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The charm quark's mass in quenched QCD*

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We present our preliminary result for the charmed quark mass, which follows from taking the D_s and K meson masses from experiment and $r_0 = 0.5\,\mathrm{fm}$ (or, equivalently $F_K = 160\,\mathrm{MeV}$) to set the scale. For the renormalization group invariant quark mass we obtain $M_c = 1684(64)\,\mathrm{MeV}$, which translates to $m_c(m_c) = 1314(40)(20)(7)\,\mathrm{MeV}$ for the running mass in the $\overline{\mathrm{MS}}$ scheme. Renormalization is treated non-perturbatively, and the continuum limit has been taken, so that the only uncontrolled systematic error consists in the use of the quenched approximation.

1. INTRODUCTION

Quark masses are among the fundamental parameters of the Standard Model, and yet not directly accessible to experimental measurement. Their determination in terms of experimental quantities requires a good quantitative control of the non-perturbative dynamics of QCD, which may be achieved by numerical simulations of the lattice regularized theory. Extending previous work on the Λ-parameter and the strange quark mass in quenched QCD [1,2] we report here on a precise determination of the charm quark mass, taking the experimental value of the D_s meson mass as essential new input. For previous work on the charm quark mass we refer to two recent papers [3] and the references given there.

2. STRATEGY

We use $\mathrm{O}(a)$ improved Wilson quarks with the Wilson gauge action. All necessary renormalization constants and improvement coefficients are known with non-perturbative accuracy in the relevant range of bare couplings. We refer to [4] for a review and further references.

2.1. Setting the parameters

In quenched QCD it is convenient to measure dimensionful quantities in units of the scale r_0 [5], since r_0/a has been determined very precisely for a large range of bare couplings [6,7]. To obtain the scale in physical units we always use $r_0 = 0.5$ fm, which is roughly equivalent to setting the scale with $F_K = 160$ MeV [2]. The renormalization group invariant (RGI) strange quark mass has been obtained in [2],

$$r_0 M_s = 0.348(13) \Rightarrow M_s = 138(6)\ \mathrm{MeV}, \qquad (1)$$

taking m_K from experiment and the mass ratio

$$M_s/\hat{M} = 24.5 \pm 1.5, \qquad \hat{M} = \tfrac{1}{2}(M_u + M_d), \qquad (2)$$

from chiral perturbation theory [8]. Up to intrinsic $\mathrm{O}(a^2)$ ambiguities, the connection between M_s and the bare strange quark mass can be established by combining the results of refs. [1,9], so that no tuning is required to satisfy the renormalization condition (1). The bare charm quark mass may then be fixed by matching directly the experimental result $m_{D_s} = 1969\,\mathrm{MeV}$ [10], which translates to $r_0 m_{D_s} = 4.98$. This is justified since electroweak effects on D meson masses are expected to be small on the scale of the expected statistical errors.

2.2. Some technical details

The simulations were done at four values of β, $6.0 \le \beta \le 6.45$, which correspond to lattice spacings in the range $0.05-0.1\,\mathrm{fm}$. Spatial lattice volumes ranged from 16^3 to 32^3, and the Euclidean time extent from 40 to 80. The linear extent of the spatial volume was never less than 1.5 fm in physical units, so that finite volume effects on the D_s meson mass can be safely neglected. Quark

*presented at the XIX International Symposium on Lattice Field Theory "Lattice 2001", August 19 – 24, 2001, Berlin, Germany propagators were computed for two mass parameters in the strange quark region, and three values around the expected charm quark mass. To compute meson masses we used correlation functions derived from the Schrödinger functional (SF) [11]. In the pseudoscalar channel these are given by

$$f_{\rm X}(x_0) = -\tfrac{1}{2}a^6 \sum_{\mathbf{y},\mathbf{z}} \left\langle \bar{c}(x)\Gamma_{\rm X} s(x)\bar{\zeta}_s(\mathbf{y})\gamma_5\zeta_c(\mathbf{z})\right\rangle , \quad (3)$$

where $\Gamma_{\rm X}$ stands for $\gamma_0\gamma_5$, γ_5 for $\rm X = A, P$ respectively. The pseudoscalar meson mass is then obtained by looking for a plateau in the effective masses,

$$am_{\rm eff}^{\rm X}(x_0 + \tfrac{1}{2}a) = \ln\left\{f_{\rm X}(x_0)/f_{\rm X}(x_0+a)\right\}, \quad (4)$$

as a function of Euclidean time x_0. Autocorrelation times were found to be completely negligible, so that the produced O(100) configurations at each β-value are statistically independent. An example for an effective mass plot is given in fig. 1. Excited states are expected to contribute significantly when either x_0 or $T - x_0$ becomes small. It is possible to estimate these contributions and thereby quantify the resulting deviation from the true plateau value. Requiring this effect to be smaller than 0.5%, the plateau region in fig. 1 was identified with the interval $25 \leq x_0/a \leq 35$; the meson mass was then estimated by averaging the effective mass in this interval.

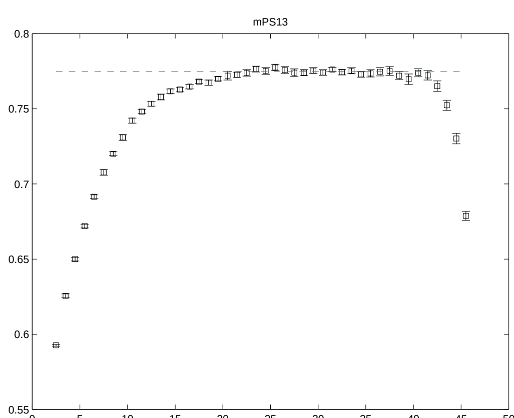

Figure 1. Effective pseudoscalar mass in lattice units vs. x_0/a at $\beta = 6.1$.

2.3. Bare and renormalized quark masses

Given the pseudoscalar masses at the chosen parameters, an interpolation in some bare charm quark mass is required to match the experimental D_s meson mass. Besides the bare mass m_0 we also consider two alternative definitions of the bare charm quark mass, which derive from the PCAC relation. First we use the heavy–light axial current and density to define,

$$m_{cs}(x_0) = \frac{\tilde{\partial}_0 f_{\rm A}(x_0) + c_{\rm A} a\partial_0^*\partial_0 f_{\rm P}(x_0)}{2f_{\rm P}(x_0)}, \quad (5)$$

which upon renormalization yields the average charm and strange quark mass. With our parameter choices yet another bare charm quark mass can be obtained from the PCAC relation involving a hypothetical strange quark, assumed to be mass degenerate with the charm quark. We denote this PCAC mass as m_{cc} but one should bear in mind that its definition does not involve flavour singlet operators.

The renormalized and O(a) improved charm quark mass $m_{\rm R}$ can now be obtained in various ways. Choosing the SF scheme we first obtain

$$m_{\rm R} = Z_{\rm P}^{-1} Z m_{\rm q}(1 + b_{\rm m} a m_{\rm q}), \quad (6)$$

where $m_{\rm q}$ is the subtracted bare charm quark mass. From the PCAC mass m_{cs}, one gets

$$m_{\rm R} + m_{{\rm R},s} = Z_{\rm P}^{-1} Z_{\rm A}\left\{1 + (b_{\rm A} - b_{\rm P})a\overline{m}_{\rm q}\right\} 2m_{cs}, \quad (7)$$

where $\overline{m}_{\rm q}$ denotes the average of the bare subtracted charm and strange quark masses, and we have assumed $x_0 = T/2$, which is kept approximately constant in physical units. Finally, we consider

$$m_{\rm R} = Z_{\rm P}^{-1} Z_{\rm A}\left\{1 + (b_{\rm A} - b_{\rm P})am_{\rm q}\right\} m_{cc}, \quad (8)$$

and convert to the RGI quark mass using the flavour-independent ratio [1]

$$M/m_{\rm R} = 1.157(15). \quad (9)$$

3. RESULTS AND CONCLUSIONS

After subtraction of the strange quark mass (1) in eq. (7) we have three definitions of $r_0 M_c$ which should coincide up to terms of O(a^2). In each case we extrapolate to the continuum with an ansatz of the form

$$r_0 M_c = c_0 + c_1 (a/r_0)^2, \qquad (10)$$

excluding the data point at the coarsest lattice spacing ($\beta = 6.0$). As can be seen in fig. 2, the results are nicely compatible with each other in the continuum limit, although the differences at finite cutoff are quite large. As our best (preliminary) estimate we quote

$$r_0 M_c = 4.26(16) \Rightarrow M_c = 1684(64)\,\mathrm{MeV}. \qquad (11)$$

Using the four-loop anomalous dimension and β-function in the $\overline{\mathrm{MS}}$ scheme [12], and $r_0\Lambda_{\overline{\mathrm{MS}}} = 0.586(48)$ [7], we obtain

$$m_c(m_c) = 1314(40)(20)(7)\,\mathrm{MeV}, \qquad (12)$$

where the first error stems from the RGI quark mass, the second is induced by the error on $\Lambda_{\overline{\mathrm{MS}}}$, while the last number is the difference between three- and four-loop perturbative evolution down to the charm scale.

We conclude that precise results can be obtained for charmed observables using the same techniques as for the light quarks, provided that the continuum extrapolation is performed. We emphasize that we have used the quenched approximation, which is known to lead to inconsistencies in the hadron spectrum at the 10% level. However, the present result constitutes a significant improvement over previous lattice estimates and will be a useful reference point for more realistic calculations involving dynamical quarks. For the time being we speculate that the mass ratio $M_c/M_s = 4.26(16)/0.348(13) \approx 12.2 \pm 1.0$ may be less sensitive to quenched scale ambiguities than the masses themselves.

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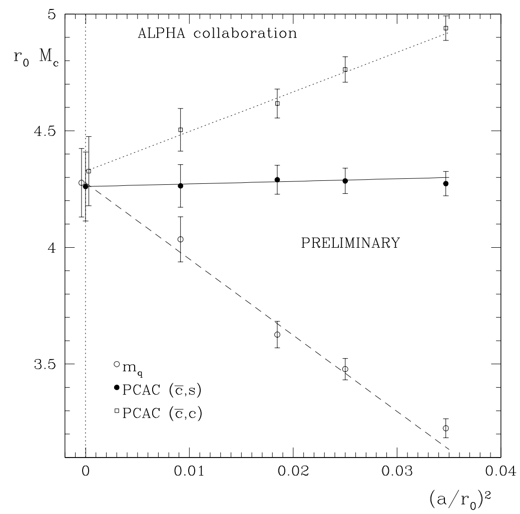

Figure 2. Continuum extrapolation of the RGI charm quark masses.

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