PRECISION TESTS OF THE ELECTROWEAK INTERACTIONS AT e^+e^- COLLIDERS

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This paper is an updated version of the lectures given at the XXIX International Meeting on Fundamental Physics in Sitges, Barcelona (February 2001). The measurements performed at LEP and SLC have substantially improved the precision of the test of the Minimal Standard Model. The precision is such that there is sensitivity to pure weak radiative corrections. This allows to indirectly determine the top mass (m_t=180±10 GeV), the W-boson mass (M_W=80.375±0.022 GeV), and to set an upper limit on the the Higgs boson mass of 196 GeV at 95% confidence level.

1 Introduction

In the context of the Minimal Standard Model (MSM), any ElectroWeak (EW) process can be computed at tree level from α (the fine structure constant measured at values of q^2 close to zero), M_W (the W-boson mass), M_Z (the Z-boson mass), and V_{jk} (the Cabbibo-Kobayashi-Maskawa flavour-mixing matrix elements).

When higher order corrections are included, any observable can be predicted in the "on-shell" renormalization scheme as a function of:

$$O_i = f_i(\alpha, \alpha_s, M_W, M_Z, M_H, m_f, V_{jk})$$

and contrary to what happens with "exact gauge symmetry theories", like QED or QCD, the effects of heavy particles do not decouple. Therefore, the MSM predictions depend on the top mass (m_t^2/M_Z^2) and to less extend to the Higgs mass ($\log(M_H^2/M_Z^2)$), or to any kind of "heavy new physics".

The subject of these lectures is to show how the high precision achieved in the EW measurements allows to test the MSM beyond the tree level predictions and, therefore, how this measurements are able to indirectly determine the value of m_t and M_W, to constrain the unknown value of M_H, and at the same time to test the consistency between measurements and theory. At present the uncertainties in the theoretical predictions are dominated by the precision on the input parameters.

1.1 Input Parameters of the MSM

The W mass is one of the input parameters in the "on-shell" renormalization scheme. It is known with a precision of about 0.04%, although the usual procedure is to take G_μ (the Fermi constant measured in the muon decay) to predict M_W as a function of the rest of the input parameters and use this more precise value.

Therefore, the input parameters are chosen[1] to be:

Input Parameter	Value	Relative Uncertainty
$\alpha^{-1}(M_Z^2) =$	128.933(21)	0.016%
$\alpha_s(M_Z^2) =$	0.118(2)	1.1%
$G_\mu =$	$1.16637(1) \times 10^{-5}$ GeV^{-2}	0.0009%
$M_Z =$	91.1875(21) GeV	0.0023%
$m_t =$	174.3(51) GeV	2.9%
$M_H >$	114.1 GeV @95% C.L.	

Notice that the less well known parameters are m_t, α_s and, of course, the unknown value of M_H. The next less well known parameter is $\alpha^{-1}(M_Z^2)$, even though its value at $q^2 \sim 0$ is known with an amazing relative precision of 4×10^{-9}, $(\alpha^{-1}(0) = 137.03599976\,(50))$.

The reason for this loss of precision when one computes the running of α,

$$\alpha^{-1}(M_Z^2) = \frac{\alpha^{-1}(0)}{1 - \Pi_{\gamma\gamma}}$$

is the large contribution from the light fermion loops to the photon vacuum polarisation, $\Pi_{\gamma\gamma}$. The contribution from leptons and top quark loops is well calculated[2]: $\Pi_{\gamma\gamma}^{\text{lepton}} = -0.031498$ and $\Pi_{\gamma\gamma}^{\text{top}} = -0.000076$ with $m_t = 174.3$ GeV. But for the light quarks non-perturbative QCD corrections are large at low energy scales. The method so far has been to use the measurement of the hadronic cross section through one-photon exchange, normalised to the point-like muon cross-section, R(s), and evaluate the dispersion integral:

$$\Re(\Pi_{\gamma\gamma}^{\text{had}}) = \frac{\alpha M_Z^2}{3\pi} \Re \left(\int \frac{R(s')}{s'(s' - M_Z^2 + i\epsilon)} ds' \right) \tag{1}$$

giving[3] $\Pi_{\gamma\gamma}^{had} = -0.02755 \pm 0.00046$, the error being dominated by the experimental uncertainty in the cross section measurements.

Recently, several new *"theory driven"* calculations[4][5] have reduced this error, by extending the regime of applicability of Perturbative QCD (PQCD).

Table 1: Relative error in units of per-mil on the MSM predictions induced by the uncertainties on the input parameters. The second column shows the present experimental errors.

	Exp. error	$\Delta m_t =$ ± 5.1 GeV	$\Delta M_H =$ [114-1000] GeV	$\Delta \alpha_s =$ ± 0.002	$\Delta \alpha^{-1} =$ ± 0.021
Γ_Z	0.9	0.5	**3.3**	0.5	-
R_b	3.0	0.8	0.1	-	-
M_W	0.4	**0.4**	**2.1**	-	-
$\sin^2 \theta_{\text{eff}}$	0.7	**0.7**	**5.2**	-	0.2

These new calculations have been validated by the most recent data at BESS II[6]. Therefore, along these lectures, the most precise value from reference [4], $\Pi_{\gamma\gamma}^{had} = -0.02763 \pm 0.00016$, will be consistently used.

1.2 What are we measuring to test the MSM?

From the point of view of radiative corrections we can divide the experimental measurements into four different groups: the Z total and partial widths, the partial width into b-quarks (Γ_b), the Z asymmetries ($\sin^2 \theta_{\text{eff}}$) and the W mass ($M_W$). For instance, the leptonic width (Γ_l) is mostly sensitive to isospin-breaking loop corrections ($\Delta\rho$), the asymmetries are specially sensitive to radiative corrections to the Z self-energy ($\Delta\kappa$), and R_b is mostly sensitive to vertex corrections (ϵ_b) in the decay $Z \rightarrow b\bar{b}$. One more parameter, Δr, is necessary to describe the radiative corrections to the relation between G_μ and M_W.

The sensitivity of these three Z observables and M_W to the input parameters is shown in table 1. The most sensitive observable to the unknown value of M_H are the Z asymmetries parametrised via $\sin^2 \theta_{\text{eff}}$. However also the sensitivity of the rest of the observables is very relevant compared to the achieved experimental precision.

2 Z lineshape

The shape of the resonance is completely characterised by three parameters: the position of the peak (M_Z), the width (Γ_Z) and the height ($\sigma_{f\bar{f}}^0$) of the resonance:

$$\sigma_{f\bar{f}}^0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_f}{\Gamma_z^2}$$ (2)

The good capabilities of the LEP detectors to identify the lepton flavours

Table 2: Average line shape parameters from the results of the four LEP experiments.

Parameter	Fitted Value	Derived Parameters
M_Z	91187.5 ± 2.1 MeV	
Γ_Z	2495.2 ± 2.3 MeV	
σ_{had}^0	41.540 ± 0.037 nb	
R_e	20.804 ± 0.050	$\Gamma_e = 83.92 \pm 0.12$ MeV
R_μ	20.785 ± 0.033	$\Gamma_\mu = 83.99 \pm 0.18$ MeV
R_τ	20.764 ± 0.045	$\Gamma_\tau = 84.08 \pm 0.22$ MeV
With Lepton Universality		
R_l	20.767 ± 0.025	$\Gamma_{had} = 1744.4 \pm 2.0$ MeV $\Gamma_l = 83.984 \pm 0.086$ MeV $\Gamma_{inv} = 499.0 \pm 1.5$ MeV

allow to measure the ratio of the different lepton species with respect to the hadronic cross-section, $R_l = \frac{\Gamma_h}{\Gamma_l}$.

About 16 million Z decays have been analysed by the four LEP collaborations, leading to a statistical precision on $\sigma_{f\bar{f}}^0$ of 0.03 % ! Therefore, the statistical error is not the limiting factor, but more the experimental systematic and theoretical uncertainties.

The error on the measurement of M_Z is dominated by the uncertainty on the absolute scale of the LEP energy measurement (about 1.7 MeV), while in the case of Γ_Z it is the point-to-point energy and luminosity errors which matter (about 1.3 MeV). The error on $\sigma_{f\bar{f}}^0$ is dominated by the theoretical uncertainty on the small angle bhabha calculations (0.06 %) and the uncertainty on the position of the inner edge of the luminometers (0.07 %).

The results of the lineshape fit are shown in table 2 with and without the hypothesis of lepton universality. From them, the leptonic widths and the invisible Z width are derived.

From the measurement of the Z invisible width, and assuming the ratio of the partial widths to neutrinos and leptons to be the MSM predictions ($\frac{\Gamma_\nu}{\Gamma_l} = 1.9912 \pm 0.0012$), the number of light neutrinos species is measured to be

$$N_\nu = 2.9841 \pm 0.0083.$$

Alternatively, one can assume three neutrino species and determine the width from additional invisible decays of the Z to be $\Delta\Gamma_{inv} < 2.0$ MeV @95% C.L.

The measurement of R_l and σ_{had}^0 are very sensitive to PQCD corrections and allow one of the most precise and clean determinations of α_s. A combined

fit to the measurements shown in table 2, and imposing m_t=174.3±5.1 GeV as a constraint gives:

$$\alpha_s(M_Z^2) = 0.117 \pm 0.003$$

in agreement with the world average[1] $\alpha_s(M_Z^2) = 0.118 \pm 0.002$.

2.1 Heavy flavour results

The large mass and long lifetime of the b and c quarks provides a way to perform flavour tagging. This allows for precise measurements of the partial widths of the decays $Z \to c\bar{c}$ and $Z \to b\bar{b}$. It is useful to normalise the partial width to Γ_h by measuring the partial decay fractions with respect to all hadronic decays

$$R_c \equiv \frac{\Gamma_c}{\Gamma_h} \ , \ R_b \equiv \frac{\Gamma_b}{\Gamma_h}.$$

With this definition most of the radiative corrections appear both in the numerator and denominator and thus cancel out, with the important exception of the vertex corrections in the Z $b\bar{b}$ vertex. This is the only relevant correction to R_b, and within the MSM basically depends on a single parameter, the mass of the top quark.

The partial decay fractions of the Z to other quark flavours, like R_c, are only weakly dependent on m_t; the residual weak dependence is indeed due to the presence of Γ_b in the denominator. The MSM predicts R_c= 0.172, valid over a wide range of the input parameters.

The combined values from the measurements of LEP and SLD gives

$$R_b = 0.21646 \pm 0.00065$$
$$R_c = 0.1719 \pm 0.0031$$

with a correlation of -14% between the two values.

3 Z asymmetries: $\sin^2\theta_{\text{eff}}$

Parity violation in the weak neutral current is caused by the difference of couplings of the Z to right-handed and left-handed fermions. If we define A_f as

$$A_f \equiv \frac{2\left(\frac{g_V^f}{g_A^f}\right)}{1 + \left(\frac{g_V^f}{g_A^f}\right)^2}, \tag{3}$$

where $g^f_{V(A)}$ denotes the vector(axial-vector) coupling constants, one can write all the Z asymmetries in terms of A_f.

Each process $e^+e^- \rightarrow Z^0 \rightarrow f\bar{f}$ can be characterised by the direction and the helicity of the emitted fermion (f). Calling forward the hemisphere into which the electron beam is pointing, the events can be subdivided into four categories: FR,BR,FL and BL, corresponding to right-handed (R) or left-handed (L) fermions emitted in the forward (F) or backward (B) direction. Then, one can write three Z asymmetries as:

$$A_{\mathrm{pol}} \equiv \frac{\sigma_{\mathrm{FR}}+\sigma_{\mathrm{BR}}-\sigma_{\mathrm{FL}}-\sigma_{\mathrm{BL}}}{\sigma_{\mathrm{FR}}+\sigma_{\mathrm{BR}}+\sigma_{\mathrm{FL}}+\sigma_{\mathrm{BL}}} = -A_{\mathrm{f}} \qquad (4)$$

$$A^{\mathrm{FB}}_{\mathrm{pol}} \equiv \frac{\sigma_{\mathrm{FR}}+\sigma_{\mathrm{BL}}-\sigma_{\mathrm{BR}}-\sigma_{\mathrm{FL}}}{\sigma_{\mathrm{FR}}+\sigma_{\mathrm{BR}}+\sigma_{\mathrm{FL}}+\sigma_{\mathrm{BL}}} = -\frac{3}{4}A_{\mathrm{e}} \qquad (5)$$

$$A_{\mathrm{FB}} \equiv \frac{\sigma_{\mathrm{FR}}+\sigma_{\mathrm{FL}}-\sigma_{\mathrm{BR}}-\sigma_{\mathrm{BL}}}{\sigma_{\mathrm{FR}}+\sigma_{\mathrm{BR}}+\sigma_{\mathrm{FL}}+\sigma_{\mathrm{BL}}} = \frac{3}{4}A_{\mathrm{e}}A_{\mathrm{f}} \qquad (6)$$

and in case the initial state is polarised with some degree of polarisation (P), one can define:

$$A_{\mathrm{LR}} \equiv \frac{1}{P}\frac{\sigma_{\mathrm{Fl}}+\sigma_{\mathrm{Bl}}-\sigma_{\mathrm{Fr}}-\sigma_{\mathrm{Br}}}{\sigma_{\mathrm{Fr}}+\sigma_{\mathrm{Br}}+\sigma_{\mathrm{Fl}}+\sigma_{\mathrm{Bl}}} = A_{\mathrm{e}} \qquad (7)$$

$$A^{\mathrm{pol}}_{\mathrm{FB}} \equiv -\frac{1}{P}\frac{\sigma_{\mathrm{Fr}}+\sigma_{\mathrm{Bl}}-\sigma_{\mathrm{Fl}}-\sigma_{\mathrm{Br}}}{\sigma_{\mathrm{Fr}}+\sigma_{\mathrm{Br}}+\sigma_{\mathrm{Fl}}+\sigma_{\mathrm{Bl}}} = \frac{3}{4}A_{\mathrm{f}} \qquad (8)$$

where r(l) denotes the right(left)-handed initial state polarisation. Assuming lepton universality, all this observables depend only on the ratio between the vector and axial-vector couplings. It is conventional to define the effective mixing angle $\sin^2\theta_{\mathrm{eff}}$ as

$$\sin^2\theta_{\mathrm{eff}} \equiv \frac{1}{4}\left(1 - \frac{g^l_V}{g^l_A}\right) \qquad (9)$$

and to collapse all the asymmetries into a single parameter $\sin^2\theta_{\mathrm{eff}}$.

3.1 Lepton asymmetries

Angular distribution

The lepton forward-backward asymmetry is measured from the angular distribution of the final state lepton. The measurement of A^l_{FB} is quite simple and robust and its accuracy is limited by the statistical error. The common systematic uncertainty in the LEP measurement due to the uncertainty on the LEP energy measurement is about 0.0003. The values measured by the LEP collaborations are in agreement with lepton universality,

$$A_{\mathrm{FB}}^{e} = 0.0145 \pm 0.0025$$
$$A_{\mathrm{FB}}^{\mu} = 0.0169 \pm 0.0013$$
$$A_{\mathrm{FB}}^{\tau} = 0.0188 \pm 0.0017$$

and can be combined into a single measurement of $\sin^2 \theta_{\mathrm{eff}}$,

$$A_{\mathrm{FB}}^{l} = 0.01714 \pm 0.00095 \implies \sin^2 \theta_{\mathrm{eff}} = 0.23099 \pm 0.00053.$$

Tau polarisation at LEP

Tau leptons decaying inside the apparatus acceptance can be used to measure the polarised asymmetries defined by equations (4) and (5). A more sensitive method is to fit the measured dependence of A_{pol} as a function of the polar angle θ :

$$A_{\mathrm{pol}}(\cos \theta) = -\frac{A_{\tau}(1 + \cos^2 \theta) + 2A_e \cos \theta}{(1 + \cos^2 \theta) + 2A_{\tau} A_e \cos \theta} \tag{10}$$

The sensitivity of this measurement to $\sin^2 \theta_{\mathrm{eff}}$ is larger because the dependence on A_l is linear to a good approximation. The accuracy of the measurements is dominated by the statistical error. The typical systematic error is about 0.003 for A_{τ} and 0.001 for A_e. The LEP measurements are:

$$A_e = 0.1498 \pm 0.0049 \implies \sin^2 \theta_{\mathrm{eff}} = 0.23117 \pm 0.00062$$
$$A_{\tau} = 0.1439 \pm 0.0043 \implies \sin^2 \theta_{\mathrm{eff}} = 0.23192 \pm 0.00054$$

3.2 $\mathrm{A_{LR}}$ **from SLD**

The linear accelerator at SLAC (SLC) allows to collide positrons with a highly longitudinally polarised electron beam (up to 77% polarisation). Therefore, the SLD detector can measure the left-right cross-section asymmetry ($\mathrm{A_{LR}}$) defined by equation (7). This observable is a factor of 4.6 times more sensitive to $\sin^2 \theta_{\mathrm{eff}}$ than, for instance, A_{FB}^{l} for a given precision. The measurement is potentially free of experimental systematic errors, with the exception of the polarisation measurement that has been carefully cross-checked at the 1% level. SLD final measurement gives

$$A_{\mathrm{LR}} = 0.1514 \pm 0.0022 \implies \sin^2 \theta_{\mathrm{eff}} = 0.23097 \pm 0.00027,$$

and assuming lepton universality it can be combined with measurements at SLD of the lepton left-right forward-backward asymmetry ($A_{\mathrm{FB}}^{\mathrm{pol}}$) defined in equation (8) to give
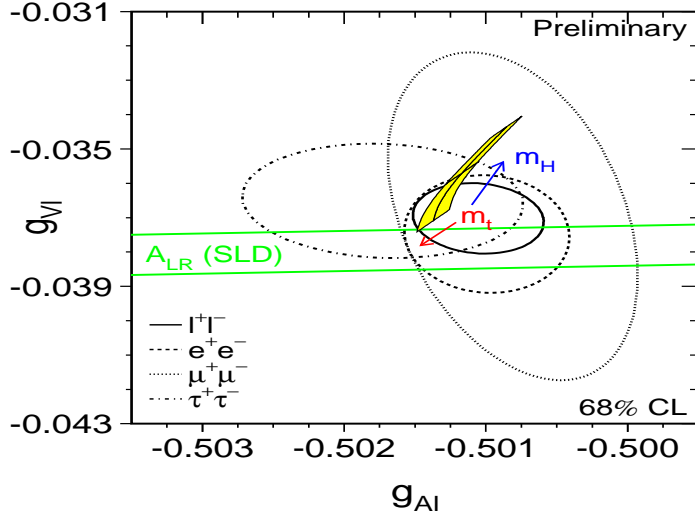
$$\sin^2 \theta_{\mathrm{eff}} = 0.23098 \pm 0.00026.$$

Figure 1: Contours of 68% probability in the $g_V^l - g_A^l$ plane. The solid contour results from a fit assuming lepton universality. Also shown is the one standard deviation band resulting from the A_{LR} measurement of SLD.

3.3 Lepton couplings

All the previous measurements of the lepton coupling (A_l) can be combined with a $\chi^2/\text{dof} = 2.6/3$ and give

$$A_l = 0.1501 \pm 0.0016 \implies \sin^2 \theta_{\text{eff}} = 0.23113 \pm 0.00021.$$

The asymmetries measured are only sensitive to the ratio between the vector and axial-vector couplings. If we introduce also the measurement of the leptonic width shown in table 2 we can fit the lepton couplings to the Z to be

$$g_V^l = -0.03783 \pm 0.00041,$$
$$g_A^l = -0.50123 \pm 0.00026,$$

where the sign is chosen to be negative by definition. Figure 1 shows the 68 % probability contours in the $g_V^l - g_A^l$ plane.

Heavy Flavour asymmetries

The inclusive measurement of the b and c asymmetries is more sensitive to $\sin^2 \theta_{\text{eff}}$ than, for instance, the leptonic forward-backward asymmetry. The reason is that A_b and A_c are mostly independent of $\sin^2 \theta_{\text{eff}}$, therefore $A_{\text{FB}}^{b(c)}$ (which is proportional to the product $A_e A_{b(c)}$) is a factor $3.3(2.4)$ more sensitive than A_{FB}^{l}. The typical systematic uncertainty in $A_{\text{FB}}^{b(c)}$ is about $0.001(0.002)$ and the precision of the measurement is dominated by statistics.

SLD can measure also the b and c left-right forward-backward asymmetry defined in equation (8) which is a direct measurement of the quark coupling A_b and A_c. The combined fit for the LEP and SLD measurements gives

$$A_{\text{FB}}^b = 0.0990 \pm 0.0017 \Longrightarrow \sin^2 \theta_{\text{eff}} = 0.23226 \pm 0.00031$$
$$A_{\text{FB}}^c = 0.0685 \pm 0.0034 \Longrightarrow \sin^2 \theta_{\text{eff}} = 0.23272 \pm 0.00079$$
$$A_b = 0.922 \pm 0.020$$
$$A_c = 0.670 \pm 0.026$$

where 15% is the largest correlation between A_{FB}^b and A_{FB}^c.

Taking the value of A_l given in section 3.3 and these measurements together in a combined fit gives

$$A_b = 0.879 \pm 0.018$$
$$A_c = 0.608 \pm 0.031$$

to be compared with the MSM predictions $A_b = 0.935$ and $A_c = 0.668$ valid over a wide range of the input parameters. The measurement of A_c is 1.9 standard deviations lower than the predicted value, while the measurement of A_b is 3.1 standard deviations lower. Notice that the direct measurements of $A_{b(c)}$ at SLD are in perfect agreement with the MSM prediction. The deviation in the combined fit of $A_{b(c)}$ and A_l, is just a reflection of the discrepancy between leptonic and hadronic measurements of $\sin^2 \theta_{\text{eff}}$, (see section 3.5).

Jet charge asymmetries

The average charge flow in the inclusive samples of hadronic Z decays is related to the forward-backward asymmetries of individual quarks:

$$\langle Q_{\text{FB}} \rangle = \sum_{\text{q}} \delta_{\text{q}} A_{\text{FB}}^{\text{q}} \frac{\Gamma_{\text{q}\bar{\text{q}}}}{\Gamma_{\text{h}}} \tag{11}$$
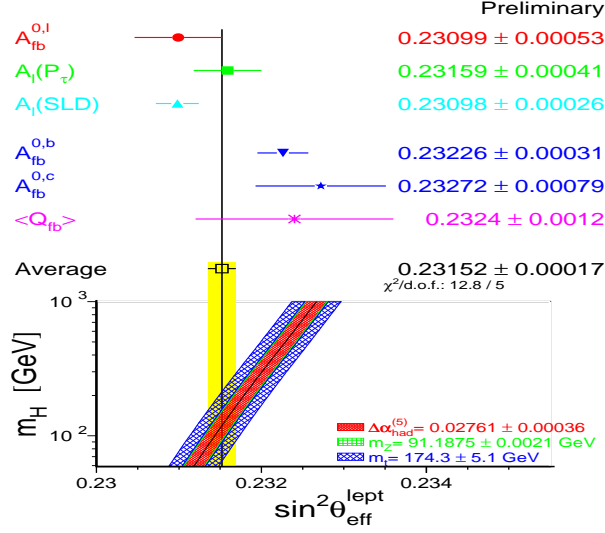
Figure 2: Comparison of several determinations of $\sin^2\theta_{\text{eff}}$ from asymmetries.

where δ_q, the charge separation, is the average charge difference between the quark and antiquark hemispheres in an event. The combined LEP value is

$$\sin^2\theta_{\text{eff}} = 0.2324 \pm 0.0012.$$

3.5   Comparison of the determinations of $\sin^2\theta_{\text{eff}}$

The combination of all the quark asymmetries shown in this section can be directly compared to the determination of $\sin^2\theta_{\text{eff}}$ obtained with leptons,

$$\sin^2\theta_{\text{eff}} = 0.23230 \pm 0.00029 \quad (\text{quark} - \text{asymmetries})$$
$$\sin^2\theta_{\text{eff}} = 0.23113 \pm 0.00021 \quad (\text{lepton} - \text{asymmetries})$$

which shows a 3.3 σ discrepancy.

Over all, the agreement is acceptable, and the combination of the individual determinations of $\sin^2\theta_{\text{eff}}$ gives

$$\sin^2\theta_{\text{eff}} = 0.23152 \pm 0.00017$$

with a $\chi^2/\text{dof} = 12.8/5$ as it is shown in figure 2.

4 W mass

Since 1996 up to 2000 LEP has been running at energies above the W-pair production treshold and about 40k W-pairs have been detected by the LEP experiments. The cross-section for the process $e^+e^- \rightarrow W^+W^-$ has been measured with a precision of 1%. The theoretical calculations have been updated to match with this precision, confirming the indirect evidence from Z physics of Gauge Boson Couplings predicted by the MSM.

More interesting in the context of these lectures, is the improvement on the W mass accuracy, previously measured in proton-proton or νN collisions.

4.1 W mass at pp colliders

At hadron colliders, the W mass is obtained from the distribution of the W transverse mass, that is the invariant mass of the W decay products evaluated in the plane transverse to the beam. This is because the longitudinal component of the neutrino momentum cannot be measured in a pp collider. On the other hand, the transverse momentum of the neutrino can be deduced from the vector sum of the transverse momentum of the charged lepton and the transverse momentum of the system recoiling against the W.

The uncertainty on the W mass is dominated by the uncertainty in the lepton energy/momentum calibration. The combination of the measurements at FERMILAB (CDF/D0), and CERN (UA2) gives:

$$M_W = 80.454 \pm 0.060 \text{ GeV}$$

where the error is dominated by the systematic uncertainty (50 MeV).

4.2 W mass from νN scattering

The ratio between the neutral and charged current interaction of neutrino on isoscalar targets, provides an indirect determination of the ratio between the W mass and the Z mass. The main experimental uncertainty comes from the model needed for the background subtraction. The results from NuTeV combined with previous CCFR measurements (asuming m_t=175 GeV and M_H=150 GeV) gives:

$$M_W = 80.25 \pm 0.11 \text{ GeV}$$

where the dependence with m_t and M_H has to be taken into account in the MSM fits in section 5.2.

4.3 **W mass at LEP**

The W-pair production cross-section near the treshold has a strong dependence on the W mass. The first data collected at LEP just above treshold has been used to get a measurement of the W mass:

$$M_W = 80.40 \pm 0.22 \text{ GeV}$$

But the most precise measurement of the W mass comes from the kinematic reconstruction of the W decay products at LEP. The precise knowledge of the c.o.m. energy is used to improve the experimental resolutions. The W mass is extracted from a comparison between data and Monte Carlo simulation for different values of the W mass giving:

$$M_W = 80.450 \pm 0.039 \text{ GeV}$$

The measurement is dominated by systematic uncertainties (30 MeV). The main systematic uncertainty is due to the hadronization model (17 MeV) and to the knowledge of the LEP c.o.m. energy (17 MeV) which affects both channels in a coherent way: 4q channel where both W's decay into quarks, and 2q channel when one of the W's decay into a lepton and a neutrino.

There are other systematic sources related to the hadronization model that only affect the 4q channel. In particular, the separation of the decay vertices is about 0.1 fm, which is small compared with the typical hadronization scale of 1 fm. This fact may lead to non-perturbative phenomena interconnecting the decays of the two W's and introducing a source of systematic uncertainties in the measurement. By comparing diferent models of Colour Reconnection (CR) that reproduce the precise LEP data, a maximum shift of 40 MeV is quoted in the 4q channel. A similar procedure is used to quote a maximum shift of 25 MeV due to possible Bose-Einstein (BE) correlations between decay products from the two W's.

As this phenomena only affects the W mass obtained from the 4q channel at LEP, it is instructive to check the consistency of this measurement with respect to the rest of the W mass measurements:

$$M_W(\text{4q LEP}) = 80.457 \pm 0.030(\text{stat.}) \pm 0.026(\text{syst.}) \pm 0.047(\text{CR/BE}) \text{ GeV}$$
$$M_W(\text{2q LEP + CDF/D0 + xsect LEP}) = 80.449 \pm 0.024(\text{stat.}) \pm 0.025(\text{syst.}) \text{ GeV}$$

which leads to a non-significative difference of $+8 \pm 39$ MeV, and supports that the quoted systematic uncertainty for these effects (47 MeV for the 4q channel) is reasonable.

All direct measurements of the W mass from LEP and TEVATRON can be combined to give a world average value of:

$$M_W = 80.451 \pm 0.033 \text{ GeV}$$

5 Consistency with the Minimal Standard Model

The MSM predictions are computed using the programs TOPAZ0 [7] and ZFITTER [8]. They represent the state-of-the-art in the computation of radiative corrections, and incorporate recent calculations such as the QED radiator function to $O(\alpha^3)$, four-loop QCD effects, non-factorisable QCD-EW corrections, and two-loop sub-leading $O(\alpha^2 m_t^2/M_Z^2)$ corrections, resulting in a significantly reduced theoretical uncertainty compared to the work summarized in reference [9].

5.1 Are we sensitive to radiative corrections other than $\Delta\alpha$?

This is the most natural question to ask if one pretends to test the MSM as a Quantum Field Theory and to extract information on the only unknown parameter in the MSM, M_H.

The MSM prediction of R_b neglecting radiative corrections is $R_b^0 = 0.2183$, while the measured value given in section 2.1 is about 2.8σ lower. From table 1 one can see that the MSM prediction depends only on m_t and allows to determine indirectly its mass to be $m_t = 155 \pm 20$ GeV, in agreement with the direct measurement ($m_t = 174.3 \pm 5.1$ GeV).

From the measurement of the leptonic width, the vector-axial coupling given in section 3.3 disagrees with the Born prediction (-1/2) by about 4.7σ, showing evidence for radiative corrections in the ρ parameter, $\Delta\rho = 0.005 \pm 0.001$.

However, the most striking evidence for pure weak radiative corrections is not coming from Z physics, but from M_W and its relation with G_μ. The value measured at LEP and TEVATRON is $M_W = 80.451 \pm 0.033$ GeV. From this measurement and through the relation

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi\alpha}{G_\mu\sqrt{2}} \left(1 + \Delta r \right) \tag{12}$$

one gets a measurement of $\Delta r = 0.032 \pm 0.002$, and subtracting the value of $\Delta\alpha$ ($\Delta\alpha = -\Pi_{\gamma\gamma}$), given in section 1.1, one obtains $\Delta r_W = \Delta r - \Delta\alpha = -0.027 \pm 0.002$, which is about 12.9σ different from zero.

Fit to the MSM predictions

Having shown that there is sensitivity to pure weak corrections with the accuracy in the measurements achieved so far, one can envisage to fit the values of the unknown Higgs mass and the less well known top mass in the context of the MSM predictions. The fit is done using the Z measurements, the W mass measurements and νN scattering measurements. The quality of the fit is acceptable, ($\chi^2/\text{dof} = 22.7/14$) and the indirect determination of the top mass gives,

$$m_t = 180^{+11}_{-9} \text{ GeV}$$

to be compared with m_t=174.3±5.1 GeV measured at TEVATRON. The result of the fit is shown in the M_H-m_t plane in figure 3. Both determinations of m_t have similar precision and are compatible. Therefore, one can constrain the previous fit with the direct measurement of m_t and obtains:

$$m_t = 175.7 \pm 4.4 \text{ GeV}$$
$$\log(M_H/\text{GeV}) = \quad 1.96 \pm 0.19 \quad (M_H = 91^{+49}_{-33} \text{ GeV})$$
$$\alpha_s = \quad 0.118 \pm 0.003$$

with a $\chi^2/\text{dof} = 23.0/15$. Most of the contribution to the χ^2 is from the discrepancy in the hadronic measurements of $\sin^2 \theta_{\text{eff}}$ mentioned in section 3.5. The distribution of the pulls of each measurement is shown in figure 4.

The best indirect determination of the W mass is obtained from the MSM fit when no information from the direct measurement is used,

$$M_W = 80.375 \pm 0.022 \text{ GeV}.$$

which is a bit low (-1.9σ) compared with the direct measurement at LEP and TEVATRON, $M_W = 80.451 \pm 0.033$ GeV. As it would become more clear in section 6.1, this is again a reflection of the discrepancy in the hadronic measurements of $\sin^2 \theta_{\text{eff}}$. The indirect prediction of the W mass not using A^q_{FB} gives: $M_W = 80.414 \pm 0.024$ GeV, in good agreement with the direct determination of the W mass.

6 Constraints on M_H

In the previous section it has been shown that the global MSM fit to the data gives

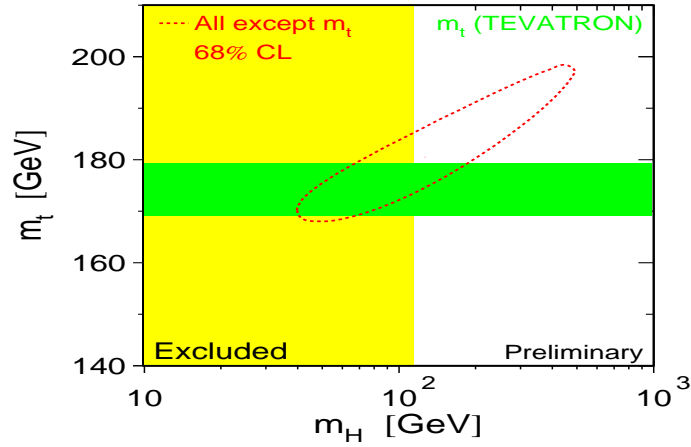$$\log(M_H/\text{GeV}) = 1.96 \pm 0.19 \ (M_H = 91^{+49}_{-33} \text{ GeV})$$

Figure 3: The 68% confidence level contour in the m_t vs M_H plane. The vertical band shows the 95% C.L. exclusion limit on M_H from direct searches.
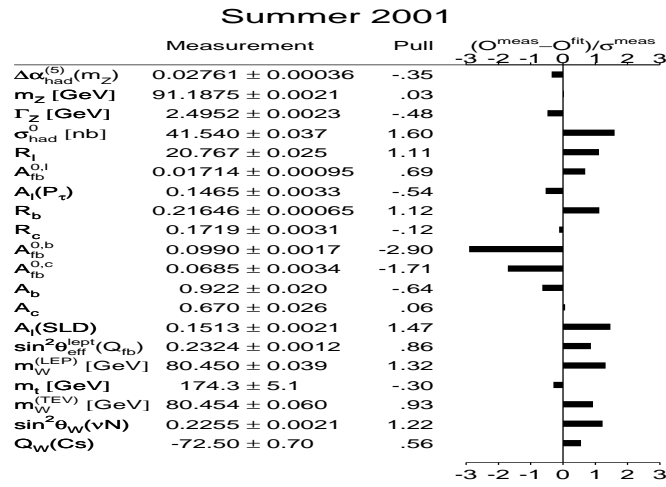

Figure 4: Pulls of the measurements with respect to the best fit results. The pull is defined as the difference of the measurement to the fit prediction divided by the measurement error.

and taking into account the theoretical uncertainties (about ± 0.05 in $\log(M_H/\text{GeV})$), this implies a one-sided 95% C.L. limit of:

$$M_H < 196 \text{ GeV @95\% C.L.}$$

which does not take into account the limits from direct searches ($M_H >$ 114.1 GeV @95% C.L.).

6.1 Consistency of the Higgs mass determination

As described in section 1.2, one can divide the measurements sensitive to the Higgs mass into three different groups: Asymmetries ($\Delta\kappa$), Widths ($\Delta\rho$) and the W mass (Δr). They test conceptually different components of the radiative corrections and it is interesting to check the internal consistency. Given the discrepancies between hadronic and leptonic measurements of the Z asymmetries, it is instructive to quote separate results for the asymmetries.

Repeating the MSM fit shown in the previous section for the three different groups of measurements with the additional constraint from [1] $\alpha_s = 0.118 \pm 0.002$ gives the results shown in the second column in table 3. The indirect determination of M_H from the Z lineshape, from the leptonic asymmetries and from the W mass are in amazing agreement, and prefer a very low value of the Higgs mass. Only the hadronic asymmetries, somehow, contradict this tendency. This is seen with more detail in figure 5, where the individual determinations of $\log(M_H/\text{GeV})$ are shown for each measurement.

From table 3 it is clear that any future improvement on the indirect determination of the Higgs mass needs a more precise determination of the Top mass.

6.2 What's next?

LHC and its multipurpose detectors (ATLAS/CMS) are the ideal laboratory to disentangle the mystery of mass generation in the MSM. It's therefore interesting to evaluate what could be the situation of the indirect determination of the Higgs mass when LHC starts sometime in 2006.

TEVATRON has started its RUN II program and CDF/D0 expect to collect a significant amount of luminosity during the coming years. In figure 6 it is shown the expected direct limit on M_H with 10 fb^{-1}. It's also shown what would be the improvement in the indirect determination when CDF/D0 are able to measure the top mass with an uncertainty of 3 GeV and improve the world average of the W mass to give an uncertainty of 20 MeV.
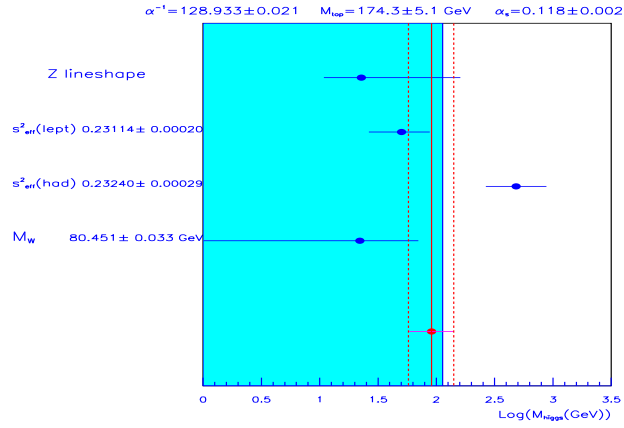
Figure 5: Individual determination of $\log(M_H/\text{GeV})$ for each of the measurements. The vertical band shows the 95% C.L. exclusion limit on M_H from direct searches.
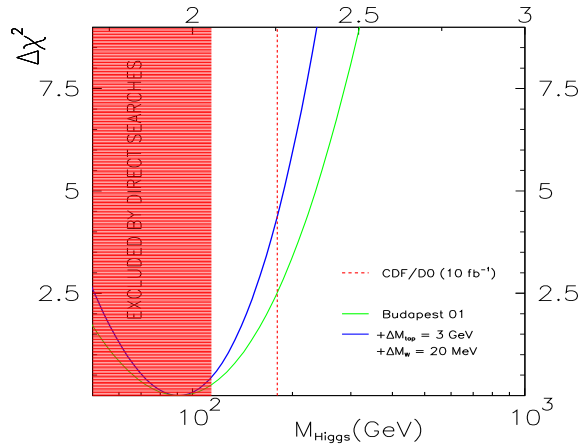

Figure 6: $\Delta\chi^2 = \chi^2 - \chi^2_{\min}$ vs. M_H curve. Different cases are considered: the present situation and the future situation when ΔM_W is measured with a precision of 20 MeV and Δm_t=3 GeV. The band shows the limit from direct searches, and the discontinous line the expected limit from TEVATRON with 10 fb^{-1}.

Table 3: Results on $\log(M_H/\text{GeV})$ for different samples of measurements. In the fit the input parameters and their uncertainties are taken to be the values presented in section 1.1. The impact of the uncertainty in each parameter is explicitly shown.

	$\log(M_H)$	$[\Delta \log(M_H)]^2$	$=$	$[\Delta_{\text{exp.}}]^2 + [\Delta m_t]^2 + [\Delta\alpha]^2 + [\Delta\alpha_s]^2$
Had. Asymm.	2.68 ± 0.26	$[0.26]^2$	$=$	$[0.22]^2 + [0.14]^2 + [0.04]^2 + [0.01]^2$
Lep. Asymm.	1.70 ± 0.26	$[0.26]^2$	$=$	$[0.22]^2 + [0.14]^2 + [0.04]^2 + [0.01]^2$
Z lineshape	$1.36^{+0.85}_{-0.32}$	$[0.46]^2$	$=$	$[0.43]^2 + [0.14]^2 + [0.02]^2 + [0.08]^2$
M_W	$1.34^{+0.50}_{-1.34}$	$[0.67]^2$	$=$	$[0.47]^2 + [0.47]^2 + [0.02]^2 + [0.00]^2$

Either the Higgs particle is found relatively soon (it may be that LEP has already seen the first hints [10] of it), or the MSM will be in real trouble to describe the precision measurements.

7   Conclusions and outlook

The measurements performed at LEP and SLC have substantially improved the precision of the tests of the MSM, at the level of O(0.1%). The effects of pure weak corrections are visible with a significance of about five standard deviations from Z observables and about twelve standard deviations from the W-boson mass.

The top mass predicted by the MSM fits, ($m_t = 180^{+11}_{-9}$ GeV) is in very good agreement with the direct measurement ($m_t = 174.3 \pm 5.1$ GeV) and of similar precision.

The W-boson mass predicted by the MSM fits, ($M_W = 80.375 \pm 0.022$ GeV) is compatible (-1.9σ) with the direct measurement ($M_W = 80.451 \pm 0.033$ GeV).

The mass of the Higgs boson is predicted to be low,

$$\log(M_H/\text{GeV}) = 1.96 \pm 0.19 \ (M_H = 91^{+49}_{-33} \text{ GeV})$$
$$M_H \quad < \quad 196 \text{ GeV @95\% C.L.}$$

All measurements are internally consistent with the predictions of the MSM and with a very low value of the Higgs mass (lower than the limit from direct searches), with the exception of the hadronic Z asymmetries that prefers a somehow larger value of the Higgs mass.

Acknowledgments

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