

# The Earth Mantle-Core Effect in Matter-Induced Asymmetries for Atmospheric Neutrino Oscillations

J. Bernabéu<sup>a,b</sup>, S. Palomares-Ruiz<sup>b</sup>, A. Pérez<sup>b</sup>, S. T. Petcov<sup>c,\*</sup>

<sup>a</sup> CERN, Geneva, Switzerland

<sup>b</sup> Departamento de Física Teórica, Universidad de Valencia, 46100 Burjassot, Valencia, Spain

<sup>c</sup> Scuola Internazionale Superiore di Studi Avanzati and Istituto Nazionale di Fisica Nucleare, I-34014 Trieste, Italy

## Abstract

Earth medium effects in the three-neutrino oscillations of atmospheric neutrinos are observable under appropriate conditions. This paper generalizes the study of the medium effects and the possibility of their observation in the atmospheric neutrino oscillations from the case of neutrinos traversing only the Earth mantle, where the density is essentially constant, to the case of atmospheric neutrinos crossing also the Earth core. In the latter case new resonance-like effects become apparent. We calculate the CPT-odd asymmetry for the survival probability of muon neutrinos and the observable muon-charge asymmetry, taking into account the different atmospheric neutrino fluxes, and show the dependence of these asymmetries on the sign of  $\Delta m_{31}^2$  and on the magnitude of the mixing angle  $\theta_{13}$ . A magnetized detector with a sufficiently good neutrino momentum resolution is required for the observation of the muon-charge asymmetry generated by the Earth mantle-core effect.

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\*Also at: Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 1784 Sofia, Bulgaria

Recently, it was shown [1] that medium effects in the three-neutrino oscillations of atmospheric neutrinos crossing the Earth mantle become observable under appropriate conditions. At the fundamental level, their study by means of a magnetized detector, able to provide muon charge discrimination [2] and energy resolution, would allow to measure the sign of the neutrino mass-squared difference,  $\Delta m_{31}^2$ , responsible for the dominant  $\nu_\mu \leftrightarrow \nu_\tau$  and  $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_\tau$  oscillations of the atmospheric  $\nu_\mu$  and  $\bar{\nu}_\mu$ . Such a study will also allow to explore the lepton mixing matrix element  $|U_{e3}| = \sin \theta_{13}$ , which “connects” the solar and atmospheric neutrino oscillations. In the limit of  $\Delta m_{21}^2 L / (2E) \ll 1$ , valid for atmospheric neutrino oscillation baselines,  $\Delta m_{21}^2 > 0$  being the neutrino mass-squared difference responsible for the oscillations of solar neutrinos,  $L$  the baseline and  $E$  the neutrino energy, the main conclusions are based on the following: i) the medium effects, which discriminate between neutrino and antineutrino propagation, determine the sign of the atmospheric  $\Delta m_{31}^2$  [3]; ii) for  $\sin \theta_{13} \equiv s_{13} = 0$  electron neutrinos decouple from the oscillations of the atmospheric neutrinos in matter, whereas they mix with the third (heaviest) mass eigenstate neutrino and take part in the atmospheric neutrino oscillations if  $s_{13} \neq 0$ , although their mixing with the first (lightest) mass eigenstate neutrino still vanishes; iii) non-resonant medium effects are already apparent in the sub-dominant channels  $\nu_e \rightarrow \nu_\mu$  and  $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ , for baselines  $L \sim 3000$  km, in both the mixing and oscillation phase shift (see also [4, 5]); iv) in order for the medium effects in the muon neutrino survival probability to be observable, the resonant MSW effect in the  $\nu_{e(\mu)} \rightarrow \nu_{\mu(e)}$  and  $\bar{\nu}_{e(\mu)} \rightarrow \bar{\nu}_{\mu(e)}$  transitions must be operational, which requires baselines larger than  $L \sim 7000$  km, the optimal baseline being a function of the value of  $\sin \theta_{13}$ ; v) taking into account the initial atmospheric  $\nu_\mu$ ,  $\bar{\nu}_\mu$ , and  $\nu_e$ ,  $\bar{\nu}_e$  fluxes and the relevant charged current neutrino-nucleon deep inelastic scattering cross-sections, it was shown that the matter-induced CPT-odd [6] and CP-odd [4, 5, 7] asymmetries are observable.

The indicated results were obtained for matter of constant density. Although there is a wide range of Nadir angles (from  $33.17^\circ$  to  $90^\circ$ ), corresponding to atmospheric neutrinos crossing the Earth mantle, and to which the results of the study [1] apply, it is of interest to extend the study to the case in which atmospheric neutrinos cross the Earth core. This is the aim of the present paper. In our analysis we use the two-layer model of the Earth density distribution (see, e.g., [8]). Detailed numerical studies [9] (see also [8, 10, 11]) showed that, for the calculation of the probabilities of interest, the two-layer model of the Earth density distribution provides a very good (in many cases excellent) approximation to the more complicated density distributions predicted by the existing models of the Earth [12]. According to the existing Earth models, neutrinos which traverse the Earth along a trajectory with a Nadir angle  $\theta_n < 33.17^\circ$  will cross the Earth core. For such trajectories, the distances of propagation in the mantle and in the core,  $L_m$  and  $L_c$ , are not independent and are given by

$$\left. \begin{aligned} L_m &= R \left( \cos \theta_n - \sqrt{\frac{r_c^2}{R^2} - \sin^2 \theta_n} \right) \\ L_c &= 2 R \sqrt{\frac{r_c^2}{R^2} - \sin^2 \theta_n} \end{aligned} \right\}, \quad \sin^2 \theta_n \leq \frac{r_c^2}{R^2} \quad (1)$$

where  $R = 6371$  km and  $r_c = 3480$  km are the radii of the Earth and of the Earth core [12], respectively. The total baseline is  $L = 2 L_m + L_c$ , with neutrinos propagating in three regions of different constant density, mantle-core-mantle, the densities of the first and third regions being identical. The probabilities of atmospheric neutrino oscillations we will consider are symmetric with respect to the interchange i) of the initial and final

points of the neutrino trajectories, located close to, or on, the Earth surface, and ii) of the initial and final flavour neutrinos. This implies the vanishing of T-odd asymmetries [13].

The effective neutrino potential differences in the Earth mantle and in the core, which in the case of the  $\nu_e \rightarrow \nu_{\mu,\tau}$ , and  $\nu_\mu \rightarrow \nu_e$  oscillations of the atmospheric neutrinos of interest have the well-known form [14, 15, 16],

$$V_{m(c)} = \sqrt{2}G_F N_{m(c)}^e, \quad (2)$$

$N_{m(c)}^e$  being the electron number density in the mantle (core)<sup>1</sup>, lead to interesting new effects of resonance-like enhancement of the  $\nu_e \rightarrow \nu_{\mu,\tau}$  and  $\nu_\mu \rightarrow \nu_e$  transitions beyond the MSW resonance of the mixing [14, 15, 18]. These effects have been well discussed in [8, 19, 20]. In the limit  $\Delta_{21} \equiv \Delta m_{21}^2 L/(2E) = 0$ , the traceless  $2 \times 2$  matrix describing the effective Hamiltonian of the two neutrino states whose evolutions are coupled (see, e.g., [21, 22]),  $\nu_e$  and  $\nu'$ ,

$$\nu' = \nu_\mu \sin \theta_{23} + \nu_\tau \cos \theta_{23}, \quad (3)$$

where  $\theta_{23}$  is the mixing angle, which in the limit of  $\theta_{13} = 0$  controls the atmospheric  $\nu_\mu \rightarrow \nu_\tau$  and  $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$  oscillations, leads to an  $2 \times 2$  unitary evolution matrix of the form (see, e.g., [23, 24])

$$U = U_m U_c U_m, \quad (4)$$

where  $U_{m(c)}$  is the evolution operator in the Earth mantle (core),

$$U_{m(c)} = e^{-i(\vec{\sigma} \hat{n}_{m(c)})\phi_{m(c)}} = \cos \phi_{m(c)} - i(\vec{\sigma} \hat{n}_{m(c)}) \sin \phi_{m(c)}. \quad (5)$$

Here

$$\phi_{m(c)} = \frac{1}{2} \Delta E_{m(c)} L_{m(c)} \quad (6)$$

is the difference between the phases of the two neutrino energy eigenstates, acquired after neutrinos have crossed the Earth mantle (core),  $\Delta E_{m(c)}$  being the difference between the energies of the two states in the mantle (core),

$$\Delta E_{m(c)} = \frac{\Delta m_{31}^2}{2E} \sqrt{\left(\cos 2\theta_{13} - \frac{2EV_{m(c)}}{\Delta m_{31}^2}\right)^2 + \sin^2 2\theta_{13}}, \quad (7)$$

and  $\hat{n}_{m(c)}$  is a real unit vector [24],

$$\hat{n}_{m(c)} = (\sin 2\theta^{m(c)}, 0, -\cos 2\theta^{m(c)}), \quad \hat{n}_{m(c)}^2 = 1, \quad (8)$$

where  $\theta^{m(c)}$  is the mixing angle in the mantle (core), which in the limit of zero neutrino effective potential difference,  $V_{m(c)} \rightarrow 0$ , coincides with  $\theta_{13}$ ,

$$\cos 2\theta^{m(c)} = \frac{1}{\Delta E_{m(c)}} \left( (\Delta m_{31}^2/2E) \cos 2\theta_{13} - V_{m(c)} \right). \quad (9)$$

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<sup>1</sup>In the two-layer model of the Earth density distribution,  $N_m^e$  and  $N_c^e$  are [8, 9] the mean electron number densities along the neutrino trajectory in the mantle and in the core, respectively. For the Earth center-crossing neutrinos, for instance, one has [12, 17]:  $N_m^e \cong 2.2 \text{ cm}^{-3} N_A$  and  $N_c^e \cong 5.4 \text{ cm}^{-3} N_A$ , where  $N_A$  is the Avogadro number.

Using eqs. (4)-(5), it is not difficult to express the evolution operator of interest  $U$  in terms of the quantities characterizing the evolution of the neutrino states in the mantle and in the core (see, e.g., [20, 23]):

$$U = e^{-i(\vec{\sigma} \hat{n})\phi} = \cos \phi - i(\vec{\sigma} \hat{n}) \sin \phi, \quad (10)$$

where

$$\cos \phi = \cos(2\phi_m) \cos \phi_c - \cos(2\theta^c - 2\theta^m) \sin(2\phi_m) \sin \phi_c, \quad (11)$$

$$\hat{n} \sin \phi = \hat{n}_m [\sin(2\phi_m) \cos \phi_c - (\hat{n}_m \cdot \hat{n}_c)(1 - \cos(2\phi_m)) \sin \phi_c] + \hat{n}_c \sin \phi_c. \quad (12)$$

The two-neutrino oscillation probability  $P(\nu_e \rightarrow \nu') = P(\nu' \rightarrow \nu_e) \equiv P_2$ , is determined by the elements of the evolution matrix  $U$ ,  $P_2 = |U_{\nu'\nu_e}|^2$ , and can be written in the form [19]:

$$P_2 = (n_1 \sin \phi)^2 + (n_2 \sin \phi)^2 = 1 - \cos^2 \phi - (n_3 \sin \phi)^2, \quad (13)$$

where  $\cos \phi$  is given by eq. (11) and  $n_3 \sin \phi$  is determined by eq. (12):

$$n_3 \sin \phi = \cos 2\theta^m [\sin \phi_c \cos 2\phi_m \cos(2\theta^c - 2\theta^m) + \cos \phi_c \sin 2\phi_m] - \sin \phi_c \sin 2\theta^m \sin(2\theta^c - 2\theta^m). \quad (14)$$

The  $\nu_e \rightarrow \nu_\mu$  and  $\nu_e \rightarrow \nu_\tau$  transition probabilities and the  $\nu_e$  survival probability of interest are related to the probability  $P_2$  (see, e.g., [21, 22]):

$$P(\nu_e \rightarrow \nu_\mu) = s_{23}^2 P_2, \quad P(\nu_e \rightarrow \nu_\tau) = c_{23}^2 P_2, \quad P(\nu_e \rightarrow \nu_e) = 1 - P_2. \quad (15)$$

where  $c_{23} = \cos \theta_{23}$  and  $s_{23} = \sin \theta_{23}$ . We also have  $P(\nu_e \rightarrow \nu_\mu) = P(\nu_\mu \rightarrow \nu_e)$ . The probabilities of oscillations of antineutrinos can be obtained from the corresponding probabilities of oscillations of neutrinos by replacing  $V_{m(c)}$  by  $-V_{m(c)}$  in the expressions for the energy differences  $\Delta E_{m(c)}$  and the mixing angles  $\theta^{m(c)}$  in the mantle and in the core, eqs. (7) and (9).

The conditions for the *absolute* ( $P_2 = 1$ ) maxima of the  $\nu_e \rightarrow \nu_\mu$ ,  $\nu_\mu \rightarrow \nu_e$  and  $\nu_e \rightarrow \nu_\tau$  transition probabilities follow from the expression (13) for the probability  $P_2$  [19]:

$$\cos \phi = 0, \quad n_3 \sin \phi = 0, \quad (16)$$

with  $\cos \phi$  and  $n_3 \sin \phi$  given by eqs. (11) and (14).

In the case of constant density, i.e.,  $V_c = V_m$  and, correspondingly,  $\Delta E_m = \Delta E_c$ ,  $\theta^c = \theta^m$ , and  $\phi = \phi_c + 2\phi_m = \Delta E_m(L_c + 2L_m)/2$ , the two conditions (16) correspond to the simultaneous requirement [1, 19] of both a maximum in the oscillating factor  $\sin^2 \phi$ , namely,  $\cos \phi = 0$ , and MSW-resonance in the amplitude of the oscillations  $\sin^2 2\theta^m$ ,  $n_3 \sin \phi = \cos 2\theta^m \sin \phi = \pm \cos 2\theta^m = 0$ . There is a crucial difference between the case of constant density and the one we are interested in. When neutrinos cross the mantle, the core and the mantle again, the oscillation phase  $\phi$  is no longer linear in the baseline  $L$ . Instead, it depends non-trivially on the distances traveled in each layer,  $L_m$  and  $L_c$ . Similarly, the condition  $n_3 \sin \phi = 0$  is a global one, without direct correspondence with the MSW-resonance condition in a given layer.

In terms of the oscillation phases in each layer,  $\phi_m$  and  $\phi_c$ , the solution of the conditions (16) for the absolute maxima of  $P_2$  can be written as

$$\tan \phi_c = \frac{1 - \tan^2 \phi_m}{2 (\hat{n}_m \hat{n}_c) \tan \phi_m} \quad , \quad \tan^2 \phi_m = \frac{1 - \frac{\Delta E_m}{\Delta E_c} (\hat{n}_m \hat{n}_c)}{1 + \frac{\Delta E_m}{\Delta E_c} (\hat{n}_m \hat{n}_c) - 2 (\hat{n}_m \hat{n}_c)^2} \quad . \quad (17)$$

They can be expressed in terms of the mixing angles in matter in each of the two layers (Earth mantle and core) [19]:

$$\begin{aligned} \tan \phi_m &= \pm \sqrt{\frac{-\cos 2\theta^c}{\cos(4\theta^m - 2\theta^c)}} \quad , \\ \tan \phi_c &= \pm \frac{\cos 2\theta^m}{\sqrt{-\cos(2\theta^c) \cos(4\theta^m - 2\theta^c)}} \quad , \end{aligned} \quad (18)$$

where the signs in (18) are correlated. Under the assumptions of  $\cos 2\theta_{13} > 0$  and  $\Delta m_{31}^2 > 0$ , together with the fact that  $V_m < V_c$ , the domain in which (18) can be fulfilled is [19]

$$\text{domain } A : \begin{cases} \cos 2\theta^c \leq 0 \\ \cos(4\theta^m - 2\theta^c) \geq 0 \end{cases} \quad (19)$$

The absolute maximum reachable in the domain (19) represents a new feature of the  $\nu_e \rightarrow \nu_{\mu(\tau)}$  and  $\nu_\mu \rightarrow \nu_e$  transition probabilities beyond the well-known MSW resonance in the mixing. The new enhancement effect disappears in the limit of constant density, when  $\theta^c = \theta^m$ , because the domain ‘‘collapses’’ just to the resonance condition for the mixing in matter. It is not guaranteed *a priori* that, for a given Nadir angle, corresponding to neutrino trajectories crossing the Earth core region, both conditions (18) can be satisfied simultaneously, since the radii of the Earth and that of the Earth core and, correspondingly,  $L_m$  and  $L_c$ , and the electron number densities in the Earth mantle and core,  $N_{m(c)}^e$ , are fixed. It was shown in [19] that solution A is realized for the  $\nu_e \rightarrow \nu_{\mu(\tau)}$  and  $\nu_\mu \rightarrow \nu_e$  transitions of the Earth core-crossing atmospheric neutrinos and a rather complete set of values of  $\Delta m_{31}^2/E$  and  $\sin^2 2\theta_{13}$ , for which both conditions in (18) hold for neutrino trajectories with Nadir angle  $\theta_n = 0^\circ; 13^\circ; 23^\circ; 30^\circ$ , was found. We will present results for the matter-induced CP-odd and CPT-odd asymmetries as a function of the Nadir angle (see below).

Besides the possible absolute maxima described by (18), with  $P_2 = 1$ , there can be alternative (local) maxima (in the variables  $\phi_m$  and  $\phi_c$ ) corresponding to [8, 19]

$$\text{i)} \quad \cos 2\phi_m = 0 \quad , \quad \sin \phi_c = 0, \quad (20)$$

$$\begin{aligned} &\text{in the ‘‘domain’’ B:} \quad \cos 2\theta^m = 0, \\ &\text{with probability} \quad P_2 = \sin^2 2\theta_m = 1; \end{aligned}$$

$$\text{ii)} \quad \sin \phi_m = 0 \quad , \quad \cos \phi_c = 0, \quad (21)$$

$$\begin{aligned} &\text{in the domain C:} \quad \cos 2\theta^c \geq 0, \\ &\text{with probability} \quad P_2 = \sin^2 2\theta^c. \end{aligned}$$

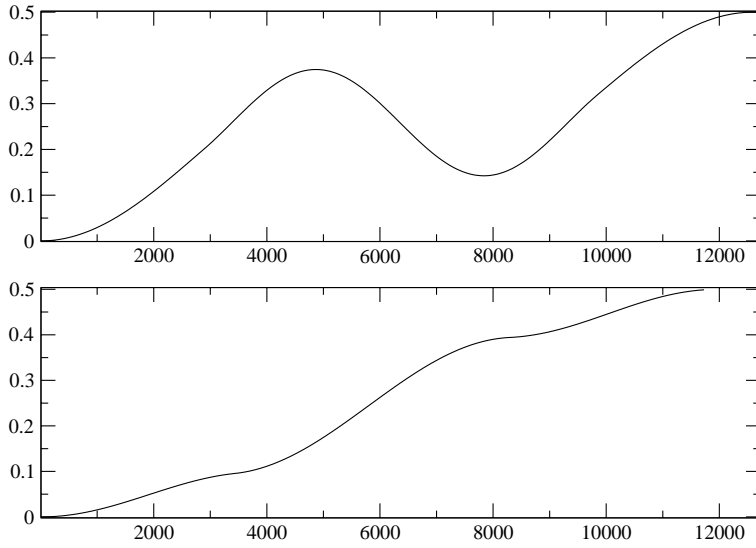


Figure 1: Constructive interference in the Earth in the channel  $\nu_e \rightarrow \nu_\mu$ :  $P(\nu_e \rightarrow \nu_\mu)$  as function of the distance (in km) traveled by the neutrinos in the Earth, for  $\Delta m_{31}^2 = 3.2 \cdot 10^{-3}$  eV<sup>2</sup> and  $\sin^2 \theta_{23} = 0.5$ . The upper panel corresponds to solution A for  $\theta_n = 0^\circ$  at  $E \simeq 6.6$  GeV and  $\sin^2 2\theta_{13} \simeq 0.15$ ; the lower panel corresponds to solution B for  $\theta_n = 23^\circ$  at  $E \simeq 5.0$  GeV and  $\sin^2 2\theta_{13} \simeq 0.05$ .

iii)

$$\cos \phi_m = 0 \quad , \quad \cos \phi_c = 0, \quad (22)$$

$$\begin{aligned} &\text{in the domain D:} && \cos(4\theta^m - 2\theta^c) \leq 0, \\ &\text{with probability} && P_2 = \sin^2(4\theta^m - 2\theta^c); \end{aligned}$$

The maxima of  $P_2$  described by (22), known as the NOLR solution [8], like the absolute maxima of solution A, eq. (18), are due to a constructive interference between the probability amplitudes of the neutrino transitions in the two different layers - the Earth mantle and the core [19]. Solution A and solution D coincide on the border line  $\cos(4\theta^m - 2\theta^c) = 0$ , which is common to both regions A and D. The building up of the indicated constructive interference is illustrated in Fig. 1, where we show  $P(\nu_e \rightarrow \nu_\mu) = s_{23}^2 P_2$  as a function of the distance traveled by the neutrinos in the Earth in two different cases, associated with solutions A and D. In both cases, we have used [25]  $\Delta m_{31}^2 = 3.2 \cdot 10^{-3}$  eV<sup>2</sup> and  $\sin^2 \theta_{23} = 0.5$ . The two panels correspond to solution A for  $\theta_n = 0^\circ$  at  $\sin^2 2\theta_{13} \cong 0.15$  and  $E \cong 6.6$  GeV, and to absolute maximum of type B (i.e., an absolute maximum at which  $\cos(4\theta^m - 2\theta^c) = 0$ ) for  $\theta_n = 23^\circ$  at  $\sin^2 2\theta_{13} \cong 0.05$  and  $E \cong 5.0$  GeV [19].

Solution B (C) defined by (20) ((21)), describes maximum conversion in the mantle (core), with no transitions taking place in the core (mantle). The region B is a line (in the  $E - \sin^2 2\theta_{13}$  plane) on which the MSW resonance condition [15] is satisfied in the Earth mantle; it lies entirely in region A [19]. On the border line of region C,  $\cos 2\theta^c = 0$ , which is also a border line of region A and on which solutions C and A coincide, the MSW resonance condition in the Earth core is fulfilled.

As emphasized in [1], in a situation in which the dominant oscillation channel is  $\nu_\mu \rightarrow \nu_\tau$ , the only method to observe the medium effects in the  $\nu_\mu$  survival probability (and thus,

reach sensitivity to the sign of  $\Delta m_{31}^2$  and the value of  $\theta_{13}$ ) is to operate under conditions in which the resonance phenomenon in the transitions  $\nu_\mu \rightarrow \nu_e$  are observable. For constant density in the mantle of the Earth, this requirement implies a baseline  $L \geq 7000$  km, i. e., a Nadir angle  $\theta_n \leq 56.68^\circ$ . The constant density approximation can only be maintained for  $\theta_n \geq 33.17^\circ$ . The question is whether for  $\theta_n < 33.17^\circ$ , one finds, for the values of  $\Delta m_{31}^2$  from the atmospheric neutrino oscillation region and of  $\theta_{13}$  from the region allowed by the CHOOZ data, solutions to the absolute maximum conditions (18) in the energy range relevant for the atmospheric neutrinos. The phenomenon of constructive interference between the neutrino transition amplitudes in the Earth mantle and in the core was shown [8, 19] to take place practically for all Nadir angles, associated with baselines crossing the Earth core, and for energies of atmospheric neutrinos, which for  $\Delta m_{31}^2 = 3.2 \times 10^{-3} \text{ eV}^2$  lie in the multi-GeV region,  $E \cong (3 - 9) \text{ GeV}$ . In Figs. 2 we show the probability  $P(\nu_e \rightarrow \nu_\mu)$  as a function of the neutrino energy  $E$  for  $\Delta m_{31}^2 = 3.2 \cdot 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta_{23} = 0.5$  and  $\sin^2 2\theta_{13} = 0.10$  (solid line),  $0.05$  (dotted line). The four panels correspond to neutrino trajectories with Nadir angles  $\theta_n = 0^\circ, 13^\circ, 23^\circ$  and  $30^\circ$ , respectively.

The two dominating maxima in the probability  $P(\nu_e \rightarrow \nu_\mu)$ , located in the energy intervals  $\sim (3 - 5) \text{ GeV}$  and  $\sim (6 - 8) \text{ GeV}$  and clearly seen in Fig. 2, are for  $\theta_n = 0^\circ; 13^\circ$ , due to absolute maxima, i.e., solutions of eq. (16) (or (18)), which take place at [19]  $\sin^2 2\theta_{13}^{max1} = 0.034; 0.039$  and  $E_{max1}^{m-c} = 4.4; 4.6 \text{ GeV}$ , and at  $\sin^2 2\theta_{13}^{max2} = 0.15; 0.17$  and  $E_{max2}^{m-c} = 6.6; 7.1 \text{ GeV}$ , respectively. One of the most important features of the results discussed is [8] that the energy of the dominating maxima of the probability  $P(\nu_e \rightarrow \nu_\mu)$ , caused by the mantle-core constructive interference effect,  $E_{max}^{m-c}$ , lies between the energies of the MSW resonance in the core and in the mantle,  $E_{res}^c$  and  $E_{res}^m$ , which, e.g., for  $\sin^2 2\theta_{13} = (0.03 - 0.04)$  read:  $E_{res}^c \cong 3.9 \text{ GeV}$  and  $E_{res}^m \cong 9.5 \text{ GeV}$ .

In order to be detected, the medium effects discussed here require a detector with a sufficiently good energy resolution. The energies at which the dominating maxima of  $P(\nu_e \rightarrow \nu_\mu)$  occur vary somewhat with the Nadir angle. Thus, the detector has to allow a sufficiently precise determination of the direction of the neutrino momentum too. It has to be able at least to identify with a rather good efficiency the events which are due to Earth core-crossing neutrinos. If these requirements are fulfilled, the matter effects under discussion might be measurable.

Once the Earth mantle-core interference effect has been studied in the sub-dominant channel  $\nu_e \rightarrow \nu_\mu$ , the most interesting implication of this effect is for the survival probability  $P(\nu_\mu \rightarrow \nu_\mu)$ . The existence of the absolute maxima in  $P(\nu_\mu \rightarrow \nu_e) = P(\nu_e \rightarrow \nu_\mu)$  will lead to an appreciable suppression of  $P(\nu_\mu \rightarrow \nu_\mu)$  if  $\Delta m_{31}^2 > 0$ , and to no effect in  $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)$ , with respect to the vacuum case. In Fig. 3 we show the dependence of the probabilities  $P(\nu_\mu \rightarrow \nu_\mu)$  and  $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)$  on  $E$  for  $\Delta m_{31}^2 = 3.2 \cdot 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta_{23} = 0.5$  and  $\sin^2 2\theta_{13} = 0.10$ . The four panels are for Nadir angles  $\theta_n = 0^\circ, 13^\circ, 23^\circ$  and  $30^\circ$ , respectively. The three lines in Figs. 3 correspond to  $\nu_\mu$  and  $\bar{\nu}_\mu$  oscillations in the Earth (solid and dashed lines, respectively) and in vacuum (dotted line).

To quantify the mantle-core interference effect in the  $\nu_\mu$  or  $\bar{\nu}_\mu$  survival probability, we construct the matter-induced CPT-odd asymmetry:

$$A_{CPT} = \frac{P(\nu_\mu \rightarrow \nu_\mu) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)}{P(\nu_\mu \rightarrow \nu_\mu) + P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)}. \quad (23)$$

The CPT-odd probability cancels other [26] matter effects common for neutrinos and antineutrinos in the dominant channel  $\nu_\mu \rightarrow \nu_\tau$ . The ‘‘theoretical’’ asymmetry (23) is shown

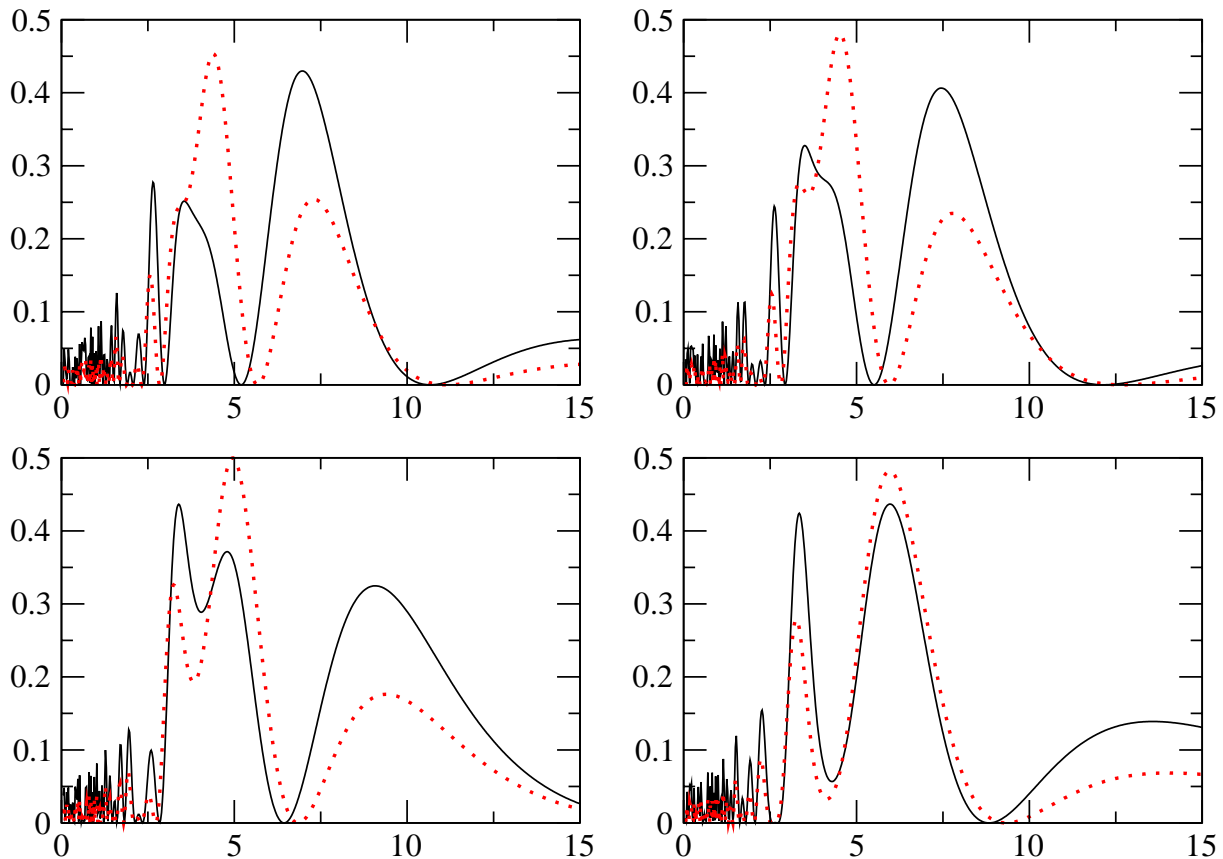


Figure 2: The probability  $P(\nu_e \rightarrow \nu_\mu)$  for neutrinos crossing the Earth core, as a function of the neutrino energy  $E$  (in GeV), for  $\Delta m_{31}^2 = 3.2 \cdot 10^{-3} \text{ eV}^2$ ,  $\sin^2 \theta_{23} = 0.5$  and  $\sin^2 2\theta_{13} = 0.10$  (solid line), 0.05 (dotted line). The upper left and right, and the lower left and right panels correspond to neutrino trajectories with Nadir angle  $\theta_n = 0^\circ$ ;  $13^\circ$ ;  $23^\circ$ ;  $30^\circ$ , respectively.

in Fig. 4 for the same values of the Nadir angle used in Fig. 3. In all cases,  $\Delta m_{31}^2 = 3.2 \cdot 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta_{23} = 0.5$ , and  $\sin^2 2\theta_{13} = 0.10$  (solid lines), and 0.05 (dotted lines). In the most relevant energy region between  $\sim 4$  and  $\sim 9$  GeV, the asymmetry is large and with a definite sign: negative for  $\Delta m_{31}^2 > 0$  and positive for  $\Delta m_{31}^2 < 0$ . There is a change of sign in the interval  $E \sim (5 - 6)$  GeV, whose width depends somewhat on the Nadir angle, but the corresponding region is rather narrow and the asymmetry in this region is relatively small, except for  $\theta_n \sim 23^\circ$ . This again indicates that to measure the effects under discussion a sufficient energy and Nadir angle resolution will be required.

The *observable* muon charge asymmetry in the muon events produced by the atmospheric neutrinos is a combination of the CPT-odd asymmetry (involving the survival probabilities  $P(\nu_\mu \rightarrow \nu_\mu)$  and  $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)$ ) and the CP-odd asymmetry (involving the appearance probabilities  $P(\nu_e \rightarrow \nu_\mu)$  and  $P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$ ). One can define

$$A = \frac{N(\mu^-; E) - \frac{\sigma_{CC}(\nu_\mu)}{\sigma_{CC}(\bar{\nu}_\mu)} N(\mu^+; E)}{N(\mu^-; E) + \frac{\sigma_{CC}(\nu_\mu)}{\sigma_{CC}(\bar{\nu}_\mu)} N(\mu^+; E)}, \quad (24)$$

where  $\sigma_{CC}(\nu_\mu)$  and  $\sigma_{CC}(\bar{\nu}_\mu)$  are the relevant charged current neutrino-nucleon cross sec-



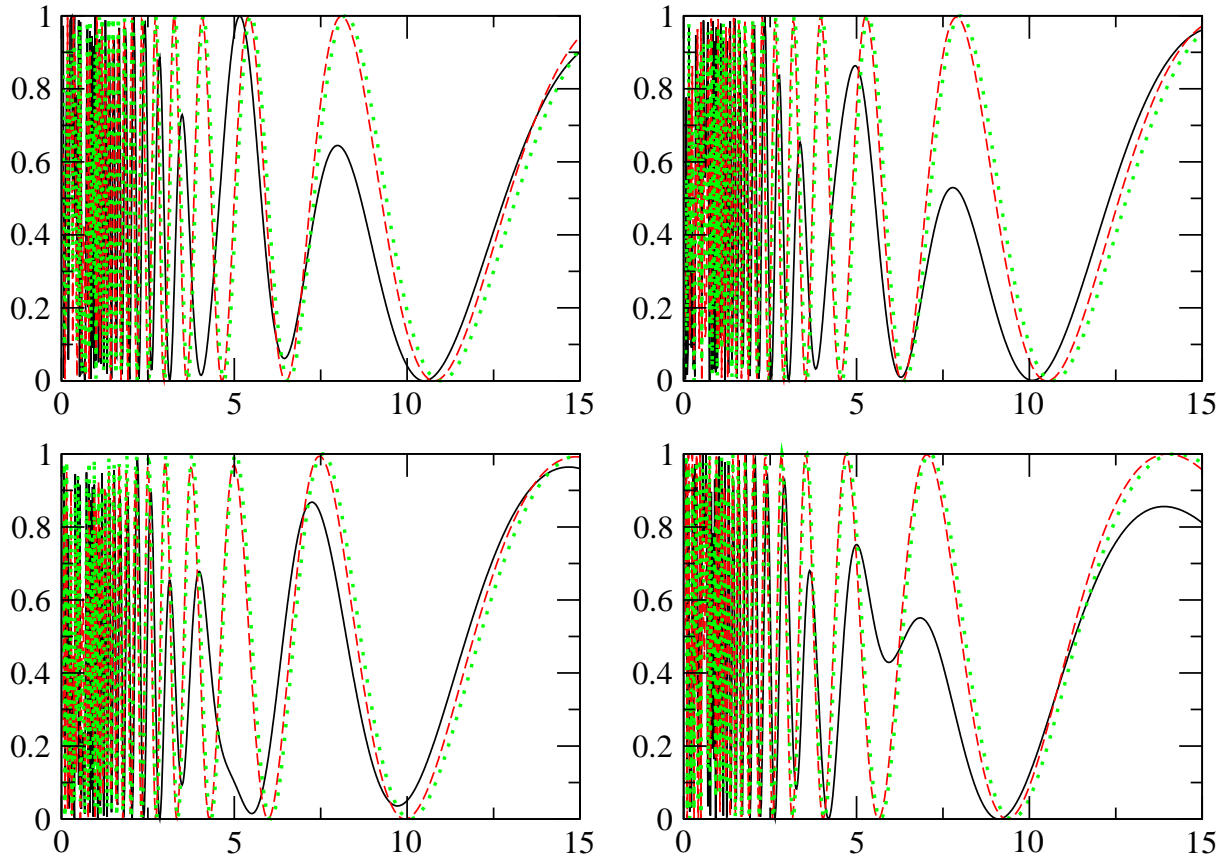


Figure 3: The survival probabilities,  $P(\nu_\mu \rightarrow \nu_\mu)$  and  $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)$ , in the case of neutrino oscillations in the Earth (solid and dashed lines, respectively) and in vacuum (dotted line), as functions of the neutrino energy  $E$  (in GeV), for  $\Delta m_{31}^2 = 3.2 \cdot 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta_{23} = 0.5$  and  $\sin^2 2\theta_{13} = 0.10$ . The upper left, right and the lower left, right panels correspond to neutrino trajectories with Nadir angle  $\theta_n = 0^\circ$ ;  $13^\circ$ ;  $23^\circ$ ;  $30^\circ$ , respectively.

tions,  $N(\mu^\pm; E)$  are the rates of  $\mu^-$  and  $\mu^+$  events produced by the atmospheric neutrinos with energy  $E$ ,

$$\begin{aligned}
 N(\mu^-; E) &= \sigma_{CC}(\nu_\mu) [\Phi^o(\nu_\mu; E) P(\nu_\mu \rightarrow \nu_\mu) + \Phi^o(\nu_e; E) P(\nu_e \rightarrow \nu_\mu)] \\
 N(\mu^+; E) &= \sigma_{CC}(\bar{\nu}_\mu) [\Phi^o(\bar{\nu}_\mu; E) P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu) + \Phi^o(\bar{\nu}_e; E) P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)]
 \end{aligned}
 \tag{25}$$

and  $\Phi^o(\nu_l; E)$  and  $\Phi^o(\bar{\nu}_l; E)$  are the initial fluxes of the atmospheric  $\nu_l$  and  $\bar{\nu}_l$ ,  $l = e, \mu$ , with energy  $E$ . These fluxes are obtained from the code explained in [27].

The charge asymmetry defined by (24) eliminates the asymmetry due to the difference of the cross sections  $\sigma_{CC}(\nu_\mu)$  and  $\sigma_{CC}(\bar{\nu}_\mu)$  which at the energies of interest, and to a good approximation [28], both depend linearly on  $E$ . Notice that the modulation produced by the neutrino fluxes leads to a result which is no longer symmetric with respect to the horizontal axis when changing the sign of  $\Delta m_{31}^2$ . The net effect, however, is the approximate displacement of the symmetry axis in the upward direction. The charge asymmetry (24) is plotted in Fig. 5 for the selected Nadir angles  $\theta_n = 0^\circ$ ,  $13^\circ$ ,  $23^\circ$  and  $30^\circ$ . The results shown in Fig. 5 can be compared with those in Fig. 4 in order to see the effects of the contributions due to the  $\nu_e \rightarrow \nu_\mu$  and  $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$  transitions. As is seen in Figs. 4 and 5, the

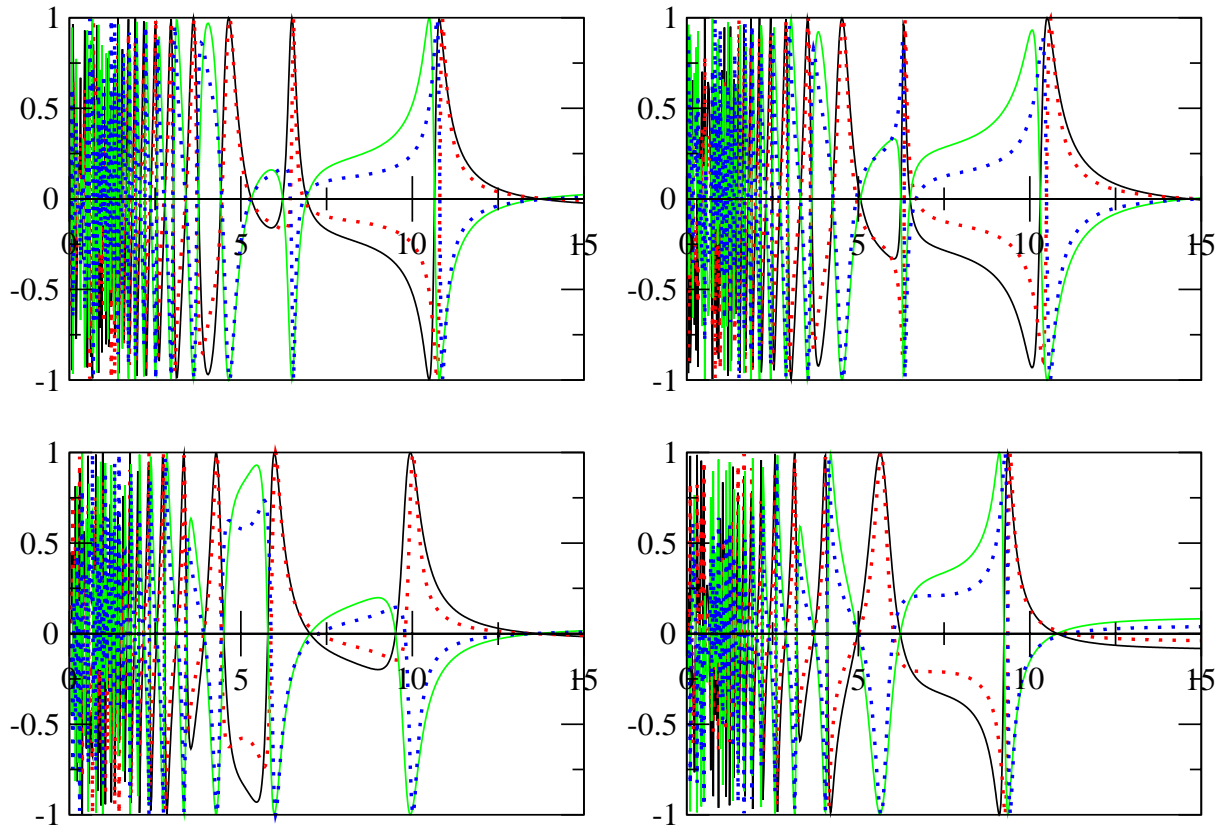


Figure 4: The CPT asymmetry  $A_{CPT}$ , eq. (23), as function of the neutrino energy  $E$  (in GeV) for  $|\Delta m_{31}^2| = 3.2 \cdot 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta_{23} = 0.5$ , and  $\sin^2 2\theta_{13} = 0.1$  (solid line),  $0.05$  (dotted line). The solid (doubly thick solid) and dotted (doubly thick dotted) lines in each panel correspond to  $\Delta m_{31}^2 > 0$  ( $\Delta m_{31}^2 < 0$ ). The upper left, right and the lower left, right panels correspond to neutrino trajectories with Nadir angle  $\theta_n = 0^\circ; 13^\circ; 23^\circ; 30^\circ$ , respectively.

main features of the asymmetry  $A_{CPT}$ , discussed earlier, are exhibited also by the charge asymmetry  $A$ .

To conclude, the propagation of the atmospheric neutrinos through the mantle and the core of the Earth leads to new resonance-like effects in the  $\nu_e(\nu_\mu) \rightarrow \nu_\mu(\nu_e)$  or  $\bar{\nu}_e(\bar{\nu}_\mu) \rightarrow \bar{\nu}_\mu(\bar{\nu}_e)$  transitions, depending on the sign of  $\Delta m_{31}^2$ . This produces observable asymmetries between, e.g., the atmospheric  $\nu_\mu$  and  $\bar{\nu}_\mu$  survival probabilities. As it was the case of the MSW-resonance in the mixing at constant density and long enough baselines, the measurement of the asymmetry between the rates of  $\mu^-$  and  $\mu^+$  events due to atmospheric neutrinos (muon charge asymmetry) can allow to determine the sign of  $\Delta m_{31}^2$ . The muon charge asymmetry is rather sensitive to the magnitude of the “connecting” mixing angle  $\theta_{13}$  as well, although the wide energy region around the resonance in the mantle, with stable matter asymmetry, disappears once the neutrino path crosses the Earth core. In addition to the muon charge discrimination, such measurement requires a sufficiently good neutrino momentum resolution, both in magnitude and direction. The implications of these results on an actual detector are under study [29].

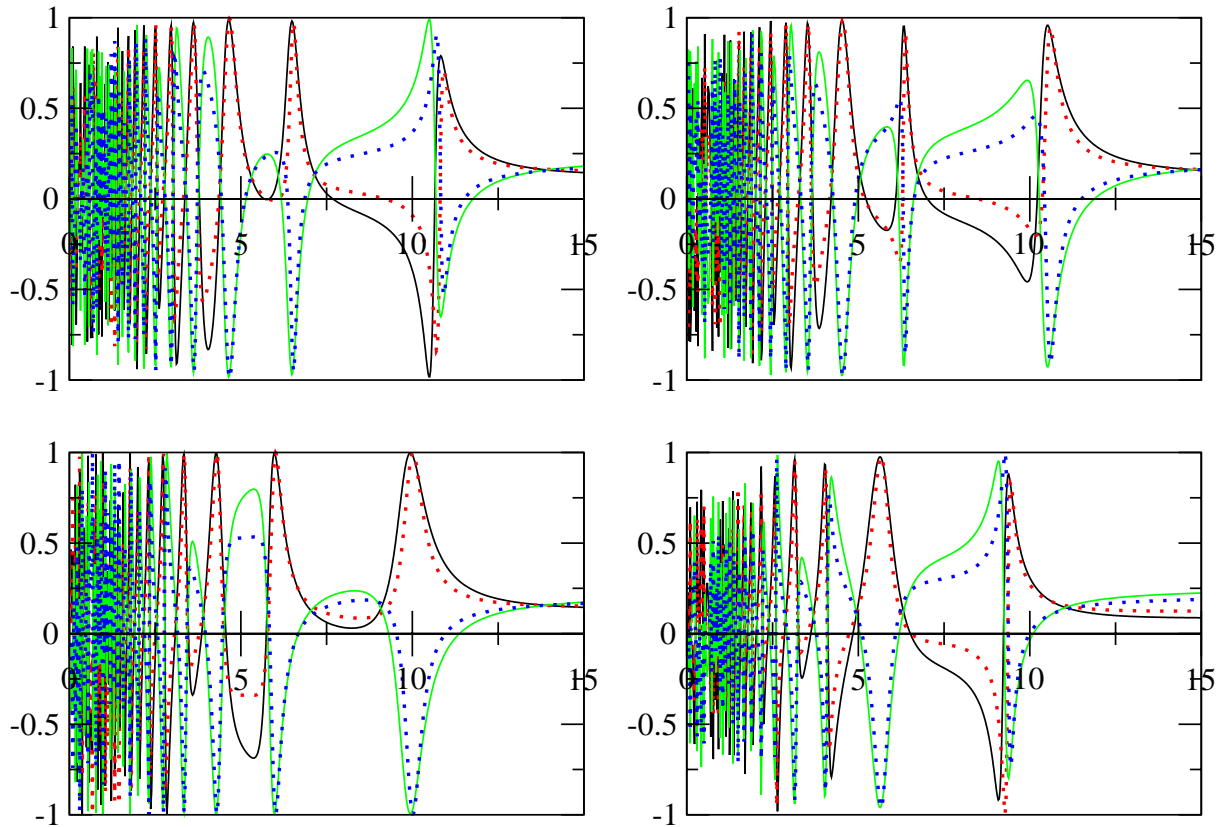


Figure 5: The muon charge asymmetry  $A$ , eq. (24), as function of the neutrino energy (GeV) for  $|\Delta m_{31}^2| = 3.2 \cdot 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta_{23} = 0.5$ , and  $\sin^2 2\theta_{13} = 0.1$  (solid line),  $0.05$  (dotted line). The solid (doubly thick solid) and dotted (doubly thick dotted) lines in each panel correspond to  $\Delta m_{31}^2 > 0$  ( $\Delta m_{31}^2 < 0$ ). The upper left, right and the lower left, right panels correspond to neutrino trajectories with Nadir angle  $\theta_n = 0^\circ; 13^\circ; 23^\circ; 30^\circ$ , respectively.

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