Screening mass responses to chemical potential at finite temperature*

QCD-TARO Collaboration: S. Choe[a], Ph. de Forcrand[b], M. García Pérez[c], S. Hioki[d], Y. Liu[a], H. Matsufuru[e], O. Miyamura[a], A. Nakamura[f], I.-O. Stamatescu[g,h], T. Takaishi[i], and T. Umeda[j]

[a] Department of Physics, Hiroshima University, Higashi-Hiroshima 739-8526, Japan
[b] Institut für Theoretische Physik, ETH-Hönggerberg,CH-8093 Zürich,Switzerland
[c] Theory Division, CERN, CH-1211 Geneva 23, Switzerland
[d] Department of Physics, Tezukayama University, Nara 631-8501, Japan
[e] Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
[f] IMC, Hiroshima University,Higashi-Hiroshima 739-8521, Japan
[g] Institut für Theoretische Physik, Universität Heidelberg, D-69120 Heidelberg, Germany
[h] FEST, Schmeilweg 5, D-69118 Heidelberg, Germany
[i] Hiroshima University of Economics, Hiroshima 731-0192, Japan
[j] Center for Computational Physics, University of Tsukuba, Tsukuba 305-8577, Japan

Responses to chemical potential of the pseudoscalar meson screening mass and the chiral condensate in lattice QCD are investigated. On a $16 \times 8^2 \times 4$ lattice with two flavors of staggered quarks the first and second responses below and above T_c are evaluated. Different behavior in the low and the high temperature phases are observed, which may be explained as a consequence of the chiral symmetry breaking and restoration.

1. Introduction

The study of finite baryon density at finite temperature is not only very useful to understand the phase transition between hadron and quark gluon plasma, but also quite important for heavy ion collision experiments which require theoretical understanding of hadronic properties at finite baryon density and temperature [1]. As the fermionic determinant at finite chemical potential is complex on the lattice, numerical simulations are very difficult but the quenched approximation at finite chemical potential can give physically misleading results [2]; simulations with dynamical fermions are therefore essential to extract the relevant physics [3].

We propose a new technique to investigate nonzero chemical potential using lattice QCD simulations. A Taylor expansion in μ is used in the vicinity of zero μ at finite temperature for evaluating masses and chiral condensate. Here we obtain the Taylor coefficients directly in a simulation at $\mu = 0$, by measuring the derivatives of the relevant observables. There is in fact much interesting physical information which can be extracted from the behavior of a system at small chemical potential. Our preliminary results have been reported in [4].

*Talk presented by Y. Liu

2. Lattice formulation

Assume that the spatial hadron correlator C(x) is dominated by a single pole contribution,

$$C(x) \equiv \sum_{y,z,t} \langle H(x,y,z,t) H(0,0,0,0)^\dagger \rangle = A(e^{-\hat{M}\hat{x}} + e^{-\hat{M}(L_x - \hat{x})}), \quad (1)$$

where $\hat{M} = aM$ and $\hat{x} = x/a$. L_x is the lattice size in the x-direction.

We take the first and second derivatives of the hadron correlator with respect to $\hat{\mu} \equiv a\mu = \mu/(N_t T)$, where μ is the chemical potential.

$$\frac{1}{C(x)}\frac{dC(x)}{d\hat{\mu}} = \frac{1}{A}\frac{dA}{d\hat{\mu}} \quad (2)$$
$$+\frac{d\hat{M}}{d\hat{\mu}}\left\{\left(\hat{x} - \frac{L_x}{2}\right)\tanh\left[\hat{M}\left(\hat{x} - \frac{L_x}{2}\right)\right] - \frac{L_x}{2}\right\},$$

and

$$\frac{1}{C(x)}\frac{d^2C(x)}{d\hat{\mu}^2} = \frac{1}{A}\frac{d^2A}{d\hat{\mu}^2} \quad (3)$$

$$+\left(\frac{2}{A}\frac{dA}{d\hat{\mu}}\frac{d\hat{M}}{d\hat{\mu}}+\frac{d^2\hat{M}}{d\hat{\mu}^2}\right)$$
$$\left\{\left(\hat{x}-\frac{L_x}{2}\right)\tanh\left[\hat{M}\left(\hat{x}-\frac{L_x}{2}\right)\right]-\frac{L_x}{2}\right\}$$
$$+\left(\frac{d\hat{M}}{d\hat{\mu}}\right)^2\left\{\left(\hat{x}-\frac{L_x}{2}\right)^2+\frac{L_x^2}{4}\right.$$
$$\left.-L_x\left(\hat{x}-\frac{L_x}{2}\right)\tanh\left[\hat{M}\left(\hat{x}-\frac{L_x}{2}\right)\right]\right\}.$$

In this work, we consider the flavor non-singlet mesons in QCD with two flavors. The hadron correlator is then given by

$$\begin{aligned}\langle H(n)H(0)^\dagger\rangle &= \langle G\rangle \qquad (4)\\ &= \langle \mathrm{Tr}\left[P(\hat{\mu}_u)_{n0}\Gamma P(\hat{\mu}_d)_{0n}\Gamma^\dagger\right]\rangle\ .\end{aligned}$$

Here $P(\hat{\mu}) = D[U;\hat{\mu}]^{-1}$ ($D[U;\hat{\mu}]$ is the Dirac operator) is the quark propagator at finite chemical potential, and Γ is the Dirac matrix which specifies the spin of the meson. In this paper we study the response to the isoscalar and isovector chemical potentials $\hat{\mu}_S, \hat{\mu}_V$ by setting

$$\hat{\mu}_S = \hat{\mu}_u = \hat{\mu}_d\ \ ;\ \ \hat{\mu}_V = \hat{\mu}_u = -\hat{\mu}_d\ . \qquad (5)$$

3. Numerical Simulations and Results

The simulations have been performed at finite temperature $T/T_c \in\sim [0.9, 1.1]$ on a $16\times 8^2\times 4$ lattice with standard Wilson gauge action and with two dynamical flavors of staggered quarks. We use the R-algorithm, with quark masses $ma =$ 0.0125, 0.017 and 0.025. We also use a corner-type wall source after Coulomb gauge fixing in each (y,z,t)-hyperplane.

The first derivative of the pseudoscalar meson correlator with respect to the isoscalar chemical potential is identically zero. For the isovector chemical potential, our simulation values for the first derivative are very small in both phases.

3.1. Response of the pseudoscalar meson to the isoscalar chemical potential

In the low temperature phase, the dependence of the mass on $\hat{\mu}_S$ is small. This behavior is to be expected, since, below the critical temperature and in the vicinity of zero $\hat{\mu}_S$, the pseudoscalar meson is still a Goldstone boson. In fact, the chiral extrapolation of the isoscalar response is consistent with zero, as shown in Figure 1. This is in contrast with the behavior above T_c, where $d^2\hat{M}^2/d\hat{\mu}^2$ seems to remain finite even in the chiral limit. In addition, our results suggest that the response of the coupling A (eq.(1)) is small below T_c.

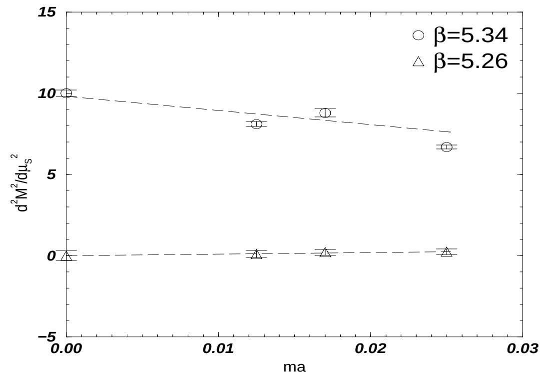

Figure 1. $d^2\hat{M}^2/d\hat{\mu}_S^2$ for the pseudoscalar meson versus ma at $T < T_c$ ($\beta = 5.26$, triangles) and $T > T_c$ ($\beta = 5.34$, circles). Extrapolation to $ma = 0$ is also shown.

Above T_c, we first note that the correlator and its response are still well fitted by the single pole formulae, Eqs. (1-3). The screening masses are manifestly larger than those below T_c. As pointed out above, the response of the mass above T_c becomes large and positive, reflecting the fact that the pion is no longer a Goldstone boson and indicating chiral symmetry restoration.

3.2. Response of the pseudoscalar meson to the isovector chemical potential

In the presence of the isovector chemical potential, π^+ and π^- may have different masses. Here we consider the π^+ ($u\bar{d}$) meson as in Eq. (4).

An interesting point in this respect is that the second derivative of the mass is negative in the low temperature phase, in contrast with the isoscalar potential case: the mass tends to decrease under the influence of the isovector chemical potential. This may be explained by the observation that, for low temperature and chemical potential above the pion mass, a Goldstone mode can appear [2]. The behavior of M is more clearly shown by a Taylor expansion: At $\beta = 5.26$ and

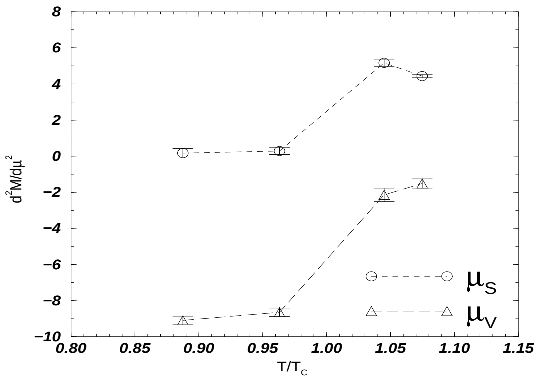

Figure 2. Second responses $d^2\hat{M}/d\hat{\mu}_S^2$ and $d^2\hat{M}/d\hat{\mu}_V^2$ of the pseudoscalar meson mass at $ma = 0.025$.

$ma = 0.017$, the data suggest

$$\frac{M(\mu_V)}{T}\Big|_{\mu_V} = 1.4024(8) - 0.0005(10)\left(\frac{\mu_V}{T}\right)$$
$$- 1.31(4)\left(\frac{\mu_V}{T}\right)^2 + O\left[\left(\frac{\mu_V}{T}\right)^3\right].$$

In the high temperature phase, the dependence of the masses on μ_V decreases. Since the pseudoscalar meson becomes heavier, the phase boundary to the pion condensate phase is further away from the $\mu_V = 0$ axis. The weaker responses may be understood from this point of view.

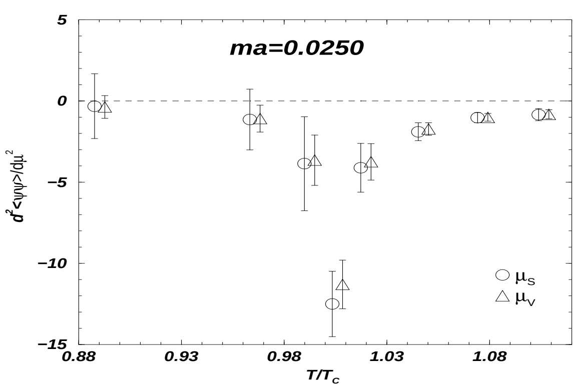

Figure 3. Second responses $d^2\langle\bar{\psi}\psi\rangle/d\hat{\mu}_S^2$ and $d^2\langle\bar{\psi}\psi\rangle/d\hat{\mu}_V^2$ at $ma = 0.025$. Note that the abscissa of coincident points have been splitted.

3.3. Responses of the chiral condensate

We also measure responses of the chiral condensate to the isoscalar and isovector chemical potentials. The first responses to both potentials are identically zero. Figure 3 shows our preliminary results for the second responses. We have found they are small and negative in both phases. Near T_c there is a large change. The critical temperature tends to decrease under the influence of $\mu_{S,V}$. Thus, in the low temperature phase, turning on the chemical potential brings the system closer to the phase transition where chiral symmetry is restored, and decreases the chiral condensate. At high temperature, because chiral symmetry is restored, responses of the chiral condensate to the isoscalar and isovector chemical potential are small.

4. Conclusions

We have developed a framework to study the response of hadrons to the chemical potential. The dependence of the pseudoscalar meson mass on μ_S in the chiral limit is consistent with zero at low temperature, reflecting the fact that at small μ_S the pion is still a Goldstone boson. For the isovector chemical potential, the $u\bar{d}$ pseudoscalar meson mass tends to decrease as a function of μ_V at a much stronger rate in the low temperature phase. This is consistent with a restoration of chiral symmetry as the isovector chemical potential increases.

REFERENCES

1. NA50 collaboration, M. C. Abreu *et al.*Phys. Lett. B**477** (2000) 28; See also H. Satz, *Lattice '00,* Nucl. Phys. B (Proc. Suppl.) **94** (2001) 204.
2. M. A. Stephanov, Phys. Rev. Lett. **76** (1996) 4472; D. T. Son *et al.*Phys. Rev. Lett. 592.
3. T. Hatsuda and S. H. Lee, Phys. Rev. C**46** (1992) 34; A. Hayashigaki, Phys. Lett. B**487** (2000) 96; CERES Collaboration, G. Agakishiev *et al.*Nucl. Phys. A**638** (1998) 159.
4. QCD-TARO Collaboration, Ph. de Forcrand *et al.Lattice '98,* Nucl. Phys. B (Proc. Suppl.) **63 A-C** (1998) 460; *Lattice '99,* Nucl. Phys. B (Proc. Suppl.) **73** (1999) 477 ; *hep-lat/0107002.*