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't Hooft loops and consistent order parameters for confinement

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We study ratios of partition functions in two types of sectors of SU(2), with fixed temporal center flux and with static fundamental charge. Both can be used as bona fide order parameters for the deconfinement transition.

1. Introduction

It is well known that the deconfinement transition which occurs at a critical temperature T_c in Yang-Mills theory corresponds to the spontaneous breaking of center symmetry. It is natural then to expect that the center degrees of freedom of the gauge field are somehow responsible for confinement and its disappearance. Indeed, numerical studies tend to show that the center vortices, extracted from the gauge field after partial gauge-fixing, confine linearly at $T \sim 0$ with the Y-M string tension [1], at least in the continuum limit [2]. Here, we show quantitatively, in a gauge-invariant way, that the free energy of a center vortex is an order parameter for the deconfinement transition, as anticipated in [3]. It jumps from 0 to $+\infty$ as the temperature is raised.

Alternatively, one should be able to use the free energy F_q of a static fundamental charge, jumping from $+\infty$ to 0 at T_c . The Polyakov loop P is commonly used for this purpose in lattice studies. If $\langle TrP \rangle = e^{-\frac{1}{T}F_q}$, the symmetric (broken) phase gives for F_q an infinite (finite) value. However, the periodic boundary conditions (b.c.) within which $\langle TrP \rangle$ is measured are incompatible with the presence of a single charge. And, like any Wilson loop, $\langle TrP \rangle$ is subject to UV-divergent perimeter terms, such that $\langle TrP \rangle = 0$ at all temperatures as the lattice spacing $a \to 0$. Here, following [4], we measure the gauge-invariant, UVregular free energy of a static fundamental charge, and show that it has the expected behaviour, dual to that of a center vortex. Moreover, the string tension below T_c and the dual (vortex) string tension above T_c [5] are quantitatively related by a universal ratio, for more details, see [6].

2. 't Hooft loop, center vortices, and t.b.c.

The 't Hooft loop \tilde{W} , defined on the dual lattice, multiplies every Wilson loop linked to it by a Z_N center element. In the case of an $R_x \times R_y$ loop in SU(2), it is constructed by flipping the sign of the coupling $\beta \to -\beta$ of the $R_x \times R_y$ (z, t)-plaquettes dual to Σ , the rectangle bounded by \tilde{W} . [To compute the 't Hooft loop, we actually flip β one plaquette at a time as in [5].] Equivalently, as shown explicitly by a change of integration variables, one could choose for Σ any other surface bounded by W. In any case, a set of plaquettes will carry the extra Z_N flux of a center vortex, whose precise location is gaugedependent. The gauge-invariant 't Hooft loop W therefore generates a fluctuating center vortex sheet bounded by \tilde{W} .

When an $L \times L$ spatial 't Hooft loop is created in an $L^3 \times N_t$ lattice, every $L \times N_t$ temporal plane dual to it must carry a Z_N flux. This is equivalent to enforcing twisted boundary conditions (t.b.c.) in this temporal plane. Calling Z_0 the periodic b.c. partition function, we measure the free energy cost $e^{-F_k} = Z_k(i)/Z_0$ of t.b.c. in i = 1, 2 and 3 temporal planes as a function of temperature T. Our results for $Z_k(1)/Z_0$ are shown in Fig. 1 for lattices of size $N_t = 4, L = 6..20$. As expected from Finite-Size Scaling (FSS), they are well described around T_c by a single function of L/ξ ,

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where the correlation length ξ diverges near T_c as $|t|^{-\nu}$, with $t = \frac{T}{T_0} - 1$ the reduced temperature and $\nu \approx 0.63$ for the 3d Ising model universality class. This is shown in Fig. 2, where the FSS variable is $x = LT_c |t|^{\nu}$. The large-x asymptotic behaviour shows an area law $exp(-\tilde{\sigma}_0 x^2)$ in the deconfined phase, and screening exp(exp(-bx))in the confined phase. In the thermodynamic limit, Z_k/Z_0 is a step function and can be used as a UV-regular order parameter as advertised. The interpretation of this behaviour is simple. Twisted b.c., or equivalently large center vortices, can be imposed at exponentially small cost when the system size becomes large compared to the confinement correlation length at $T \sim 0$. At finite temperature, the spread of the center vortex in a temporal plane is now limited by the compact dimension 1/T. Above T_c , the Polyakov loop acquires a non-zero expectation value. Temporally twisted b.c., which enforce antiperiodicity on the Polyakov loop, then create an interface of minimal area L^2 separating positive and negative Polyakov loops. Such interfaces and the corresponding center vortices are exponentially suppressed by the dual-string or interface tension. T.b.c. in 2 and 3 temporal planes modify the minimal area of the interface to $L^2\sqrt{2}$ and $L^2\sqrt{3}$ respectively. We observe that the dual string tension $\tilde{\sigma}_0$ is multiplied by precisely $\sqrt{2}$ and $\sqrt{3}$.

3. Confinement/screening of fundamental charge: electric flux

As seen above, temporally (say, (z, t)) twisted b.c. enforce antiperiodicity on the Polyakov loop: $TrP^{\dagger}(\vec{x})P(\vec{x}+L\hat{z}) = -1$, instead of +1 with periodic b.c. Consider then the expectation value of this binary observable, in an enlarged partition function Z_{enl} which sums over all possible temporal b.c., periodic or twisted in each plane. $Z_{enl} = Z_0 + 3Z_k(1) + 3Z_k(2) + Z_k(3)$. Contrary to Z_0, Z_{enl} allows for the consistent introduction of a static fundamental charge, which is neutralized by its image in the twisted direction (say \hat{z}). Keeping track of the (z, t)-flux for each b.c. [6],

$$\langle TrP^{\dagger}(\vec{x})P(\vec{x}+L\hat{z})\rangle \equiv Z_{e}(1)$$
 (1)
= $\frac{1}{Z_{enl}}(Z_{0}+Z_{k}(1)-Z_{k}(2)-Z_{k}(3))$

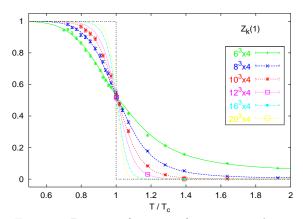


Figure 1. Partition function of one temporal twist as a function of temperature for various volumes.

Similar expressions (in fact, Z_2 Fourier transforms of the Z_k 's) follow for 2 and 3 orthogonal 'electric' fluxes [4,7]. Fig. 3 shows $Z_e(1)$. Its behaviour is dual to that of $Z_k(1)$. It also is a function of the FSS variable x, and it shows screening $exp(exp(-\tilde{b}x^2))$ above T_c and confinement $exp(-\sigma_0 x)$ below T_c , see Fig. 4. Moreover, because Z_e and Z_k are simply related by Fourier transform, one obtains that $\tilde{b} = \tilde{\sigma}_0$ and $b = \sigma_0$.

Universality states that the FSS functions measured here for SU(2) are the same as for the 3*d* Ising model after a single rescaling of *x*. This allows a precise determination of the large-*x* coefficients $\sigma_0, \tilde{\sigma}_0$ by matching only the small-*x* part of the FSS functions between the two models [8]. Another universal ratio is given by

$$\tilde{\sigma}_0/\sigma_0^2 = \tilde{\sigma}(T_c^+) / \left(\frac{\sigma(T_c^-)}{T_c^-}\right)^2 \approx 0.41.$$
⁽²⁾

Our data is consistent with this prediction, which relates the two sides of the phase transition.

4. Summary

Let us give an intuitive description of the deconfinement transition in Yang-Mills theory: • At low temperature, center vortices can spread to lower their free energy. Their proliferation dis-

orders the Wilson loop and makes the average Polyakov loop traceless, reflecting confinement.

• As the temperature is raised, the squeezing of center vortices in the compact 1/T dimen-

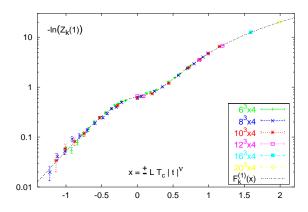


Figure 2. Free energy of one temporal twist as a function of the FSS variable $x \ge 0$ for $T \ge T_c$.

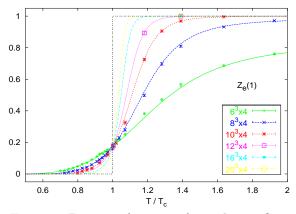


Figure 3. Partition function of one electric flux.

sion prevents them from arbitrarily lowering their free energy. Eventually, they cannot disorder the Polyakov loop anymore. Macroscopic regions of Polyakov loops of a definite center sector appear, separated by interfaces whose tension exponentially suppresses center vortices. Confinement has disappeared. This squeezing does not affect center flux in spatial planes, so that the spatial string tension persists at all temperatures.

Because the free energy F_q of a fundamental charge results from the cancellation of partition functions with different b.c., $Z_e = e^{-\frac{1}{T}F_q}$ is exponentially small in the confined phase, and ~ 1 in the deconfined phase. We have shown that the ratios of partition functions Z_k/Z_0 or Z_e/Z_0 , which are well-defined continuum quantities, can both be used as order parameters for the transition.

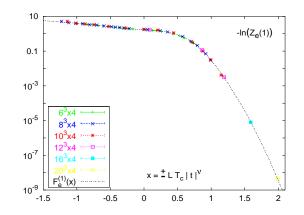


Figure 4. Free energy of one electric flux as a function of the FSS variable $x \ge 0$ for $T \ge T_c$.

If dynamical fundamental charges (quarks) are introduced in the system, the picture becomes murky, reflecting the change from a phase transition to a crossover. To start with, the 't Hooft loop now depends on the particular surface Σ along which center flux is introduced. Its relevance in full QCD remains to be studied.

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