Rare Kaon Decays – Overview

GERHARD BUCHALLA

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract

The theory of rare K decays is reviewed, emphasizing short-distance processes and the prospects to probe the physics of flavour. A brief overview of the subject is presented, along with a more detailed discussion of the theory of $K \to \pi \nu \bar{\nu}$ decays.

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1 Introduction and Overview

Rare decays of kaons [1] probe the details of weak interactions at the quantum level. They can be sensitive to energy scales much higher than the kaon mass itself and can thus yield fundamental insights into physics at very short distances. A remarkable historical example is the suppression of flavour-changing neutral currents, implied by the fact that $B(K_L \to \mu^+ \mu^-) = 7 \cdot 10^{-9}$ while $B(K^+ \to \mu^+ \nu) = 0.64$, which led to the GIM mechanism and the concept of charm. Another famous case is the $K_L - K_S$ mass difference ΔM_K , which arises through $K - \bar{K}$ mixing, a second-order weak process with high sensitivity to the charm-quark mass m_c . The analysis of this rare transition by Gaillard and Lee in 1974 gave a correct estimate of $m_c \sim 1.5$ GeV, prior to the discovery of charm. Today the focus has shifted to modes that depend on much higher energy scales, related to CP violation, the top quark and, in general, new degrees of freedom from physics beyond the standard model. However, the spirit of the approach is still very much the same.

The field of rare K decays is rich and varied. Three broad classes may be distinguished:

- Long-distance dominated rare or radiative decays such as $K^+ \to \pi^+ l^+ l^-$, $K_L \to \pi^0 \gamma \gamma$, $K_S \to \gamma \gamma$ or $K_L \to \mu^+ \mu^-$. Although not immediately useful to obtain short-distance information, they are still important to learn about low-energy QCD dynamics, largely relying on chiral perturbation theory. In this way long-distance "background" can be better controlled in cases where it is more difficult to disentangle short- and long-distance dynamics.
- Short-distance dominated decays as $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$ can provide excellent tools to test the standard model with high precision. $K_L \to \pi^0 e^+ e^-$ partly belongs to this class as well, but long-distance physics plays a non-negligible role in this case.
- Decay modes that are forbidden in the standard model could be dramatic indicators of new physics. Examples are $K_L \to \mu e$, $K^+ \to \pi^+ \mu^+ e^-$ and $K_L \to \pi^0 \mu e$, where stringent upper limits on the branching ratios exist of $4.7 \cdot 10^{-12}$ [2], $2.8 \cdot 10^{-11}$ [3] and $4.4 \cdot 10^{-10}$ [4], respectively.

In the following section we will very briefly discuss the most important theoretical methods needed to describe rare K decays. Subsequently we shall focus on the "golden modes" $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$. We review in particular the status of theoretical uncertainties, especially in the charm sector of $K^+ \to \pi^+ \nu \bar{\nu}$, which is the most critical issue. For a more detailed account of other important subjects in the field of rare kaon decays we refer the reader to the corresponding articles in these proceedings. Specific overviews are given by D'Ambrosio (long-distance modes), Silvestrini (new physics) and Littenberg (experiment).

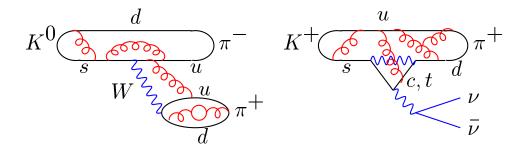


Figure 1: QCD effects in weak decays.

2 Theoretical Methods

The essential problem in computing weak decays of hadrons is the influence of strong interactions, which need to be properly accounted for to extract the underlying flavour physics at the quark level (Fig. 1). For kaon decays two different, complementary approaches are at our disposal: The framework of effective weak hamiltonians and chiral perturbation theory (ChPT).

The effective weak hamiltonian has the form

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_i C_i(\mu) Q_i \tag{1}$$

Here V_{CKM} is a CKM factor. The C_i are Wilson coefficients comprising the short-distance contributions from scales $> \mu \sim 1$ GeV. They can be calculated perturbatively from the underlying fundamental theory, i.e. the standard model or one of its extensions. The Q_i are dimension-6 operators, typically of the 4-fermion type such as e.g. $(\bar{s}u)_{V-A}(\bar{u}d)_{V-A}$. Their matrix elements between hadronic states contain the nonperturbative, long-distance dynamics of the weak amplitude.

- The effective hamiltonian formalism is based on an operator product expansion (corresponding to integrating out heavy fields such as W or top), which achieves a systematic factorization of short-distance (C_i) and long-distance $(\langle Q_i \rangle)$ contributions.
- The disadvantage is that using \mathcal{H}_{eff} the QCD dynamics is still formulated in terms of quarks and gluons. The matrix elements of the Q_i , involving quarks, must be calculated between hadronic states. In general, this is a very complicated task.
- The advantage is that the short-distance information (dependence on top, weak phases, new physics parameters), which we are primarily interested in, is explicit.

In *chiral perturbation theory* the strong and weak interactions are formulated in terms of the meson fields

$$\Sigma = \exp\left(\frac{2i}{f}\Phi\right) \tag{2}$$

where

$$\Phi \equiv T^a \pi^a = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$
(3)

 Σ transforms as $\Sigma \to L\Sigma R^{\dagger}$ under $SU(3)_L \otimes SU(3)_R$ chiral rotations and f is the generic decay constant for the light pseudoscalars (in a normalization in which $f_{\pi} = 131 \,\mathrm{MeV}$). The lagrangians of strong (QCD) and weak $(\Delta S = 1)$ interactions are organized in powers of derivatives, corresponding to momenta, and quark masses:

$$\mathcal{L}^{QCD} = \mathcal{L}_2^{QCD} + \mathcal{L}_4^{QCD} + \dots \tag{4}$$

$$\mathcal{L}^{\Delta S=1} = \mathcal{L}_2^{\Delta S=1} + \mathcal{L}_4^{\Delta S=1} + \dots$$
 (5)

The lowest order terms read explicitly

$$\mathcal{L}_{2}^{QCD} = \frac{f^{2}}{8} \operatorname{tr} \left[D_{\mu} \Sigma D^{\mu} \Sigma^{\dagger} + 2B_{0} (\mathcal{M} \Sigma^{\dagger} + \Sigma \mathcal{M}^{\dagger}) \right]$$
 (6)

$$\mathcal{L}_{2}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} |V_{us}^* V_{ud}| g_8 \frac{f^4}{4} \operatorname{tr} \lambda_6 D_\mu \Sigma D^\mu \Sigma^\dagger$$
 (7)

Here $\mathcal{M} = \operatorname{diag}(m_u, m_d, m_s)$ is the quark mass matrix, f and B_0 are the two parameters entering \mathcal{L}_2^{QCD} , and g_8 is the dominant weak coupling in ChPT at second order in momenta (for simplicity we have omitted a second term in $\mathcal{L}_2^{\Delta S=1}$, proportional to a coupling g_{27} , which is suppressed numerically as a consequence of the $\Delta I = 1/2$ rule).

- ChPT is a low-energy effective theory based on the chiral symmetries of QCD combined with an expansion in p^2/Λ^2 , where p represents the (small) momenta of the light mesons and $\Lambda \sim 1$ GeV is the hadronic scale. This approach is model-independent, since all possible interaction terms with the correct properties under chiral symmetry have to be included. Each term is multiplied with an a priori unknown parameter (or coupling constant). The framework becomes predictive because only a finite number of parameters appears to any fixed order in the momentum expansion. Once they are extracted from a corresponding number of measurements, further predictions can be made.
- The advantage of ChPT is that the QCD dynamics is already expressed in terms of hadrons (Φ) .
- The disadvantage is that the short-distance physics is implicit. It is hidden in the counterterms, i.e. the coupling constants of the chiral lagrangian, which are renormalized by loop contributions.

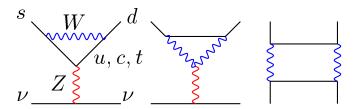


Figure 2: The leading order electroweak diagrams contributing to $K \to \pi \nu \bar{\nu}$ in the standard model.

In principle the "dual" pictures of $\mathcal{L}^{\Delta S=1}$ and \mathcal{H}_{eff} describe the same physics. However, establishing a direct link between them would require the computation of the matrix elements $\langle Q_i \rangle$ and a comparison of the resulting $\langle \mathcal{H}_{eff} \rangle$ with the amplitudes from $\mathcal{L}^{\Delta S=1}$. For a general kaon decay this is not possible at present because of our poor control of nonperturbative QCD in terms of quarks and gluons. In the meantime, the two pictures approach the problem of weak amplitudes from opposite directions: \mathcal{H}_{eff} starting from high energies, $\mathcal{L}^{\Delta S=1}$ from low energies. From the characteristic advantages and disadvantages listed above it is clear that \mathcal{H}_{eff} is more useful for applications where short-distance dynamics is essential, such as short-distance dominated rare decays or CP violation $(\varepsilon, \varepsilon'/\varepsilon)$. On the other hand, ChPT is the method of choice for processes controlled by long-distance physics.

An important special case is given by the modes $K \to \pi \nu \bar{\nu}$. Here the hadronic matrix element of the quark-level operator is particularly simple and known from $K \to \pi l \nu$. In this situation the effective hamiltonian approach can solve the problem completely as we will further discuss in the following section. It is interesting to note that a complementary example exists as well: $K_S \to \gamma \gamma$ can be computed by a finite one-loop calculation based on (7). In that case a parameter-free prediction is obtained once g_8 is fixed from $K \to \pi \pi$ (see the talk by D'Ambrosio for more details).

3 The Golden Modes:

$$K^+ \to \pi^+ \nu \bar{\nu}$$
 and $K_L \to \pi^0 \nu \bar{\nu}$

3.1 Basic Properties and Results

The decays $K \to \pi \nu \bar{\nu}$ proceed through flavour-changing neutral currents, which arise at one loop in the standard model (Fig. 2). The GIM structure of the amplitude can be written as

$$\sum_{i=u,c,t} \lambda_i F(x_i) = \lambda_c \left(F(x_c) - F(x_u) \right) + \lambda_t \left(F(x_t) - F(x_u) \right) \tag{8}$$

with $\lambda_i = V_{is}^* V_{id}$ and $x_i = m_i^2 / M_W^2$. The first important point is the characteristic hard GIM cancellation pattern, which means that the function F depends as a power on the internal mass scale

$$F(x_u) \sim \frac{\Lambda_{QCD}^2}{M_W^2} \sim 10^{-5} \ll F(x_c) \sim \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c} \sim 10^{-3} \ll F(x_t) \sim 1$$
 (9)

The up-quark contribution is a long-distance effect, determined by the scale Λ_{QCD} . As an immediate consequence, top and charm contribution with their hard scales m_t , m_c dominate the amplitude, whereas the long-distance part $F(x_u)$ is negligible. Note that the charm contribution, $\lambda_c F(x_c) \sim 10^{-1} \cdot 10^{-3}$, and the top contribution, $\lambda_t F(x_t) \sim 10^{-4} \cdot 1$, have the same order of magnitude when the CKM factors are included. The short-distance dominance of the $s \to d\nu\bar{\nu}$ transition next implies that the process is effectively semileptonic, because a single, local operator $(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$ describes the interaction at low-energy scales. Hence the amplitude has the form

$$A(K^+ \to \pi^+ \nu \bar{\nu}) \sim G_F \alpha (\lambda_c F_c + \lambda_t F_t) \langle \pi^+ | (\bar{s}d)_V | K^+ \rangle (\bar{\nu} \nu)_{V-A}$$
 (10)

The coefficient function $\lambda_c F_c + \lambda_t F_t$ is calculable in perturbation theory. The hadronic matrix element can be extracted from $K^+ \to \pi^0 e^+ \nu$ decay via isospin. The $K^+ \to \pi^+ \nu \bar{\nu}$ amplitude is then completely determined, and with good accuracy.

The neutral mode proceeds through CP violation in the standard model and has the form

$$A(K_L \to \pi^0 \nu \bar{\nu}) \sim \text{Im} \lambda_t F_t + \text{Im} \lambda_c F_c$$
 (11)

where

$$\operatorname{Im} \lambda_t F_t \sim 10^{-4} \cdot 1 \gg \operatorname{Im} \lambda_c F_c \sim 10^{-4} \cdot 10^{-3}$$
 (12)

The $K \to \pi \nu \bar{\nu}$ modes have been studied in great detail over the years to quantify the degree of theoretical precision. Important effects come from short-distance QCD corrections. These were computed at leading order in [5]. The complete next-to-leading order calculations [6, 7, 8] reduce the theoretical uncertainty in these decays to $\sim 5\%$ for $K^+ \to \pi^+ \nu \bar{\nu}$ and $\sim 1\%$ for $K_L \to \pi^0 \nu \bar{\nu}$. This picture is essentially unchanged when further small effects are considered, including isospin breaking in the relation of $K \to \pi \nu \bar{\nu}$ to $K^+ \to \pi^0 l^+ \nu$ [9], long-distance contributions [10, 11], the CP-conserving effect in $K_L \to \pi^0 \nu \bar{\nu}$ in the standard model [10, 12], two-loop electroweak corrections for large m_t [13] and subleading-power corrections in the OPE in the charm sector [14].

While already $K^+ \to \pi^+ \nu \bar{\nu}$ can be reliably calculated, the situation is even better for $K_L \to \pi^0 \nu \bar{\nu}$. Since only the imaginary part of the amplitude contributes, the charm sector, in $K^+ \to \pi^+ \nu \bar{\nu}$ the dominant source of uncertainty, is completely negligible for $K_L \to \pi^0 \nu \bar{\nu}$ (0.1% effect on the branching ratio). Long distance contributions ($\lesssim 0.1\%$) and also the indirect CP violation effect ($\lesssim 1\%$) are likewise negligible. The total theoretical uncertainties, from perturbation theory in the top sector and in the isospin breaking corrections, are safely

Table 1: Compilation of important properties and results for $K \to \pi \nu \bar{\nu}$.

| | $K^+ \to \pi^+ \nu \bar{\nu}$ | $K_L 	o \pi^0 u \bar{ u}$ |
|-----------------|---|--|
| | CP conserving | CP violating |
| CKM | V_{td} | ${\rm Im}V_{ts}^*V_{td} \sim J_{CP} \sim \eta$ |
| contributions | top and charm | only top |
| scale dep. (BR) | $\pm 20\%$ (LO) | $\pm 10\% \text{ (LO)}$ |
| | $\rightarrow \pm 5\% \text{ (NLO)}$ | $\rightarrow \pm 1\% \text{ (NLO)}$ |
| BR (SM) | $(0.8 \pm 0.3) \cdot 10^{-10}$ | $(2.6 \pm 1.2) \cdot 10^{-11}$ |
| exp. | $\left(1.5^{+3.4}_{-1.2}\right) \cdot 10^{-10} \text{ BNL } 787 \text{ [15]}$ | $< 5.9 \cdot 10^{-7} \text{ KTeV } [16]$ |

Table 2: Relative uncertainties in $|V_{td}|$ from $K^+ \to \pi^+ \nu \bar{\nu}$. The errors shown added in quadrature amount to a total of $\Delta |V_{td}|/|V_{td}| = \pm 12\%$.

| $B(K^+ \to \pi^+ \nu \bar{\nu})$ | | $\mu_t/{\rm GeV}$ | $\mu_c/{\rm GeV}$ |
|----------------------------------|--|------------------------------------|----------------------------------|
| $(1.0 \pm 0.1) \cdot 10^{-10}$ | | 100 - 300 | 1 - 3 |
| ±6.8% | | $\pm 0.5\%$ | $\pm 4.5\%$ |
| | | | |
| V_{cb} | $\Lambda_{\overline{\rm MS}}^{(4)}/{ m GeV}$ | $m_t/{ m GeV}$ | $m_c/{ m GeV}$ |
| V_{cb} 0.040 ± 0.002 | $\Lambda_{\overline{\rm MS}}^{(4)}/{\rm GeV}$ 0.325 ± 0.080 | $\frac{m_t/\text{GeV}}{166 \pm 5}$ | $m_c/{\rm GeV}$ 1.3 ± 0.1 |

below 3% for $B(K_L \to \pi^0 \nu \bar{\nu})$. This makes this decay mode truly unique and very promising for phenomenological applications.

In Table 1 we have summarized some of the main features of $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$. Note that the ranges given as the standard model predictions in Table 1 arise from our, at present, limited knowledge of standard model parameters (CKM), and not from intrinsic uncertainties in calculating the branching ratios.

3.2 Theoretical Uncertainties in $K^+ \to \pi^+ \nu \bar{\nu}$ and $|V_{td}|$

Table 2 displays the various sources of uncertainty for determining $|V_{td}|$ from $K^+ \to \pi^+ \nu \bar{\nu}$, where a measured branching ratio of $(1.0 \pm 0.1) \cdot 10^{-10}$ is assumed for illustration. Uncertainties from input parameters are shown in the lower half. The residual dependences on the renormalization scales (μ_t and μ_c for the top-sector and the charm-sector, respectively) are used to estimate the uncertainty intrinsic to the theoretical calculation itself, which is necessarily approximate and relies here on NLO perturbation theory.

The most critical issue is clearly the charm sector. After all, $m_c \equiv \bar{m}_c(\bar{m}_c) = 1.3 \,\text{GeV}$ is not extremely large compared to Λ_{QCD} and the applicability of the OPE and perturbation theory at the charm scale has to be decided on a case-by-case basis. Concerning the reliability of perturbation theory in the present case, the following checks can be made.

 \bullet The μ_c -dependence is reduced from $\pm 28\%$ at LO down to $\pm 13\%$ at NLO

in RG improved perturbation theory. (Here and in the following discussion these are relative uncertainties referring to the charm amplitude alone. Due to the existence of the large top-quark contribution, their impact is reduced in the $|V_{td}|$ determination. For instance, the NLO scale dependence of $\pm 13\%$ corresponds to the $\pm 4.5\%$ variation in $|V_{td}|$ shown in Table 2.)

- The NLO result is within the range estimated from scale dependence at LO.
- The difference between the LO and the NLO result is about 10%, hence a very moderate correction.
- The LO terms have the form $x \ln x (\alpha_s \ln x)^n$, resummed to all orders n, and the NLO corrections are $x(\alpha_s \ln x)^n$. (Here $x \equiv x_c$; due to the smallness of x only first-order terms need to be retained.) At $\mathcal{O}(\alpha_s)$ (n = 1) these terms are of the order $\alpha_s x \ln^2 x$ and $\alpha_s x \ln x$, respectively. The (unresummed) term of order $\alpha_s x$ contributes only at NNLO in the charm sector and is not included in the usual NLO results. However, this term is known from the full $\mathcal{O}(\alpha_s)$ calculation in the top sector and can be used to estimate the truncation error, independently of the standard procedure using residual scale dependence. In fact, the $\mathcal{O}(\alpha_s x)$ term is about 10% for charm, fully compatible with the $\pm 13\%$ uncertainty estimated from NLO scale dependence.

These observations demonstrate that perturbation theory is well behaved for the charm contribution and that the error estimate is under control.

So far we have considered the uncertainty due to truncation of the resummed perturbative series. In addition there are also power corrections. One source are the long-distance contributions related to up-quark loops. They are of order Λ_{QCD}^2/m_c^2 and were estimated to be below 5% [11]. The second type of power corrections comes from higher orders in the OPE, in the process where the charm quark is integrated out. The leading corrections are of order m_K^2/m_c^2 . They were recently estimated to be again at the level of about 5% [14]. These effects are safely below the perturbative uncertainty.

We conclude that the charm contribution to $K^+ \to \pi^+ \nu \bar{\nu}$ can be reliably computed based on OPE and RG improved perturbation theory, and that the uncertainty can be assessed with confidence.

3.3 Further Phenomenological Applications

With a measurement of $B(K^+ \to \pi^+ \nu \bar{\nu})$ and $B(K_L \to \pi^0 \nu \bar{\nu})$ available very interesting phenomenological studies could be performed. For instance, $B(K^+ \to \pi^+ \nu \bar{\nu})$ and $B(K_L \to \pi^0 \nu \bar{\nu})$ together determine the unitarity triangle (Wolfenstein parameters ϱ and η) completely (Fig. 3). The expected accuracy with $\pm 10\%$ branching ratio measurements is comparable to the one that can be achieved by CP violation studies at B factories before the LHC era [17].

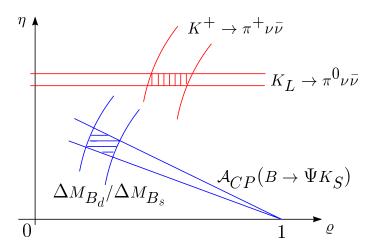


Figure 3: Schematic determination of the unitarity triangle vertex (ϱ, η) from $K \to \pi \nu \bar{\nu}$ (vertically hatched) and from the B system (horizontally hatched). Both determinations can be performed with small theoretical uncertainty and any discrepancy between them would indicate new physics, as illustrated in this hypothetical example.

The quantity $B(K_L \to \pi^0 \nu \bar{\nu})$ by itself offers probably the best precision in determining $\text{Im} V_{ts}^* V_{td}$ or, equivalently, the Jarlskog parameter

$$J_{CP} = \operatorname{Im}(V_{ts}^* V_{td} V_{us} V_{ud}^*) = \lambda \left(1 - \frac{\lambda^2}{2}\right) \operatorname{Im} \lambda_t$$
 (13)

The prospects here are even better than for B physics at the LHC. As an example, let us assume the following results will be available from B physics experiments

$$\sin 2\alpha = 0.40 \pm 0.04$$
 $\sin 2\beta = 0.70 \pm 0.02$ $V_{cb} = 0.040 \pm 0.002$ (14)

The small errors quoted for $\sin 2\alpha$ and $\sin 2\beta$ from CP violation in B decays require precision measurements at the LHC. In the case of $\sin 2\alpha$ we have to assume in addition that the theoretical problem of 'penguin-contamination' can be resolved. These results would then imply $\operatorname{Im} \lambda_t = (1.37 \pm 0.14) \cdot 10^{-4}$. On the other hand, a $\pm 10\%$ measurement $B(K_L \to \pi^0 \nu \bar{\nu}) = (3.0 \pm 0.3) \cdot 10^{-11}$ together with $m_t(m_t) = (170 \pm 3) GeV$ would give $\operatorname{Im} \lambda_t = (1.37 \pm 0.07) \cdot 10^{-4}$. If we are optimistic and take $B(K_L \to \pi^0 \nu \bar{\nu}) = (3.0 \pm 0.15) \cdot 10^{-11}$, $m_t(m_t) = (170 \pm 1) GeV$, we get $\operatorname{Im} \lambda_t = (1.37 \pm 0.04) \cdot 10^{-4}$, a remarkable accuracy. The prospects for precision tests of the standard model flavour sector will be correspondingly good.

The future experimental prospects for $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$ are discussed in the talks by Bryman, Cox, Inagaki, Muramatsu and Ramberg.

Recent work on new-physics effects in $K \to \pi \nu \bar{\nu}$ can be found in [18].

4 Summary

Decays of kaons have played a key role in the development of the standard model. Currently, flavour physics is entering a new era of intense and promising experimental investigation. In this context rare K decays in particular will continue to provide excellent opportunities.

The search for decay modes forbidden in the standard model $(K_L \to \mu e, K \to \pi \mu e)$ probes physics at very short distances and possible exotic scenarios with impressive sensitivity. Tests of chiral perturbation theory, beyond their intrinsic interest, help to develop our theoretical understanding of long-distance background to new physics effects, within a model-independent approach to low-energy QCD. Processes of interest here are $K^+ \to \pi^+ l^+ l^-$, $K_L \to \pi^0 \gamma \gamma$, $K_S \to \gamma \gamma$ among many others. Specific further opportunities are given by searching for violations of discrete symmetries (CP, T) in $K_L \to \pi^0 e^+ e^-$ or $K^+ \to \pi^0 \mu^+ \nu$ (μ -polarization). Of particular importance are standard model precision tests with the golden modes $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$.

The main goals of flavour physics will be accurate and decisive tests of the CKM mechanism and the search for new phenomena. In this respect kaon physics can contribute unique information, complementary to physics with B and D mesons. At present, the standard model appears to work well, also in the flavour sector, and has passed already a number of nontrivial tests. Therefore theoretically clean, high-precision observables, such as those offered by rare K decays, will become even more valuable and important.

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