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Critical behaviour of the Ginzburg-Landau model in the type II region

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We study the critical behaviour of the three-dimensional U(1) gauge+Higgs theory (Ginzburg-Landau model) at large scalar self-coupling λ ("type II region") by measuring various correlation lengths as well as the Abrikosov-Nielsen-Olesen vortex tension. We identify different scaling regions as the transition is approached from below, and carry out detailed comparisons with the criticality of the 3d O(2) symmetric scalar theory.

The three-dimensional (3d) U(1) gauge+Higgs theory (Ginzburg-Landau (GL) model, scalar electrodynamics) is an effective theory for phase transitions in superconductors, liquid crystals, and possibly cosmology. The Lagrangian is

$$\mathcal{L}_{GL} = \frac{1}{4}F_{ij}^2 + |D_i\phi|^2 + m_3^2|\phi|^2 + \lambda_3|\phi|^4\,. \qquad (1)$$

Here ϕ is a complex scalar field, $F_{ij} = \partial_i A_j - \partial_j A_i$, and $D_i = \partial_i + ie_3 A_i$. The parameters m_3, e_3^2, λ_3 have the dimension GeV. Physics then depends on the dimensionless ratios

$$x \equiv \lambda_3/e_3^2, \qquad y \equiv m_3^2/e_3^4.$$

The GL model has no local order parameters. However, there are non-local quantities, like the photon mass m_γ, and the vortex tension T [1], which vanish in the symmetric and are non-zero in the broken phase. The schematic phase diagram is shown in Fig. 1. At small x (type I region) the transition is of the first order, while at large x (type II region) the transition is believed to be of the second order, with a (presumably) tricritical point in between, at $x \approx 0.3$ [2].

In this work we study the phase transition deep in the type II region, at $x = \lambda_3/e_3^2 = 16$, shown as a thick vertical line in Fig. 1, and compare the scaling behaviour with that of the 3d XY model.

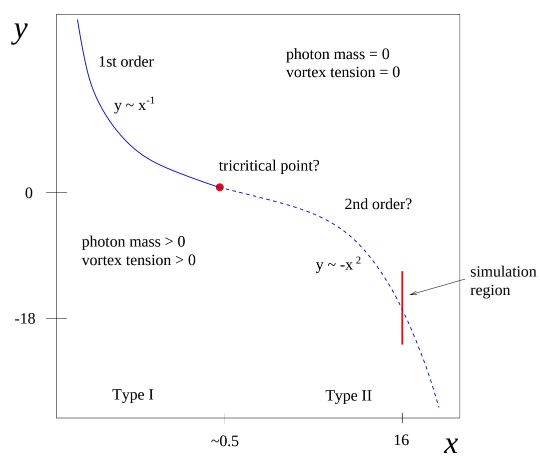

Figure 1. The schematic phase diagram of the GL model.

The universality class of the transition in the type II domain has not been previously resolved unambiguously with lattice simulations.

The scaling behaviour of the system is determined by the longest correlation lengths, or the lightest "masses". We shall consider the photon and scalar masses m_γ and m_s as functions of $\tau \equiv (y - y_c)$. The relation between masses varies strongly as τ is varied; consequently, in the broken phase, one can argue that there are three distinct possibilities:

1. Mean field: The textbook case, valid if $\tau \ll 0$, leading to $m_\gamma \sim (|\tau|/x)^{1/2} \ll m_s \sim (2|\tau|)^{1/2}$.
2. XY scaling: Here scaling is dominated by the dynamics of the scalar field; indeed, since $\lambda_3 \gg e_3^2$, it is not unreasonable for this to happen. Such scaling is supported by RG arguments [3], and has been observed experimentally [4]. In this domain $m_s \sim |\tau|^{\nu_{\rm xy}}$, $m_\gamma \sim |\tau|^{\nu_{\rm xy}/2}$, with $\nu_{\rm xy} \approx 0.67$, and we expect $m_s < 2m_\gamma$, otherwise the scalar would decay into two photons.

However, we do not expect this scaling to remain valid very close to the critical point: the XY model has a massless Goldstone at $\tau \leq 0$, whereas the GL model has a massless photon only when $\tau \geq 0$. Moreover, due to logarithmic confinement in 3d U(1) gauge theory, one can expect the scalar to remain massive at the transition point.
3. Inverted XY scaling: Here the GL "temperature" parameter τ is supposed to map to the *inverted* temperature $(-\tau)$ of the 3d XY model, and the broken/symmetric phases of the GL model are supposed to correspond to the symmetric/broken phases of the XY model. This scenario is supported by duality arguments [5]. When $\tau < 0$ the GL model in (2+1)d has two massive photon polarisation states, which become critical at $\tau = 0$. At $\tau > 0$, these d.o.f's become the massless photon, and a massive vector "resonance". These counts match exactly the two massive scalars in the symmetric phase of the XY model, which, in the broken phase, become the massless Goldstone and the massive radial mode.

Identification of the photon with the critical degree of freedom of the XY model suggests $m_\gamma \sim |\tau|^{\nu_{\rm xy}}$; however, other estimates in the literature suggest $m_\gamma \sim |\tau|^{0.5\ldots0.67}$ [5,6]. The scalar remains massive, since all diverging d.o.f's have been accounted for. (Alternatively, the scalar mass could vanish, but with an exponent smaller than the photon one).

Duality arguments also relate the Abrikosov-Nielsen-Olesen vortex tension T to the scalar mass in the dual theory, giving $T \sim |\tau|^{\nu_{\rm xy}}$.

We use the standard lattice discretization of Eq. (1), with a non-compact U(1) gauge action. For details, see [1]. We choose a fixed lattice spacing ($e_3^2 a = 1$); observing the universal behaviour does not require taking the continuum limit.

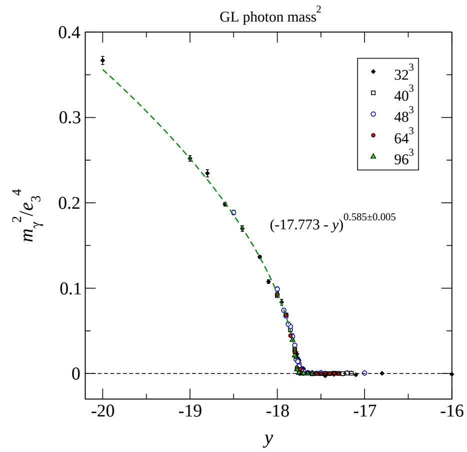

Figure 2. The photon (mass)^2 in the GL model.

Unfortunately, in the GL model there is no gauge invariant local observable which would correspond to magnetization. Vortex percolation has often been used to probe the critical behaviour [7]; however, on the lattice this is not rigorous [8]. We shall here only measure quantities such as the specific heat, scalar and photon correlation lengths, and the vortex tension.

In this paper we report on correlation lengths. In the GL model, these are measured using the following operators (for concreteness, measurements taken along the x_3-direction):

Scalar	$S = \phi^*\phi$,
Photon (plaquette)	$B_3 = \epsilon_{123}F_{12} = \Box_{12}$,
Vector	$V_i = {\rm Im}\,\phi^* D_i \phi\,.$

The vector and photon operators are measured at finite "transverse momentum" to, say, the x_1-direction. Additionally, each channel contains states with multiple photons, complicating the scalar channel measurement in particular. In the symmetric phase the scalar is always a resonance, decaying into two or more photons. For each operator we use several levels of *blocking + smearing* to reduce noise, and we diagonalize the full cross-correlation matrix between the blocked operators, as described, e.g., in [9].

In Fig. 2 the photon mass squared is shown. The critical exponent is $2\nu \approx 0.59$. Inverted XY scaling is usually thought to imply a larger exponent, $1.0 \lesssim 2\nu \lesssim 2\nu_{\rm xy} \approx 1.34$ (see above). Thus, we conclude that inverted XY scaling, at least in

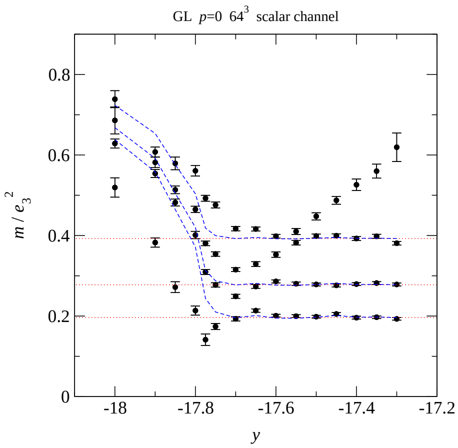

Figure 3. The four lowest mass states in the scalar channel. Horizontal dotted lines: masses of the three lowest two-photon states, with (single photon) $m_\gamma = 0$. Dashed lines: masses of two-photon states, using m_γ from Fig. 2 as input.

this form, is not valid for these $\tau = y - y_c$.

In the scalar channel we include up to four lowest two-photon states at zero total transverse momentum in the cross-correlation analysis, requiring diagonalization of matrices up to 14^2, and improve thus on earlier work at $x = 2$ [10]. In Fig. 3 we show measurements on 64^3 lattices. Three of the states clearly correspond to two photons, and the scalar ($\phi^*\phi$) is a resonance in the symmetric phase.

The scalar (resonance) mass is shown separately in Fig. 4. Approaching y_c from the broken phase, the mass appears to vanish with a critical exponent $m_s \propto |\tau|^{0.48}$. The value does not agree well with "standard" XY scaling, $\nu_{\rm xy} \approx 0.67$, but it is marginally compatible with the inverted XY scaling scenario, where the scalar mass should either remain finite at y_c, or at least have a critical exponent smaller than $\nu_{\rm xy}$.

In conclusion, the photon correlation length does not display (at least with the signatures commonly suggested) inverted XY scaling, as predicted by duality arguments. In addition, the critical exponent for the vortex tension T is several standard deviations off from the inverted XY scaling prediction (to be reported elsewhere).

Of course, one possible reason for the discrepancy is that at $x = 16$ the scaling window may be an extremely narrow band around y_c, rendering it almost impossible to observe the true scaling in Monte Carlo simulations.

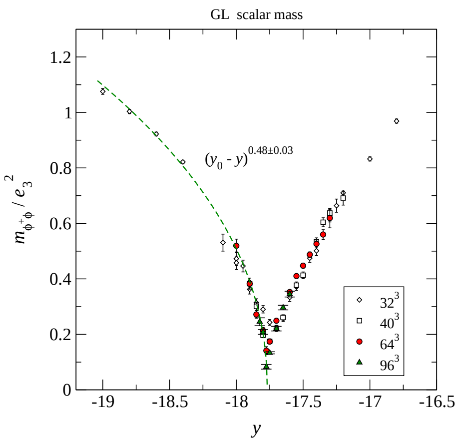

Figure 4. The scalar resonance mass.

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