

Static three- and four-quark potentials

C. Alexandrou ^{a *}, Ph. de Forcrand ^{b,c}, and A. Tsapalis ^{d †}

^a Department of Physics, University of Cyprus, CY-1678 Nicosia, Cyprus

^b Institut für Theoretische Physik, ETH-Hönggerberg, CH-8093 Zürich, Switzerland

^c Theory Division, CERN, CH-1211 Geneva 23, Switzerland

^d Department of Physics, University of Athens, Athens, Greece

We present results for the static three- and four- quark potentials in $SU(3)$ and $SU(4)$ respectively. Using a variational approach, combined with multi-hit for the time-like links, we determine the ground state of the baryonic string with sufficient accuracy to test the Y - and Δ - ansätze for the baryonic Wilson area law. Our results favor the Δ ansatz, where the potential is the sum of two-body terms.

1. Introduction

There are many lattice studies of the $q\bar{q}$ potential emphasizing the important role it plays in our understanding of the structure of mesons. Despite the equally important role that the three-quark potential plays in the understanding of baryon structure it has, until recently, received little attention in lattice QCD studies. Now, two lattice studies of the three quark potential have appeared during the last year [1,2], which reach different conclusions for the area law behaviour of the baryonic Wilson loop. It must be stressed that the main difficulty to resolve the dominant area law for the baryonic potential is the fact that the maximal difference between the two ansätze is a mere 15% for $SU(3)$. In this work we reexamine the baryonic potential using state of the art lattice techniques [3]. Since the same issues arise for any gauge group $SU(N)$ we corroborate our conclusions by also studying $SU(4)$, choosing lattice geometries which maximize the difference between the two ansätze to the 20% level.

2. Baryon Wilson loop

In $SU(N)$ we create a gauge invariant N -quark state at time $t = 0$ which is annihilated at a later time T . In $SU(3)$ the baryon Wilson loop W_{3q} , shown in Fig. 1, is given by

$$\frac{1}{3!} \epsilon^{abc} \epsilon^{a'b'c'} U(\mathbf{x}, \mathbf{y}, 1)^{aa'} U(\mathbf{x}, \mathbf{y}, 2)^{bb'} U(\mathbf{x}, \mathbf{y}, 3)^{cc'} \quad (1)$$

*Talk presented by C. Alexandrou.

† Acknowledges funding from the University of Cyprus and the European network ESOP (HPRN-CT-2000-00130).

where

$$U(x, y, j) = P \exp \left[ig \int_{\Gamma(j)} dx^\mu A_\mu(x) \right] \quad , \quad (2)$$

P is the path ordering and $\Gamma(j)$ denotes the path from x to y for quark line j .

The N -quark potential is then extracted from the long time behaviour of the Wilson loop:

$$V_{Nq} = - \lim_{T \rightarrow \infty} \frac{1}{T} \ln \langle W_{Nq} \rangle \quad . \quad (3)$$

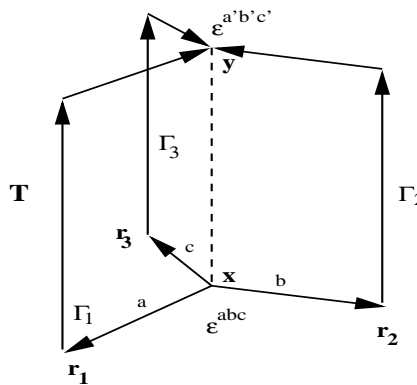


Figure 1. The baryonic Wilson loop in $SU(3)$. The quarks are located at positions \mathbf{r}_1 , \mathbf{r}_2 and \mathbf{r}_3 .

3. Geometries in $SU(3)$ and $SU(4)$

Two ansätze exist in the literature regarding the area law behaviour of the baryon Wilson loop:

- *Y*-ansatz: In the strong coupling limit, minimization of the static energy gives the flux tubes of shortest total length L_Y joining the quarks. For $SU(3)$, if the three quarks are at positions $\mathbf{r}_1, \mathbf{r}_2$ and \mathbf{r}_3 , the flux tubes in that configuration will meet at an interior point [4], known as the Steiner point, where the angles between the flux tubes are 120° independently of the vectors \mathbf{r}_k , provided that none of the angles of the triangle formed by the three quarks exceeds 120° . [Otherwise, the configuration of minimal total length is made of two flux tubes meeting at the third quark location.] Time evolution of the general case produces a three-bladed area similar to Fig. 1, known as the *Y*- area law.

For $SU(4)$, minimization of the static energy leads to two possibilities as shown in Fig. 2: namely, one configuration with two Steiner points, A and B (*Y*-ansatz) and one with a single Steiner point (*X*-ansatz). If, for simplicity, we assume that the double string between the two Steiner points has the same tension as the other four, single strings, then the *Y*-ansatz always has lower energy. In fact, the tension of the double string is 1.357(29) times greater [5]. Since this further increases the potential of the *Y*-ansatz, it will turn out to have no bearing on our conclusions. The two Steiner points are obtained by an iterative numerical procedure.

- Δ -ansatz: The second possibility [6] for the relevant area dependence of the baryonic Wilson loop is that it is given by the sum of the minimal areas A_{ij} spanning quark lines i and j which because of its shape in $SU(3)$ is known as the Δ - area law, with L_Δ the length of all interquark distances. The potential is then a sum of two-body potentials.

For $SU(3)$, the maximal difference of 15% between the two proposed area laws is obtained when the three quarks form an equilateral triangle.

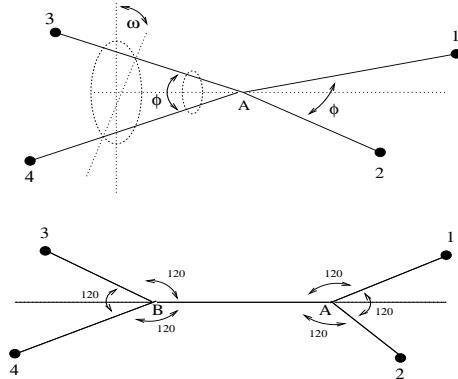


Figure 2. Flux tube configurations for four quarks at positions $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3$ and \mathbf{r}_4 . The upper graph (*X*) shows the local minimum of the energy with one Steiner point A, and the lower (*Y*) is the minimum with two Steiner points A and B.

For $SU(4)$, it turns out that the relative difference between the *Y*- energy and the two-body law is also maximal for the configuration of maximal symmetry among the four quarks. This entails putting the quarks on the vertices of a regular tetrahedron, which gives a relative difference of $\approx 22\%$ between the two ansätze. We make no attempt here to distinguish between the *Y*- and *X*- ansätze, since for the highly symmetric geometries that we choose the difference is on the few percent level.

The lattice results are thus compared to the two expected forms of the baryonic potential which in $SU(N)$ are

$$V_{Nq}(\mathbf{r}_1, \dots, \mathbf{r}_N) = \frac{N}{2}V_0 - \frac{1}{N-1} \sum_{j < k} \frac{g^2 C_F}{4\pi r_{jk}} + \sigma \left\{ \frac{L_\Delta}{N-1} \right\} \quad (4)$$

with $C_F = (N^2 - 1)/2N$ and σ the string tension of the $q\bar{q}$ potential. The factor of $1/(N-1)$ in the Δ - ansatz makes $L_\Delta/(N-1) < L_Y$. Note that, in contrast to Ref. [2], we do not vary σ . In addition, the three- or four-quark potential is compared directly with the sum of two-body potentials measured on the same gauge configurations, with no adjustable parameters.

4. Lattice techniques

We use the multi-hit procedure for the time-like links. For $SU(3)$, the mean-link integral is

obtained analytically, whereas for $SU(4)$ a Monte Carlo integration is performed [3]. In addition, we use a variational approach, and consider M different levels of APE smearing for the spatial links, with optimized parameters as in ref. [7]. Therefore, we obtain an $M \times M$ correlation matrix $C(t)$ of Wilson loops. To obtain the groundstate potential, we solve the generalized eigenvalue problem $C(t)v_k(t) = \lambda_k(t)C(t_0)v_k(t)$, taking $t_0/a = 1$. In the first variant the potential levels are extracted via $aV_k = \text{Lim}_{t \rightarrow \infty} -\ln\left(\frac{\lambda_k(t+a)}{\lambda_k(t)}\right)$ by fitting to the plateau. In the second variant we consider the projected Wilson loops $W_P(t) = v_0^T(t_0)C(t)v_0(t_0)$ and fit aV_0 to the plateau value of $-\ln\left(W_P(t+1)/W_P(t)\right)$. Both procedures give consistent results. The energy for the first excited state was used to check, in the extraction of the ground state, that the contamination is less than e^{-2} .

5. Results

For the baryonic loop in $SU(3)$ we used 220 configurations at $\beta = 5.8$ and 200 at $\beta = 6.0$ for a lattice of size $16^3 \times 32$ from the NERSC archive. For $SU(4)$ we generated 100 configurations at $\beta = 10.9$, which gives a similar string tension σa^2 as $SU(3)$, $\beta = 6.0$.

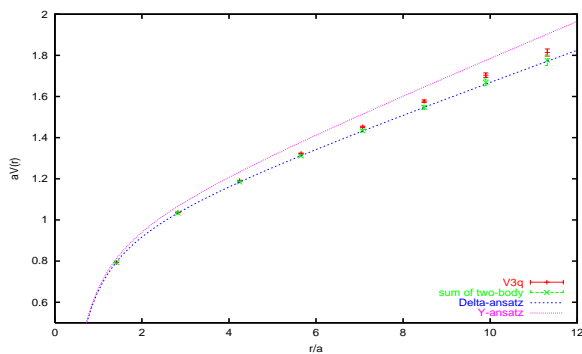


Figure 3. The static baryonic potential at $\beta = 6.0$ (pluses). The crosses show the sum of the static $q\bar{q}$ potentials. The curves for the Δ and Y ansätze are also displayed. The quarks are located at $(l, 0, 0)$, $(0, l, 0)$, $(0, 0, l)$, and $r = l\sqrt{2}$.

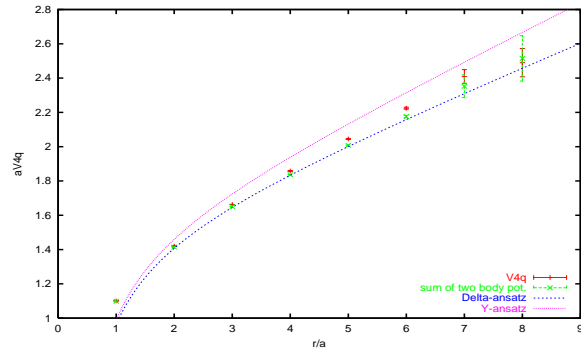


Figure 4. Same as in Fig.3, for the static $SU(4)$ baryonic potential. The quarks are located at $(r, 0, 0)$, $(0, r, 0)$, $(0, 0, r)$ and $(0, 0, 0)$.

6. Conclusions

Our results for the static three- and four-quark potentials in $SU(3)$ and $SU(4)$ are consistent with the sum of two-body potentials up to a distance of about 0.8 fm, and inconsistent with the Y -ansatz. For larger distances, where our statistical and systematic errors both become appreciable, there appears to be a small enhancement due to an admixture of a many-body component. Nevertheless, for the distances up to 1.2 fm that we were able to probe in this work, the Δ area law gives the closest description of our data. More refined noise-reduction techniques, such as the Lüscher-Weisz [8] algorithm, will be needed in order to clarify whether a genuine many-body component is present at large distances.

REFERENCES

1. G. S. Bali, Phys. Rept. **343** (2001) 1-136.
2. T. T. Takahashi *et al.*, Phys. Rev. Lett. **86** (2001) 18.
3. C. Alexandrou, Ph. de Forcrand and A. Tsapalis, hep-lat/0107006.
4. J. Carlson, J. Kogut and V. R. Pandharipande, Phys. Rev. **D27** (1983) 233.
5. B. Lucini, M. Teper, hep-lat/0107007
6. J. M. Cornwall, Phys. Rev. **D54** (1996) 6527.
7. M. Guagnelli, R. Sommer and H. Wittig, Nucl. Phys. **B535** (1998) 389.
8. M. Lüscher, P. Weisz, hep-lat/0108014.