

Conservation Laws and Particle Production in Heavy Ion Collisions*

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We discuss the role of the conservation laws related with U(1) internal symmetry group in the statistical model description of particle productions in ultrarelativistic heavy ion collisions. We derive and show the differences in particle multiplicities in the canonical and the grand canonical formulation of quantum number conservation. The time evolution and the approach to chemical equilibrium in the above ensembles is discussed in terms of kinetic master equation. The application of the statistical model to the description of (multi)strange particle yields at GSI/SIS and the SPS energies is also presented.

I. INTRODUCTION

There are in general two approaches to describe integrated particle yields measured in ultrarelativistic heavy ion collisions: (i) the microscopic transport models [1] and (ii) the macroscopic statistical thermal models [2–6]. In this article we will discuss the statistical approach and show that it provides a very satisfactory description of experimental data. We will emphasize the importance of the conservation laws in the particular strangeness conservation when modelling particle chemical freezeout conditions.

Within a statistical approach, the production of particles is commonly described using the grand canonical (GC) ensemble, where the charge conservation is controlled by the related chemical potential. In this description a net value of a given U(1) charge is conserved on the average. The (GC) approach can be only valid if the total number of particles carrying quantum number related with this symmetry is very large. In the opposite limit of a small particle multiplicities, conservation laws must be implemented exactly and locally, i.e., the canonical (C) ensemble for conservation laws must be used [7,8]. The local conservation of quantum numbers in the canonical approach severely reduces the phase space available for particle productions. This treatment of charge conservation is of crucial importance in the description of particle multiplicities in proton induced processes [9,10], in e^+e^- [9] as well as in central heavy ion collisions at low beam energies [11].

In this article we describe the exact strangeness conservation in the context of relativistic statistical thermodynamics. A kinetic theory for the time evolution of particle production and the approach to the grand canonical and the canonical equilibrium distribution will be also introduced. Finally the example of the applications of the statistical model in (C) ensemble is presented in the context of low energy central as well as in high energy peripheral heavy ion collisions.

II. STATISTICAL MODEL AND PARTICLE MULTIPLICITY

The exact treatment of quantum numbers in statistical mechanics has been well established for some time now [7,8,11]. It is in general obtained by projecting the partition function onto the desired values of the conserved charges by using group theoretical methods. For our purpose we shall only consider the conservation laws related to the abelian U(1) symmetry group. In particular, we concentrate on strangeness conservation.

The basic quantity in the statistical mechanics describing a thermal properties of a system is the partition function $Z(T, V)$. In the (GC) ensemble,

$$Z^{GC}(T, V, \mu_Q) \equiv Tr[e^{-\beta(H - \mu_Q Q)}] \quad (1)$$

where Q is the conserved charge, H the hamiltonian of the system, μ_Q is the chemical potential which plays the role of the Lagrange multiplier which guarantees that the charge Q is conserved on the average in the whole system. Finally $\beta = 1/T$ is the inverse temperature.

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In the (C) ensemble the charge Q is conserved exactly. Thus there is no more chemical potential under the trace and instead we calculate the partition function summing only these states which are carrying exactly the quantum number Q , that is

$$Z_Q^C(T, V) \equiv \text{Tr}_Q[e^{-\beta H}] \quad (2)$$

The canonical and the grand canonical partition functions are related through the following cluster decomposition,

$$Z^{GC}(T, V, \lambda) = \sum_{Q=-\infty}^{+\infty} \lambda^Q Z_Q^C(T, V) \quad (3)$$

where the fugacity $\lambda \equiv \exp(\beta\mu_Q)$ and the sum is taken over all possible values of the charge Q .

For the (GC) partition function, which is well behaving analytic function of the fugacity λ , the above relation can be inverted and the canonical partition function with a given value of the charge Q reads,

$$Z_Q = \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{-iQ\phi} \tilde{Z}(T, V, \phi) \quad (4)$$

where the generating function \tilde{Z} is obtained from the grand canonical partition function replacing the fugacity parameter λ by the factor $e^{i\phi}$,

$$\tilde{Z}(T, V, \phi) \equiv Z^{GC}(T, V, \lambda \rightarrow e^{i\phi}) \quad (5)$$

The form of the generating function \tilde{Z} in the above equation is model dependent. Having in mind the application of the statistical description to particle production in heavy ion collisions we calculate \tilde{Z} in the ideal gas approximation, however, including all particles and resonances [11]. This is not an essential restriction, because, describing the freeze-out conditions we are dealing with a dilute system where the interactions should not influence particle production anymore. We neglect any medium effects on particle properties. In general, however, already in the low-density limit, the modifications of resonance width or particle dispersion relations could be of importance [12]. For the sake of simplicity, we use classical statistics, i.e. we assume temperature and density regime so that all particles can be treated using Boltzmann distributions.

In nucleus-nucleus collisions the absolute values of the baryon number, electric charge and strangeness are fixed by the initial conditions. Modelling particle production in statistical thermodynamics would in general require the canonical formulation of all these quantum numbers. From the previous analysis [7,11], however, it is clear, that in heavy ion collisions only strangeness should be treated exactly, whereas the conservation of baryon and electric charges can be described by the appropriate chemical potentials in the grand canonical ensemble.

Within the approximations described above and neglecting first the contributions from multi-strange baryons, the generating function in equation (4) has the following form,

$$\tilde{Z}(T, V, \mu_Q, \mu_B, \phi) = \exp(N_{s=0} + N_{s=1}e^{i\phi} + N_{s=-1}e^{-i\phi}) \quad (6)$$

where $N_{s=0,\pm 1}$ is defined as the sum over all particles and resonances having strangeness 0, ± 1 ,

$$N_{s=0,\pm 1} = \sum_k Z_k^1 \quad (7)$$

and Z_k^1 is the one-particle partition function defined as

$$Z_k^1 \equiv \frac{V g_k}{2\pi^2} m_k^2 T K_2(m_k/T) \exp(b_k \mu_B + q_k \mu_q) \quad (8)$$

with the mass m_k , spin-isospin degeneracy factor g_k , particle baryon number b_k and electric charge q_k . The volume of the system is V and the chemical potentials related with the electric charge and the baryon number are determined by μ_q and μ_B respectively.

With the particular form of the generating function equations (6,7,8) the ϕ -integration in equation (4) can be done analytically giving the canonical partition function for a gas with total strangeness S [11]:

$$Z_S(T, V, \mu_B, \mu_Q) = Z_0(T, V, \mu_B, \mu_Q) I_S(x) \quad (9)$$

where $Z_0 = \exp(N_{S=0})$ is the partition function of all particles having zero strangeness and the argument of the Bessel function

$$x \equiv 2\sqrt{S_1 S_{-1}}. \quad (10)$$

with $S_{\pm 1} \equiv N_{s=\pm 1}$. The parameter x thus measures the total number of strange particles in thermal fireball.

The calculation of the particle density n_k in the canonical formulation is straightforward. It amounts to the replacement

$$Z_k^1 \mapsto \lambda_k Z_k^1 \quad (11)$$

of the corresponding one-particle partition function in equation (7) and taking the derivative of the canonical partition function equation (4) with respect to λ_k

$$n_k^C \equiv [\lambda_k \frac{\partial}{\partial \lambda_k} \ln Z_Q(\lambda_k)]_{\lambda_k=1} \quad (12)$$

As an example, we quote the result for the density of thermal kaons in the canonical formulation assuming that the total strangeness of the system $S = 0$,

$$n_K^C = \frac{Z_K^1}{V} \frac{S_1}{\sqrt{S_1 S_{-1}}} \frac{I_1(x)}{I_0(x)} \quad (13)$$

Comparing the above formula with the result for thermal kaons density in the grand canonical ensemble, $n_K^{GC} = (Z_K^1/V) \exp(\mu_s/T)$, one can see that the canonical result can be obtained from the grand canonical one replacing the strangeness fugacity $\lambda_S \equiv \exp(\mu_S/T)$ in the following way:

$$n_K^C = n_K^{GC} \left(\lambda_S \mapsto \frac{S_1}{\sqrt{S_1 S_{-1}}} \frac{I_1(x)}{I_0(x)} \right) \quad (14)$$

In the limit of large x that is large volume and/or temperature the canonical and the grand canonical formulation are equivalent. For a small number of strange particles in a system, however, the differences are large. This can be seen in the most transparent way when comparing two limiting situations: the large and small x limit of the above equation. In the limit $x \rightarrow \infty$ we have

$$\lim_{x \rightarrow \infty} \frac{I_1(x)}{I_0(x)} \rightarrow 1 \quad (15)$$

and the kaon density is independent of the volume of the system as expected in the grand canonical ensemble. On the other hand in the limit of a small x we have

$$\lim_{x \rightarrow 0} \frac{I_1(x)}{I_0(x)} \rightarrow \frac{x}{2} \quad (16)$$

and the particle density is linearly dependent on the volume. It is thus clear, that the major difference between the canonical and the grand canonical treatment of the conservation laws appears through different volume dependence of strange particle densities as well as strong suppression of thermal particle phase space. The relevant parameter, F_S , which measures the suppression of particle multiplicities from their grand canonical result is determined by the ratio of the Bessel functions

$$F_S \equiv \frac{I_1(x)}{I_0(x)} \quad (17)$$

with the argument x defined in equation (10).

In Fig. 1 we show the canonical suppression factor $F_S(x)$ as a function of the argument x . To relate the initial volume of the system with the number of participant in A-A collisions one uses the approximate relation $V \sim 1.9\pi A_{part}$. The corresponding value of x at SIS, AGS and SPS energy is calculated with the baryon chemical potential and temperature extracted from the measured particle multiplicity ratios [11]. The results in Fig. 1 shows the importance of the canonical treatment of strangeness conservation at SIS energy. Here, the canonical suppression factor can be even larger than an order of magnitude.

For central Au-Au collisions at AGS or SPS energy this suppression is not relevant any more and the (GC)-formalism is adequate. In general, one expects, that the statistical interpretation of particle production in central heavy ion collisions requires the canonical treatment of strangeness conservation if the CMS collision energy $\sqrt{s} < 2-3\text{GeV}$. This is mainly because at these energies the freeze-out temperature is still too low to maintain large argument expansion of the Bessel functions in equation (15). The canonical description of strangeness conservation can be, however, also of importance at the SPS energy were $\sqrt{s} \sim 18\text{GeV}$ when one considers the peripheral heavy ion collisions. This is particularly true for multistrange particle production since the canonical suppression of the thermal particle phase-space increases with strangeness content of the particle [10].

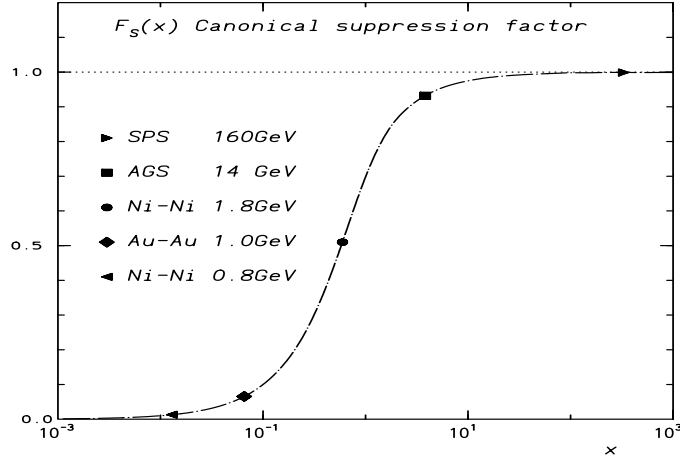


FIG. 1. Canonical strangeness suppression factor (see text).

A. Multistrange particle multiplicities

The extension of the canonical description to multi-strange particle multiplicities is straightforward. One needs first to extend the generating functional in Eq. 6 by including the contributions of multistrange baryons. In this case the canonical partition function constraint by the strangeness neutrality condition reads [10,13],

$$Z_{S=0}^C = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi \exp\left(\sum_{s=-3}^3 S_s e^{is\phi}\right), \quad (18)$$

where $S_s \equiv \sum_i Z_i^1$ and the sum is taken over all particles and resonances carrying strangeness s . The one-particle partition function Z_i^1 is defined in Eq.(8).

With the particular form of the partition function given by Eq. (18) the density n_s of particle i with strangeness s in volume V is obtained by the replacement $Z_i \mapsto \lambda_i Z_i$ in Eq. (18) and then taking an appropriate derivative [7,10]:

$$n_{\pm s} = \frac{\langle N_{\pm s} \rangle}{V} \equiv \left[\frac{\lambda_i}{V} \frac{\partial \ln Z_0}{\partial \lambda_i} \right]_{\lambda_i=1} \simeq Z_{\pm s} \frac{(S_{\pm 1})^s}{(S_{+1} S_{-1})^{s/2}} \{ I_s(x_1) I_0(x_2) + \sum_{m=1}^{\infty} I_m(x_2) [I_{2m+|s|}(x_1) A^{m/2} + I_{2m-|s|}(x_1) A^{-m/2}] \} / Z_{S=0} \quad (19)$$

where

$$x_k \equiv 2\sqrt{S_k S_{-k}} \quad , \quad A \equiv \frac{S_{-1}^2 S_2}{S_1^2 S_{-2}}, \quad (20)$$

and the partition function

$$Z_{S=0} \simeq I_0(x_1)I_0(x_2) + \sum_{m=1}^{\infty} I_{2m}(x_1)I_m(x_2)[A^{m/2} + A^{-m/2}]. \quad (21)$$

In the derivation of Eq.(19) we have neglected, after differentiation over particle fugacity, the term $S_{\pm 3}$. This approximation, however, due to small value of $S_{\pm 3}$ coefficients in comparison with $S_{\pm 1}$ and $S_{\pm 2}$ is quite satisfactory. The complete expression for the partition function without any approximation can be found in reference [13].

In the large system like Pb-Pb and for large collision energy, required to reach high T , the density n_s of particle carrying strangeness s is V independent. In the opposite limit, however, this dependence is changed to $n_s \sim V^s$, which can be verified from Eq.(19). Indeed for small x_i we have approximately:

$$n_{\pm s} \simeq Z_{\pm s} \frac{(S_{\pm 1})^s}{(S_{+1}S_{-1})^{s/2}} \frac{I_s(x_1)}{I_0(x_1)}. \quad (22)$$

and when expanding the Bessel functions $I_n(x) \sim x^n$ in Eq.(22) we see that $n_s \sim V^s$.

From the above expression it is clear that strangeness suppression, which is measured by the ration I_s/I_0 , is increasing with the strangeness content of the particle. Thus, there are two important ingredients of canonical modifications of multistrange particle density with respect to their (GC) value: (i) the density is volume dependent that is also centrality dependent and (ii) the thermal phase is suppressed and this suppression increases with strangeness content of the particle.

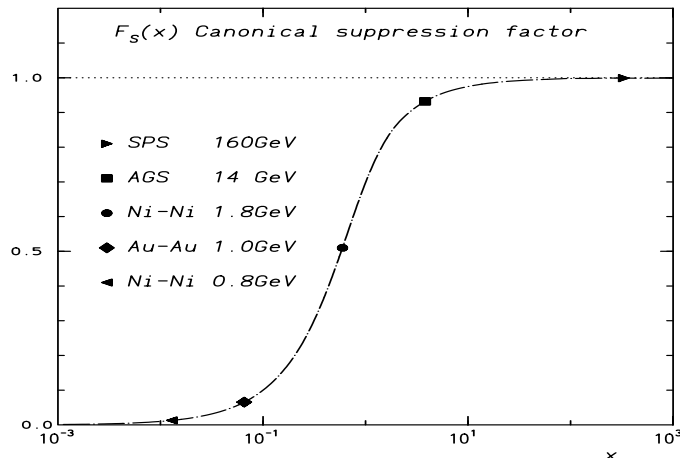


FIG. 2. Measured K^+ multiplicity per participant A_{part} as a function of A_{part} for Au-Au collisions at 1 A/GeV together with the canonical model results

III. TIME EVOLUTION AND STRANGENESS EQUILIBRATION

In the last section we have formulated the statistical model for strange particle multiplicities $\langle N_S \rangle$ assuming that the system is in thermal and chemical equilibrium. We have shown that dependently on the total number of strange particles there are two distinct equilibrium limits [8,11]: if $\langle N_s \rangle$ is small then we are in the canonical regime and in the opposite limit the canonical and grand canonical description coincides. In this section we consider the time evolution of the multiplicity of strange particles and formulate a kinetic master equations which distinguish between these two equilibrium limits. For a sake of illustration we consider a simple example of K^+K^- production in the environment of thermal pions in volume V and temperature T due to the following binary process, $\pi^+\pi^- \rightarrow K^+K^-$. We formulate for this example the kinetics for the time evolution of kaon multiplicities and their approach to chemical equilibrium .

In the standard formulation [15], the rate equation for this binary process is described by the following population equation:

$$\frac{d \langle N_K \rangle}{d\tau} = \frac{G}{V} \langle N_{\pi^+} \rangle \langle N_{\pi^-} \rangle - \frac{L}{V} \langle N_{K^+} \rangle \langle N_{K^-} \rangle, \quad (23)$$

where $G \equiv \langle \sigma_G v \rangle$ and $L \equiv \langle \sigma_L v \rangle$ give the momentum-averaged cross sections for the gain $\pi^+ \pi^- \rightarrow K^+ K^-$ and the loss $K^+ K^- \rightarrow \pi^+ \pi^-$ process respectively. The value of $\langle N_K \rangle$ represents the total number of produced kaons.

To include the possible correlations between the production of K^+ and K^- [8], let us define $P_{i,j}$ as the probability to find i number of K^+ and j number of K^- in an event. We also denote by P_i as the probability to find i number of K pairs in an event. The average number of K per event is defined as:

$$\langle N_K \rangle = \sum_{i=0}^{\infty} i P_i. \quad (24)$$

We can now write the following general rate equation for the average kaon multiplicities:

$$\frac{d\langle N_K \rangle}{d\tau} = \frac{G}{V} \langle N_{\pi^+} \rangle \langle N_{\pi^-} \rangle - \frac{L}{V} \sum_{i,j} ij P_{i,j}. \quad (25)$$

Due to the local conservation of quantum numbers, we have:

$$\begin{aligned} P_{i,j} &= P_i \delta_{ij}, \\ \sum_{i,j} ij P_{i,j} &= \sum_i i^2 P_i \equiv \langle N^2 \rangle = \langle N \rangle^2 + \langle \delta N^2 \rangle, \end{aligned} \quad (26)$$

where $\langle \delta N^2 \rangle$ represents the event-by-event fluctuation of the number of $K^+ K^-$ pairs. Note that we always consider abundant π^+ and π^- so that we can neglect the number fluctuation of these particles and the change of their multiplicities due to the considered processes.

Following Eqs.(25-26) the general rate equation for the average number of $K^+ K^-$ pairs can be written as:

$$\frac{d\langle N_K \rangle}{d\tau} = \frac{G}{V} \langle N_{\pi^+} \rangle \langle N_{\pi^-} \rangle - \frac{L}{V} \langle N_K^2 \rangle. \quad (27)$$

For abundant production of $K^+ K^-$ pairs where $\langle N_K \rangle \gg 1$,

$$\langle N_K^2 \rangle \approx \langle N_K \rangle^2, \quad (28)$$

and Eq.(27) obviously reduces to the standard form:

$$\frac{d\langle N_K \rangle}{d\tau} \approx \frac{G}{V} \langle N_{\pi^+} \rangle \langle N_{\pi^-} \rangle - \frac{L}{V} \langle N_K \rangle^2. \quad (29)$$

However, for rare production of $K^+ K^-$ pairs where $\langle N_K \rangle \ll 1$, the rate equations (23) and (29) are no longer valid. We have instead

$$\langle N_K^2 \rangle \approx \langle N_K \rangle, \quad (30)$$

which reduces Eq.(27) to the following form [8]:

$$\frac{d\langle N_K \rangle}{d\tau} \approx \frac{G}{V} \langle N_{\pi^+} \rangle \langle N_{\pi^-} \rangle - \frac{L}{V} \langle N_K \rangle. \quad (31)$$

Thus, in the limit where $\langle N_K \rangle \ll 1$, the absorption term depends on the pair number only linearly, instead of quadratically for the limit of $\langle N_K \rangle \gg 1$. Thus, it is clear that the time evolutions and equilibrium values for kaon multiplicities are obviously different in the above limiting situations.

In the limit of large $\langle N_K \rangle$ the equilibrium value for the number of $K^+ K^-$ pairs, which coincides with the multiplicity of K^+ and K^- , is obtained from Eq.(29) as,

$$\langle N_K \rangle_{\text{eq}}^{\text{GC}} = \frac{V}{2\pi^2} m_K^2 T K_2(M_K/T) \quad (32)$$

thus, it is described by the (GC) result with vanishing chemical potential due to strangeness neutrality condition.

In the opposite limit where $\langle N_K \rangle \ll 1$, the time evolution is described by Eq.(31), which has the following equilibrium solution:

$$N_{\text{eq}}^{\text{C}} = \left[\frac{V}{2\pi^2} M_{K^+}^2 T K_2(M_{K^+}/T) \right] \left[\frac{V}{2\pi^2} M_{K^-}^2 T K_2(M_{K^-}/T) \right]. \quad (33)$$

The above equation demonstrates the locality of strangeness conservation. With each K^+ the K^- is produced in the same event in order to conserve strangeness exactly and locally. This is the result expected from the (C) formulation of the conservation laws as described in the previous section. We note that Eq.(33) is just the leading term in the expansion of the canonical result for multiplicities of particles which are carrying $U(1)$ charges. The general expression is given by Eq.(13) and Eq.(20).

Comparing Eq.(32) and Eq.(33), we first find that, for $\langle N_K \rangle \ll 1$, the equilibrium value in the canonical formulation is far smaller than what is expected from the grand canonical result as

$$\langle N_K \rangle_{\text{eq}}^{\text{C}} = [\langle N_K \rangle_{\text{eq}}^{\text{GC}}]^2 \ll \langle N_K \rangle_{\text{eq}}^{\text{GC}}. \quad (34)$$

This shows the importance of the canonical description of quantum number conservation when the multiplicity of particles carrying non-zero $U(1)$ charges is small. We also note that the volume dependence in the two cases differs. The particle density in the GC limit is independent of V whereas in the opposite canonical limit the density scales linearly with V . Secondly, we note that the relaxation time for a canonical system is far shorter than what is expected from the grand canonical result [8]. It is also clear from Eq.(26) that (C) and (GC) limits are essentially determined by the size of $\langle \delta N^2 \rangle$, the event-by-event fluctuation of the number of K^+K^- pairs. The grand canonical results correspond to small fluctuations, i.e., $\langle \delta N^2 \rangle / \langle N \rangle^2 \ll 1$; while the canonical description is necessary in the opposite limit.

IV. MODEL PREDICTIONS VERSUS EXPERIMENTAL DATA

In heavy ion collisions the number of produced strange particles depends on the collision energy and the centrality of these collisions. At low collision energies like eg. in GSI/SIS the freezeout temperature is relatively low being of the order of 50 – 80 MeV. Consequently the number of strange particles in the final state is very small. Following our discussion in the last sections it is clear that the statistical description require here the canonical approach. In Fig. 2 we show the experimental data on K^+ yield per participant A_{part} as a function of A_{part} measured in Au-Au collisions at $E_{\text{lab}} \sim 1$ A/GeV [14].

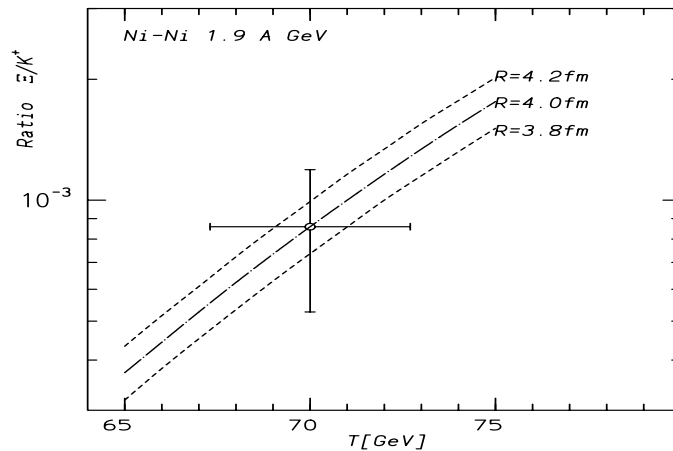


FIG. 3. The ratio of Ξ to K^+ multiplicity as a function of temperature. The point with errors indicates the predictions of the thermal model for Ni-Ni collisions at 1.9 A/GeV. The lines corresponds to canonical calculations for different centrality measured by the initial radius R of the system.

The volume parameter in the statistical model scale with the number of participant. We can thus directly compare the model with data by fixing thermal parameters, the temperature and the baryon chemical potential, such that to reproduce the measured particle multiplicity ratios [11]. In Fig. 2 the results of the canonical model is shown

by the full line. The results clearly indicates that strong almost quadratic dependence of kaon yield on the number of participant is well reproduced by the model. The quadratic dependence of particle multiplicity in the canonical regime is the basic property of the model. In Fig. 3 we calculate the ratio Ξ/K^+ multiplicities for central $Ni - Ni$ collisions in the temperature range which corresponds to the collision energy of $E_{lab} \sim 2$ A/GeV.

The yield of Ξ is seen to be substantially smaller then the yield of K^+ . This result is not only related with the differences in particle masses but particularly it appears due to the canonical suppression. Since Ξ carry strangeness minus two it has to be produced together with two K^+ to locally and exactly neutralized strangeness. It is also interesting to note that the Ξ/K^+ ratio is independent on the baryon chemical potential. This is because K^+ appears together with Λ , thus it contains the same dependence on μ_B as multistrange baryons.

The importance of the canonical treatment of strangeness conservation is also seen in higher collision energies like at the SPS when considering centrality dependance of multistrange baryons. In peripheral collisions the yield of strange particles is small such that also here the canonical description should be applied. The canonical suppression of thermal particle phase-space increases with strangeness content of the particle. The exact conservation of strangeness requires that each particle carrying strangeness \bar{s} has to appear e.g. with s other particles of strangeness one to satisfy strangeness neutrality condition.

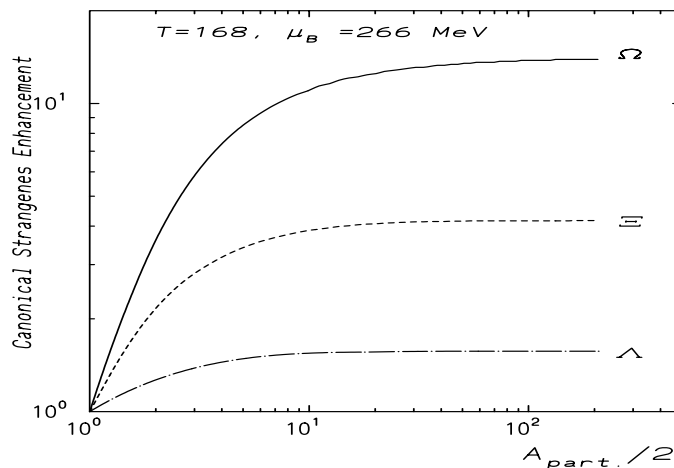


FIG. 4. Particle multiplicities per participant normalized to its value in p+p system as a function of the number of participants $A_{part.}$ calculated in the statistical model in the canonical ensemble.

In Fig. 4 we calculate the multiplicity/participant of $\Omega, \Xi,$ and Λ relative to its value in a small system with only two participants [10]. Thermal parameters were assumed here to be A_{part} independent. Fig. 4 shows that the statistical model in (C) ensemble reproduces the basic features of WA97 data [16]: the enhancement pattern and enhancement saturation for large A_{part} indicating here that (GC) limit is reached. Fig. 4 also demonstrate different A_{part} dependence of strange and multistrange baryons. For small A_{part} this dependence is power like as describe by Eq.(22). The quantitative comparison of the model with the experimental data would require an additional assumption on the variation of μ_B with centrality to account for larger value of \bar{B}/B ratios in p+A than in Pb+Pb collisions [10,16]. The most recent results of NA57 [17], showing an abrupt change of the enhancement for Ξ are, however, very unlikely to be reproducible in terms of the canonical approach.

V. SUMMARY AND CONCLUSIONS

We have discussed the importance of the conservation laws in the application of the statistical model to the description of strangeness production in heavy ion collisions. We have presented the arguments that the more general treatment of strangeness conservation based on the canonical ensemble is required if one compares the model with experimental data for particle yields obtained in central A-A collisions at SIS energies or peripheral collisions at the SPS. In both situations the number of produced strange particles per event is still too small to use the asymptotic grand canonical ensemble. The time evolution of strangeness production and the approach to chemical equilibrium limit was discussed in the context of a kinetic approach. We have shown on few examples that the statistical model

predictions are consistent with the experimental data. A more complete presentation of the model versus data can be found in [2–6,9–11].

VI. ACKNOWLEDGMENTS

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