)\ 10^{-3}$

In order to investigate the implications for the next generation of linear colliders, all previous exercises are repeated for a center-of-mass energy of $\sqrt{s} = 500$ GeV. Since the sensitivity at these energies is expected to be at least one order of magnitude larger than at $\sqrt{s} = 200$ GeV [**?**], the values $h_j^Z, f_j^Z = 0.025, 0.05, 0.1, 0.2$ are considered. Cross section differences are shown in Table 3, and the shifts due to shape distribution discrepancies are presented in Figures 4-5. It is evident that the differences between off-shell and on-shell treatments are extremely small in all cases.

Finally, we should be concerned about the influence of an off-shell treatment on the h_j^V limits obtained at Tevatron [**?**, **?**]. At $\mathrm{p\bar{p}}$ colliders the requirements of consistency with the Z mass are either loose (CDF) or somehow indirect (D0 and $\mathrm{Z} \rightarrow \nu\bar{\nu}$). In order to estimate the effect of looser cuts, the h_j^Z differences between on-shell and off-shell approaches have been estimated for an invariant mass cut of $\mid m_{\mathrm{f\bar{f}}} - m_{\mathrm{Z}} \mid < 30$ GeV. Similar results ($\mid \Delta h \mid < 0.01$ in all cases) are obtained. Our conclusion is that off-shell effects in $\mathrm{q\bar{q}} \rightarrow \mathrm{Z}\gamma$ should be also negligible.

5 An alternative view. Redefinition of the h_2^V and h_4^V convention

Actually, the problem with the convention in Equations 1-2 can be solved at the "construction" level, just by imposing Bose-Einstein symmetry and electromagnetic gauge invariance as constraints.

Let us first consider the f_5^Z case. What is relevant in the definition is the basic P-violating structure $i\ \epsilon^{\alpha\beta\mu\rho}\ q_{1\rho}$. On it we have to impose Bose-Einstein symmetry for the three Z bosons. It can be seen that a symmetrization of $i\ \epsilon^{\alpha\beta\mu\rho}\ (q_{1\rho} - q_{2\rho})$ leads to a trivial vanishing result. Therefore, one has to multiply it by a momentum-dependent scalar factor (corresponding to a higher dimension term):

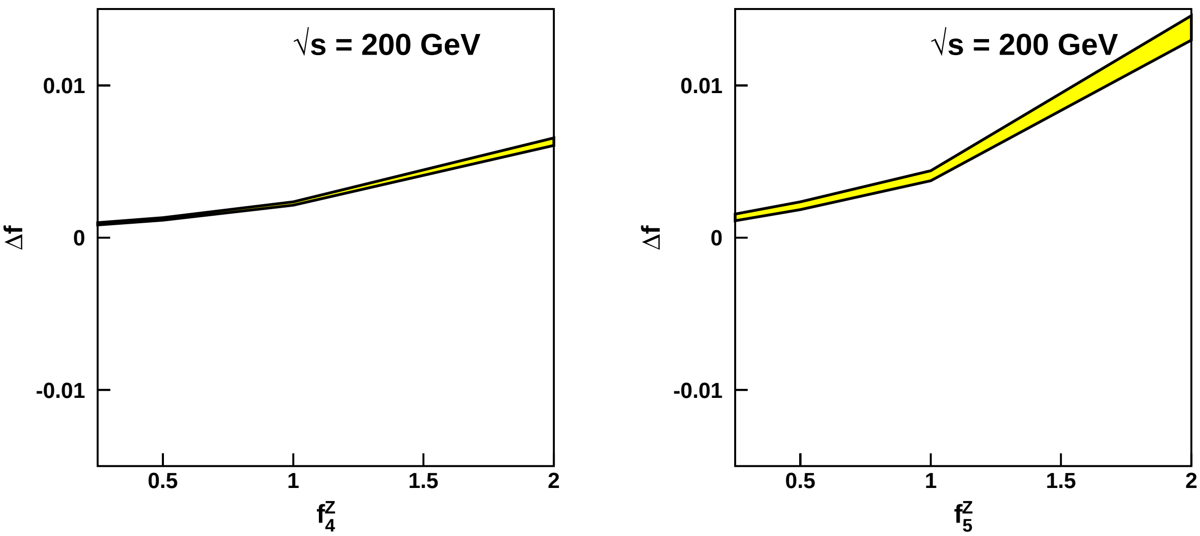

Figure 3: Differences, Δf, between off-shell and on-shell measurements of the anomalous gauge couplings f_4^Z (left) and f_5^Z (right). Measurements are derived from the mean values of the O_2 distribution (for f_4^Z) and of the O_1 distribution (for f_5^Z). The analyzed process is $\mathrm{e^+e^-} \to (\mathrm{Z}/\gamma)^*(\mathrm{Z}/\gamma)^* \to \mathrm{f\bar{f}f'\bar{f}'}$ at $\sqrt{s} = 200$ GeV.

Table 3: Relative difference in the number of expected events, $\Delta N/N$, between off-shell and on-shell analyses at $\sqrt{s} = 500$ GeV. Different values of the h_j^Z and f_j^Z anomalous couplings are considered. Cuts on the fermion-pair invariant mass, $\mid m_{\mathrm{f\bar{f}}} - m_{\mathrm{Z}} \mid < 10$ GeV, and on the photon polar angle, $\mid \cos\theta_\gamma \mid < 0.9$, are applied.

Coupling value	$\frac{\Delta N}{N}$
$h_1^Z = 0.025$	$(0.4 \pm 0.1)\ 10^{-5}$
$h_1^Z = 0.05$	$(0.9 \pm 0.2)\ 10^{-5}$
$h_1^Z = 0.1$	$(1.5 \pm 0.3)\ 10^{-5}$
$h_1^Z = 0.2$	$(1.8 \pm 0.3)\ 10^{-5}$
$h_3^Z = 0.025$	$(0.3 \pm 0.1)\ 10^{-5}$
$h_3^Z = 0.05$	$(0.9 \pm 0.2)\ 10^{-5}$
$h_3^Z = 0.1$	$(1.4 \pm 0.3)\ 10^{-5}$
$h_3^Z = 0.2$	$(1.7 \pm 0.3)\ 10^{-5}$
$f_4^Z = 0.025$	$-(0.8 \pm 0.9)\ 10^{-6}$
$f_4^Z = 0.05$	$-(1.6 \pm 1.6)\ 10^{-6}$
$f_4^Z = 0.1$	$-(2.1 \pm 2.0)\ 10^{-6}$
$f_4^Z = 0.2$	$-(2.3 \pm 2.2)\ 10^{-6}$
$f_5^Z = 0.025$	$(0.7 \pm 0.1)\ 10^{-5}$
$f_5^Z = 0.05$	$(1.5 \pm 0.2)\ 10^{-5}$
$f_5^Z = 0.1$	$(1.9 \pm 0.3)\ 10^{-5}$
$f_5^Z = 0.2$	$(2.0 \pm 0.3)\ 10^{-5}$

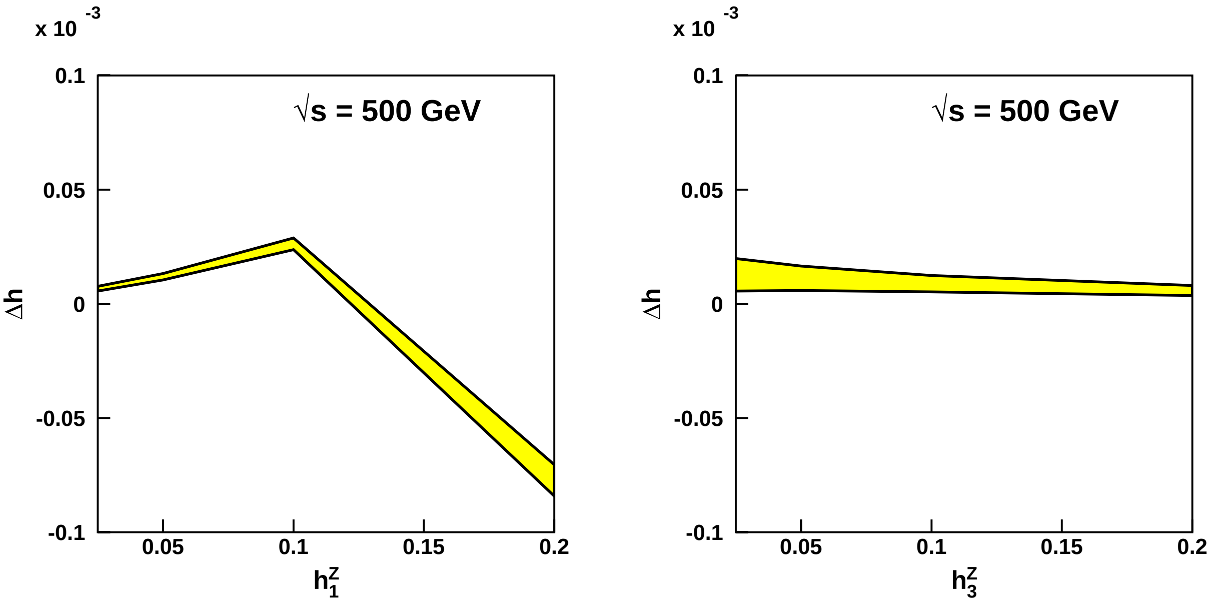

Figure 4: Differences, Δh, between off-shell and on-shell measurements of the anomalous gauge couplings h_1^Z (left) and h_3^Z (right). Measurements are derived from the mean values of the O_2 distribution (for h_1^Z) and of the O_1 distribution (for h_3^Z). The analyzed process is $\mathrm{e^+e^-} \rightarrow (\mathrm{Z}/\gamma)^*\gamma \rightarrow \mathrm{f\bar{f}}\gamma$ at $\sqrt{s} = 500$ GeV.

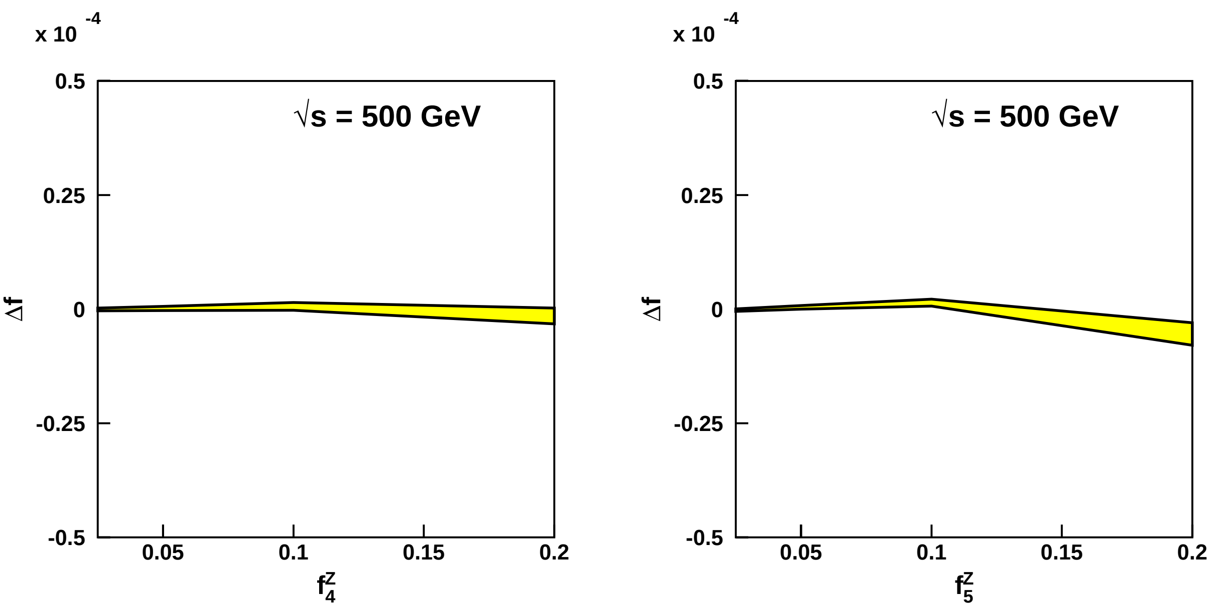

Figure 5: Differences, Δf, between off-shell and on-shell measurements of the anomalous gauge couplings f_4^Z (left) and f_5^Z (right). Measurements are derived from the mean values of the O_2 distribution (for f_4^Z) and of the O_1 distribution (for f_5^Z). The analyzed process is $\mathrm{e^+e^-} \rightarrow (\mathrm{Z}/\gamma)^*(\mathrm{Z}/\gamma)^* \rightarrow \mathrm{f\bar{f}f'\bar{f}'}$ at $\sqrt{s} = 500$ GeV.

$$ie\; f_5^Z\; \epsilon^{\alpha\beta\mu\rho}\; (q_{1\rho} - q_{2\rho}) \;\;\rightarrow\;\; ie\; f_5^Z\; \frac{q_3^2}{m_Z^2}\; \epsilon^{\alpha\beta\mu\rho}\; (q_{1\rho} - q_{2\rho}) \tag{29}$$

It is the symmetrization of this last expression [4)] which leads to the off-shell equation 11. A second example concerns the h_1^γ coupling. In this case, we have to impose not only Bose-Einstein symmetry, but electromagnetic gauge invariance on the P-conserving term $i\,(q_2^\mu g^{\alpha\beta} - q_2^\alpha g^{\beta\mu})$. This last requirement reads: $q_{3\mu}\Gamma^{\alpha\beta\mu}_{Z\gamma\gamma} = q_{2\beta}\Gamma^{\alpha\beta\mu}_{Z\gamma\gamma} = 0$, but, since terms proportional to $q_{2\beta}, q_{3\mu}$ are neglected, the right expressions to use are: $q_{2\beta}\Gamma^{\alpha\beta\mu}_{Z\gamma\gamma} \propto q_2^2$, $q_{3\mu}\Gamma^{\alpha\beta\mu}_{Z\gamma\gamma} \propto q_3^2$. The two constraints are satisfied by the following modification:

$$ie\; h_1^\gamma\; (q_2^\mu g^{\alpha\beta} - q_2^\alpha g^{\beta\mu}) \;\;\rightarrow\;\; ie\; h_1^\gamma\; \frac{q_3^2}{m_Z^2} (q_2^\mu g^{\alpha\beta} - q_2^\alpha g^{\beta\mu}) \tag{30}$$

Again it is the symmetrization of this last expression which leads to the off-shell equation 18.

Let us now discuss the issue of anomalous couplings proceeding via higher dimension Lagrangians. It has been shown in Reference **?** that more off-shell structures, not covered by the on-shell studies of h_2^V and h_4^V on-shell structures, are possible. Our opinion is that the experimental sensitivity to those new terms must be extremely low. Besides the fact that they correspond to effects from terms of higher dimension, they vanish exactly for Z, γ on-shell production, whereas a reasonable rate of off-shell boson production is required in order to perform a sensible measurement.

Therefore, only terms associated in the on-shell limit to h_2^V and h_4^V structures will be considered. Imposing Bose-Einstein symmetry and electromagnetic gauge invariance the following expressions are obtained:

$$\begin{aligned}
\Gamma^{\alpha\beta\mu}_{Z\gamma Z} \rightarrow \quad & ie\, \tfrac{h_2^Z}{m_Z^4} \quad \left[q_3^2\; q_3^\alpha\; (q_2 q_3\; g^{\beta\mu} - q_2^\mu\; q_3^\beta) + q_1^2\; q_1^\mu\; (q_2 q_1\; g^{\alpha\beta} - q_2^\alpha\; q_1^\beta)\right] \\
& + ie\, \tfrac{h_2^Z}{2m_Z^2} \quad \left[(q_3^2 - q_1^2)\; (q_2^\mu g^{\alpha\beta} - q_2^\alpha g^{\beta\mu})\right] && (31) \\
\Gamma^{\alpha\beta\mu}_{Z\gamma\gamma} \rightarrow \quad & ie\, \tfrac{h_2^\gamma}{m_Z^4} \quad \left[(q_3^\alpha q_3^2 + q_2^\alpha q_2^2)\; (q_2 q_3\; g^{\beta\mu} - q_2^\mu\; q_3^\beta)\right] && (32) \\
\Gamma^{\alpha\beta\mu}_{Z\gamma Z} \rightarrow \quad & ie\, \tfrac{h_4^Z}{m_Z^4} \quad \left[q_3^2 q_3^\alpha\; \epsilon^{\mu\beta\rho\sigma}\; q_{3\rho}\; q_{2\sigma} + q_1^2 q_1^\mu\; \epsilon^{\alpha\beta\rho\sigma}\; q_{1\rho}\; q_{2\sigma}\right] \\
& + ie\, \tfrac{h_4^Z}{2m_Z^2} \quad \left[(q_3^2 - q_1^2)\; \epsilon^{\alpha\beta\mu\rho}\; q_{2\rho}\right] && (33) \\
\Gamma^{\alpha\beta\mu}_{Z\gamma\gamma} \rightarrow \quad & ie\, \tfrac{h_4^\gamma}{m_Z^4} \quad \left[(q_3^\alpha q_3^2 - q_2^\alpha q_2^2)\; \epsilon^{\mu\beta\rho\sigma}\; q_{3\rho}\; q_{2\sigma}\right] && (34)
\end{aligned}$$

In the on-shell limit the corresponding structures in Equation 1 are obtained. Let us comment at this point that the original proposal for h_2^Z and h_4^Z couplings [**?**] was ill-defined from the point of view of Bose-Einstein symmetry. Imposing this symmetry on it forces the inclusion of redundant structures of the h_1^Z and h_3^Z type, as it can be easily confirmed by visual inspection of Equations 31 and 33. This particular feature was also observed in Reference **?**. Probably it would have been more sensible to define those couplings with a pure dimension-eight content, without this redundant dimension-six mixing.

[4)] Scalar factors like $(q_1^2 + q_2^2)$ and $(q_1 q_2)$ lead to an equivalent result.

6 $SU(2)_L \times U(1)_Y$ symmetry

So far, we have only considered the case in which h_j^V and f_j^V couplings are studied separately in $\mathrm{e^+e^-} \to (\mathrm{Z}/\gamma)^*\gamma$ and $\mathrm{e^+e^-} \to (\mathrm{Z}/\gamma)^*(\mathrm{Z}/\gamma)^*$ events. However, once off-shell effects are included, combined searches may become a complicated issue. An example is the search for f_j^V couplings in the $\mathrm{e^+e^-} \to (\mathrm{Z}/\gamma)^*(\mathrm{Z}/\gamma)^*$ sample. In this case deviations due to the simultaneous presence of h_j^V couplings (affecting the non-resonant $\mathrm{e^+e^-} \to \mathrm{Z}^*\gamma^*$ component) may arise.

The previous situation could become natural in the future. Given the good agreement between present data with SM predictions, any signal of new physics from a large scale Λ will most probably respect the underlying $SU(2)_L \times U(1)_Y$ symmetry of the SM at the electroweak scale. This condition leads to a physics situation in which only two operators can contribute at the lowest dimension (eight) [**?**]:

$$\mathcal{O}_8 = i\tilde{B}^{\mu\nu}(\partial^\sigma B_{\sigma\mu})(\Phi^+ D_\nu \Phi) \quad (35)$$

$$\tilde{\mathcal{O}}_8 = iB^{\mu\nu}(\partial^\sigma B_{\sigma\mu})(\Phi^+ D_\nu \Phi) \quad (36)$$

where $B^{\mu\nu}$ is the tensor field associated to the $U(1)_Y$ group, Φ is the Higgs field and D denotes the covariant derivative. The first operator, $\mathcal{O}_8$, conserves CP whereas $\tilde{\mathcal{O}}_8$ violates CP. Since $B^\mu = \cos\theta_w A^\mu - \sin\theta_w \mathrm{Z}^\mu$, being θ_w the Weinberg angle and A_μ, Z_μ the photon and Z fields, the net effect of this reduction of possibilities is a set of constraints among all the couplings previously discussed [**?**]:

$$f_5^Z = -f_5^\gamma \tan\theta_w = h_3^Z \tan\theta_w = -h_3^\gamma \tan^2\theta_w \quad (37)$$

$$f_4^Z = -f_4^\gamma \tan\theta_w = h_1^Z \tan\theta_w = -h_1^\gamma \tan^2\theta_w \quad (38)$$

Although tiny effects are expected in regions in which the the matrix element ratio $\left|\frac{M(\mathrm{e^+e^-} \to \mathrm{Z}^*\gamma^*)}{M(\mathrm{e^+e^-} \to \mathrm{Z}^*\mathrm{Z}^*)}\right|$ is small, a full off-shell treatment is advisable in general.

7 Conclusions

We have analyzed the experimental consequences of including a proper off-shell treatment in the searches for anomalous NTGCs. We find that the quantitative differences between on-shell and off-shell treatments are negligible, provided that the $\mathrm{e^+e^-} \to \mathrm{Z}\gamma$ and $\mathrm{e^+e^-} \to \mathrm{ZZ}$ analyses are performed in regions where Z resonant production is dominant. This conclusion is also valid for future $\mathrm{e^+e^-}$ studies at higher energies. Present on-shell studies guarantee a coverage of all physics deviations for which a reasonable experimental sensitivity is expected. Just for theoretical consistency [**?**], and in order to avoid misleading results in off-resonance studies, we advocate the use of the new vertex functions presented in Equations 11-18 and 31-34.