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## D-brane models with non-linear supersymmetry

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### Abstract

We study a class of type I string models with supersymmetry broken on the world-volume of some D-branes and vanishing tree-level potential. Despite the non-supersymmetric spectrum, supersymmetry is non-linearly realized on these D-branes, while it is spontaneously broken in the bulk by Scherk-Schwarz boundary conditions. These models can easily accommodate 3-branes with interesting gauge groups and chiral fermions. We also study the effective field theory and in particular we compute the four-fermion couplings of the localized Goldstino with the matter fermions on the brane.

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# 1 Introduction

One of the most challenging problems in string theory is the mechanism of supersymmetry breaking. A particularly attractive possibility is by the process of compactification, using Scherk-Schwarz boundary conditions [1]. It consists of imposing non-periodic boundary conditions on the higher-dimensional fields using an exact (discrete) R-symmetry of the theory. As a result, the Kaluza-Klein spectrum of different components of the superfields exhibit mass-shifts according to their R-charges and the supersymmetry breaking scale is set by the compactification scale [2, 3]. This was one of the main motivations for large internal dimensions in string theory with size of order  $\text{TeV}^{-1}$  [4]. Their phenomenological consequences have been extensively studied during the last years [5].

Type I string theory offers new realizations of the Scherk-Schwarz mechanism, due to the presence of D-branes. Indeed, for D-branes transverse to the direction used by the Scherk-Schwarz deformation, the massless spectrum on their world-volume remains supersymmetric (at lowest order), while supergravity is spontaneously broken in the bulk [6, 7]. This is the phenomenon of “brane supersymmetry”, present also when breaking supersymmetry in M-theory [8] along the eleventh dimension [9]. It is a generalisation of a similar effect present in orbifold compactifications of the heterotic string, where twisted fields localized at the orbifold fixed points do not feel the supersymmetry breaking.

Another attractive possibility in type I string model building is the mechanism of “brane supersymmetry breaking” [10] generalising the stable non-BPS brane construction of refs. [11, 12]. Supersymmetry is now primordially broken on the world-volume of some D-branes while it remains unbroken (to lowest order) in the closed string bulk. If the Standard Model is localized on these non-supersymmetric D-branes, the string scale must be at TeV energies in order to protect the gauge hierarchy. The observed weakness of gravity can then be explained by introducing some extra large dimensions in the bulk, of size that can reach a millimeter [13]. In this case, the spectrum localized on these D-branes is not supersymmetric, but still supersymmetry is non-linearly realized [14]. In particular, there is a massless Goldstino, localized on their world-volume, transforming non-linearly under supersymmetry [15]. On the other hand, radiative corrections are expected to generate a “tiny” supersymmetry breaking in the bulk which should vanish in the large volume limit. One of the main difficulties of these models is the presence of a non-vanishing tree-level potential giving rise to tadpoles for the dilaton and other NS-NS (Neveu-Schwarz) scalars, which make any quantitative study of these models questionable.

In this work, we construct a class of models exhibiting brane supersymmetry breaking with vanishing (global) tadpoles. This is achieved by introducing at the same time Scherk-Schwarz boundary conditions along some directions orthogonal to the non-supersymmetric branes, which produce an additional (small) supersymmetry breaking in the bulk, that vanishes in the decompactification limit. As we show, in explicit examples, these models can easily accommodate 3-branes with interesting gauge groups and chiral fermions. Thus, supersymmetry is broken at the string scale on the branes, although it remains non-linearly realized, while supergravity is spontaneously broken at the compactification scale in the bulk. If the string scale is at the TeV [13, 16], the gravitino mass varies between  $10^{-3}$  eV to 10 MeV, for two up to six extra large transverse dimensions, respectively.

We also study the effective interactions of the Goldstino with the matter fields living on the brane. In particular, we focus on the dimension eight four-fermion terms which can be arranged in two different operators. These were analysed in the past in refs. [17, 18] using non-linear supersymmetry, that fixes the coupling of one particular operator but leaves the other undetermined. In this work, we perform a string computation of both couplings and we determine in particular the unknown coefficient. In the simplest case, where matter fermions correspond to open strings with both ends on the same set of branes, we find that the Goldstino decay constant is given by the total tension of the corresponding D-branes, while the unknown coefficient has two possible values, depending on whether the matter fermions have the same or different internal helicity with the Goldstino.

The paper is organised as follows. In section 2, we present the effective field theory describing the Goldstino interactions and we determine the corresponding couplings by a string computation, using the results of ref. [19]. In section 3, we describe the essential points of our construction using three ingredients: Scherk-Schwarz deformation in the bulk, orientifold planes at the boundaries and appropriate introduction of D-branes on top of the orientifolds. We also discuss the general properties of the resulting models. In section 4, we present the simplest model in nine dimensions, using D8-branes, which contains however the essential properties of our construction. In section 5, we give a six-dimensional chiral example using D9 and D5 branes. Supersymmetry is non-linearly realized on the world-volume of D5-branes, while is spontaneously broken by the Scherk-Schwarz boundary conditions in the closed string sector and on the D9-branes in the bulk. In section 6, we present a four-dimensional example with chiral fermions and Pati-Salam type gauge group, having the same characteristics with the six-dimensional example. Finally, section 7 contains our conclusions. The results of sections 4,5 and 6 are summarized at the end of section 3, so that the reader who

is not familiar with string theory can skip them and go directly to the conclusions. We have also included four appendices containing notations and technical details used in sections 4-6.

## 2 Interaction of Goldstino with matter

The spontaneous breaking of continuous symmetries lead to the appearance of Nambu-Goldstone fields. These represent the parameters of (non-linear) transformations realizing the broken symmetries in the vacuum. For the case of supersymmetry, the Nambu-Goldstone field  $\chi^\alpha$  has spin-1/2 and is called Goldstino [14]. If the broken supersymmetry is local then the Goldstino mixes with the gravitino, giving rise to the longitudinal components of the massive spin-3/2 particle.

In this section, we describe the leading interaction terms involving two Goldstinos with two matter fermions. We will restrict our analysis to the case of  $\mathcal{N} = 1$  supersymmetry in four dimensions, the generalisation to other dimensions or higher supersymmetries being straightforward. In superstring models, these amplitudes are easy to compute and allow to extract the value of the Goldstino decay constant. On the other hand, the non-observation of effects corresponding to the production of two Goldstinos in the interaction of matter fermions has been used to derive experimental bounds on the supersymmetry breaking scale for a class of models where the gravitino mass is much smaller than all other sparticle masses [20]. In fact, in this case, at low energy, the interactions of gravitinos with matter are dominated by the interaction with their spin 1/2 components, i.e. the Goldstino.

The breaking of supersymmetry is characterized on the one hand by an order parameter  $v$ , fixing the Goldstino decay constant, associated to the primordial breaking scale, and on the other hand by the mass-splittings of the supermultiplets,  $\Delta m_i^2 = \lambda_i v^2$ , with  $\lambda_i$  the corresponding superpotential couplings. In the case of spontaneous breaking by a  $D$ -term,  $v^2 = D$ , while for a breaking through a non-vanishing  $F$ -term,  $v^2 = \sqrt{2}F$ . In the low energy limit, when all mass-splittings go to infinity with  $v$  fixed, the non-linear supersymmetry transformations of the Goldstino fields are given by:

$$\begin{aligned}\delta^{SUSY} \chi^\alpha &= v^2 \xi^\alpha - \frac{i}{v^2} \left( \chi^\beta \sigma^\mu \bar{\xi}_\beta - \xi_\beta \sigma^\mu \bar{\chi}^\beta \right) \partial_\mu \chi^\alpha \\ \delta^{SUSY} \bar{\chi}_{\dot{\alpha}} &= v^2 \bar{\xi}_{\dot{\alpha}} - \frac{i}{v^2} \left( \chi^\beta \sigma^\mu \bar{\xi}_\beta - \xi_\beta \sigma^\mu \bar{\chi}^\beta \right) \partial_\mu \bar{\chi}_{\dot{\alpha}}\end{aligned}\quad (1)$$

where  $\xi^\alpha, \bar{\xi}_{\dot{\alpha}}$  are (two-component) spinorial transformation parameters.

The lowest dimensional operators describing the interaction of two matter fermions  $f$  with two Goldstinos  $\chi$  have dimension eight and can be written

as [17, 18]:

$$\mathcal{L} = \int d^4x \left[ -\frac{1}{2v^4} \left( \chi \overleftrightarrow{\partial}_\mu \sigma^\nu \bar{\chi} \right) \left( f \overleftrightarrow{\partial}_\nu \sigma^\mu \bar{f} \right) + \frac{2C_f}{v^4} (f \partial^\mu \chi) (\bar{f} \partial_\mu \bar{\chi}) \right]. \quad (2)$$

The first term corresponds to the coupling of the energy-momentum tensor of matter fermions with the Goldstino and is model independent. Its normalization is fixed by the choice of canonical kinetic terms for the matter fermions. In contrast, the real coefficient  $C_f$  is model dependent<sup>1</sup> and describes a possible coupling of matter fermions to a non-trivial torsion term of the Goldstino [17]. While in the low energy effective field theory context, the value of  $C_f$  is an arbitrary unknown parameter, it can be computed explicitly in the fundamental (string) theory.

The values of the two parameters  $v$  and  $C_f$  can be extracted from the analysis of the amplitude  $f\bar{f} \rightarrow \chi\bar{\chi}$ . The four-dimensional momenta corresponding to the particles  $f, \bar{f}, \chi, \bar{\chi}$  are chosen all directed inward and denoted by  $k_i$ . We use the Mandelstam variables (kinematical invariants)  $\{s, t, u\}$  defined as:

$$s = -(k_1 + k_2)^2, \quad t = -(k_2 + k_3)^2, \quad u = -(k_1 + k_3)^2. \quad (3)$$

The different helicity amplitudes for the process  $f\bar{f} \rightarrow \chi\bar{\chi}$  can be easily extracted from the effective Lagrangian (2):

$$\begin{aligned} \mathcal{A}(f_L \bar{f}_R \rightarrow \chi_L \bar{\chi}_R) &= -\frac{2}{v^4} (t u + C_f s u), \\ \mathcal{A}(f_L \bar{f}_R \rightarrow \chi_R \bar{\chi}_L) &= \frac{2}{v^4} (t u + C_f s t), \end{aligned} \quad (4)$$

where the subscripts  $L$  and  $R$  label the left-handed and right-handed helicities respectively.

In the following, we consider a scenario where matter fields are localized on a collection of non supersymmetric D-branes embedded in a higher-dimensional bulk. We will work in the limit where the volume of the bulk (transverse volume) is large compared to the string length  $l_s \equiv M_s^{-1}$ . As we discuss below, in this limit, the effects of supersymmetry breaking in the bulk are negligible and we can restrict our analysis to the breaking of global supersymmetry on the brane.

As we describe in sections 4-6, supersymmetry breaking on the world-volume of D-branes is achieved by placing them together with appropriate

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<sup>1</sup>The parameter  $C_f$  is related to the parametrisation of other authors by  $C_f = -\frac{\alpha}{4}$ [17] =  $\frac{C_{ff}}{2}$ [18].

orientifold planes and applying the corresponding orbifold projections. In the simplest case and in the large transverse volume limit, the gauge group is orthogonal (symplectic) while fermions transform in the symmetric (antisymmetric) representation. This representation is reducible and its singlet component can be identified with the Goldstino [15, 21, 22]. Thus, the spectrum is not supersymmetric on the brane, although there is exact supergravity in the bulk, but supersymmetry is non-linearly realized with a massless localized Goldstino. At finite volume, the gravitino obtains a mass via Scherk-Schwarz boundary conditions, and the gauginos appear as odd open string winding modes, transforming in the adjoint representation of the gauge group; in the large volume limit, they become superheavy and decouple from the spectrum.

We will consider a system of at most two type of branes:  $Dp$ -branes and  $Dq$ -branes with  $p - q = 0 \pmod{4}$ . In particular we will deal with systems of D9 branes or D9–D5 branes which can be mapped through  $T$ -duality to a system of D3 or D7–D3 branes. The six-dimensional internal space is compactified on a six-dimensional torus. The world-volume of a  $Dp$ -brane is here extending along the four-dimensional Minkowski space as well as along a volume  $V_p$  in the internal compact space<sup>2</sup>. For the sake of simplicity, we will ignore here the effects of the presence of orbifolds and orientifolds which lead to model dependent projections of some (or all) goldstinos away from the massless spectrum.

In these models, the Goldstino is identified with a massless mode of an open string with both ends located on parallel D-branes. Such strings are denoted as “DD strings” as the associated world-sheet fields satisfy DD (Dirichlet-Dirichlet) boundary conditions along all transverse directions. If instead the open strings are stretched between non-parallel branes (or between  $Dp$  and  $Dq$  branes with  $p \neq q$ ), they are denoted as “ND strings” as the the associated world-sheet fields satisfy now ND (Neumann-Dirichlet) boundary conditions. Their massless modes appear as localized states living at the corresponding brane intersections.

The scattering amplitudes of open string modes are described, at the lowest order, by correlation functions of the corresponding vertex operators on a two-dimensional surface with the topology of a disk. Each end of the open string carries a charge which is representing the transformation under the gauge symmetry group  $G = U(N_p)$ ,  $SO(N_p)$  or  $USp(N_p)$ , where  $N_p$  is given by the number of parallel  $Dp$ -branes stacked together. The transformation of the vertex operators under the gauge group  $G$  is encoded in Chan-Paton matrices  $\lambda$ . The form of vertex operators is given for instance in ref. [19],

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<sup>2</sup>Here  $V_p$  denotes the volume along the  $p - 3$  compact directions of the  $D_p$ -brane.

where we choose the normalisation of Chan-Paton matrices of matter fields as  $Tr(\lambda^a \lambda^b) = \delta_{ab}$ . Since the Goldstinos are gauge singlets, their corresponding Chan-Paton matrices are  $\lambda_p^{(a)} = \frac{\mathbf{1}_{N_p}}{\sqrt{N_p V_p}}$  with  $\mathbf{1}_{N_p}$  the identity matrix of rank  $N_p$ .

On the world-volume of each brane, the space-time fermions transform as spinors of  $SO(10)$  and they are labeled by their helicity with respect to the maximal subgroup  $SO(2)^5$  as  $\frac{1}{2}(\pm \pm \pm \pm \pm)$ , the first two (three)  $SO(2)$ 's represent the four-dimensional space-time (six-dimensional D5-brane worldvolume) helicities.<sup>3</sup> Each  $Dp$ -brane breaks spontaneously half of the bulk supersymmetries. The corresponding Goldstinos are the gauginos of the  $U(1)$  gauge boson appearing on the world-volume of the brane. In the case of  $N_p$  parallel branes, the Goldstinos are the gauginos of the overall  $U(1)$  with the Chan-Paton matrices given above. In our computations we choose the Goldstino helicity to be  $\frac{1}{2}(++++)$ .

We will first consider the case where there is only one type of brane and compute the four-fermion interaction involving two Goldstinos and two matter fermions. This is obtained from the correlation function of four vertex operators representing the emission (absorption) of DD open strings associated with the two Goldstinos and two fermions. For the two Goldstinos and conjugate vertices we choose the helicities  $\frac{1}{2}(++++)$  and  $\frac{1}{2}(----)$  which leaves us with two choices for the matter fermions helicities:

- case  $(I)_{DD}$  where the fermions having six-dimensional internal space helicity as the Goldstinos, corresponding to  $\frac{1}{2}(- - + + +)$  and  $\frac{1}{2}(- + - - -)$ .
- case  $(II)_{DD}$  where, with respect of the Goldstinos, the fermions have opposite helicity in the internal six-dimensional space, corresponding for instance to  $\frac{1}{2}(- + + + -)$  and  $\frac{1}{2}(- - - - +)$ .

Here, the (four-dimensional) space-time helicities are chosen according to the chiralities of the field theory amplitudes (4) that we want to study.

The corresponding scattering amplitudes are straightforward to obtain. They can be computed for instance using the results of ref. [23, 19]. The total scattering amplitude  $\mathcal{A}_{total}(1, 2, 3, 4)$  is obtained by summing over the six possible ordered amplitudes:  $A(1, 2, 3, 4)$  and the five other permutations of the vertex operators. It is useful to define  $\mathcal{A}(1, 2, 3, 4) = A(1, 2, 3, 4) + A(4, 3, 2, 1)$  as the two amplitudes have the same traces on Chan-Paton ma-

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<sup>3</sup>In fact, the space-time fermions transform as spinors of  $SO(1, 9)$ , but it is easier to work in the Euclidean version.

trices, so that:

$$\mathcal{A}_{total}(\tilde{1}, \tilde{2}, 3, 4) = \mathcal{A}(\tilde{1}, \tilde{2}, 3, 4) + \mathcal{A}(\tilde{1}, 3, \tilde{2}, 4) + \mathcal{A}(\tilde{1}, \tilde{2}, 4, 3) \quad (5)$$

where the tilde stands for Goldstinios.

- In the  $(I)_{DD}$  configuration

$$\begin{aligned} \mathcal{A}(\tilde{1}, \tilde{2}, 3, 4) &= -2g_s l_s^2 \text{tr}(\tilde{\lambda}^1 \tilde{\lambda}^2 \lambda^3 \lambda^4 + \lambda^4 \lambda^3 \tilde{\lambda}^2 \tilde{\lambda}^1) \int_0^1 dx x^{-1-s l_s^2} (1-x)^{-1-t l_s^2} \\ &\quad (\tilde{v}^{(1)} \gamma_\mu \tilde{u}^{(2)} \bar{v}^{(4)} \gamma^\mu u^{(3)} (1-x) - \tilde{v}^{(1)} \gamma_\mu v^{(4)} \bar{u}^{(3)} \gamma^\mu \tilde{u}^{(2)} x) \end{aligned} \quad (6)$$

which leads after performing the sum over the permutations to:

$$\begin{aligned} \mathcal{A}(f_L \bar{f}_R \rightarrow \chi_L \bar{\chi}_R) &= -\frac{4}{N_p V_p} g_s \left[ \left( -\frac{2t}{s} - \frac{2t}{t} \right) \mathcal{F}(s, t) + \frac{2t}{t} \mathcal{F}(t, u) + \frac{2t}{s} \mathcal{F}(u, s) \right] \\ &\simeq -\frac{4\pi^2}{N_p V_p M_s^4} g_s u^2 \end{aligned} \quad (7)$$

where  $g_s$  is the type  $I$  string coupling and  $M_s = l_s^{-1}$  is the string scale. The symbol “ $\simeq$ ” means that we have taken the leading order for  $|s l_s^2|, |t l_s^2|, |u l_s^2| \ll 1$  which corresponds to the field theory limit and used:

$$\mathcal{F}(x, y) \simeq 1 - \frac{\pi^2}{6} \frac{xy}{M_s^4}. \quad (8)$$

- In the  $(II)_{DD}$  case we have instead

$$\begin{aligned} \mathcal{A}(\tilde{1}, \tilde{2}, 3, 4) &= -2g_s l_s^2 \text{tr}(\tilde{\lambda}^1 \tilde{\lambda}^2 \lambda^3 \lambda^4 + \lambda^4 \lambda^3 \tilde{\lambda}^2 \tilde{\lambda}^1) \int_0^1 dx x^{-1-s l_s^2} (1-x)^{-1-t l_s^2} \\ &\quad (\tilde{v}^{(1)} \gamma_\mu \tilde{u}^{(2)} \bar{v}^{(4)} \gamma^\mu u^{(3)} (1-x) - 2\tilde{v}^{(1)} v^{(4)} \bar{u}^{(3)} \tilde{u}^{(2)} x) \end{aligned} \quad (9)$$

which leads to the total amplitude

$$\begin{aligned} \mathcal{A}(f_L \bar{f}_R \rightarrow \chi_R \bar{\chi}_L) &= -\frac{4}{N_p V_p} g_s \left[ \left( -\frac{2t}{s} - \frac{2t}{t} \right) \mathcal{F}(s, t) + \frac{2t}{t} \mathcal{F}(t, u) + \frac{2t}{s} \mathcal{F}(u, s) \right] \\ &\simeq \frac{4\pi^2}{N_p V_p M_s^4} g_s u t \end{aligned} \quad (10)$$



We observe that there are no poles (only presence of contact terms) and the dominant contribution comes from dimension eight operator as expected. Comparing eqs. (7) and (10) with eq. (4), we can identify  $v^4$  and the coefficients  $C_f^{DD,I}$  and  $C_f^{DD,II}$  corresponding to fermions from DD strings with the same or different six-dimensional internal helicities with the Goldstinos, respectively. We obtain:

$$C_f^{DD,I} = 1, \quad C_f^{DD,II} = 0, \quad \frac{v^4}{2} = N_p V_p \frac{M_s^4}{4\pi^2 g_s} = N_p V_p \tau_3, \quad (11)$$

where we have identified the D3-brane tension  $\tau_3 = \frac{M_s^4}{4\pi^2 g_s}$ . By performing an appropriate T-duality transformation along the directions transverse to the four space-time directions, the volume factor  $V_p$  disappears and thus Dp-branes can be viewed as D3-branes. Thus, in four dimensions, the Goldstino decay constant is identified with the D3-brane tension times the RR (Ramond) charge  $N_p$ , while the parameter  $C_f$  takes two possible values, depending on whether the fermions have the same or opposite internal helicity than the Goldstinos.

We consider now a system with two types of branes, for instance D5 or D $\bar{5}$  and D9 branes. We can decompose the  $SO(2)^5$  helicities as six-dimensional and internal through  $SO(2)^5 = SO(2)^3 \otimes SO(2)^2$ , such that the helicity in the six-dimensional D5 or D $\bar{5}$  branes world-volume is given by the product of the first three signs. Each type of branes breaks spontaneously 16 supersymmetries out of the 32 present in the bulk. The corresponding Goldstinos can be assembled into two sets of 8-component spinors having the same six-dimensional and internal helicities:

$$\begin{aligned} \text{D9} &\rightarrow 8_{++} + 8'_{--} \\ \text{D5} &\rightarrow 8_{++} + 8''_{+-} \\ \text{D}\bar{5} &\rightarrow 8'''_{-+} + 8'_{--} \end{aligned} \quad (12)$$

where the first and second signs correspond to the six-dimensional and internal helicities, respectively. Note that the D5 and D $\bar{5}$  branes together break all supersymmetries as we retrieve all the 32 supercharges among the Goldstinos. On the other hand the D5–D9 or D $\bar{5}$ –D9 systems preserve 8 supercharges each. In each of these two cases, 8 supercharges are broken by both branes and the corresponding Goldstinos appear as  $U(1)$  gauginos in six dimensions.

We consider the case of Goldstino with helicity  $\frac{1}{2}(++++)$ , thus among the  $8_{++}$  spinors. The Goldstino  $\chi$  cannot be only seen as a 55 or 99 state associated with open strings with both ends on D5 or D9 branes respectively,

but it is a linear combination of the two. It is given by:

$$\chi = \sqrt{\frac{N_9 V_9}{N_5 V_5 + N_9 V_9}} \chi_9 + \sqrt{\frac{N_5 V_5}{N_5 V_5 + N_9 V_9}} \chi_5 \quad (13)$$

Only this combination leads only to dimension eight operators. The 59 strings have no charges under the corresponding linear combination for  $U(1)$  gauge bosons.

In a similar way than the case of a single type of brane, we will choose for the Goldstinos and for its conjugate, the helicities  $\frac{1}{2}(++++)$  and  $\frac{1}{2}(----)$ . There are then two cases:

- case  $(I)_{ND}$  where the fermions have six-dimensional space-time helicity opposite to the one of the Goldstinos and correspond to  $\frac{1}{2}(-++)$  and  $\frac{1}{2}(---)$ .
- case  $(II)_{ND}$  where the fermions have the same six-dimensional space-time helicities with the Goldstinos, and correspond to  $\frac{1}{2}(-+-)$  and  $\frac{1}{2}(-+-)$ .

A straightforward computation leads to:

- in the  $(I)_{ND}$  one

$$\begin{aligned} \mathcal{A}(f_{L,Ii} \bar{f}_{R,Jj} \rightarrow \chi_R \bar{\chi}_L) &= \frac{8}{N_5 V_5 + N_9 V_9} g_s \delta_{IJ} \delta_{ij} \\ &\times \left[ \left( \frac{2t}{s} - \frac{t}{t} \right) \mathcal{F}(s, t) - \frac{2t}{s} \mathcal{F}(s, u) + \frac{t}{t} \mathcal{F}(t, u) \right] \\ &\simeq - \frac{4\pi^2}{(N_5 V_5 + N_9 V_9) M_s^4} g_s \delta_{IJ} \delta_{ij} t^2 \end{aligned} \quad (14)$$

- for the  $(II)_{ND}$  case

$$\begin{aligned} \mathcal{A}(f_{L,Ii} \bar{f}_{R,Jj} \rightarrow \chi_L \bar{\chi}_R) &= - \frac{16\pi^2}{3(N_5 V_5 + N_9 V_9) M_s^4} g_s \delta_{IJ} \delta_{ij} \\ &\times [B(-sl_s^2, 1 - tl_s^2) + B(1 - sl_s^2, \frac{1}{2} - tl_s^2) \\ &- B(-sl_s^2, 1 - ul_s^2) - B(1 - ul_s^2, \frac{1}{2} - tl_s^2)] \\ &\simeq - \frac{16\pi^2}{3(N_5 V_5 + N_9 V_9) M_s^4} (1 - w) g_s \delta_{IJ} \delta_{ij} \\ &\left[ ut + \frac{\frac{1}{2} + 2w}{1 - w} us \right] \end{aligned} \quad (15)$$

where  $w = \frac{3\delta}{\pi^2} \simeq 0.373$  (see below for the definition of  $\delta$ ) and the function  $B(a, b)$  is defined as :

$$B(a, b) = \int_0^1 dx x^{a-1} (1-x)^{b-1} = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}. \quad (16)$$

It is related to the string form factor  $\mathcal{F}(s, t)$  by :

$$\begin{aligned} B(-sl_s^2, -tl_s^2) &= \frac{u}{stl_s^2} \mathcal{F}(s, t) \\ B(-sl_s^2, 1 - tl_s^2) &= -\frac{1}{sl_s^2} \mathcal{F}(s, t) \end{aligned} \quad (17)$$

and is expanded in Taylor series around  $x, y = 0$  as:

$$\begin{aligned} B(1-x, \frac{1}{2} - y) &\simeq 2 + 4y + \delta x \\ B(-x, 1-y) &\simeq -\frac{1}{x} + \frac{\pi^2}{6} y \end{aligned} \quad (18)$$

where  $\delta$  is given in term of the Euler's constant  $\gamma \simeq 0.577$  and digamma function value  $\psi(\frac{3}{2}) \simeq 0.036$  with  $\psi(z) = \frac{\Gamma'(z)}{\Gamma(z)}$ :

$$\delta = 2(\gamma + \psi(\frac{3}{2})) \simeq 1.227 \quad (19)$$

Comparing with the field theory result, we thus get:

$$\begin{aligned} \frac{v_{ND,I}^4}{2} &= (N_5 V_5 + N_9 V_9) \tau_3 & C_f^{ND,I} &= 1 \\ \frac{v_{ND,II}^4}{2} &= \frac{3}{4(1-w)} (N_5 V_5 + N_9 V_9) \tau_3 & C_f^{ND,II} &= \frac{\frac{1}{2} + 2w}{1-w} \end{aligned} \quad (20)$$

Note that in the second case, where the matter fermions have the same six-dimensional space-time helicities with the Goldstino, one obtains a different result for both the decay constant and  $C_f$ . We believe that this is due to the effects of the width of the brane intersection which becomes important in this case.

### 3 Building non-supersymmetric brane-worlds

Before providing explicit examples in diverse dimensions, we will present here the main ingredients of the construction and the main properties of string models with non-linear brane supersymmetry and vanishing (tree-level) cosmological constant.

### 3.1 Construction

To make the construction simple, we consider one large extra dimension (used by the Scherk-Schwarz deformation), and follow three steps: (i) coordinate dependent compactification on  $S^1/\mathbb{Z}_2$ ; (ii) introduction of orientifold planes at the boundaries of the segment  $S^1/\mathbb{Z}_2$ ; (iii) placing D-branes on top of the orientifolds.

(i) *Coordinate dependent compactification on  $S^1/\mathbb{Z}_2$*

Although the results on the Scherk-Schwarz breaking of supersymmetry presented here are known [1], we recall their main features in order to make this section self-contained.

We start with a supersymmetric string vacuum in  $D + 1$  dimensions. The space-time is spanned<sup>4</sup> by the coordinates  $X^M$ . The  $(D + 1)$ -dimensional theory has a linearly realized supersymmetry between the vielbein  $e_a^M$  and the gravitino and between bosons  $\Phi$  and their spin 1/2 fermionic partners  $\Psi$  which can be formally written as:

$$\begin{aligned}\delta_\eta \Phi &\sim \bar{\eta} \Psi \\ \delta_\eta \Psi &\sim \eta \not{\partial} \Phi \\ \delta_\eta e_a^M &\sim \bar{\eta} \Gamma_a \Psi^M \\ \delta_\eta \Psi^M &\sim D^M \eta + \dots\end{aligned}\tag{21}$$

where  $\eta(x^M)$  are local supersymmetry transformation parameters and  $D^M$  is the appropriate covariant derivative. The dots represent contributions of other fields ( $p$ -forms) that are needed to close the supersymmetry algebra in  $D + 1$  dimensions. In eq. (21) we neglected bilinear and higher order terms in the fermions.

The space-like  $x^D = y$  coordinate is compactified on a segment  $S^1/\mathbb{Z}_2$  of length  $\pi R$ . The bosonic fields  $\Phi$  and  $e_a^M$  satisfy periodic conditions along  $S^1$  while the fermions are anti-periodic:

$$\begin{aligned}y &\rightarrow y + 2\pi R : \\ \{\Phi, e_a^M, \dots\}(x^\mu, y) &= \{\Phi, e_a^M, \dots\}(x^\mu, y + 2\pi R) \\ \{\Psi, \Psi^M\}(x^\mu, y) &= -\{\Psi, \Psi^M\}(x^\mu, y + 2\pi R)\end{aligned}\tag{22}$$

where the dots stand for the additional  $p$ -form fields.

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<sup>4</sup>For curved indices in  $D + 1$  dimensions we use capital letters from the middle of Latin alphabet  $M = 0, \dots, D$ , while letters from the beginning of the alphabet are used for flat tangent space coordinates. We use corresponding Greek letters for  $D$ -dimensional indices.

Under the  $\mathbb{Z}_2$  orbifold action, the bosonic fields can be decomposed into even (labeled by  $e$ ) and odd parts (labeled by  $o$ ):

$$\begin{aligned} y \rightarrow -y : \\ \{\Phi_e, e_a^\mu, e_D^D\}(x^\mu, y) &= \{\Phi_e, e_a^M, e_D^D\}(x^\mu, -y) \\ \{\Phi_o, e_D^\mu, e_\mu^D\}(x^\mu, y) &= -\{\Phi_o, e_D^\mu, e_\mu^D\}(x^\mu, -y) \end{aligned} \quad (23)$$

where the complex scalar fields  $\Phi$  have been splitted into even and odd parts:  $\Phi = \Phi_e + \Phi_o$ . The fields  $\Phi_e$  and  $\Phi_o$  can be decomposed into Fourier modes:

$$\begin{aligned} \Phi_e(x^\mu, y) &= \sum_{n=0}^{\infty} \Phi_e^{(n)}(x^\mu) \cos\left(\frac{n}{R}y\right) \\ \Phi_o(x^\mu, y) &= \sum_{n=0}^{\infty} \Phi_o^{(n)}(x^\mu) \sin\left(\frac{n}{R}y\right) \end{aligned} \quad (24)$$

and similarly for the vielbein and the various  $p$ -forms. The modes  $n = 0$  correspond to massless states which remain in the  $D$ -dimensional effective Lagrangian.

The spinors  $\Psi$  and  $\Psi^M$  are decomposed into two components in  $D$  dimensions:

$$\Psi = \begin{pmatrix} \psi_e \\ \psi_o \end{pmatrix}; \quad \Psi^\mu = \begin{pmatrix} \psi_e^\mu \\ \psi_o^\mu \end{pmatrix}; \quad \Psi^D = \begin{pmatrix} \psi_o^D \\ \psi_e^D \end{pmatrix} \quad (25)$$

which satisfy the following parity transformations:

$$\begin{aligned} y \rightarrow -y : \\ \left( \begin{array}{l} \{\psi_e, \psi_e^\mu\}, \{\psi_o^D\} \\ \{\psi_o, \psi_o^\mu\}, \{\psi_e^D\} \end{array} \right) (x^\mu, y) &= \left( \begin{array}{l} \{\psi_e, \psi_e^\mu\}, -\{\psi_o^D\} \\ -\{\psi_o, \psi_o^\mu\}, \{\psi_e^D\} \end{array} \right) (x^\mu, -y). \end{aligned} \quad (26)$$

This implies that the fermion  $\Psi$  has the following Fourier decomposition :

$$\begin{aligned} \psi_e(x^\mu, y) &= \sum_{n=0}^{\infty} \psi_e^{(n)}(x^\mu) \cos\left(\frac{n+1/2}{R}y\right) \\ \psi_o(x^\mu, y) &= \sum_{n=0}^{\infty} \psi_o^{(n)}(x^\mu) \sin\left(\frac{n+1/2}{R}y\right) \end{aligned} \quad (27)$$

and in a similar way for the gravitino components:

$$\psi_e^M(x^\mu, y) = \sum_{n=0}^{\infty} \psi_e^{M(n)}(x^\mu) \cos\left(\frac{n+1/2}{R}y\right)$$

$$\psi_o^M(x^\mu, y) = \sum_{n=0}^{\infty} \psi_o^{M(n)}(x^\mu) \sin\left(\frac{n+1/2}{R}y\right). \quad (28)$$

Note that there are no massless fermions in  $D$ -dimensional space, as all Kaluza-Klein (KK) modes have mass-shift by half a unit.

Consider now the action of the  $\mathbb{Z}_2$  orbifold on the supersymmetric transformations. The spinorial parameter  $\eta$  of the transformation is also decomposed in two parts, an even and an odd one:

$$\eta = \begin{pmatrix} \eta_e \\ \eta_o \end{pmatrix}. \quad (29)$$

The bosonic fields are periodic and should remain as such after supersymmetric transformations. As a result, the supersymmetry transformation parameter should be anti-periodic and can be Fourier decomposed as:

$$\begin{aligned} \eta_e(x^\mu, y) &= \sum_{n=0}^{\infty} \eta_e^{(n)}(x^\mu) \cos\left(\frac{n+1/2}{R}y\right) \\ \eta_o(x^\mu, y) &= \sum_{n=0}^{\infty} \eta_o^{(n)}(x^\mu) \sin\left(\frac{n+1/2}{R}y\right) \end{aligned} \quad (30)$$

A few important remarks follow from the above analysis:

- At  $y = 0$  the odd component of  $\eta$  vanishes:  $\eta_o = 0$ . Half of the original supersymmetry transformations, associated with the spinor  $\eta_e(x^\mu, 0) \equiv \eta_e(x^\mu)$ , remain. The associated gravitino  $\psi_e^\mu$  survives and can have localized supersymmetric coupling with matter fields at  $y = 0$ . The other gravitino  $\psi_o^\mu$  has vanishing wave function and can have only  $y$ -derivative couplings at  $y = 0$ . On the boundary at  $y = \pi R$ , the other half of supersymmetry associated with  $\eta_o(x^\mu, \pi R) \equiv \eta_o(x^\mu)$  is preserved. We will denote by  $Q_e$  and  $Q_o$  the supersymmetry generators associated with  $\eta_e(x^\mu)$  and  $\eta_o(x^\mu)$ , respectively.
- The fermion  $\eta$  has no massless modes in  $D$  dimensions. In the Fourier expansion, all modes  $\eta_e^{(n)}$  and  $\eta_o^{(n)}$  are massive. Thus, in the  $D$ -dimensional theory, there is no supersymmetric transformation leftover, i.e. supersymmetry is totally broken.

## (ii) Appearance of orientifolds

The above discussion is generic to all string vacua. We will now consider the case of weakly coupled type IIB orientifolds [24, 25]. These contain

two types of extended objects, orientifold planes ( $O_p$ -planes) and  $Dp$ -branes, whose world-volume is extending in  $p + 1$  dimensions. They carry two types of charges Ramond–Ramond (RR) and Neveu–Schwarz–Neveu–Schwarz (NS–NS) charges as listed<sup>5</sup> in Table 1. For the purpose of our discussion it is important to remind that:

	Orientifolds				D-branes	
Symbol	$O_p$	$\overline{O}_p$	$O'_p$	$\overline{O}'_p$	$Dp$	$\overline{D}p$
RR charge	–	+	+	–	+	–
NS–NS charge	–	–	+	+	+	+

Table 1: The RR and NS–NS charges of orientifolds and D-branes.

- The tension of these extended objects is proportional to their NS–NS charge. This implies in particular that the orientifold  $O_p$  and  $\overline{O}_p$  planes have negative tensions and therefore they are not dynamical objects, i.e. they do not have massless fluctuations. Requiring vanishing tree-level cosmological constant amounts in our construction to impose zero total NS–NS charge.
- The total RR charge of the system in the compact internal space should vanish by Gauss law (see tadpole cancellation conditions in section 4).
- Each brane and orientifold plane breaks by itself half of the bulk supersymmetry. The conserved (broken) half is linearly (non-linearly) realized on the world-volume of the D-brane. In our notations,  $O_p$ ,  $O'_p$  and  $Dp$  conserve the supersymmetries associated to  $Q_e$ , while  $\overline{O}_p$ ,  $\overline{O}'_p$  and  $\overline{D}p$  conserve  $Q_o$ .

Since the orientifold planes act as mirrors, changing the orientation of strings when going through them, they can appear only at the boundaries of the compactification interval  $S^1/Z_2$ . We will consider here configurations where an  $O_{D-1}$  and an  $\overline{O}_{D-1}$  planes are sitting at  $y = 0$  and  $y = \pi R$ , respectively. Note, that these do not break any further the part of supersymmetry leftover by the orbifold projection.

### (iii) Adding D-branes

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<sup>5</sup> $O_p$  and  $O'_p$  are often denoted in the literature as  $O_p^-$  and  $O_p^+$ , respectively.

We can now introduce D-branes at the boundaries. The previous choice of orientifold planes carrying negative tension allows to compensate the vacuum energy arising from the tensions of the D-branes, in order to keep the tree-level cosmological constant vanishing. To avoid appearance of a global RR charge we can only add pairs of  $Dp\text{--}\overline{D}p$  branes. We will consider two possibilities for the positions of these branes:

- Model I: the  $Dp$ -branes are put at  $y = 0$  while the antibranes  $\overline{D}p$  are put at  $y = \pi R$ . This ensures local cancellation of all tree-level tadpoles and leads to a gauge theory with linearly realized supersymmetries associated to  $Q_e$  and  $Q_o$  on the branes at  $y = 0$  and  $y = \pi R$ , respectively. The Scherk-Schwarz boundary conditions allow supersymmetry in the bulk to interpolate between  $Q_e$  and  $Q_o$  when going from one boundary to the other. Such a brane model has been studied in detail in refs. [6, 26].
- Model II: The other possibility is to put the  $Dp$ -branes on top of an  $\overline{O}_p$  at  $y = \pi R$  and the  $\overline{D}p$ -branes at  $y = 0$ . At each boundary, the branes and orientifolds preserve a different half of the original supersymmetry. Thus their superposition breaks it totally both at  $y = 0$  and  $y = \pi R$ . The supersymmetries associated with  $Q_e$  and  $Q_o$  are non-linearly realized on the world volumes of  $\overline{D}p$  and  $Dp$  branes, respectively. In fact, it is possible to identify a gauge singlet fermion on each boundary with the Goldstino and the scale of supersymmetry breaking corresponds to the one computed in section 2.

This construction is summarized in figure 1. We will discuss explicit realizations of this scenario in the following. Here, we would like to comment about the origin of non-linear supersymmetry on the boundaries. Consider first a stack of  $N$  coincident (anti) D-branes in the bulk (away from the boundaries) with a  $U(N)$  Yang-Mills theory on their world-volume. These are solitonic objects which break spontaneously half of the bulk supersymmetry. Thus, the leftover supersymmetry is realized linearly while the broken half becomes non-linear. The corresponding Goldstino can be identified with the abelian gaugino of the  $U(1)$  factor in  $U(N)$  [21]. When this collection of (anti) D-branes is put on top of an (orientifold) anti-orientifold O-plane, say at the origin  $y = 0$ , the orientifold projection breaks explicitly the linear supersymmetry on the branes as it acts differently on fermions and bosons. However, the non-linear supersymmetry on their world-volume is preserved.

Another aspect of the non-linear realization of global supersymmetry is the presence of a vacuum energy. This is related to the Goldstino decay



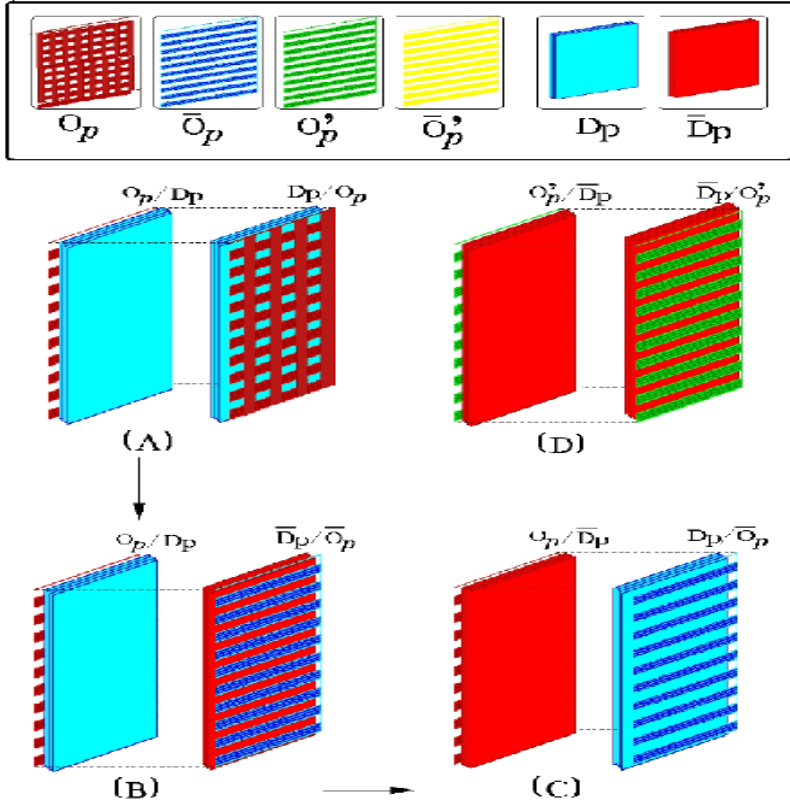


Figure 1: (A) *Supersymmetric configuration* (B) *a  $\mathbb{Z}_2$  Scherk-Schwarz deformation leads to supersymmetry breaking in the bulk* (C) *Switching the positions of branes and anti-branes leads to supersymmetry breaking also on the branes.* The case (D) corresponding to previously considered non-supersymmetric brane-worlds [10, 12] is displayed for comparison.

constant which was computed in section 2 and (for D3-branes) is given by  $v^4/2$ . When coupled to supergravity with vanishing cosmological constant, the gravitino mass is given by  $m_{3/2} = v^2/M_{pl}$ , where  $M_{pl}$  is the four-dimensional Planck mass [27]. Note that this relation does not hold in our case, since there is an additional source of supersymmetry breaking due to the Scherk-Schwarz boundary conditions in the bulk and the (tree-level) gravitino mass is given by  $1/2R$ . In fact, in our model the tree-level contribution to the vacuum energy from the supersymmetry breaking is zero as a result of cancellations of different contributions among (positive tension) branes and (negative tension) orientifolds.

$y = 0$		Bulk	$y = \pi R$	
DD	DN	closed/NN	$\overline{\text{DN}}$	$\overline{\text{DD}}$
$A_\mu$		$e_a^M$		$A_\mu$
$\lambda_+$		$\psi_+^\mu \quad \psi_-^\mu$		$\lambda_-$
$\psi_-$		$\psi_-^D \quad \psi_+^D$		$\psi_+$
$\phi$		$A_M^{(b)}$		$\phi$
	$\psi_-'$	$\lambda_+^{(b)} \quad \lambda_-^{(b)}$	$\psi_+'$	
	$\phi'$	$\psi_-^{(b)} \quad \psi_+^{(b)}$	$\phi'$	
		$\phi^{(b)}$		
Linear $Q_e$			Linear $Q_o$	
Model I				

$y = 0$		Bulk	$y = \pi R$	
$\overline{\text{DD}}$	$\overline{\text{DN}}$	closed/NN	DN	DD
$A_\mu$		$e_a^M$		$A_\mu$
$\lambda_-$		$\psi_+^\mu \quad \psi_-^\mu$		$\lambda_+$
$\psi_+$		$\psi_-^D \quad \psi_+^D$		$\psi_-$
$\phi$		$A_M^{(b)}$		$\phi$
	$\psi_-'$	$\lambda_+^{(b)} \quad \lambda_-^{(b)}$	$\psi_+'$	
	$\phi'$	$\psi_-^{(b)} \quad \psi_+^{(b)}$	$\phi'$	
		$\phi^{(b)}$		
Non-L $Q_e$			Non-L $Q_o$	
Model II				

Table 2: Comparison of Model I and Model II. In the first,  $Q_e$  and  $Q_o$  supersymmetries are linearly realized at  $y = 0$  and  $y = \pi R$ , respectively. In the second they are non-linearly realized. The labels + and - represent the chiralities,  $\lambda_\pm$  are fermions in the adjoint representations, gauginos in Model I. The label b stands for bulk states.

### 3.2 Main features of the construction

The first important remark we should make is about presence of possible tachyons. The branes and anti-branes attract each other, leading at short distances to the appearance of tachyonic modes for open strings stretched between the two sets. To avoid them, it is necessary that the dimension employed by the Scherk-Schwarz deformation is much larger than the string scale. This is fortunately the regime where the effective field theory analysis carried above is valid.

Another important issue is stability of masses under radiative corrections. At tree-level, the KK spectrum of bulk fields is the same for both models I and II. In the explicit examples given in section 4, this is manifest from the fact that the torus and Klein bottle amplitudes are the same for the two configurations. At one-loop, in model II, new contributions to the gravitino and other masses are expected to arise from radiative corrections of the boundary states that are not supersymmetric. Naively, these are of the order of  $M_s^2/M_{pl}$ , although a careful analysis is needed to include the effects of all KK excitations, as well as the contributions of both boundaries. Note that

$M_s^2/M_{pl}$  is of the same order as the tree-level masses given by the compactification scale  $1/R$  only for the case of two transverse dimensions. For more than two,  $1/R$  is dominant, while for one  $M_s^2/M_{pl}$  is larger. However, the case of bulk propagation in one dimension is subtle because of the expected large corrections growing linearly with the radius, originating from local tadpoles [28]. A similar question concerns the value of the Goldstino mass due to radiative corrections from the bulk, where local supersymmetry is spontaneously broken by the Scherk-Schwarz boundary conditions. We plan to return to this issues in a future work.

Let us also discuss some phenomenological applications of these models. If the Standard Model is localized on the world-volume of non-supersymmetric D-branes considered here, then the gauge hierarchy problem requires that the string scale should lie in the TeV region. It is then preferable to choose the extra-dimension separating the brane-antibrane sets to be part of the gravitational bulk with a size as large as a millimeter [13].

In order to evade the above constraint on the string scale, we need to embed the Standard Model gauge group on other branes with supersymmetric massless spectrum. Examples of such configurations are illustrated in figure 2. The non-supersymmetric branes act then as “hidden sectors” and bulk fields mediate the supersymmetry breaking to the Standard Model branes. The study of the issues of radiative corrections discussed above is now necessary to fix the desired values for the string and compactification scales [29].

### 3.3 Summary of the explicit examples

In the next section we will present explicit realizations of the construction described here. We will first consider the simplest case with a single compact dimension  $S^1/\mathbb{Z}_2$  and only one type of branes and orientifolds, D8 and  $O_8$ . We will show that in the presence of a Scherk-Schwarz breaking of supersymmetry in the bulk, the tadpole cancellation conditions have two solutions. In the first solution, the boundary massless states form supersymmetric multiplets while in the second they don't. More precisely, on each of the boundaries lives an  $SO(16)$  Yang-Mills theory. In the first model, the fermions are in the anti-symmetric representation of  $SO(16)$  and form vector supermultiplets with the gauge bosons, while in the second the fermions are in the symmetric representations and do not have massless supersymmetric partners. In order to illustrate that these constructions realize model I and II described above, we show that one can go from one solution to the other by interchanging the positions of branes on the two boundaries. We will also show that the RR charges at the boundaries vanish in the first solution while they have opposite

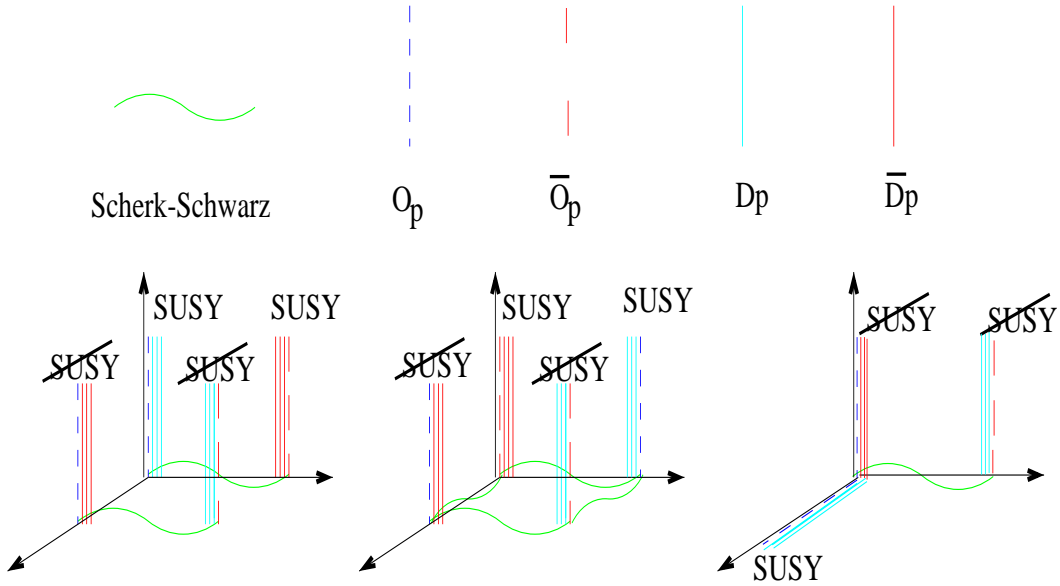


Figure 2: *Constructions containing “tree-level supersymmetric sectors”.* The configurations with branes on top of orientifolds with the same RR charge are non-supersymmetric while those with opposite RR charges lead to supersymmetric massless spectrum. In the first figure, the Scherk-Schwarz boundary condition is on one direction, while in the second figure, it is imposed in two directions.

values in the second. This realizes the above description in terms of an  $O_8-\overline{O}_8$  system on which there are stacked  $D8-\overline{D}8$  branes. In both cases, the bulk states contain only the ten-dimensional supergravity multiplet where the fermions, gravitino and dilatino have mass shifts by  $1/2R$ .

A simple toroidal compactification of the above nine-dimensional example does not lead to four-dimensional chiral fermions. To obtain chiral four-dimensional models, we need to perform further orbifolding. A simple example consider here is given by  $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_3) \equiv T^6/\mathbb{Z}'_6$  and will be constructed in two steps: first we obtain a chiral six-dimensional model through compactification on  $T^4/\mathbb{Z}_2$ , and second we perform a further compactification on  $T^2$  followed by a  $\mathbb{Z}_3$  projection to obtain a chiral four-dimensional theory.

The six-dimensional example obtained by compactification on  $T^4/\mathbb{Z}_2$  with a Scherk-Schwarz breaking of supersymmetry along one of the  $T^4$  compact directions has some interesting features. First, it contains two types of branes  $D5$  and  $D9$ . The  $D9$  branes extend in the whole space-time and give rise to Yang-Mills theory coupled with matter in the bulk while the  $D5$ 's give rise to chiral gauge theories localized on the boundaries. It provides then

an example with the most generic bulk+boundaries content as described in Table 2. In the bulk, on one hand the closed string modes lead to massless graviton, anti-symmetric tensor, dilaton and sixteen scalars. On the other hand, the open string modes from D9 branes give rise to the gauge bosons of  $U(16)$  and four scalars in the  $\mathbf{120} + \overline{\mathbf{120}}$  representations. The fermionic supersymmetric partners of these bulk states are massive because of the Scherk-Schwarz boundary conditions. On the boundaries, the closed string twisted states form supersymmetric multiplets. These represent sixteen neutral hypermultiplets localized at each of the sixteen  $T^4/\mathbb{Z}_2$  fixed points; half of them located at  $y = 0$  have negative chirality, while the other half located at  $y = \pi R$  have positive chirality. The open string states lead to  $U(8)$  gauge theories on the boundaries. The matter content of Model II is then listed in Table 3. It differs from the spectrum of Model I first in the chirality assignments as explained in Table 2, and second because the fermions are in symmetric representations in Model I and in anti-symmetric ones in Model II. The symmetric representation 36 of  $U(8)$  can in fact be decomposed into the irreducible components  $35 + 1$ , where the singlet is identified with the Goldstino of the supersymmetry which becomes non-linearly realized in Model II<sup>6</sup>.

From the six-dimensional example, it is easy to construct a chiral four-dimensional descendant on  $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_3)$  orbifold. The  $\mathbb{Z}_3$  orbifolding does not lead to new open string states but acts only as a projection on the six-dimensional spectrum. The resulting open string massless modes are listed in table 4. The four-dimensional Goldstino of the non-linearly realized supersymmetry is also easily identified with the  $\mathbb{Z}_3$ -invariant component of the corresponding six-dimensional spinor. It is interesting to note that without much efforts, we obtain in this way a two-family version of Pati-Salam model localized on the boundaries.

The construction presented here can be modified in several ways. The models we describe have a symmetry between theories living on the two boundaries. There are mirror worlds related by a chirality flip. However, as in the Hořava–Witten compactification of M-theory, this symmetry could be broken upon compactification.

## 4 Non-supersymmetric D8-branes

The construction of the nine-dimensional model that we describe here as an orientifold from type IIB is quite simple and was given in ref. [6]. The generic

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<sup>6</sup>More precisely, in the presence of 59 strings, the Goldstino is a linear combination of 99 and 55 states, as we described in section 2.

type I string partition function is given as the sum of four contributions corresponding to the diagrams represented by the torus, the Klein bottle, the annulus and the Möbius worldsheet surfaces [24]. The corresponding amplitudes are denoted by the symbols  $\mathbf{T}$ ,  $\mathbf{K}$ ,  $\mathbf{A}$  and  $\mathbf{M}$ , respectively. The general expressions for these amplitudes together with our notations are given in the appendices A and B.

The simplest example of string models with non-linear brane supersymmetry and vanishing tree-level vacuum energy is obtained as an orientifold of type IIB string compactified on  $S^1$  of radius  $R$ . It can also be described in its  $T$ -dual version as a compactification of the ten-dimensional type IIA string on  $S^1/\mathbb{Z}_2$  of radius  $\tilde{R} = 1/R$ . Below, we will often make use of the type IIA description which has natural geometrical interpretation in the region  $\tilde{R} > l_s$  in terms of D8-branes, while the partition function will be given for convenience in terms of type IIB variables.

We consider a compactification of type IIA string on an orbifold  $S^1/\mathbb{Z}_2$  parametrized by the coordinate  $y \in [0, \pi\tilde{R}]$ . We will require the presence of :

- *At  $y = 0$ :*  $n_{0\epsilon}$  D8-branes, each carrying a RR charge  $\epsilon = \pm 1$ , stacked on an  $O_8$  orientifold plane.
- *At  $y = \pi\tilde{R}$ :*  $n_{\pi\epsilon}$  of D8-branes, each carrying a RR charge  $-\epsilon$ , stacked on an  $\overline{O}_8$  orientifold plane.
- *Along  $y \in [0, \pi\tilde{R}]$ :* a Scherk-Schwarz deformation shifting all bulk fermions masses by  $1/2R$ .

The perturbative spectrum of the model can be read off from the one-loop partition function. Using type IIB variables, it can be written as:

$$\frac{1}{2}\mathbf{T} + \mathbf{K} + \mathbf{A} + \mathbf{M} = \int \left( \frac{1}{2}\mathcal{T}(\tau, \bar{\tau}) + \mathcal{K}(2i\tau_2) + \mathcal{A}\left(\frac{it}{2}\right) + \mathcal{M}\left(\frac{it}{2} + \frac{1}{2}\right) \right) \quad (31)$$

where the integration measure for torus, Klein bottle, annulus and Möbius contributions are defined by:

$$\begin{aligned} \mathbf{T} &= \frac{1}{(4\pi^2\alpha')^{\frac{9}{2}}} \int \frac{d\tau d\bar{\tau}}{(\text{Im } \tau)^{11/2}} \left| \frac{1}{\eta^7} \right|^2 \mathcal{T} , \\ \mathbf{K} &= \frac{1}{(4\pi^2\alpha')^{9/2}} \int_0^\infty \frac{d\tau_2}{\tau_2^{11/2}} \frac{1}{\eta^7} \mathcal{K} , \\ \mathbf{A} &= \frac{1}{(8\pi^2\alpha')^{9/2}} \int_0^\infty \frac{dt}{t^{11/2}} \frac{1}{\eta^7} \mathcal{A} \\ \mathbf{M} &= \frac{1}{(8\pi^2\alpha')^{9/2}} \int_0^\infty dt t^{11/2} \frac{1}{\eta^7} \mathcal{M} . \end{aligned} \quad (32)$$

The integrands  $\mathcal{T}$ ,  $\mathcal{K}$ ,  $\mathcal{A}$  and  $\mathcal{M}$  can be written in a compact way as:

$$\begin{aligned}
\mathcal{T} &= E_0 (V_8 \overline{V}_8 + S_8 \overline{S}_8) + O_0 (I_8 \overline{I}_8 + C_8 \overline{C}_8) - E_{\frac{1}{2}} (V_8 \overline{S}_8 + S_8 \overline{V}_8) \\
&\quad - O_{\frac{1}{2}} (I_8 \overline{C}_8 + C_8 \overline{I}_8) \\
\mathcal{K} &= \frac{1}{2}(V_8 - S_8) \sum_m Z_{2m} + \frac{1}{2}(I_8 - C_8) \sum_m Z_{2m+1} \tag{33} \\
\mathcal{A} &= \left[ \frac{n_{0+}^2 + n_{\pi+}^2 + n_{0-}^2 + n_{\pi-}^2}{2} (V_8 - S_8) + (n_{0+}n_{0-} + n_{\pi+}n_{\pi-})(I_8 - C_8) \right] \sum_m Z_{2m} \\
&\quad + [(n_{0+}n_{\pi-} + n_{0-}n_{\pi+})(V_8 - S_8) + (n_{0+}n_{\pi+} + n_{0-}n_{\pi-})(I_8 - C_8)] \sum_m Z_{2(m+\frac{1}{2})} \\
\mathcal{M} &= \sum_m \left[ -\frac{n_{0+} + n_{\pi+}}{2} (\hat{V}_8 - (-1)^m \hat{S}_8) - \frac{n_{0-} + n_{\pi-}}{2} (\hat{V}_8 + (-1)^m \hat{S}_8) \right] Z_{2m}
\end{aligned}$$

in terms of  $SO(8)$  characters and lattice sums as defined in the appendix A.

The cancellation of global tadpoles (vanishing of the total NS-NS and RR charges) requires the coefficients of the massless modes contributing to the characters  $V_8$  and  $S_8$  in the transverse closed string channel to vanish. This implies:

$$(n_{0+} + n_{\pi+}) + (n_{0-} + n_{\pi-}) = 32, \quad (n_{0+} - n_{\pi+}) - (n_{0-} - n_{\pi-}) = 0 \tag{34}$$

Only the annulus and Möbius are sensitive to the RR charges of the branes. For instance, the torus amplitude in eq. (33) is directly obtained from the supersymmetric expression by a simple deformation corresponding to shifting the masses of all fermionic KK modes of closed strings, due to the Sherck-Schwarz breaking along the tenth coordinate, and complete the partition function in the non-trivial winding sector using modular invariance [3].

The simultaneous presence of branes and anti-branes leads to instability that manifests in the appearance of tachyons associated to the identity character  $I_8$  in eq. (33). All these tachyonic modes acquire positive mass-squared in the limit  $R \rightarrow 0$  ( $\tilde{R} \rightarrow \infty$ ) except for the ones appearing in the sector  $I_8 Z_{2m}$  for  $m = 0$ . To remove this tachyon we choose  $n_{0\epsilon} = n_{\pi\epsilon} = 0$  for either  $\epsilon = +$  or  $\epsilon = -$ . This condition amounts to require that only branes (or only anti-branes) are present at  $y = 0$ , while only anti-branes (or only branes) carrying the opposite RR charge are present at  $y = \pi R$ . As a result, there are two possible choices:

$$\text{Model I :} \quad n_{0+} = n_{\pi+} = 16, \quad n_{0-} = n_{\pi-} = 0 \tag{35}$$

$$\text{Model II :} \quad n_{0+} = n_{\pi+} = 0, \quad n_{0-} = n_{\pi-} = 16 \tag{36}$$

In Model I, studied in great detail in ref. [6], supersymmetry is broken at tree-level only in the bulk, while on the boundaries the massless states form supersymmetric multiplets: a vector and a spinor in the adjoint of  $SO(16) \otimes SO(16)$ . On one boundary, D8-branes are stacked on top of an  $O_8$  plane, while on the other boundary,  $\overline{D8}$  branes sit on top of an  $\overline{O}_8$ . Note however that open string winding modes are not supersymmetric, since they are obviously sensible in global effects. In particular, fermions with odd winding numbers in each of the two boundaries transform in the symmetric representation of the gauge group, while bosons remain in the adjoint.

In contrast to Model I, in Model II, even the massless states on the boundaries are non-supersymmetric: they form vectors in the adjoint representation  $(1, 120) + (120, 1)$  with a spinor in the symmetric representation  $(135, 1) + (1, 1)$  and  $(1, 135) + (1, 1)$  of  $SO(16) \otimes SO(16)$ . The model corresponds to  $\overline{D8}$ -branes sitting on top of  $O_8$  on one boundary, and D8-branes sitting on top of  $\overline{O}_8$  on the other side. The gauge singlet fermions are identified with the two localized Goldstinos in each boundary, while the corresponding decay constant can be computed as explained in section 2. It is given by  $16\tau_8$ , with  $\tau_8 = \frac{1}{(2\pi)^7} \frac{M_s^9}{g_s}$  the D8-brane tension. Note that in this case, odd winding modes are supersymmetric.

## 4.1 Interpolating between the two models

In this section we will provide further evidence to the geometrical interpretation of the models presented above. We will first show that one can obtain model I from model II, and vice-versa, by interchanging the branes between the two boundaries. We will then compute the NS-NS and RR charges to verify the presence of orientifolds and branes with opposite charges in the two ends, as described in section 3.

For this purpose, it is useful to consider the generic situation where the D8 (or  $\overline{D8}$ ) branes are not sitting at the orbifold fixed points. Half of the branes are moved away from the origin  $y = 0$ , at a distance  $2\pi\tilde{R}\vec{a} = (2\pi\tilde{R}a_1, \dots, 2\pi\tilde{R}a_{16})$ , while the other half are away from  $y = \pi\tilde{R}$  by  $2\pi\tilde{R}\vec{a}' = (2\pi\tilde{R}a'_1, \dots, 2\pi\tilde{R}a'_{16})$ .

In the type IIB representation, the brane separations are described by Wilson lines appearing as shifts in the KK momenta of open strings. These



contribute to the annulus and Möbius amplitudes which become:

$$\begin{aligned}
\mathcal{A} &= \frac{1}{2}(V_8 - S_8) \left[ \sum_{i,j,m} Z_{2(m+a_i+a_j)} + \sum_{i,j,m} Z_{2(m+a'_i+a'_j)} \right] \\
&\quad + (I_8 - C_8) \sum_{i,j,m} Z_{2(m+\frac{1}{2}+a_i+a'_j)} \\
-2\mathcal{M} &= \sum_{i,m} Z_{2(m+2a_i)}(\hat{V}_8 - (-1)^m \hat{S}_8) + \sum_{i,m} Z_{2(m+2a'_i)}(\hat{V}_8 - (-1)^m \hat{S}_8)
\end{aligned} \tag{37}$$

As the torus and Klein bottle amplitudes (in the direct channel) get contributions only from closed strings, they are not affected. The two cases of interest are:

- $\underline{\vec{a}} = \underline{\vec{a}'} = \vec{0}$  which leads to:

$$\begin{aligned}
\mathcal{A} &= 2^8(V_8 - S_8) \sum_m Z_{2m} + 2^8(I_8 - C_8) \sum_m Z_{2(m+\frac{1}{2})} \\
\mathcal{M} &= -2^4 \sum_m Z_{2m}(\hat{V}_8 - (-1)^m \hat{S}_8)
\end{aligned} \tag{38}$$

and reproduces the corresponding spectrum of Model I.

- $\underline{\vec{a}} = \underline{\vec{a}'} = \frac{\vec{1}}{2}$  which leads instead to:

$$\begin{aligned}
\mathcal{A} &= 2^8(V_8 - S_8) \sum_m Z_{2m} + 2^8(I_8 - C_8) \sum_m Z_{2(m+\frac{1}{2})} \\
\mathcal{M} &= -2^4 \sum_m Z_{2m}(\hat{V}_8 + (-1)^m \hat{S}_8)
\end{aligned} \tag{39}$$

describing Model II.

This shows that the two models are related through a shift by  $\pi\tilde{R}$  in the positions (i.e. an interchange) of the D-branes.

In the expression of the transverse channel for the one-loop amplitudes, the coefficients of the characters  $V_8$  and  $S_8$  give the local NS-NS and R-R tadpoles, respectively. Summing up the Klein, annulus and Möbius contributions we obtain:

$$\begin{aligned}
&\frac{R}{2^{\frac{3}{2}}} V_8 \sum_n \left[ \{32 - 2(trW^{2n} + trW'^{2n})\} \tilde{Z}_{2n} + \frac{1}{32} (trW^n + (-1)^n trW'^n)^2 \tilde{Z}_n \right] \\
&- \frac{R}{2^{\frac{3}{2}}} S_8 \sum_n \left[ \{32 + 2(trW^{2n+1} + trW'^{2n+1})\} \tilde{Z}_{2n+1} + \frac{1}{32} (trW^n - (-1)^n trW'^n)^2 \tilde{Z}_n \right]
\end{aligned}$$

where the phases

$$\begin{aligned} W &= \text{diag}(e^{2i\pi a_1}, \dots, e^{2i\pi a_{16}}) \\ W' &= \text{diag}(e^{2i\pi a'_1}, \dots, e^{2i\pi a'_{16}}) \end{aligned} \quad (40)$$

contain the effects of displacement of the branes from the boundaries, so that  $W = W' = 1$  reproduces Model I while  $W = W' = -1$  is associated with Model II.

After some straightforward manipulations, we can express the NS-NS tadpole as :

$$\frac{R}{2^{\frac{13}{2}}} V_8 \sum_n \left\{ \left[ \text{tr}(2 - W^{2n} - W'^{2n}) \right]^2 \tilde{Z}_{2n} + \left[ \text{tr}(W^{2n+1} - W'^{2n+1}) \right]^2 \tilde{Z}_{2n+1} \right\} \quad (41)$$

which cancels for both  $W = W' = 1$  and  $W = W' = -1$ . On the other hand, the RR tadpole is given by:

$$-\frac{R}{2^{\frac{13}{2}}} S_8 \sum_n \left\{ \left[ \text{tr}(2 - W^{2n+1} - W'^{2n+1}) \right]^2 \tilde{Z}_{2n+1} + \left[ \text{tr}(W^{2n} - W'^{2n}) \right]^2 \tilde{Z}_{2n} \right\} \quad (42)$$

which vanishes for  $W = W' = 1$  but not for  $W = W' = -1$ , leading:

$$-\frac{R}{2^{\frac{3}{2}}} S_8 (4 \times 32 \sum_n \tilde{Z}_{2n+1}). \quad (43)$$

It follows that while in model I branes are sitting on top of orientifolds with opposite RR charges, in Model II the RR charge of the branes and orientifolds are the same.

In eq. (43), only RR states with odd KK momenta feel the presence of charges which implies that the charges at the boundaries are opposite. In fact, let us denote by  $S$  the corresponding RR field. It can be Fourier expanded as:

$$S(x^\mu, y) = \sum_{n=0}^{\infty} S_+^{(n)}(x^\mu) \cos\left(\frac{n}{\tilde{R}}y\right) + \sum_{n=1}^{\infty} S_-^{(n)}(x^\mu) \sin\left(\frac{n}{\tilde{R}}y\right) \quad (44)$$

The coupling of the RR field to charges  $q_0$  at  $y = 0$  and  $q_\pi$  at  $y = \pi\tilde{R}$  can be written as:

$$\begin{aligned} \int d^{10}x \mathcal{L}_{RR} &= \int d^9x \int dy \{ \delta(y) q_0 S(x^\mu, y) + \delta(y - \pi\tilde{R}) q_\pi S(x^\mu, y) \} \\ &= \int d^9x \sum_{n=0}^{\infty} \left[ S_+^{(2n)}(x^\mu) (q_0 + q_\pi) + S_+^{(2n+1)}(x^\mu) (q_0 - q_\pi) \right]. \end{aligned} \quad (45)$$

Thus, the absence of  $\tilde{Z}_{2n}$  in (43) corresponds to  $q_0 = -q_\pi$ .

## 5 Chiral six-dimensional example

Our second example is a six-dimensional type IIB model with D5,  $\overline{\text{D5}}$  and D9 branes. Compared to the nine-dimensional example, two new features appear here: the chirality of Majorana-Weyl spinors in six dimensions allows to have smaller number of supersymmetry charges, and Yang-Mills fields are now present on both boundaries and in the bulk.

The model is obtained through compactification of type IIB on a  $T^4/\mathbb{Z}_2$  orbifold. The  $\mathbb{Z}_2$  action on the four coordinates  $x^6, x^7, x^8$  and  $x^9 \equiv y$  compactified on circles of radii  $R^1, R^2, R^3$  and  $R^4 = R$ , respectively, is given by:

$$x^6, x^7, x^8, y \rightarrow -x^6, -x^7, -x^8, -y \quad (46)$$

and leads to  $2^4$  fixed points where orientifolds are localized. In the compact direction  $y$ , all bulk fermions  $\Psi$  are chosen to satisfy anti-periodic boundary conditions:

$$\Psi(x^\mu, y + 2\pi R) = -\Psi(x^\mu, y) \quad (47)$$

which lead to a Scherk-Schwarz breaking of supersymmetry in six dimensions [6].

The knowledge of the one-loop torus, Klein bottle, annulus and Möbius amplitudes:

$$\begin{aligned} \mathbf{T} &= 1(4\pi^2\alpha')^3 \int \frac{d\tau d\bar{\tau}}{(\text{Im } \tau)^4} |1\eta^4|^2 \mathcal{T}, \\ \mathbf{K} &= 1(4\pi^2\alpha')^3 \int_0^\infty d\tau_2 \tau_2^4 1\eta^4 \mathcal{K}, \\ \mathbf{A} &= 1(8\pi^2\alpha')^3 \int_0^\infty dt t^4 1\eta^4 \mathcal{A}, \\ \mathbf{M} &= 1(8\pi^2\alpha')^3 \int_0^\infty dt t^4 1\eta^4 \mathcal{M}, \end{aligned} \quad (48)$$

allows to derive the spectrum of the model. In the model under study, the torus and Klein bottle amplitudes are the same as in the model of ref. [6] by construction, as we do not affect the bulk states by switching the positions of the D5 and  $\overline{\text{D5}}$  branes.

The torus amplitude is given by  $\mathcal{T} = \mathcal{T}_U + \mathcal{T}_T$  with:

$$\begin{aligned}\mathcal{T}_U &= \Lambda^{(3,3)} 2 \left[ E'_0 (|V_8|^2 + |S_8|^2) + O'_0 (|I_8|^2 + |C_8|^2) \right. \\ &\quad \left. - E'_{1/2} (V_8 \bar{S}_8 + S_8 \bar{V}_8) - O'_{1/2} (I_8 \bar{C}_8 + C_8 \bar{I}_8) \right] \\ \mathcal{T}_T &= \frac{1}{4} (|Q_O - Q_V|^2 + |Q'_O - Q'_V|^2) |4\eta^2 \theta_2^2|^2 + \frac{1}{4} (|Q_S + Q_C|^2 + |Q'_S + Q'_C|^2) |4\eta^2 \theta_4^2|^2 \\ &\quad + \frac{1}{4} (|Q_S - Q_C|^2 + |Q'_S - Q'_C|^2) |4\eta^2 \theta_3^2|^2\end{aligned}$$

while the Klein bottle amplitude in the direct channel is  $\mathcal{K} = \mathcal{K}_U + \mathcal{K}_T$  with:

$$\begin{aligned}\mathcal{K}_U &= 14 \left[ (V_8 - S_8) \left( \sum_m Z_m \Lambda^{(3)} + \sum_n \tilde{Z}_{2n} \tilde{\Lambda}^{(3)} \right) + (I_8 - C_8) \sum_n \tilde{Z}_{2n+1} \tilde{\Lambda}^{(3)} \right] \\ \mathcal{K}_T &= (Q_S + Q_C + Q'_S + Q'_C) \eta^2 \theta_4^2\end{aligned}\tag{50}$$

Here  $\mathcal{T}_U$  and  $\mathcal{K}_U$  represent the contributions of untwisted closed strings, while  $\mathcal{T}_T$  and  $\mathcal{K}_T$  represent the corresponding twisted closed string sectors. All the untwisted massless fermions acquire masses  $1/2R$  due to the Scherk-Schwarz boundary conditions. Thus, the (bosonic) massless untwisted closed string states contain the graviton, the anti-symmetric tensor, the dilaton and sixteen additional scalars. The twisted sector does not feel the Scherk-Schwarz breaking at tree-level and forms supersymmetric multiplets. These represent sixteen neutral hypermultiplets localized at each of the sixteen  $T^4/\mathbb{Z}_2$  fixed points; half of them located at  $y = 0$  have negative chirality, while the other half located at  $y = \pi R$  have positive chirality. The change of chirality arises from the Scherk-Schwarz boundary conditions, which change half of the orientifolds  $O_5$ -planes to  $\overline{O}_5$ -planes as we explained in section 3.

The absence of tachyons is insured by separating the branes from the anti-branes, thus taking the limit  $RM_s \gg 1$ <sup>7</sup>. We consider the Model II described in section 3, obtained by putting  $\overline{D}5$ -branes on top of the  $O_5$  at  $y = 0$  and an equal number of  $D5$ -branes on top of  $\overline{O}_5$  at  $y = \pi R$ . The one-loop amplitudes sensitive to the resulting breaking of supersymmetry on the boundaries are the annulus and Möbius strip. The first is given in the

<sup>7</sup>To be contrasted with the previous section, where in the type IIB language  $RM_s \ll 1$ , here we have  $D5$ -branes of IIB, while before we had  $D8$ -branes of IIA.

direct (open string) channel by:

$$\begin{aligned}
\mathcal{A} = & n_N^2 4(V_8 \sum_m Z_{2m} - S_8 \sum_m Z_{2(m+1/2)}) \Lambda^{(3)} \\
& + 14 \left[ (n_{D_1}^2 + n_{D_2}^2)(V_8 - S_8) \sum_n \tilde{Z}_{2n} + 2n_{D_1} n_{D_2} (I_8 - C_8) \sum_n \tilde{Z}_{2(n+1/2)} \right] \tilde{\Lambda}_0^{(3)} \\
& + 14 [2n_N n_{D_1} (Q_S + Q_C) + 2n_N n_{D_2} (Q'_S + Q'_C)] \eta^2 \theta_4^2 \\
& + [12R_N^2 (Q_O - Q_V + Q'_O - Q'_V) + R_{D_2}^2 (Q_O - Q_V) + R_{D_1}^2 (Q'_O - Q'_V)] \eta^2 \theta_2^2 \\
& + 14 [2R_N R_{D_2} (Q'_S - Q'_C) + 2R_N R_{D_1} (Q_S - Q_C)] \eta^2 \theta_3^2 \quad (51)
\end{aligned}$$

while the Möbius amplitude in the direct channel reads:

$$\begin{aligned}
\mathcal{M} = & -n_N 4(\hat{V}_8 \sum_m Z_{2m} - \hat{S}_8 \sum_m Z_{2(m+1/2)}) \Lambda^{(3)} \\
& - n_{D_1} + n_{D_2} 4 \sum_n (\hat{V}_8 \tilde{Z}_{2n} + \hat{S}_8 (-1)^n \tilde{Z}_{2n}) \tilde{\Lambda}_0^{(3)} \\
& + \left[ n_N (\hat{V}_4 \hat{I}_4 - \hat{I}_4 \hat{V}_4) + n_{D_1} (\hat{Q}_O - \hat{Q}_V) + n_{D_2} (\hat{Q}'_O - \hat{Q}'_V) \right] \frac{\hat{\eta}^2}{\hat{\theta}_2^2} \quad (52)
\end{aligned}$$

where  $\Lambda_0^{(3)}$  denotes the origin of the Narain's lattice.

While the effect of the Scherk-Schwarz breaking of supersymmetry in the bulk was to transform the chirality of half of the twisted states into the opposite chirality through  $Q_{S,C} \rightarrow Q'_{S,C}$  [6], the effect of switching the position of the branes with that of the anti-branes corresponds, first, to a permutation  $R_{D_1} \leftrightarrow R_{D_2}$  and  $n_{D_1} \leftrightarrow n_{D_2}$ , and second, to shift  $n$  to  $n + 1$  in  $\tilde{Z}_{2n}$  of the Möbius amplitude, which changes the sign of  $\hat{S}_8 (-1)^n \tilde{Z}_{2n}$ .

The tadpole cancellation conditions can be obtained from the sum of the Klein bottle, annulus and Möbius amplitudes expressed in the transverse channel (see appendix A). There are two types of RR and NS-NS fields, the twisted and the untwisted ones. From the former we get:

$$R_N = R_{D_1} = R_{D_2} = 0, \quad (53)$$

while from the untwisted tadpoles we obtain:

$$n_N = 32, \quad n_{D_1} = 16 \quad n_{D_2} = 16. \quad (54)$$

The conditions (53) imply that the gauge groups are unitary and their Chan-Paton charge spaces can be described through ‘‘complex’’ charges:

$$\begin{aligned}
n_N = n + \bar{n}, & \quad R_N = i(n - \bar{n}), \\
n_{D_1} = m_1 + \bar{m}_1, & \quad n_{D_2} = m_2 + \bar{m}_2, \\
R_{D_1} = i(m_1 - \bar{m}_1), & \quad R_{D_2} = i(m_2 - \bar{m}_2). \quad (55)
\end{aligned}$$

The tadpole cancellation conditions (54) lead then to:

$$n = 16, \quad m_1 = 8, \quad m_2 = 8. \quad (56)$$

The massless open string spectrum can be read from the one-loop amplitude:

$$\begin{aligned} \mathcal{A}_0 + \mathcal{M}_0 = & (n\bar{n} + m_1\bar{m}_1 + m_2\bar{m}_2)V_4I_4 \\ & + \left( \frac{n(n-1)}{2} + \frac{\bar{n}(\bar{n}-1)}{2} + \frac{m_1(m_1-1)}{2} \right. \\ & + \left. \frac{\bar{m}_1(\bar{m}_1-1)}{2} + \frac{m_2(m_2-1)}{2} + \frac{\bar{m}_2(\bar{m}_2-1)}{2} \right) I_4V_4 \\ & - \left( \frac{m_1(m_1+1)}{2} + \frac{\bar{m}_1(\bar{m}_1+1)}{2} + m_2\bar{m}_2 \right) C_4C_4 \\ & - \left( \frac{m_2(m_2+1)}{2} + \frac{\bar{m}_2(\bar{m}_2+1)}{2} + m_1\bar{m}_1 \right) S_4S_4 \\ & + (n\bar{m}_1 + \bar{n}m_1)Q_s + (n\bar{m}_2 + \bar{n}m_2)Q'_s \end{aligned} \quad (57)$$

Hence, the gauge group is  $U(16)_9 \otimes U(8)_5 \otimes U(8)_{\bar{5}}$ . The  $U(16)$  arises on D9 branes while the  $U(8)_5$  and  $U(8)_{\bar{5}}$  live on D5-branes and  $\bar{D}5$ -branes located at  $y = \pi R$  and  $y = 0$ , respectively. The massless spectrum is given in Table 3.

	Representation of $U(8)_{\bar{5}} \otimes U(16)_9 \otimes U(8)_5$
gauge bosons:	$(\mathbf{64}, 1, 1) + (1, \mathbf{256}, 1) + (1, 1, \mathbf{64})$
fermions:	$(\mathbf{64}, 1, 1)_- + (1, 1, \mathbf{64})_+$ $(\mathbf{36}, \mathbf{1}, \mathbf{1})_+ + (\overline{\mathbf{36}}, \mathbf{1}, \mathbf{1})_+ + (1, 1, \mathbf{36})_- + (1, 1, \overline{\mathbf{36}})_-$ $(\bar{\mathbf{8}}, \mathbf{16}, \mathbf{1})_- + (1, \overline{\mathbf{16}}, \mathbf{8})_+$
scalars:	$4 \times [(1, \mathbf{120}, 1) + (1, \overline{\mathbf{120}}, 1)]$ $4 \times [(\mathbf{28}, \mathbf{1}, \mathbf{1}) + (\overline{\mathbf{28}}, \mathbf{1}, \mathbf{1}) + (1, 1, \mathbf{28}) + (1, 1, \overline{\mathbf{28}})]$ $2 \times [(\bar{\mathbf{8}}, \mathbf{16}, \mathbf{1}) + (1, \overline{\mathbf{16}}, \mathbf{8})]$

Table 3: Massless spectrum in six dimensions. The indices  $+$  and  $-$  of fermions represent the six-dimensional chiralities.

Let us enumerate the main features of the spectrum. First, we discuss the quantum numbers of the D9 brane fermions. These are not modified by the interchange of branes with anti-branes. As discussed in section 3, the

Scherk-Schwarz boundary conditions keep only those with one chirality at  $y = 0$  and those with the opposite chirality at  $y = \pi R$ . Our convention is to denote the chirality of (even) gauginos at  $y = 0$  by  $+$  and the one of (odd) gauginos at  $y = \pi R$  by  $-$ , as done in Tables 2 and 3. The supersymmetry transformations allow to derive the chiralities of all other bulk fermions, that can be also read off from the partition function. They are listed in Table 2. As a consequence of the fact that hypermultiplets and gauginos should have opposite chirality, the ND fermions have chirality  $-$  at  $y = 0$  and  $+$  at  $y = \pi R$ . Note also that these ND states appear as half of hypermultiplets because of the pseudo-reality condition.

Second, we discuss the quantum numbers of massless states living on the D5 and  $\overline{\text{D5}}$  branes. The chirality flip of the fermions when the position of the branes and anti-branes are interchanged is described in Table 2. There is no more (linear) supersymmetric couplings between the bulk states (as the gravitinos) and those on the D5 and  $\overline{\text{D5}}$  branes. A more striking signal of supersymmetry breaking is that matter fermions and scalars do not form anymore hypermultiplets. While the scalars still transform in the anti-symmetric representation of the gauge group, the fermions transform now in the symmetric representation. This is a consequence of a sign change in the Möbius amplitude due to the new orientifold projection.

Third, the supersymmetries  $Q_e$  and  $Q_o$  are non-linearly realized on the  $\overline{\text{D5}}$  and D5 branes located at  $y = 0$  and  $y = \pi R$ , respectively. We focus here on  $y = 0$  boundary, the situation at  $y = \pi R$  being similar and can be obtained by a chirality flip. Note first that at  $y = 0$ ,  $Q_o$  is broken by the orbifold and does not play any role. Note also that although there are fermions in the adjoint representation of  $U(8)$ , these do not have the proper chirality to be the gauginos superpartners of the gauge bosons of  $U(8)$  under  $Q_e$ . In fact, their representation is reducible, as  $64 = 63 + 1$  which corresponds to the gauge group decomposition  $U(8) \rightarrow SU(8) \otimes U(1)$ . The singlet fermion can be identified with the Goldstino of the non-linearly realized supersymmetry  $Q_e$  on the  $\overline{\text{D5}}$  world-volume, from the point of view of the 55 matter fermions living on the 5-branes. Its associated Chan-Paton factor is given by  $\frac{1}{\sqrt{8V_5}} \mathbf{1}_8$  with  $\mathbf{1}_8$  representing the identity matrix of rank 8. The decay constant of the Goldstino can be computed from the scattering of two Goldstinos with two matter fermions on the  $\overline{\text{D5}}$  branes and equals  $8\tau_5$  with  $\tau_5 = \frac{1}{16\pi^4} \frac{M_s^6}{g_s}$  the  $\overline{\text{D5}}$  brane tension. When the matter fermions are 95 strings (localized on the brane intersection), the Goldstino appears as a linear combination of 99 and 55 states, as we described in section 2.

Finally, as a consistency check, one can compute the anomaly polynomial:

$$A = \frac{1}{4}(tr F_5^2 - tr F_5^2)(tr F_9^2 - \frac{1}{2}tr R^2) \quad (58)$$

whose factorized form allows for a generalized Green-Schwarz anomaly cancellation mechanism [30, 31, 32], where the gauge anomaly is cancelled by adding a Wess-Zumino counterterm in the Lagrangian [31].

## 6 Chiral four-dimensional example

Starting from the six-dimensional model of section 5, it is possible to construct a chiral four-dimensional descendant. As an example, a compactification on  $T^2$  followed by an additional orbifolding by a  $\mathbb{Z}_3$  discrete symmetry leads to a chiral type IIB orientifold on  $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_3) \equiv T^6/\mathbb{Z}'_6$  [7] whose the Yang-Mills and matter field content will be derived below.

Consider the ten-dimensional type IIB string theory compactified on  $T^6/\mathbb{Z}'_6$ . The internal  $T^6$  dimensions  $\{x^4, \dots, x^9 \equiv y\}$  are paired into three complex variables,  $z_1 = x^4 + ix^5$ ,  $z_2 = x^6 + ix^7$  and  $z_3 = x^8 + iy$ . The actions of the discrete groups  $\mathbb{Z}_2$  and  $\mathbb{Z}_3$  on the space-time coordinates are defined as :

$$\mathbb{Z}_2 : \quad z_1 \rightarrow z_1, \quad z_2 \rightarrow -z_2, \quad z_3 \rightarrow -z_3 \quad (59)$$

$$\mathbb{Z}_3 : \quad z_1 \rightarrow \omega^{-1}z_1, \quad z_2 \rightarrow \omega z_2, \quad z_3 \rightarrow z_3 \quad (60)$$

where  $\omega = e^{\frac{2i\pi}{3}}$ . The  $\mathbb{Z}_2$  action leads to the six-dimensional model of section 5. Since the additional  $\mathbb{Z}_3$  projection does not introduce any new D-brane sector, the four-dimensional open string spectrum can be obtained by an appropriate truncation of the spectrum of the six-dimensional  $T^4/\mathbb{Z}_2$  model, we constructed in the previous section, upon a further toroidal compactification on  $T^2$  to four dimensions. Besides its space-time action (60),  $\mathbb{Z}_3$  acts also on the Chan-Paton factors in a way dictated by tadpole cancellation conditions [33, 7].

On the one hand, the model contains D9 branes that extend in the whole bulk. The associated DD string fermionic modes feel the Scherk-Schwarz breaking and acquire mass shifts by  $1/2R$ . On the other hand, it contains  $\overline{\text{D5}}$  and D5 branes located at  $y = 0$  and  $y = \pi R$  respectively, with world-volumes transverse to the  $y$  direction. As a consequence, the massless modes arising from open strings with at least one end on the D5 or  $\overline{\text{D5}}$  branes are unaffected at tree-level. Performing a  $T$ -duality along the coordinate  $z_1$ , this system is transformed into a configuration of D7-branes extending in  $z_2$



and  $z_3 = x^8 + iy$ , together with  $\overline{\text{D3}}$  and D3 branes located at  $y = 0$  and  $y = \pi R$ , respectively. The Scherk-Schwarz breaking acts now, besides the closed string bulk, on the fermions propagating on the world-volume of the D7-branes.

From the  $\mathbb{Z}_3$  action on the Chan-Paton factors, we can derive its action on the fundamental representations of  $U(16)$  and  $U(8)$  and furthermore on all states. Indeed, all other representations can be obtained from tensor products of fundamentals with anti-fundamentals. Labelling the representations with a subscript  $\omega^k$ ,  $k = 0, \pm 1$  that indicates how they transform under  $\mathbb{Z}_3$ , we have:

$$\begin{aligned} \mathbf{16} &= (\mathbf{4}, \mathbf{1}, \mathbf{1})_\omega + (\mathbf{1}, \mathbf{4}, \mathbf{1})_{\omega^{-1}} + (\mathbf{1}, \mathbf{1}, \mathbf{8})_1 \\ \mathbf{8} &= (\mathbf{2}, \mathbf{1}, \mathbf{1})_\omega + (\mathbf{1}, \mathbf{2}, \mathbf{1})_{\omega^{-1}} + (\mathbf{1}, \mathbf{1}, \mathbf{4})_1 \end{aligned} \quad (61)$$

The tensor product  $8 \otimes \overline{8}$  leads then to the following decomposition of the adjoint and 2-index symmetric and antisymmetric representations:

$$\begin{aligned} \mathbf{64} &= (\mathbf{4}, \mathbf{1}, \mathbf{1})_1 + (\mathbf{1}, \mathbf{4}, \mathbf{1})_1 + (\mathbf{1}, \mathbf{1}, \mathbf{16})_1 + (\mathbf{2}, \overline{\mathbf{2}}, \mathbf{1})_{\omega^{-1}} + (\overline{\mathbf{2}}, \mathbf{2}, \mathbf{1})_\omega + (\mathbf{2}, \mathbf{1}, \overline{\mathbf{4}})_\omega \\ &\quad + (\overline{\mathbf{2}}, \mathbf{1}, \mathbf{4})_{\omega^{-1}} + (\mathbf{1}, \mathbf{2}, \overline{\mathbf{4}})_{\omega^{-1}} + (\mathbf{1}, \overline{\mathbf{2}}, \mathbf{4})_\omega \\ \mathbf{28} &= (\mathbf{1}, \mathbf{1}, \mathbf{1})_{\omega^{-1}} + (\mathbf{1}, \mathbf{1}, \mathbf{1})_\omega + (\mathbf{1}, \mathbf{1}, \mathbf{6})_1 + (\mathbf{2}, \mathbf{2}, \mathbf{1})_1 + (\mathbf{2}, \mathbf{1}, \mathbf{4})_\omega + (\mathbf{1}, \mathbf{2}, \mathbf{4})_{\omega^{-1}} \\ \mathbf{36} &= (\mathbf{3}, \mathbf{1}, \mathbf{1})_{\omega^{-1}} + (\mathbf{1}, \mathbf{3}, \mathbf{1})_\omega + (\mathbf{1}, \mathbf{1}, \mathbf{10})_1 + (\mathbf{2}, \mathbf{2}, \mathbf{1})_1 + (\mathbf{2}, \mathbf{1}, \mathbf{4})_\omega + (\mathbf{1}, \mathbf{2}, \mathbf{4})_{\omega^{-1}} \end{aligned}$$

Combining these transformations with the  $\mathbb{Z}_3$  action on the space-time quantum numbers (60), we can identify the massless spectrum as the set of states left invariant by the product of the two actions. We find that, the  $\mathbb{Z}_3$  action breaks each of the six-dimensional gauge group factors into three subgroups:

$$\begin{aligned} \overline{\text{D5}} : \quad & U(8)_{\overline{5}} \rightarrow U(2)_{\overline{5}} \otimes U(2)_{\overline{5}} \otimes U(4)_{\overline{5}} \\ \text{D9} : \quad & U(16)_9 \rightarrow U(4)_9 \otimes U(4)_9 \otimes U(8)_9 \\ \text{D5} : \quad & U(8)_5 \rightarrow U(2)_5 \otimes U(2)_5 \otimes U(4)_5 \end{aligned} \quad (62)$$

The full open string massless spectrum is listed in Table 4. Note that each 5-brane sector contains a Pati-Salam type gauge group with two chiral families with the quantum numbers of quarks and leptons and two electroweak Higgs bosons. A three family model can be obtained by breaking for instance the three  $U(4)$  factors  $U(4)_{\overline{5}} \otimes U(4)_9 \otimes U(4)_9$  into the diagonal  $U(4)$  subgroup, using the scalar bi-fundamental representations available in the massless spectrum. As a result, one obtains an additional chiral family from the  $\overline{5}_9$  sector.

On the other hand, the massless closed string spectrum contains besides the graviton, dilaton and axion, 5 untwisted complex scalars, 18  $\mathbb{Z}_3$  twisted complex scalars and 24 twisted chiral multiplets localized in the  $y$ -direction.

$\bar{55}$	Representation of $U(2)_{\bar{5}} \otimes U(2)_{\bar{5}} \otimes U(4)_{\bar{5}}$
gauge bosons:	$(\mathbf{4}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{4}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{16})$
fermions:	$(\mathbf{4}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{4}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{16})$ $(\mathbf{3}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{10}) + (\mathbf{1}, \mathbf{1}, \bar{\mathbf{10}})$ $(\mathbf{2}, \mathbf{1}, \bar{\mathbf{4}}) + (\bar{\mathbf{2}}, \mathbf{1}, \bar{\mathbf{4}}) + (\mathbf{1}, \mathbf{2}, \mathbf{4}) + (\mathbf{1}, \bar{\mathbf{2}}, \mathbf{4})$ $(\mathbf{2}, \mathbf{2}, \mathbf{1}) + (\bar{\mathbf{2}}, \mathbf{2}, \mathbf{1}) + (\bar{\mathbf{2}}, \bar{\mathbf{2}}, \mathbf{1})$
scalars:	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \bar{\mathbf{1}}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{6}) + (\mathbf{1}, \mathbf{1}, \bar{\mathbf{6}})$ $(\mathbf{2}, \mathbf{2}, \mathbf{1}) + (\bar{\mathbf{2}}, \bar{\mathbf{2}}, \mathbf{1}) + (\mathbf{1}, \mathbf{2}, \mathbf{4}) + (\bar{\mathbf{2}}, \mathbf{1}, \bar{\mathbf{4}})$
$\bar{59}$	Representations of $U(2)_{\bar{5}} \otimes U(2)_{\bar{5}} \otimes U(4)_{\bar{5}} \otimes U(4)_9 \otimes U(4)_9 \otimes U(8)_9$
“Chiral multiplets”:	$(\mathbf{1}, \mathbf{2}, \mathbf{1}; \mathbf{1}, \mathbf{4}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{4}; \mathbf{4}, \mathbf{1}, \mathbf{1}) + (\mathbf{2}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \mathbf{8})$ $(\bar{\mathbf{2}}, \mathbf{1}, \mathbf{1}; \bar{\mathbf{4}}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \bar{\mathbf{4}}; \mathbf{1}, \bar{\mathbf{4}}, \mathbf{1}) + (\mathbf{1}, \bar{\mathbf{2}}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \bar{\mathbf{8}})$
99	Representation of $U(4)_9 \otimes U(4)_9 \otimes U(8)_9$
gauge bosons:	$(\mathbf{16}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{16}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{64})$
scalars:	$(\mathbf{6}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \bar{\mathbf{6}}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{28}) + (\mathbf{1}, \mathbf{1}, \bar{\mathbf{28}})$ $(\mathbf{4}, \mathbf{1}, \bar{\mathbf{8}}) + (\bar{\mathbf{4}}, \mathbf{1}, \bar{\mathbf{8}}) + (\mathbf{1}, \mathbf{4}, \mathbf{8}) + (\mathbf{1}, \bar{\mathbf{4}}, \mathbf{8})$ $(\mathbf{4}, \mathbf{4}, \mathbf{1}) + (\bar{\mathbf{4}}, \mathbf{4}, \mathbf{1}) + (\bar{\mathbf{4}}, \bar{\mathbf{4}}, \mathbf{1})$

Table 4: Massless spectrum in four dimensions. All the scalars are complex and the fermions left-handed. The representations with a bar on top have opposite  $U(1)$  charges to those without bar. The 55 and 59 states are deduced by simple conjugation from the  $\bar{55}$  and  $\bar{59}$  representations, respectively.

We discuss now the main aspects of supersymmetry breaking in these models. As we explained above, the effect of  $\mathbb{Z}_3$  orbifolding amounts to an additional projection that breaks half of the non-linear (or linear) supersymmetry of the six-dimensional model, in the presence (or absence) of the Scherk-Schwarz deformation. The non-linear realization of supersymmetry on the boundaries is therefore inherited from the six-dimensional model. Let us denote by  $Q_e^{(6)}$  and  $Q_o^{(6)}$  the two supersymmetric generators of the six dimensional supersymmetries non-linearly realized at  $y = 0$  and  $y = \pi R$ , respectively. Each of them has 8 spinorial degrees of freedom. The  $\mathbb{Z}_3$  action projects away half and we are left with two four-dimensional supercharges,  $Q_e^{(4)}$  and  $Q_o^{(4)}$  at  $y = 0$  and  $y = \pi R$ , respectively. This is described as:

$$Q_e^{(6)} = Q_{e,1}^{(4)} + Q_{e,2}^{(4)} \xrightarrow{\mathbb{Z}_3} Q_{e,1}^{(4)} \equiv Q_e^{(4)} \quad (63)$$

$$Q_o^{(6)} = Q_{o,3}^{(4)} + Q_{o,4}^{(4)} \xrightarrow{\mathbb{Z}_3} Q_{o,3}^{(4)} \equiv Q_o^{(4)} \quad (64)$$

It is important to stress that the non-linearly realized supersymmetries on the world-volumes of the  $\overline{\text{D5}}$  and the D5 branes located at  $y = 0$  and  $y = \pi R$ , respectively, are different:  $Q_e^{(4)} \neq Q_o^{(4)}$ . Also, as in the six-dimensional case, the fermions in the adjoint representations can not be identified with gauginos as they hold wrong chiralities.

It is now straightforward to identify the Goldstino field associated with the non-linear realization of these supersymmetries on the boundaries. For simplicity, we will restrict our discussion to the boundary  $y = 0$ . From eq. (63), we see that the Goldstino of  $Q_e^{(4)}$  is a component of the six-dimensional Goldstino of  $Q_e^{(6)}$ .

First, we remind that in six dimensions the Goldstino is identified as the singlet component in the decomposition  $64 = 63 + 1$  of the adjoint representation of  $U(8) = SU(8) \otimes U(1)$ . We denote this singlet as  $\lambda_-^{(6)}$ . Under  $\mathbb{Z}_3$ , this is decomposed as:

$$\lambda_-^{(6)} = \lambda_{-,P}^{(4)} + \lambda_{+,I}^{(4)} \quad (65)$$

where the component  $\lambda_{-,I}^{(4)}$  is projected out while  $\lambda_{-,P}^{(4)} \equiv \chi$  remains and it is identified with the Goldstino in four dimensions.

Second, note that the overall  $U(1)$  factor in six dimensions is decomposed into three  $U(1)$ 's by  $\mathbb{Z}_3$ , as:

$$\mathbf{64} \longrightarrow (\mathbf{3} + \mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{3} + \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{15} + \mathbf{1}) \quad (66)$$

where we identify three singlets  $\lambda_{\overline{3},i}^{(4)}$ ,  $i = 1, 2, 3$

$$\mathbf{4} = \mathbf{3} + (\mathbf{1} \leftarrow \lambda_{\overline{3},1}^{(4)}, \lambda_{\overline{3},2}^{(4)}) \quad (67)$$

$$\mathbf{16} = \mathbf{15} + (\mathbf{1} \leftarrow \lambda_{\overline{3},3}^{(4)}) \quad (68)$$

One linear combination of the singlets corresponds to the Goldstino field  $\chi = \lambda_{-,P}^{(4)}$ . Taking into account the normalization of the Chan-Paton generators as in section 2, the Goldstino is given by<sup>8</sup>:

$$\chi = \frac{1}{2}[\lambda_1^{(4)} + \lambda_2^{(4)} + \sqrt{2}\lambda_3^{(4)}]. \quad (69)$$

Finally, let us discuss the issue of  $U(1)$  anomalies in this model. There are nine  $U(1)$  factors whose associated charges are denoted in a self-explanatory notation by  $Q_i^\alpha$ , where  $\alpha = \bar{5}, 9, 5$  labels the different kind of branes and  $i = 1, \dots, 3$  counts the  $U(1)$ 's for given  $\alpha$ . These  $U(1)$ 's have mixed anomalies from triangular diagrams with two non-abelian gauge bosons from  $SU(2)$  or  $SU(4)$  factors. Following ref. [34], we denote by  $A_{ij}^{\alpha\beta}$  the associated anomaly coefficient  $Tr(Q_i^\alpha T_j^\beta T_j^\beta)$ , with  $T_j^\beta$  the generator of the non-abelian group  $j$  from branes of the type  $\beta$ . These anomalies can be collected in a matrix:

$$\mathbb{A} = (A_{ij}^{\alpha\beta})_{\substack{\alpha, \beta = \bar{5}, 9, 5 \\ 1 \leq i, j \leq 3}} = \begin{pmatrix} 1 & 1 & -4 & -2 & 0 & 4 & 0 & 0 & 0 \\ -1 & -1 & 4 & 0 & 2 & -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & -2 \\ 0 & 1 & -2 & 0 & 0 & 0 & 0 & -1 & 2 \\ 1 & -1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & -4 & -1 & -1 & 4 \\ 0 & 0 & 0 & 0 & -2 & 4 & 1 & 1 & -4 \\ 0 & 0 & 0 & -2 & 2 & 0 & 0 & 0 & 0 \end{pmatrix}$$

From the matrix anomaly  $\mathbb{A}$ , we can deduce the anomaly free  $U(1)$ 's which are given in an appropriate basis by the linear independent combinations:

$$\begin{aligned} & Q_1^{\bar{5}} + Q_2^{\bar{5}} + \frac{1}{2}Q_3^{\bar{5}} \\ & Q_1^9 + Q_2^9 + \frac{1}{2}Q_3^9 \\ & Q_1^5 + Q_2^5 + \frac{1}{2}Q_3^5 \\ & Q_1^{\bar{5}} - Q_2^{\bar{5}} + Q_1^5 - Q_2^5 \\ & Q_1^{\bar{5}} + Q_2^{\bar{5}} - 4Q_3^{\bar{5}} + Q_1^9 + Q_2^9 - 4Q_3^9 + Q_1^5 + Q_2^5 - 4Q_3^5 \end{aligned}$$

The remaining  $U(1)$ 's are anomalous and can be expressed in a convenient basis by:

$$\begin{aligned} & Q_1^{\bar{5}} - Q_2^{\bar{5}} - Q_1^5 + Q_2^5 \\ & Q_1^9 - Q_2^9 \\ & Q_1^{\bar{5}} + Q_2^{\bar{5}} - 4Q_3^{\bar{5}} - Q_1^5 - Q_2^5 + 4Q_3^5 \\ & Q_1^{\bar{5}} + Q_2^{\bar{5}} - 4Q_3^{\bar{5}} - 2Q_1^9 - 2Q_2^9 + 8Q_3^9 + Q_1^5 + Q_2^5 - 4Q_3^5 \end{aligned}$$

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<sup>8</sup>Moreover, one should take the linear combination of 99 and 55 states, as discussed in section 2.

All these anomalies are expected to be cancelled by a generalized Green-Schwarz mechanism, which introduces 4 axions transforming non-trivially under the corresponding anomalous gauge symmetries.

## 7 Conclusion

A simple and elegant way to break supersymmetry on D-branes is to place them on top of orientifold planes that preserve different supersymmetries. The resulting spectra are not superymmetric but supersymmetry is still realized non-linearly with a Goldstino living on the D-brane world-volume. A particular advantage of this method is the absence of tachyons. However, previous implementations of this idea suffered from a non-vanishing tree-level cosmological constant of the order of the (string scale)<sup>4</sup>, which makes any quantitative prediction and computation questionable.

In this work, we propose a solution to this problem. Our construction relies on the possibility to use at the same time a Scherk-Schwarz deformation, which introduces an additional “tiny” breaking of supersymmetry in the bulk, that vanishes in the decompactification limit. An immediate consequence of the Scherk-Schwarz boundary conditions is the introduction of an additional set of anti-branes with orientifolds, so that the total contribution to the vacuum energy vanishes.

In this paper, we have computed the four-fermion effective interaction of the Goldstino with matter fermions. In the simplest case, where matter fermions correspond to open strings with both ends on the same set of branes, we found that the Goldstino decay constant is given by the total tension of the corresponding D-branes. The 4-fermion interactions involve also a dimensionless parameter that takes two different values, depending on whether the matter fermions have the same or different internal helicity with the Goldstino.

As we have shown in explicit examples, our construction allows to obtain non-supersymmetric models with chiral spectrum and interesting gauge groups, such as Pati-Salam type with three generations of quarks and leptons. These constructions are very useful in the context of low-scale string theory with a string tension lying in the TeV region. On the other hand, more general constructions allow for the simultaneous presence of branes with non-supersymmetric world-volume and others with (tree-level) supersymmetric massless modes. In these models, there is another option that the Standard Model may reside on one of the supersymmetric branes, in which case the string scale should be much larger than the TeV, for instance at intermediate energy scales.

Our construction also provides a consistent framework for investigating properties of non-supersymmetric brane-worlds, such as threshold corrections to gauge couplings, and mediation of supersymmetry breaking. We plan to return to these issues in the near future.

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## A Notations for one-loop amplitudes

For  $10-d$  compact space dimensions, the one-loop string amplitudes  $\mathbf{T}$ ,  $\mathbf{K}$ ,  $\mathbf{A}$ ,  $\mathbf{M}$  corresponding to the torus, Klein bottle, annulus and Möbius diagrams, respectively, are given by:

$$\begin{aligned}
\mathbf{T} &= 1(4\pi^2\alpha')^{d2} \int \frac{d\tau d\bar{\tau}}{(\text{Im } \tau)^{1+d2}} \left| \frac{1}{\eta(\tau)} \right|^{2d-4} \mathcal{T}, \\
\mathbf{K} &= 1(4\pi^2\alpha')^{d2} \int_0^\infty d\tau_2 \tau_2^{1+d2} \frac{1}{[\eta(2i\tau_2)]^{d-2}} \mathcal{K} \\
&= 1(4\pi^2\alpha')^{d2} \int_0^\infty dl \frac{1}{\eta(il)^{d-2}} \tilde{\mathcal{K}}, \\
\mathbf{A} &= 1(8\pi^2\alpha')^{d2} \int_0^\infty dt t^{1+d2} \frac{1}{[\eta(\frac{it}{2})]^{d-2}} \mathcal{A} \\
&= 1(8\pi^2\alpha')^{d2} \int_0^\infty dl \frac{1}{\eta(il)^{d-2}} \tilde{\mathcal{A}}, \\
\mathbf{M} &= 1(8\pi^2\alpha')^{d2} \int_0^\infty dt t^{1+d2} \frac{1}{[\hat{\eta}(\frac{it}{2} + \frac{1}{2})]^{d-2}} \mathcal{M} \\
&= 1(8\pi^2\alpha')^{d2} \int_0^\infty dl \frac{1}{\hat{\eta}(il + \frac{1}{2})^{d-2}} \tilde{\mathcal{M}}
\end{aligned} \tag{70}$$

where  $\alpha' = M_s^{-2}$ . The integral over the modular parameter  $\tau = \tau_1 + i\tau_2$  in the torus amplitude is performed over the fundamental domain:

$$-\frac{1}{2} \leq \tau_1 \leq \frac{1}{2}, \tau_2 \geq 0, |\tau| \geq 1 \tag{71}$$

The direct (open string) channel amplitudes  $\mathcal{T}$ ,  $\mathcal{K}$ ,  $\mathcal{A}$  and  $\mathcal{M}$  are given as functions of the conformal field theory characters and the compactification lattice sums defined below. The corresponding expressions in the transverse (closed string) channel  $\tilde{\mathcal{K}}$ ,  $\tilde{\mathcal{A}}$  and  $\tilde{\mathcal{M}}$  are obtained by the following transformations on the integration variables

$$\mathbf{K} : \quad 2\tau_2 \xrightarrow{S} \frac{1}{2\tau_2} \equiv l \quad (72)$$

$$\mathbf{A} : \quad \frac{t}{2} \xrightarrow{S} \frac{2}{t} \equiv l \quad (73)$$

$$\mathbf{M} : \quad it2 + 12 \xrightarrow{P} i2t + 12 \equiv il + 12 . \quad (74)$$

and allow to express one-loop open string diagrams as tree-level closed string ones. The conformal field theory characters are given by

$$\chi_r = q^{h_r - c24} \sum_{n=0}^{\infty} d_n^r q^n , \quad (75)$$

where  $h_r$  is the conformal weight,  $c$  is the central charge of the conformal field theory and the  $d_n^r$  are positive integers. The hatted characters are defined as:

$$\hat{\chi}_r \left( il + \frac{1}{2} \right) = e^{-i\pi h_r} \chi_r \left( il + \frac{1}{2} \right) . \quad (76)$$

The above characters can be expressed using the Dedekind  $\eta$  function

$$\eta(\tau) = q^{124} \prod_{n=1}^{\infty} (1 - q^n) , \quad (77)$$

and the Jacobi  $\theta$  theta functions with general characteristic  $(\alpha, \beta)$ :

$$\vartheta[\alpha\beta](z, \tau) = \sum_{n \in \mathbb{Z}} e^{i\pi\tau(n-\alpha)^2} e^{2\pi i(z-\beta)(n-\alpha)} , \quad (78)$$

where  $q = e^{2\pi i\tau}$ . We use the notation:

$$\vartheta_1(z, \tau) \equiv \vartheta[1212](z, \tau), \quad \vartheta_2(z, \tau) \equiv \vartheta[120](z, \tau), \quad (79)$$

$$\vartheta_3(z, \tau) \equiv \vartheta[00](z, \tau), \quad \vartheta_4(z, \tau) \equiv \vartheta[012](z, \tau) \quad (80)$$

In the orbifold models we consider in this work, the partition function can be expressed in terms of the  $SO(2n)$  characters

$$\begin{aligned} I_{2n} &= 12\eta^n(\theta_3^n + \theta_4^n) , & V_{2n} &= 12\eta^n(\theta_3^n - \theta_4^n) , \\ S_{2n} &= 12\eta^n(\theta_2^n + i^n\theta_1^n) , & C_{2n} &= 12\eta^n(\theta_2^n - i^n\theta_1^n) \end{aligned} \quad (81)$$

At the lowest level,  $I_{2n}$  represents a scalar,  $V_{2n}$  a vector, while  $S_{2n}, C_{2n}$  represent spinors of opposite chiralities. The transformations  $S$  and  $P$  defined in eq. (74) from direct to transverse channels act on the vector  $\{I_{2n}, V_{2n}, S_{2n}, C_{2n}\}$  through the matrices:

$$S_{(2n)} = 12 \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & i^{-n} & -i^{-n} \\ 1 & -1 & -i^{-n} & i^{-n} \end{pmatrix}, \quad P_{(2n)} = \begin{pmatrix} c & s & 0 & 0 \\ s & -c & 0 & 0 \\ 0 & 0 & \zeta c & i\zeta s \\ 0 & 0 & i\zeta s & \zeta c \end{pmatrix} \quad (82)$$

where  $c = \cos(n\pi/4)$ ,  $s = \sin(n\pi/4)$  and  $\zeta = e^{-in\pi/4}$ .

Useful combinations of these characters in the case of compactifications on  $T^4/\mathbb{Z}_2$  are:

$$\begin{aligned} Q_o &= V_4 I_4 - C_4 C_4, & Q_v &= I_4 V_4 - S_4 S_4, \\ Q_s &= I_4 C_4 - S_4 I_4, & Q_c &= V_4 S_4 - C_4 V_4, \end{aligned} \quad (83)$$

and

$$\begin{aligned} Q'_o &= V_4 I_4 - S_4 S_4, & Q'_v &= I_4 V_4 - C_4 C_4, \\ Q'_s &= I_4 S_4 - C_4 I_4, & Q'_c &= V_4 C_4 - S_4 V_4, \end{aligned} \quad (84)$$

Here the first factor refers to the six-dimensional space-time (in the light-cone gauge), while the second refers to the internal compact space.

## B Compactification lattice summations

The contributions to the one-loop amplitudes from the compactification lattice can be expressed as function of the series  $Z_{m+a, n+b}$ . For instance, the torus amplitude  $\mathcal{T}$  can be expressed as function of:

$$Z_{m+a, n+b}(\tau, \bar{\tau}) = 1 |\eta(\tau)|^2 q^{(m+aR+n+bR2)^2} \bar{q}^{(m+aR-n+bR2)^2} \quad (85)$$

The other one-loop diagrams are expressed as functions of Kaluza-Klein and winding lattice summations

$$Z_{m+a}(\tau) = \frac{q^{12(\frac{m+a}{R})^2}}{\eta(\tau)}, \quad \tilde{Z}_{n+b}(\tau) = \frac{q^{12((n+b)R2)^2}}{\eta(\tau)}. \quad (86)$$

Under Poisson resummation we have

$$\sum_m e^{2i\pi mb} Z_{m+a}(-1\tau) = R e^{-2i\pi ab} \sum_n e^{-2i\pi na} \tilde{Z}_{2n+2b}(\tau) \quad (87)$$



$$\sum_{m,n} Z_{m,n}(\tau, \bar{\tau}) = R\tau_2^{-1/2} \sqrt{2} |\eta(\tau)|^2 \sum_{\tilde{m},n} e^{-\pi R^2 2\tau_2 |\tilde{m}+n\tau|^2} \quad (88)$$

It is sometimes convenient to use the projected lattice sums

$$\begin{aligned} E_0 &= \sum_{m,n} 1 + (-1)^m 2Z_{m,n} , & O_0 &= \sum_{m,n} 1 - (-1)^m 2Z_{m,n} , \\ E_{1/2} &= \sum_{m,n} 1 + (-1)^m 2Z_{m,n+1/2} , & O_{1/2} &= \sum_{m,n} 1 - (-1)^m 2Z_{m,n+1/2} , \end{aligned}$$

and

$$\begin{aligned} E'_0 &= \sum_{m,n} 1 + (-1)^n 2Z_{m,n} , & O'_0 &= \sum_{m,n} 1 - (-1)^n 2Z_{m,n} , \\ E'_{1/2} &= \sum_{m,n} 1 + (-1)^n 2Z_{m+1/2,n} , & O'_{1/2} &= \sum_{m,n} 1 - (-1)^n 2Z_{m+1/2,n} , \end{aligned}$$

where  $E$  ( $E'$ ) and  $O$  ( $O'$ ) refer correspondingly to even and odd KK momenta (windings) and the subscripts 0 and 1/2 refer to unshifted and shifted winding (KK momenta). The primed sums are obtained from the unprimed ones through interchange of  $m$  and  $n$ . In the case of  $D$  compact dimensions we will use the notation:

$$\begin{aligned} \Lambda^{(D)} &= \prod_{i=1}^D \sum_{m_i} Z_{m_i} & \tilde{\Lambda}^{(D)} &= \prod_{i=1}^D \sum_{n_i} \tilde{Z}_{n_i} \\ \Lambda^{(D,D)} &= \prod_{i=1}^D \sum_{m_i, n_i} Z_{m_i, n_i} \end{aligned} \quad (89)$$

## C Supersymmetric one-loop partition functions in 9D and 6D

For comparison, we provide the reader with the partition functions in the supersymmetric cases in 9D and 6D. With our conventions, the amplitudes for the supersymmetric nine-dimensional theory obtained by compactification of the  $SO(32)$  type I strings on a circle are given by:

$$\begin{aligned} \mathcal{T} &= |V_8 - S_8|^2 \sum_{m,n} Z_{m,n} , & \mathcal{K} &= 12(V_8 - S_8) \sum_m Z_m , \\ \mathcal{A} &= 12(V_8 - S_8) \sum_{i,j,m} Z_{2(m+a_i+a_j)} , & \mathcal{M} &= -12(\hat{V}_8 - \hat{S}_8) \sum_{i,m} Z_{2(m+2a_i)} . \end{aligned}$$

which corresponds to two  $O8$  orientifold planes with Ramond-Ramond (RR) charge  $-16$  and  $32$   $D8$ -branes each carrying a unit of RR charge.

For the supersymmetric six-dimensional  $T^4/\mathbb{Z}_2$  we have:

$$\begin{aligned}
\mathcal{T} &= 12\Lambda^{(4,4)}|V_8 - S_8|^2 + \frac{1}{2}|Q_o - Q_v|^2 \left| \frac{2\eta}{\theta_2} \right|^4 + \frac{1}{2}|Q_s + Q_c|^2 \left| \frac{2\eta}{\theta_4} \right|^4 \\
&\quad + \frac{1}{2}|Q_s - Q_c|^2 \left| \frac{2\eta}{\theta_3} \right|^4, \\
\mathcal{K} &= \frac{1}{4} \left\{ (Q_o + Q_v)(\Lambda^{(4)} + \tilde{\Lambda}^{(4)}) + 32(Q_s + Q_c) \left( \frac{\eta}{\theta_4} \right)^2 \right\}, \\
\mathcal{A} &= \frac{1}{4} \left\{ (V_4 O_4 + O_4 V_4 - C_4 C_4 - S_4 S_4) \left[ n_N^2 \Lambda^{(4)} + n_D^2 \tilde{\Lambda}^{(4)} \right] \right. \\
&\quad + (V_4 O_4 - O_4 V_4 - C_4 C_4 + S_4 S_4) \left( \frac{2\eta}{\vartheta_2} \right)^2 (R_N^2 + R_D^2) \\
&\quad + 2(O_4 C_4 + V_4 S_4 - S_4 O_4 - C_4 V_4) \left( \frac{\eta}{\vartheta_4} \right)^2 n_N n_D \\
&\quad \left. + 2(O_4 C_4 - V_4 S_4 - S_4 O_4 + C_4 V_4) \left( \frac{\eta}{\vartheta_3} \right)^2 R_N R_D \right\}, \\
\mathcal{M} &= -\frac{1}{4} \left\{ (\hat{V}_4 \hat{O}_4 + \hat{O}_4 \hat{V}_4 - \hat{C}_4 \hat{C}_4 - \hat{S}_4 \hat{S}_4) \left[ n_N \Lambda^{(4)} + n_D \tilde{\Lambda}^{(4)} \right] \right. \\
&\quad \left. - (\hat{V}_4 \hat{O}_4 - \hat{O}_4 \hat{V}_4 - \hat{C}_4 \hat{C}_4 + \hat{S}_4 \hat{S}_4) \left( \frac{2\hat{\eta}}{\hat{\vartheta}_2} \right)^2 (n_N + n_D) \right\}
\end{aligned}$$

where the tadpole cancellation requires  $n_N = n_D = 32$  and  $R_N = R_D = 0$ .

## D One-loop vacuum amplitudes of the 6D model in the transverse channel

The one-loop Klein bottle amplitude in the transverse channel is given by :

$$\begin{aligned}
\tilde{\mathcal{K}} &= \tilde{\mathcal{K}}_0 + \frac{2^5}{4} (\sqrt{v_4})^2 \left[ \sum_n \tilde{Z}_{2n} \tilde{\Lambda}_e^{(3)} \right]' (V_4 I_4 + I_4 V_4 - S_4 S_4 - C_4 C_4) \\
&\quad + \frac{2^5}{4} \left( \frac{1}{\sqrt{v_4}} \right)^2 \left[ \sum_n Z_{2n} \Lambda_e^{(3)} \right]' (V_4 I_4 + I_4 V_4) \\
&\quad - \frac{2^5}{4} \left( \frac{1}{\sqrt{v_4}} \right)^2 \left[ \sum_n Z_{2n+1} \Lambda_e^{(3)} \right]' (S_4 S_4 + C_4 C_4)
\end{aligned}$$

with [ ]' means that the Narain's lattice is not included. The  $v_4$  is the volume of the compact space, in  $\Lambda_e^{(3)}$  and  $\tilde{\Lambda}_e^{(3)}$  the lattice sums are restricted to even values of momenta and winding, respectively. The contribution  $\tilde{\mathcal{K}}_0$  from the origin of Narain lattice is :

$$\begin{aligned}\tilde{\mathcal{K}}_0 &= \frac{2^5}{4}(\sqrt{v_4} + \frac{1}{\sqrt{v_4}})^2(V_4 I_4(I_4 I_4)_B + I_4 V_4(V_4 V_4)_B) \\ &\quad + \frac{2^5}{4}(\sqrt{v_4} - \frac{1}{\sqrt{v_4}})^2(V_4 I_4(V_4 V_4)_B + I_4 V_4(I_4 I_4)_B) \\ &\quad - \frac{2^5}{4}(\sqrt{v_4})^2(S_4 S_4 + C_4 C_4)(I_4 I_4 + V_4 V_4)_B\end{aligned}$$

where the characters with label  $B$  correspond to:

$$\begin{aligned}(I_4 I_4 + V_4 V_4)_B &= \left[ \sum_n \tilde{Z}_{2n} \tilde{\Lambda}_e^{(3)} \right]_0 = \left[ \sum_n Z_{2n} \Lambda_e^{(3)} \right]_0, \\ (I_4 I_4 - V_4 V_4)_B &= 4 \frac{\eta^2}{\theta_2^2}, \quad (Q_S + Q_C)_B = 4 \frac{\eta^2}{\theta_4^2}, \quad (Q_S - Q_C)_B = 4 \frac{\eta^2}{\theta_3^2}\end{aligned}$$

In the transverse channel, the annulus amplitude is :

$$\begin{aligned}2^7 \tilde{\mathcal{A}} &= 2^7 \tilde{\mathcal{A}}_0 + (\sqrt{v_4})^2 n_N^2 (V_4 I_4 + I_4 V_4 - S_4 S_4 - C_4 C_4) \left[ \sum_n \tilde{Z}_{2n} \tilde{\Lambda}^{(3)} \right]' \\ &\quad + (\sqrt{v_4})^2 n_N^2 (I_4 I_4 + V_4 V_4 - S_4 C_4 - C_4 S_4) \left[ \sum_n \tilde{Z}_{2n+1} \tilde{\Lambda}^{(3)} \right]' \\ &\quad + \left( \frac{1}{\sqrt{v_4}} \right)^2 (n_{D_1} + n_{D_2})^2 (V_4 I_4 + I_4 V_4) \left[ \sum_n \tilde{Z}_{2n} \tilde{\Lambda}^{(3)} \right]' \\ &\quad - \left( \frac{1}{\sqrt{v_4}} \right)^2 (n_{D_1} - n_{D_2})^2 (S_4 S_4 + C_4 C_4) \left[ \sum_n \tilde{Z}_{2n} \tilde{\Lambda}^{(3)} \right]' \\ &\quad + \left( \frac{1}{\sqrt{v_4}} \right)^2 (n_{D_1} - n_{D_2})^2 (V_4 I_4 + I_4 V_4) \left[ \sum_n \tilde{Z}_{2n+1} \tilde{\Lambda}^{(3)} \right]' \\ &\quad - \left( \frac{1}{\sqrt{v_4}} \right)^2 (n_{D_1} + n_{D_2})^2 (S_4 S_4 + C_4 C_4) \left[ \sum_n \tilde{Z}_{2n+1} \tilde{\Lambda}^{(3)} \right]'\end{aligned}$$

where  $\tilde{\mathcal{A}}_0$  corresponds to the contribution from the origin of the Narain's lattice :

$$\begin{aligned}
2^7 \tilde{\mathcal{A}}_0 &= (\sqrt{v_4} n_N + \frac{n_{D_1} + n_{D_2}}{\sqrt{v_4}})^2 (V_4 I_4 (I_4 I_4)_B + I_4 V_4 (V_4 V_4)_B) \\
&+ (\sqrt{v_4} n_N - \frac{n_{D_1} + n_{D_2}}{\sqrt{v_4}})^2 (V_4 I_4 (V_4 V_4)_B + I_4 V_4 (I_4 I_4)_B) \\
&- (\sqrt{v_4} n_N + \frac{n_{D_1} - n_{D_2}}{\sqrt{v_4}})^2 (C_4 C_4 (I_4 I_4)_B + S_4 S_4 (V_4 V_4)_B) \\
&- (\sqrt{v_4} n_N - \frac{n_{D_1} - n_{D_2}}{\sqrt{v_4}})^2 (C_4 C_4 (V_4 V_4)_B + S_4 S_4 (I_4 I_4)_B) \\
&+ \left[ \frac{(R_N + 4R_{D_1})^2}{4} + \frac{7R_N^2}{4} \right] (Q_S Q_{SB} + Q_C Q_{CB}) \\
&+ \left[ \frac{(R_N - 4R_{D_1})^2}{4} + \frac{7R_N^2}{4} \right] (Q_S Q_{CB} + Q_C Q_{SB}) \\
&+ \left[ \frac{(R_N + 4R_{D_2})^2}{4} + \frac{7R_N^2}{4} \right] (Q'_S Q_{SB} + Q'_C Q_{CB}) \\
&+ \left[ \frac{(R_N - 4R_{D_2})^2}{4} + \frac{7R_N^2}{4} \right] (Q'_S Q_{CB} + Q'_C Q_{SB})
\end{aligned}$$

Furthermore, for the Möbius transverse channel, we found :

$$\begin{aligned}
-2\tilde{\mathcal{M}} &= -2\tilde{\mathcal{M}}_0 + n_N v_4 (\hat{V}_4 \hat{I}_4 + \hat{I}_4 \hat{V}_4) \left[ \sum_n (\tilde{Z}_{4n} + \tilde{Z}_{4n+2}) \tilde{\Lambda}_e^{(3)} \right]' \\
&- n_N v_4 (\hat{S}_4 \hat{S}_4 + \hat{C}_4 \hat{C}_4) \left[ \sum_n (\tilde{Z}_{4n} - \tilde{Z}_{4n+2}) \tilde{\Lambda}_e^{(3)} \right]' \\
&+ \frac{n_{D_1} + n_{D_2}}{v_4} (\hat{V}_4 \hat{I}_4 + \hat{I}_4 \hat{V}_4) \left[ \sum_n Z_{2n} \Lambda_e^{(3)} \right]' \\
&+ \frac{n_{D_1} + n_{D_2}}{v_4} (\hat{S}_4 \hat{S}_4 + \hat{C}_4 \hat{C}_4) \left[ \sum_n Z_{2n+1} \Lambda_e^{(3)} \right]'
\end{aligned}$$

with the contribution from the origin  $\tilde{\mathcal{M}}_0$  given by :

$$\begin{aligned}
-2\tilde{\mathcal{M}}_0 &= (\sqrt{v_4} + \frac{1}{\sqrt{v_4}})(\sqrt{v_4}n_N + \frac{n_{D_1} + n_{D_2}}{\sqrt{v_4}})(\hat{V}_4\hat{I}_4(\hat{I}_4\hat{I}_4)_B + \hat{I}_4\hat{V}_4(\hat{V}_4\hat{V}_4)_B) \\
&+ (\sqrt{v_4} - \frac{1}{\sqrt{v_4}})(\sqrt{v_4}n_N - \frac{n_{D_1} + n_{D_2}}{\sqrt{v_4}})(\hat{V}_4\hat{I}_4(\hat{V}_4\hat{V}_4)_B + \hat{I}_4\hat{V}_4(\hat{I}_4\hat{I}_4)_B) \\
&- \sqrt{v_4}(\sqrt{v_4}n_N - \frac{n_{D_1} - n_{D_2}}{\sqrt{v_4}})(\hat{C}_4\hat{C}_4(\hat{V}_4\hat{V}_4)_B + \hat{S}_4\hat{S}_4(\hat{I}_4\hat{I}_4)_B) \\
&- \sqrt{v_4}(\sqrt{v_4}n_N + \frac{n_{D_1} - n_{D_2}}{\sqrt{v_4}})(\hat{C}_4\hat{C}_4(\hat{I}_4\hat{I}_4)_B + \hat{S}_4\hat{S}_4(\hat{V}_4\hat{V}_4)_B).
\end{aligned}$$

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