Transport theory and low energy properties of colour superconductors

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The one-loop polarisation tensor and the propagation of "in-medium" photons of colour superconductors in the 2SC and CFL phase is discussed. For a study of thermal corrections to the low energy effective theory in the 2SC phase, a classical transport theory for fermionic quasiparticles is invoked.

1. Introduction

Quantum chromodynamics (QCD) under extreme conditions displays a very rich phase structure. At low temperature and high baryonic density, quarks form Cooper pairs due to attractive interactions amongst them. The diquark condensate modifies the ground state of QCD and leads to the phenomenon of colour superconductivity [1]. This phase is typically characterised by the Anderson-Higgs mechanism and an energy gap associated to the fermionic quasiparticles. It is expected that a colour superconducting state of matter is realized in compact stars.

For two light quark flavours the diquark condensate breaks the $SU_c(3)$ group down to $SU_c(2)$ (2SC phase) [1]. Five gluons acquire a mass, while three gluons remain massless and exhibit confinement. Furthermore, quarks of one colour remain ungapped, as they do not participate in the condensation. A global axial $U_A(1)$ is broken at asymptotically large densities, leading to a pseudo Nambu-Goldstone mode, similar to the η' meson. For three light quark flavours the condensates lock the colour and flavour symmetry (CFL phase) [1]. Colour, chiral and baryon number symmetry are spontaneously broken. All gluons become massive and all quarks are gapped. The light degrees of freedom are the Nambu-Goldstone bosons – eight associated to the breaking of chiral symmetry, one associated to the breaking of baryon number symmetry, and, at asymptotically large densities, an extra pseudo Nambu-Goldstone boson associated to the breaking of $U_A(1)$ – which dominate the long distance physics of the superconductor.

In this contribution, we discuss results obtained in collaboration with C. Manuel in [2,3]. First, we include electromagnetic interactions and study the one-loop photon polarisation tensor in colour superconductors [2]. Second, we discuss a semi-classical transport theory for quark quasiparticles in the 2SC phase [3]. This is a convenient starting point for both the construction of low energy effective theories or the computation of transport coefficients of dense quark matter.

2. Photons in colour superconductors

A diquark condensate breaks, apart from the colour and flavour symmetry, also the $U(1)$ symmetry of electromagnetism [1]. However, the linear combination $\widetilde{A}$ of the standard photon and the eighth gluon remains massless,

$$\widetilde{A} = -\sin\theta\, G^8 + \cos\theta\, A\,, \qquad \widetilde{G}^8 = \cos\theta\, G^8 + \sin\theta\, A\,, \qquad \widetilde{e} = e\cos\theta\,. \tag{1}$$

The field $\widetilde{A}$ plays the role of the "in-medium" photon in the superconductor, with the in-medium gauge coupling $\widetilde{e}$. Hence, the usual $U(1)$ charge generator Q is replaced by the in-medium generator $\widetilde{Q}$ [1]. Notice also that the rotation angle θ differs for the 2SC and CFL phases,

$$\cos\theta_{\rm CFL} = g\,(g^2 + \tfrac{4}{3}e^2)^{-1/2}\,, \qquad \cos\theta_{\rm 2SC} = g\,(g^2 + \tfrac{1}{3}e^2)^{-1/2}\,. \tag{2}$$

In the CFL phase, the charged particles are the Goldstone bosons $\pi^\pm$, $K^\pm$, four gluons, and four quarks, all with $\widetilde{Q}$-charge $\pm\widetilde{e}$. The condensates are $\widetilde{Q}$-neutral. The spectrum contains the same number of positively and negatively charged particles. In consequence, quark matter in the CFL phase is $\widetilde{Q}$-neutral. This holds also in the presence of K^0 condensates. CFL matter is also Q-neutral, even for a non-vanishing strange quark mass or chemical potential [4]. No further charge carriers are needed to make the system electrically neutral. In the 2SC phase, four gluons and four quarks have half integer charges $\pm\frac{1}{2}\widetilde{e}$, while the up quark which is not participating in the condensate has the $\widetilde{Q}$-charge $\widetilde{e}$. Additional heavy strange quarks have the $\widetilde{Q}$-charge $-\frac{1}{2}\widetilde{e}$. Again, the condensates are electrically neutral. $\widetilde{Q}$-neutrality of the full system requires additional negative charges, either as a background of heavy strange quarks or electrons [1].

The photon self energy, to one-loop order, is given by all one-loop diagrams with two external photon lines and internal loops of charged particles. The finite parts of heavy gluon loops are suppressed in the infrared, as are those from the light pions and kaons. Hence, photon polarisation effects are entirely dominated in the infrared limit by the quark loops [2]. A simple physical picture has emerged from the explicit computation of the photon polarisation tensor [2]. For intermediate photon momenta $\Delta \ll p_0, p \ll \mu$, the polarisation tensor due to gapped quarks reduces to its well-known hard-dense-loop counterpart in the normal phase. This is easily understood. The photon wavelength is larger than the size of the Cooper pair. Hence, the photons scatter off the gapped quarks just as they would do in the phase without condensation. They are Landau damped, static magnetic fields remain unscreened while static electric fields are effectively Debye screened with a mass of the order $\sim \widetilde{e}\mu$, due to all charged quarks. In the 2SC phase, an additional contribution to the Debye mass arises for non-vanishing electron chemical potential. For photons in this momentum regime, and apart from numerical differences for the value of the Debye mass and the couplings, the 2SC and CFL phases look qualitatively the same.

The situation changes significantly for photons with low momenta $p_0, p \ll \Delta$. Here, their Compton wave length is too large to resolve the condensates. The condensates are electrically neutral, but they have an electrical dipole moment and hence modify the dielectric constant $\widetilde{\epsilon}$ of the medium. Condensates with vanishing spin and angular momentum ($J = 0$) have no magnetic moment. Therefore, the magnetic permeability remains unchanged. To leading order in $p_0, p \ll \Delta$, one finds for the longitudinal and transversal components of the polarisation tensor $\Pi^{\rm gap}$ arising from the gapped quarks [2]

$$\Pi_L^{\rm gap}(p_0, \mathbf{p}) = -\widetilde{\kappa}\ p^2\,, \qquad \Pi_T^{\rm gap}(p_0, \mathbf{p}) = -\widetilde{\kappa}\ p_0^2\,, \qquad \widetilde{\kappa} = \frac{c}{18\pi^2}\frac{\widetilde{e}^2\mu^2}{\Delta^2}\,. \tag{3}$$

The coefficient c counts the charges squared of all gapped quarks participating in the condensation: $c_{\rm 2SC} = 1$ and $c_{\rm CFL} = 4$. The dielectric constant of the medium is

$$\widetilde{\epsilon} = 1 + \frac{c}{18\pi^2}\frac{\widetilde{e}^2\mu^2}{\Delta^2} > 1\ . \tag{4}$$

For realistic values of the gauge coupling, the gap and the quark chemical potential, $\widetilde{\kappa}$ is of order one. It becomes very large for asymptotic large densities and fixed $\widetilde{e}$.

In the CFL phase, the low momentum limit of the polarisation tensor is solely given by (3). Due to the absence of ungapped quarks, electric fields are no longer Debye screened in the infrared limit. Since all $\widetilde{Q}$-charged hadronic excitations in the CFL phase acquire a gap for non-vanishing quark masses, the photon cannot scatter if its energy is below the energy of the lightest charged mode: CFL matter is a transparent insulator [1]. The polarisation of the medium implies that photons propagate with a velocity smaller than the velocity of light in vacuum. The refraction index of CFL matter with the normal phase is $\widetilde{n} = \widetilde{\epsilon}^{1/2} > 1$.

In the 2SC phase, the low momentum limit of the polarisation tensor is given by (3) plus an additional contribution from the charged ungapped quark and, possibly, electrons. In the static limit, they combine into $\Pi_L = -(\widetilde{m}^2 + \widetilde{\kappa}\ p^2)$ with a Debye mass due to the ungapped quark and the electrons. Therefore, photons still scatter from the charged fermions, and Debye screening, though now with a different Debye mass, persists even for low momenta: 2SC matter is an opaque conductor.

To conclude, we have seen that the propagation of photons in a colour superconducting phase is very different from the normal phase, and also different for the different colour superconducting phases. This result is an important input for a quantitative study of transport coefficients like the electrical or thermal conductivity of the medium. Although our results are based only on a one-loop computation, the corrections to the dielectric constant are small for realistic values of the parameters. Therefore, one may expect that higher loop orders do not change the qualitative picture discussed here.

3. Transport in colour superconductors

Next, we study thermal corrections to the low energy physics of a two-flavour colour superconductor [3]. For $T = 0$, an infrared effective theory has been given recently for the massless $SU(2)_c$ gauge fields [5]. The gauge field dynamics differs from the vacuum theory, because the condensates, although neutral with respect to the unbroken $SU(2)$, polarise the medium [6]. This is fully analogous to what has been found above for the photon interactions. For momenta $q \ll \Delta$, the effective theory is

$$S_{\rm eff}^{T=0}[A] = \int d^4x \left(\frac{\epsilon}{2}\,\mathbf{E}_a \cdot \mathbf{E}_a - \frac{1}{2}\,\mathbf{B}_a \cdot \mathbf{B}_a\right)\ , \tag{5}$$

where $E_i^a \equiv F_{0i}^a$ and $B_i^a \equiv \frac{1}{2}\epsilon_{ijk}F_{jk}^a$ are the $SU(2)$ electric and magnetic fields. Here, $\epsilon = 1 + g^2\mu^2/(18\pi^2\Delta^2)$ is the colour dielectric susceptibility [5]. The colour magnetic susceptibility remains unchanged. This theory is confining with a highly reduced scale of confinement $\Lambda'_{\rm QCD} \sim \Delta \exp\left(-\frac{2\sqrt{2}\pi}{11}\frac{\mu}{g\Delta}\right)$. Due to asymptotic freedom, it is expected that perturbative computations are reliable for energy scales larger than $\Lambda'_{\rm QCD}$.

At non-vanishing temperature, thermal excitations modify the low energy physics. The condensate melts at the critical temperature $T_c \approx 0.57\Delta_0$, where Δ_0 is the gap at vanishing temperature. We restrict the discussion to temperatures within $\Lambda'_{\rm QCD} \ll T < T_c$. In this regime, the main contribution stems from the thermal excitations of the gapped quarks. Those of the massless gauge fields are of the order g^2T^2 and subleading for sufficiently large μ, and the gapless quarks and the η meson do not couple to the $SU(2)$ gauge fields. We introduce a transport equation for the quark quasiparticles, which are described in terms of an on-shell one-particle phase space density $f(x, \mathbf{p}, Q)$, $x^\mu = (t, \mathbf{x})$. The distribution function depends on time, the phase space variables position $\mathbf{x}$, momentum $\mathbf{p}$, and on $SU(2)$ colour charges Q_a, with the colour index $a = 1, 2$ and 3. The quasiparticles carry $SU(2)$ colour charges simply because the constituents of the condensate do. Using natural units $k_B = \hbar = c = 1$, the on-shell condition for massless quarks $m_q = 0$ relates the energy of the quasiparticle excitation to the chemical potential and the gap as $p_0 \equiv \epsilon_p$, with

$$\epsilon_p = \sqrt{(p-\mu)^2 + \Delta^2(T)}\,, \qquad \mathbf{v}_p \equiv \frac{\partial \epsilon_p}{\partial \mathbf{p}} = \frac{|p-\mu|}{\sqrt{(p-\mu)^2+\Delta^2(T)}}\,\hat{\mathbf{p}}\,. \tag{6}$$

Hence, the quasiparticle velocity $\mathbf{v}_p$ depends on both the chemical potential and the gap. In the presence of the gap, their propagation is suppressed, $|\mathbf{v}_p| < 1$. The one-particle distribution function obeys the transport equation

$$\left[D_t + \mathbf{v}_p \cdot \mathbf{D} - gQ_a\left(\mathbf{E}^a + \mathbf{v}_p \times \mathbf{B}^a\right)\frac{\partial}{\partial \mathbf{p}}\right] f = C[f]\,. \tag{7}$$

Here, we have introduced the short-hand notation $D_\mu f \equiv [\partial_\mu - g\epsilon^{abc}Q_c A^\mu_b \partial^Q_a] f$ for the covariant derivative acting on f. The first two terms on the left-hand side of (7) combine to a covariant drift term $v^\mu_p D_\mu$, where $v^\mu_p = (1, \mathbf{v}_p)$ and $D_\mu = (D_t, \mathbf{D})$. The terms proportional to the colour electric and magnetic fields provide a force term. The right-hand side of (7) contains an unspecified collision term $C[f]$. Eq. (7) is very similar to transport equations used earlier for hot or dense QCD in the normal phase [7,8]. The main difference is the non-trivial dispersion relation (6), which leads to modified Wong equations in the present case. The thermal quasiparticles carry a $SU(2)$ charge, and provide a $SU(2)$ colour current. We introduce the colour density

$$J_a(x, \mathbf{p}) = g\int dQ Q_a f(x, \mathbf{p}, Q)\ , \tag{8}$$

where an implicit sum over species or helicity indices on f is understood. The colour measure obeys $\int dQ Q_a = 0$ and $\int dQ Q_a Q_b = C_2 \delta_{ab}$ ($C_2 = \frac{1}{2}$ for quarks in the fundamental) [8]. The induced colour current of the medium follows from (8) as $J^\mu_a(x) = \int \frac{d^3p}{(2\pi)^3} v^\mu_p J_a(x, \mathbf{p})$. It is covariantly conserved.

As an application of (7), we study the collisionless dynamics $C[f] = 0$ close to thermal equilibrium and to leading order in the gauge coupling. Consider the distribution function $f = f^{\rm eq} + gf^{(1)}$ where $f^{\rm eq} = (\exp\epsilon_p/T + 1)^{-1}$ is the fermionic equilibrium distribution function and $gf^{(1)}$ a slight deviation from equilibrium. Expanding the transport equation (7) in g, and taking the two helicities per quasiparticle into account, we find

$$[D_t + \mathbf{v}_p \cdot \mathbf{D}]\, J(x,\mathbf{p}) = g^2 N_f\, \mathbf{v}_p \cdot \mathbf{E}(x)\, \frac{df^{\rm eq}}{d\epsilon_p}\ . \tag{9}$$

This equation can be solved for $J(x,\mathbf{p})$ and gives the full colour current as a functional of the gauge fields only, $J(x) = J[A](x)$ [3]. It defines the "hard superconducting loop" effective action $\Gamma_{\rm HSL}[A]$ through $J[A] = -\delta\Gamma_{\rm HSL}[A]/\delta A$, in full analogy to the derivation of the hard thermal loop effective theory within classical transport theory [8]. Hence, the low energy effective theory for modes with $q \ll \Delta$ in the 2SC phase at finite temperature is $S^T_{\rm eff}[A] = S^{T=0}_{\rm eff}[A] + \Gamma_{\rm HSL}[A]$ to leading order in g.

Consider the thermal polarisation tensor which follows from $J[A]$ [3]. It describes Landau damping and Debye screening of the $SU(2)$ gauge fields in the 2SC phase. For low momenta, it agrees with the findings of [6], based on a field-theoretical derivation. In contrast to the normal phase, the dispersion relations for longitudinal and transverse gluons are modified. For example, the Debye mass is $m_D^2 = (2\Delta/\pi^2 T)^{1/2} g^2\mu^2 \exp -\Delta/T$ in the limit of large density and low temperature. In the general case, the Debye mass can only be computed numerically. An approximate expression for the polarisation tensor could be given in terms of an effective mean squared velocity v_* for the quasiparticles, similar to the approximation used in [9] for the normal phase.

In conclusion, we have discussed a very simple classical transport equation for quark quasiparticles in the 2SC phase. To leading order in g, it leads to the same result as a full field theoretical computation. Due to its simplicity, the present approach seems to be a good starting point for the computation of transport coefficients in the 2SC phase, like the colour conductivity. Here, the techniques of classical transport theory can be exploited [7,8,10]. It would also be interesting to provide a similar approach for the CFL phase, where the low energy degrees of freedom are substantially different.

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