# Constraints from Accelerator Experiments on the Elastic Scattering of CMSSM Dark Matter 

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#### Abstract

We explore the allowed ranges of cross sections for the elastic scattering of neutralinos $\chi$ on nucleons in the constrained minimal supersymmetric extension of the Standard Model (CMSSM), in which scalar and gaugino masses are each assumed to be universal at some input grand unification scale. We extend previous calculations to larger $\tan \beta$ and investigate the limits imposed by the recent LEP lower limit on the mass of the Higgs boson and by $b \rightarrow s \gamma$, and those suggested by $g_{\mu}-2$. The Higgs limit and $b \rightarrow s \gamma$ provide upper limits on the cross section, particularly at small and large $\tan \beta$, respectively, and the value of $g_{\mu}-2$ suggests a lower limit on the cross section for $\mu>0$. The spin-independent nucleon cross section is restricted to the range $6 \times 10^{-8} \mathrm{pb}>\sigma_{S I}>2 \times 10^{-10} \mathrm{pb}$ for $\mu>0$, and the spin-dependent nucleon cross section to the range $10^{-5} \mathrm{pb}>\sigma_{S D}>2 \times 10^{-7} \mathrm{pb}$. Lower values are allowed if $\mu<0$.


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One of the front-running candidates for cold dark matter is the lightest supersymmetric particle (LSP), which is often taken to be the lightest neutralino $\chi$ [罒]. Several experiments looking for the scattering of cold dark matter particles on nuclear targets [2] have reached a sensitivity to a spin-independent elastic cross section $\sigma_{S I}$ of the order of $10^{-5} \mathrm{pb}$ for $m_{\chi} \sim 100 \mathrm{GeV}$ [3], and one experiment has reported a possible positive signal [4]. A new generation of more sensitive experiments is now being prepared and proposed, with sensitivities extending as low as $3 \times 10^{-9} \mathrm{pb}$ [5]. It is therefore important to update theoretical predictions for the elastic scattering cross section, including the spin-dependent component, $\sigma_{S D}$, as well as the spin-independent part, $\sigma_{S I}$.

The cross-section ranges allowed in the general minimal supersymmetric extension of the Standard Model (MSSM) are quite broad, being sensitive to the Higgs and squark masses, in particular [6, (7). It is common to focus attention on the constrained MSSM (CMSSM), in which all the soft supersymmetry-breaking scalar masses $m_{0}$ are required to be equal at an input superysmmetric GUT scale, as are the gaugino masses $m_{1 / 2}$ and the trilinear soft supersymmetry-breaking parameters $A$. These assumptions yield well-defined relations between the various sparticle masses, and correspondingly more definite predictions for the elastic $\chi$-nucleon scattering cross sections as functions of $m_{\chi}$ [8]. This paper is devoted to an updated discussion of $\sigma_{S I}$ and $\sigma_{S D}$ in the CMSSM as functions of $m_{0}, m_{1 / 2}$, and $\tan \beta$ for $A=0$.

This is timely in view of two significant experimental developments since our previous analysis [7]. One has been the improvement in the experimental lower limit from LEP on the mass of the lightest MSSM Higgs boson $h$ [9, which is now $m_{h}>114.1 \mathrm{GeV}$ in the context of the CMSSM $\ddagger$. The second major experimental development has been the report of a possible 2.6- $\sigma$ discrepancy between the measured and Standard Model values of the anomalous magnetic moment of the muon, $a_{\mu} \equiv\left(g_{\mu}-2\right) / 2: a_{\mu}=(43 \pm 16) \times 10^{-10}$ [10], which we interpret as $11 \times 10^{-10}<a_{\mu}<75 \times 10^{-10}$. The supersymmetric interpretation 111, 12] of this result is not yet established: it could be that strong-interaction uncertainties in the Standard Model prediction have been underestimated, or there might have been a statistical fluctuation in the data. Even if the discrepancy is confirmed, it might be evidence for some other type of physics beyond the Standard Model. Nevertheless, we are tempted to explore its possible consequences for dark matter scattering within the CMSSM context [13].

Theoretically, there have also been improvements recently in the calculations in the CMSSM of the supersymmetric relic density $\Omega_{\chi} h^{2}$ for large values of the ratio $\tan \beta$ of

[^0]Higgs vacuum expectation values [14]. These define better the interesting region of CMSSM parameter space where the relic density may fall within the range $0.1<\Omega_{\chi} h^{2}<0.3$ preferred by astrophysics and cosmology (15].

We find that the expected ranges of both the spin-independent cross sections $\sigma_{S I}$ and the spin-dependent cross sections $\sigma_{S D}$ in the CMSSM are quite restricted (see also [16]). The LEP Higgs limit [9] sets upper bounds on $\sigma_{S I}$ and $\sigma_{S D}$, not only via the direct contribution of Higgs exchange to the scattering matrix element, but also because it provides a strong lower limit on $m_{\chi}$ at low $\tan \beta$, in particular [17]. 2 At high $\tan \beta$, the observed rate of $b \rightarrow s \gamma$ also provides [18] an important lower limit on $m_{\chi}$ and hence an upper limit on $\sigma_{S I, S D}$ [19]. In view of these upper limits, we are unable to provide a CMSSM interpetation of the DAMA signal [4]. More excitingly for prospective experiments, the range $11 \times 10^{-10}<a_{\mu}<75 \times 10^{-10}$ would imply important upper limits on sparticle masses, and hence a lower limit: $\sigma_{S I}>2 \times 10^{-10} \mathrm{pb}$. Putting together all the constraints, we find for $\mu>0$ a relatively narrow band $6 \times 10^{-8} \mathrm{pb}$ $>\sigma_{S I}>2 \times 10^{-10} \mathrm{pb}$. The allowed range is typically broadest at large $\tan \beta$. Lower cross sections are possible if $\mu<0$.

As has been discussed in detail elsewhere, the regions of $m_{1 / 2}, m_{0}$ plane where the relic density falls within the preferred range $0.1<\Omega_{\chi} h^{2}<0.3$ can be divided into four generic parts, whose relative significances depend on $\tan \beta$. There is a 'bulk' region at moderate $m_{1 / 2}$ and $m_{0}[]$, where supersymmetry is relatively easy to detect at colliders and as dark matter. Then, extending to larger $m_{1 / 2}$, there is a 'tail' of the parameter space where the LSP $\chi$ is almost degenerate with the next-to-lightest supersymmetric particle (NLSP), and efficient coannihilations [20] keep $\Omega_{\chi} h^{2}$ in the preferred range, even for larger values of $m_{\chi}$ [21]. At larger $m_{0}$, close to the boundary where electroweak symmetry breaking is no longer possible, there is the 'focus-point' region where the LSP has a more prominent Higgsino component and $m_{\chi}$ is small enough for $\Omega_{\chi} h^{2}$ to be acceptable [22]. Finally, extending to larger $m_{1 / 2}$ and $m_{0}$ at intermediate values of $m_{1 / 2} / m_{0}$, there may be a 'funnel' of CMSSM parameter space where rapid direct-channel annihilations via the poles of the heavier Higgs bosons $A$ and $H$ may keep $\Omega_{\chi} h^{2}$ in the preferred range [14, 23]. In this paper, we focus on the case $A=0$ and use the SSARD code to calculate the relic density [24]. The precise values of $m_{1 / 2}$ and $m_{0}$ in the 'focus-point' and 'funnel' regions are quite sensitive to the precise values and treatments of the input CMSSM and other parameters [25, 26]. These regions are not emphasized in the following discussion, but are commented upon where appropriate.

The code we use to calculate the elastic dark matter scattering cross sections $\sigma_{S I, S D}$ was

[^1]documented in [8, 7, together with the ranges of values of the hadronic matrix elements that we use. The cross sections for protons and neutrons are similar within the quoted uncertainties in these matrix elements. Codes are available 27 that include additional contributions to the scattering matrix elements, but a recent comparison [28] shows that the improvements are not essential in the CMSSM parameter space that we explore here. Fig. प displays contours of the spin-independent cross section for the elastic scattering of the LSP $\chi$ on protons in the $m_{1 / 2}, m_{0}$ planes for (a) $\tan \beta=10, \mu<0$, (b) $\tan \beta=10, \mu>0$, (c) $\tan \beta=35, \mu<0$, and (d) $\tan \beta=50, \mu>0$. The latter are close to the largest values of $\tan \beta$ for which we find generic solutions to the electroweak symmetry-breaking conditions for $\mu<0$ and $>0$, respectively [14]. The double dot-dashed (orange) lines are contours of the spin-independent cross section, and we have indicated the contours $\sigma_{S I}=10^{-9} \mathrm{pb}$ in panels (a, d) and $\sigma_{S I}=10^{-12} \mathrm{pb}$ in panels ( $\mathrm{b}, \mathrm{c}$ ). The other bolder contours are for cross sections differing by factors of 10 , and the finer contours for cross sections differing by interpolating factors of 3 (in order to ensure clarity, not all of the interpolating contours are displayed).

These cross-section contours are combined in Fig. 1 with other information on the CMSSM parameter space. The lower right-hand corners of the panels are excluded because there the LSP is the lighter $\tilde{\tau}_{1}$. The light (turquoise) shaded regions are those with $0.1<\Omega_{\chi} h^{2}<0.3$ [15]. The 'bulk' regions are clearly visible in panels (a,b) and (d), and coannihilation 'tails' in all panels [14]. For our default choices $A=0, m_{t}($ pole $)=175 \mathrm{GeV}$ and $m_{b}\left(m_{b}\right)_{S M}^{\overline{M S}}=4.25 \mathrm{GeV}$, the 'focus-point' regions [22] are at larger values of $m_{0}$ than are shown in any of the panels. Rapid-annihilation 'funnels' are visible in panels (c) and (d): that in the former panel bisects the 'bulk' region. The near-vertical dashed (black) lines at small $m_{1 / 2}$ are the chargino-mass contours $m_{\chi^{ \pm}}=103.5 \mathrm{GeV}$ [29], and the near-vertical dotted (red) lines at larger $m_{1 / 2}$ are the contours $m_{h}=114.1 \mathrm{GeV}$ [9], as calculated using FeynHiggs [30]. The large medium (green) shaded regions in panels (a) and (c) are those excluded by $b \rightarrow s \gamma$ [18]: smaller excluded regions are also visible in panels (b) and (d) at small $m_{1 / 2}$. Finally, the sloping shaded (pink) regions in panels (b) and (d) delineate the $\pm 2-\sigma$ ranges of $g_{\mu}-2$ [12], which are absent for the $\mu<0$ panels (a) and (c).

The LEP lower limits on $m_{h}$ and $m_{\chi^{ \pm}}$, as well as the experimental measurement of $b \rightarrow s \gamma$ for $\mu<0$, tend to bound the cross sections from above, as we discuss later in more detail. Generally speaking, the spin-independent cross section is relatively large in the 'bulk' region, but falls off in the 'tail' and 'funnel' regions. In the focus-point regions, the spin-independent cross section is relatively independent of $m_{\chi}$ and for $\tan \beta=10$, takes a value between $10^{-9}$ and $10^{-8} \mathrm{pb}$ [22, 28]. Also, we note also that there is a strong cancellation in the spin-


Figure 1: Spin-independent cross sections in the $\left(m_{1 / 2}, m_{0}\right)$ planes for (a) $\tan \beta=10, \mu<0$, (b) $\tan \beta=10, \mu>0$, (c) $\tan \beta=35, \mu<0$ and (d) $\tan \beta=50, \mu>0$, assuming $A_{0}=$ $0, m_{t}=175 \mathrm{GeV}$ and $m_{b}\left(m_{b}\right)_{S M}^{\overline{M S}}=4.25 \mathrm{GeV}$ [14]. The double dot-dashed (orange) curves are contours of the spin-independent cross section, differing by factors of 10 (bolder) and interpolating factors of 3 (finer - when shown). For example, in (b), the curves to the right of the one marked $10^{-9} \mathrm{pb}$ correspond to $3 \times 10^{-10} p b$ and $10^{-10} p b$. The near-vertical lines are the LEP limits $m_{\chi^{ \pm}}=103.5 \mathrm{GeV}$ (dashed and black) [29], $m_{h}=114.1 \mathrm{GeV}$ (dotted and red) [9]. In the dark (brick red) shaded regions, the LSP is the charged $\tilde{\tau}_{1}$, so this region is excluded. The light (turquoise) shaded areas are the cosmologically preferred regions with $0.1 \leq \Omega_{\chi} h^{2} \leq 0.3$ [14]. The medium (dark green) shaded regions that are most prominent in panels (a) and (c) are excluded by $b \rightarrow s \gamma$ [18]. The sloping shaded (pink) regions in panels (b) and (d) delineate the $\pm 2-\sigma$ ranges of $g_{\mu}-2$ [12].
independent cross section when $\mu<0$ [8, 7, as seen along strips in panels (a, c) of Fig. [1 where $m_{1 / 2} \sim 500,1100 \mathrm{GeV}$, respectively. In the cancellation region, the cross section drops lower than $10^{-14} \mathrm{pb}$. All these possibilities for suppressed spin-independent cross sections are disfavoured by the data on $g_{\mu}-2$ [10, 11, (12], which favour values of $m_{1 / 2}$ and $m_{0}$ that are not very large, as well as $\mu>0$, as seen in panels (b, d) of Fig. Thus $g_{\mu}-2$ tends to provide a lower bound on the spin-independent cross section.

Fig. 2 displays contours of the spin-dependent cross section in the $m_{1 / 2}, m_{0}$ planes for (a) $\tan \beta=10, \mu<0$, (b) $\tan \beta=10, \mu>0$, (c) $\tan \beta=35, \mu<0$, and (d) $\tan \beta=$ $50, \mu>0$. The dot-dashed (blue) lines are those of the spin-dependent cross section, and the other notation is as in Fig. 11. The bolder lines are contours differing by factors of 10 from the indicated ones, and the finer lines, when shown, differ by interpolating factors of 3. We note again that the cross section is generically larger in the 'bulk' region and smaller in the coannihilation 'tail' and rapid-annihilation 'funnel' regions. In the focus-point regions, the spin-dependent cross-section is also relatively constant and for $\tan \beta=10$ takes values between $10^{-5}$ and $10^{-4} \mathrm{pb}$ [22, 28]. Unlike the spin-independent case, there are no cancellations in the spin-dependent cross section.

Fig. 回illustrates the effect on the cross sections of each of the principal phenomenological constraints, in the particular cases $\tan \beta=10$ and (a, b) $\mu>0$, (c, d) $\mu<0$, (e) $\tan \beta=$ $35, \mu<0$ and (f) $\tan \beta=50, \mu>0$. The solid (blue) lines mark the bounds on the cross sections allowed by the relic-density constraint $0.1<\Omega_{\chi} h^{2}<0.3$ alone [15. For any given value of $m_{1 / 2}$, only a restricted range of $m_{0}$ is allowed. Therefore, only a limited range of $m_{0}$, and hence only a limited range for the cross section, is allowed for any given value of $m_{\chi}$. The thicknesses of the allowed regions are due in part to the assumed uncertainties in the nuclear inputs. These have been discussed at legnth in [7, 8] and we refer the reader there for details. In the case (e) of $\tan \beta=35, \mu<0$ and (f) of $\tan \beta=50, \mu>0$, two or three different narrow ranges of $m_{0}$ may be allowed for the same value of $m_{1 / 2}$, but they have quite similar cross sections, as seen already in Figs. $1(c, d)$ and $2(c, d)$. On the other hand, a broad range of $m_{\chi}$ is allowed, when one takes into account the coannihilation 'tail' region at each $\tan \beta$ and the rapid-annihilation 'funnel' regions for $\tan \beta=35,50$ [14] The dashed (black) lines in Fig. 3 display the range allowed by the $b \rightarrow s \gamma$ constraint 18] alone, which is more important for $\mu<0$. In this case, a broader range of $m_{0}$ and hence the spin-independent cross section is possible for any given value of $m_{\chi}$. The impact of the constraint due to $m_{h}$ is shown by the dot-dashed (green) lines in Fig. 3. We implement this constraint by requiring that $m_{h}>114.1 \mathrm{GeV}$ when calculated using the FeynHiggs code 30.

[^2]

Figure 2: Spin-dependent cross sections in the ( $m_{1 / 2}, m_{0}$ ) planes for (a) $\tan \beta=10, \mu<$ 0 , (b) $\tan \beta=10, \mu>0$, (c) $\tan \beta=35, \mu<0$ and (d) $\tan \beta=50, \mu>0$, assuming $A_{0}=0, m_{t}=175 \mathrm{GeV}$ and $m_{b}\left(m_{b}\right)_{S M}^{\overline{M S}}=4.25 \mathrm{GeV}$ [14]. The dot-dashed (blue) lines are contours of the spin-dependent cross section, differing by factors of 10 (bolder) and interpolating factors of 3 (finer - when shown). The near-vertical dashed lines are the LEP limits $m_{\chi^{ \pm}}=103.5 \mathrm{GeV}$ (black) [2G], $m_{h}=114.1 \mathrm{GeV}$ (red) [G]. In the dark (brick red) shaded regions, the $L S P$ is the charged $\tilde{\tau}_{1}$, so this region is excluded. The light (turquoise) shaded areas are the cosmologically preferred regions with $0.1 \leq \Omega_{\chi} h^{2} \leq 0.3$ [15]. The medium (dark green) shaded regions are excluded by $b \rightarrow s \gamma$ 18].

Comparing with the previous constraints, we see that a region at low $m_{\chi}$ is excluded by $m_{h}$, strengthening significantly the previous upper limit on the spin-independent cross section. Finally, the dotted (red) lines in Fig. 3 show the impact of the $g_{\mu}-2$ constraint [12]. This imposes an upper bound on $m_{1 / 2}$ and hence $m_{\chi}$, and correspondingly a lower limit on the spin-independent cross section.

We emphasize again the important impacts of the updated LEP limits on the chargino and (particularly) Higgs masses. Significantly smaller LSP masses and correspondingly larger cross sections could be found if one used earlier, weaker LEP limits.

The shaded (pale blue) regions in panels (a,b,f) of Fig. 3 show the ranges of $m_{\chi}$ and the cross sections that survive all the phenomenological constraints. We find for $\tan \beta=10$,

$$
\begin{equation*}
135 \mathrm{GeV} \lesssim m_{\chi} \lesssim 180 \mathrm{GeV} \text { for } \mu>0 \tag{1}
\end{equation*}
$$

 and there is no such upper bound for $\mu<0$, unless one interprets the LEP 'hint' as a real Higgs signal [9], and imposes $m_{h}<117 \mathrm{GeV}$, in which case one finds $m_{\chi} \lesssim 370 \mathrm{GeV}$. The ranges of cross sections corresponding to (1) are

$$
\begin{align*}
& 5 \times 10^{-10} \mathrm{pb} \sigma_{S I}  \tag{2}\\
& 1 \times 3 \times 10^{-9} \mathrm{pb},  \tag{3}\\
& 1 \times 10^{-6} \mathrm{pb} \lesssim \sigma_{S D} \\
& 4 \times 10^{-6} \mathrm{pb},
\end{align*}
$$

for $\tan \beta=10$ and $\mu>0$, and we find

$$
\begin{align*}
\sigma_{S I} & \lesssim 2 \times 10^{-11} \mathrm{pb}  \tag{4}\\
\sigma_{S D} & \lesssim 1 \times 10^{-6} \mathrm{pb} \tag{5}
\end{align*}
$$

for $\tan \beta=10$ and $\mu<0$. No lower limits for the spin-independent cross section are possible with the constraints considered above, both because the $g_{\mu}-2$ constraint is inapplicable and must be discarded if this sign of $\mu$ is to be considered at all, and also because of the cancellation in $\sigma_{S I}$ that is visible in panels (a) and (c) of Fig. 1]. Even if we take the LEP 'hint' of a signal for a Higgs boson, and impose the upper limit $m_{h}<117 \mathrm{GeV}$, because the bound on $m_{\chi}$ is past the cancellation region, we find no useful lower bound for $\tan \beta=10$ and $\mu<0$. For the spin-dependent cross section, a lower limit due to the relic density is determined by the endpoint of the coannihilation region, namely $\sigma_{S D} \gtrsim 2 \times 10^{-8} \mathrm{pb}$. A Higgs mass bound of 117 GeV in this case would impose $\sigma_{S D} \gtrsim 10^{-7} \mathrm{pb}$.

We see in panel (f) of Fig. 3 that the spin-independent cross section for $\mu>0$ may be rather larger for $\tan \beta=50$ than for $\tan \beta=10$ [16], as shown in panel (a). This analysis is


Figure 3: Allowed ranges of the cross sections for $\tan \beta=10$ and ( $a, b$ ) $\mu>0$, ( $c, d$ ) $\mu<0$, for ( $a, c$ ) spin-independent and ( $b, d$ ) spin-dependent elastic scattering. Panel (e) shows the spin-independent cross section for $\tan \beta=35$ and $\mu<0$, and panel ( $f$ ) the spin-independent cross section for $\tan \beta=50$ and $\mu>0$. The solid (blue) lines indicate the relic density constraint [15], the dashed (black) lines the $b \rightarrow s \gamma$ constraint [18], the dot-dashed (green) lines the $m_{h}$ constraint [G], and the dotted (red) lines the $g_{\mu}-2$ constraint [19]. The shaded (pale blue) region is allowed by all the constraints.
extended in panels (a) and (c) of Fig. $\tan ^{\text {to }}$ all the values $8<\tan \beta \leq 55$ (below $\tan \beta \simeq 8$ it is not possible to satisfy both the Higgs mass and $g-2$ constraints [11, 12], and above $\tan \beta \simeq 55$ we no longer find consistent CMSSM parameters), and we find overall that

$$
\begin{align*}
& 2 \times 10^{-10} \mathrm{pb} \lesssim \sigma_{S I}  \tag{6}\\
& 2 \times 6 \times 10^{-8} \mathrm{pb}  \tag{7}\\
& 2 \times 10^{-7} \mathrm{pb} \lesssim \sigma_{S D} \\
& 10^{-5} \mathrm{pb}
\end{align*}
$$

for $\tan \beta \leq 55$ and $\mu>0$. As we see in panels (a) and (c) of Fig. ( 4 , for $\mu>0, m_{h}$ provides the most important upper limit on the cross sections for $\tan \beta<23$, and $b \rightarrow s \gamma$ for larger $\tan \beta$, with $g_{\mu}-2$ always providing a more stringent lower limit than the relic-density constraint. The relic density constraint shown is evaluated at the endpoint of the coannihilation region. At large $\tan \beta$, we have not considered moving far out into the Higgs funnels or the focuspoint regions, as their locations are very sensitive to input parameters and calculational details [25]. In the case $\mu<0$, there is no lower limit on the spin-independent cross section, for the reasons discussed earlier. We find

$$
\begin{align*}
& \sigma_{S I} \lesssim 2 \times 10^{-10} \mathrm{pb},  \tag{8}\\
& 2 \times 10^{-8} \mathrm{pb} \lesssim  \tag{9}\\
& \sigma_{S D} \lesssim 2 \times 10^{-6} \mathrm{pb}
\end{align*}
$$

for $\mu<0$ and $5<\tan \beta \leq 35$ (below $\tan \beta \simeq 5$ it is not possible to satisfy both the Higgs mass and relic density constraints [17, and above $\tan \beta \simeq 35$ we no longer find consistent CMSSM parameters), with the upper limits being imposed by $m_{h}$ for $\tan \beta<12$ and by $b \rightarrow s \gamma$ for larger $\tan \beta$, as seen in panels (b) and (d) of Fig. ©. The relic-density constraint imposes an interesting lower limit on $\sigma_{S D}$, but not on $\sigma_{S I}$, as discussed above. Again, requiring $m_{h}<117 \mathrm{GeV}$ would impose a lower limit $\sigma_{S D} \gtrsim 3 \times 10^{-7} \mathrm{pb}$, and since a 117 GeV bound would cut out the cancellation region, we can obtain a lower bound on the spin-independent cross section, $\sigma_{S I} \gtrsim 10^{-11}$ for $\tan \beta=35$ and $\mu<0$.

We conclude that the available experimental constraints on CMSSM model parameters greatly restrict the allowed ranges of elastic scattering cross sections for supersymmetric dark matter. Upper limits are imposed on both $\sigma_{S I}$ and $\sigma_{S D}$ by both the LEP Higgs constraint and $b \rightarrow s \gamma$. If one takes at face value the $g_{\mu}-2$ constraint, in addition to requiring $\mu>0$, it also imposes lower limits on both $\sigma_{S I}$ and $\sigma_{S D}$, providing experiments with a plausible sensitivity to aim for. On the other hand, if one drops the $g_{\mu}-2$ constraint and tolerates $\mu<0$, there is no useful lower limit on $\sigma_{S I}$. A lower bound on $\sigma_{S D}$ is possible if one imposes $m_{h}<117 \mathrm{GeV}$, motivated by the LEP Higgs 'hint'. The LEP constraints are now stable, but the situation with $g_{\mu}-2$ can be expected to clarify soon. If the apparent deviation from


Figure 4: The allowed ranges of $(a, b)$ the spin-independent cross section and $(c, d)$ the spin-dependent cross section, for ( $a, c$ ) $\mu>0$ and (b, d) $\mu<0$. The darker solid (black) lines show the upper limits on the cross sections obtained from $m_{h}$ and $b \rightarrow s \gamma$, and (where applicable) the lighter solid (red) lines show the lower limits suggested by $g_{\mu}-2$ and the dotted (green) lines the lower limits from the relic density.
the Standard Model [10] is confirmed, direct searches for supersymmetric dark matter may have bright prospects, at least within the CMSSM framework studied here.

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[^0]:    ${ }^{1}$ In the general MSSM, $m_{h}$ could be as low as $\sim 90 \mathrm{GeV}$, but this is only possible for variants in which the $Z-Z-h$ coupling is suppressed to an extent that does not occur within the CMSSM as studied here.

[^1]:    ${ }^{2}$ Apart from cancellations that occur in $\sigma_{S D}$ when $\mu<0$, the elastic cross sections are monotonically decreasing functions of $m_{\chi}$ in the CMSSM [8].

[^2]:    ${ }^{3}$ We do not show predictions in the 'focus-point' region (22].

