Lower Limits on Soft Supersymmetry-Breaking Scalar Masses

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Abstract

Working in the context of the CMSSM, we argue that phenomenological constraints now require the universal soft supersymmetry-breaking scalar mass m_0 be non-zero at the input GUT scale. This conclusion is primarily imposed by the LEP lower limit on the Higgs mass and the requirement that the lightest supersymmetric particle not be charged. We find that $m_0 > 0$ for all $\tan\beta$ if $\mu < 0$, and $m_0 = 0$ may be allowed for $\mu > 0$ only when $\tan\beta \sim 8$ and one allows an uncertainty of 3+ GeV in the theoretical calculation of the Higgs mass. Upper limits on flavour-changing neutral interactions in the MSSM squark sector allow substantial violations of non-universality in the m_0 values, even if their magnitudes are comparable to the lower limit we find in the CMSSM. Also, we show that our lower limit on m_0 at the GUT scale in the CMSSM is compatible with the no-scale boundary condition $m_0 = 0$ at the Planck scale.

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Motivated by the naturalness of the gauge hierarchy [1], TeV-scale supersymmetry is, perhaps, the most plausible scenario for low-energy physics beyond the Standard Model. Here we study the minimal supersymmetric extension of the Standard model (MSSM). Some of the greatest puzzles of supersymmetry are associated with its breaking. There is no consensus on the origin of supersymmetry breaking, even within string (or M) theory, and we do not know what fixes the scale of supersymmetry breaking (and how). Within this general area of puzzles, there are minor puzzles, such as questions whether soft supersymmetry-breaking scalar and gaugino masses, m_0 and $m_{1/2}$, respectively, are universal. In particular, generation-dependent scalar masses would threaten the observed suppression of flavour-changing neutral interactions (FCNI) [2], whereas differences between the scalar masses of sparticles with different gauge quantum numbers would be less problematic. In other words, one needs $m_0^{e_L} = m_0^{\mu_L} = m_0^{\tau_L}$ to a very good approximation, and similarly for the ℓ_R and for the squarks. On the other hand, there is no strong phenomenological reason why $m_0^{\ell_L} = m_0^{\ell_R}$, or why squark and slepton masses should be equal. For the moment, however, we work in the context of the constrained MSSM (CMSSM), where this extended universality is assumed.

Some proposed mechanisms for supersymmetry breaking in string theory yield generation-dependent scalar masses, for example because they depend on moduli characterizing the string vacuum, whereas other mechanisms are naturally generation-independent. The former are *a priori* in conflict with the constraints imposed by FCNI. Many of the latter mechanisms achieve consistency with these limits by resuscitating no-scale gravity [3], in which the soft supersymmetry-breaking scalar masses *vanish at the input supersymmetric grand-unification (GUT) scale.* These input values are renormalized by gauge and Yukawa interactions at lower scales. The renormalizations by gauge interactions are generation-independent, whereas those by Yukawa interactions break universality by amounts related to quark and lepton masses, which may be phenomenologically acceptable. Therefore, no-scale supergravity and its possible string antecedents are experiencing some sort of phenomenological renaissance.

Is the no-scale hypothesis italicized in the previous paragraph actually excluded by the continuing absence of sparticles and by other experimental limits? Or does it require some reformulation? If so, are the FCNI constraints endangered? These are the issues addressed in this paper.

As we have indicated, we restrict our attention to the CMSSM which imposes universal gaugino masses $m_{1/2}$, scalar masses m_0 (including those of the Higgs multiplets) and trilinear supersymmetry breaking parameters A_0 are input at the supersymmetric grand unification scale. In this framework, the Higgs mixing parameter μ can be derived (up to a sign) from the other MSSM parameters by imposing the electroweak vacuum condi-

tions for any given value of $\tan\beta$. Thus, given the set of input parameters determined by $\{m_{1/2}, m_0, A_0, \tan\beta, sgn(\mu)\}$, the entire spectrum of sparticles can be derived. In our analysis, we consider the following experimental limits. (1) Lower limits on slepton masses from LEP, in particular the bound $m_{\tilde{e}_R} > 99$ GeV [4]. (2) The LEP lower limit on mass of the lightest Higgs boson: $m_h > 114.1$ GeV [5] - where we discuss and take into account theoretical uncertainties in the mass calculation [6]. (3) The experimental range for $b \to s\gamma$ decay [7]. (4) The recent BNL E821 measurement of the anomalous magnetic moment of the muon [8] - however, we are reluctant to rely strongly on this latter constraint until the theoretical uncertainties in the Standard Model prediction are more widely understood. (5) The requirement that the lightest supersymmetric particle (LSP) be the lightest neutralino χ [10], rather than the lightest slepton $\tilde{\tau}_1$. (6) The lower limit $\Omega_\chi h^2 > 0.1$ on the relic LSP density, which would apply if the LSP constitutes most of the cold dark matter in the Universe, though we allow this condition to be relaxed.

Since the last two constraints are less directly related to particle experiments, we motivate them in more detail. Recent observations in cosmology and particle physics strengthen the expectation that supersymmetry plays a fundamental rôle in the structure of the Universe. The observation of the first three acoustic peaks in the Cosmic Microwave Background (CMB) radiation anisotropies [11] not only indicates clearly that we are living in (or very near) a flat $k = 0$ or $\Omega = 1$ Universe, as indicated by the position of the first peak, but also confirms that the mass-energy density of the Universe is mostly non-baryonic, as indicated by the ratios of the heights of the even and odd peaks. These indicate that the baryon density $\Omega_b h^2 \sim 0.021$, with an uncertainty of 10 to 20%, and is in good agreement with the entirely independent estimates from Big-Bang nucleosynthesis [12] based on the abundance of D/H in quasar absorption systems [13]. The combination of CMB measurements together with other astrophysical data independently require a much larger total for the matter density, $\Omega_m \sim 0.3$, also in agreement with previous independent estimates. The case for non-baryonic dark matter has therefore been greatly strengthened by these recent measurements of the CMB.

It is well known that low-energy supersymmetry: $m_{susy} = \mathcal{O}(1)$ TeV, as motivated independently by the gauge hierarchy problem [1] and the unification of gauge couplings [9], provides an excellent candidate for this non-baryonic dark matter, namely the LSP [10], as long as R parity is conserved, as we assume in this paper [1]. However, this LSP cannot be charged, or it would conflict strongly with upper limits on charged relics from the Big Bang. This motivates requirement (5) above, $\tilde{\tau}_1 > m_\chi$. The lightest neutralino has all the

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[1]FCNI pose even more challenges for R-violating theories.

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properties desired for non-baryonic cold dark matter, which should have $\Omega_{CDM}h^2 > 0.1$, although there could be other components, in which case $\Omega_\chi h^2 < 0.1$ might be possible. If the LSP is the dominant component of the cold dark matter, the allowed range for its relic density: $0.1 \leq \Omega_\chi h^2 \leq 0.3$ provides stringent constraints on the sparticle masses and thus on the soft supersymmetry-breaking parameters. In particular, neutralino annihilation depends on the sfermion masses, and hence on the soft scalar masses m_0. Hence, the lower bound $\Omega_\chi h^2 > 0.1$ could be translated into an upper bound on the $\chi - \chi$ annihilation cross section, which in turn would imply a lower bound on m_0, as mentioned in point (6) above. In fact, as we show below, there is a non-trivial lower bound on m_0 in the CMSSM, even if one only imposes the weaker requirement $\tilde{\tau}_1 > m_\chi$.

A first example of the interplay between these constraints is shown in Fig. 1(a). Here $m_0 = 0$ is assumed, as are $\tan\beta = 10, \mu > 0$ and $m_t = 175$ GeV. The left-hand vertical axis shows the value of m_h, and the right-hand side shows the ratio $m_{\tilde{\tau}_1}/m_\chi$. We indicate the impact of constraint (1) above, namely that $m_{e_R} > 99$ GeV [4] by the vertical thin dashed line. For $m_0 = 0$ and $\tan\beta = 10$, the selectron mass limit implies $m_{1/2} > 230$ GeV. The impact of constraint (2) above depends on the codes used to evaluate m_h. We show as a solid (red) line the value calculated with the `FeynHiggs` code [6], and as a dashed (green) line the value calculated with the program of [14], hereafter referred to as `HHH`. They differ little for $m_{1/2} < 200$ GeV and/or $m_h < 110$ GeV, but disagree by up to ~ 3 GeV at large $m_{1/2}$. We find that the experimental limit $m_h > 114.1$ GeV imposes $m_{1/2} > 330(465)$ GeV if we use the `FeynHiggs` (`HHH`) code. There is no significant constraint (3) from $b \to s\gamma$ for this value of $\tan\beta = 10$ and sign of μ. The measured value (4) of $g_\mu - 2$ favours the range 175 GeV $< m_{1/2} < 450$ GeV (195 GeV $< m_{1/2} < 290$ GeV) at the two- (one-) σ level. The constraint (5) $m_{\tilde{\tau}_1}/m_\chi > 1$ imposes $m_{1/2} < 210$ GeV. Combining all these calculations, we find *no range* of $m_{1/2}$ for which all these constraints are satisfied. Specifically, the upper limit (5) from $m_{\tilde{\tau}_1}/m_\chi$: $m_{1/2} < 210$ GeV is in *prima facie* contradiction with the lower limit (2) from m_h: $m_{1/2} > 330(465)$ GeV. Phrased another way: $m_{\tilde{\tau}_1}/m_\chi > 1$ only for values of $m_{1/2}$ corresponding to $m_h < 110$ GeV, which is excluded for the CMSSM discussed here.

Since calculations of m_h are well known to be very sensitive to m_t, with $\partial m_h/\partial m_t > 0$, we show in Fig. 1(b) the equivalent of Fig. 1(a) for $m_t = 180$ GeV. We see that the lower limit on $m_{1/2}$ becomes $250(315)$ GeV for $m_h > 114.1$ GeV (2), whereas the other constraints are essentially unchanged. Phrased another way: the lower limit on m_h imposes $m_{\tilde{\tau}_1}/m_\chi < 0.94$, even in the more conservative `FeynHiggs` calculation [6]. *We conclude that $m_0 = 0$ is not possible for* $\tan\beta = 10$ *and* $\mu > 0$.

More general views of the interplays between the different constraints (1) to (6) above as

functions of $\tan\beta$ are shown in Fig. 2. Panel (a) is for $\mu > 0$ and $A_0 = 0$, with $m_t = 175$ GeV. We restrict the analysis to $\tan\beta \leq 55$, which is close to the largest value for which we find generic regions of parameter space with consistent electroweak symmetry breaking [15]. Four lower limits on m_0 are plotted, corresponding to different implementations of the constraints. The (red) solid line assumes $m_h > 113.5$ GeV [2], which allows some safety factor compared with the experimental lower limit of 114.1 GeV, and employs the weaker cosmological constraint (5) $m_{\tilde{\tau}_1} > m_\chi$ (i.e., it ignores the constraint (6)). The (green) dashed line also uses $m_h > 113.5$ GeV, but imposes the stronger cosmological constraint (6) $\Omega_\chi h^2 > 0.1$. We see in both cases that $m_0 \neq 0$: the absolute lower limits are

$$m_0 \quad \gtrsim \quad 40 \text{ GeV for } m_{\tilde{\tau}_1} > m_\chi, \tag{1}$$

$$m_0 \quad \gtrsim \quad 65 \text{ GeV for } \Omega_\chi h^2 > 0.1, \tag{2}$$

both attained for $\tan\beta \sim 8$ to 10. The rise in the lower bound on m_0 for smaller $\tan\beta$ reflects the impact of the m_h constraint. This constraint is also important for $\tan\beta \lesssim 15$ to 20, but the lower limit for larger $\tan\beta$ reflects the impact of the $b \to s\gamma$ constraint combined with the weaker or stronger cosmological constraint. The darker (black) dotted line employs $m_{\tilde{\tau}_1} > m_\chi$ and the weaker constraint $m_h > 110.5$ GeV, which allows a generous safety theoretical factor compared with the available codes FeynHiggs and HHH. We recall - see Fig. 1 - that these codes agree rather well for small m_0 and $m_{1/2}$ and $\tan\beta \sim 10$. This is the only case where $m_0 = 0$ may be permitted, and only for $6 \lesssim \tan\beta \lesssim 9$. If one strengthens the cosmological constraint to require $\Omega_\chi h^2 > 0.1$, $m_0 = 0$ is disallowed even for the weak Higgs constraint, as shown by the lighter (blue) dotted curve. The convergence of the curves corresponding to the weaker and stronger Higgs constraints for $\tan\beta \gtrsim 20$ reflects the dominance of the $b \to s\gamma$ constraint at larger $\tan\beta$.

Panel (b) of Fig. 2 is obtained by varying $A_0 \neq 0$. When looking for the minimum of m_0, $m_t = 180$ GeV was chosen, as that always weakens the Higgs mass constraint on $m_{1/2}$, and m_0 is then generally allowed to be smaller, as seen in Fig. 1. Then A_0 was varied so as to minimize the allowed value of m_0. When $\tan\beta$ is small, $m_{1/2}$ is nevertheless forced to be high, approaching the end of the coannihilation region [16]. In this case, the minimum and maximum values of m_0 become the same. As $\tan\beta$ is raised, $m_{1/2}$ continues to be constrained by the lower limit on m_h, but the lower limit generally decreases. Increasing A_0 enhances this tendency, but A_0 cannot be made arbitrarily large, for fear of driving some scalars tachyonic. As $\tan\beta$ increases and $m_{1/2}$ is lowered, the value of A_0 used also drops. At intermediate values of $\tan\beta$, the effect of A_0 on $m_{\tilde{\tau}_1}$ also becomes relevant. Increasing A_0

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[2] Calculated (conservatively) with the FeynHiggs code [6].

tends to reduce $m_{\tilde{\tau}_1}$, which must be compensated by raising m_0. Hence, there is competition between wanting A_0 large so as to increase m_h and small so as to increase $m_{\tilde{\tau}_1}$. For $\tan\beta \sim 8$, the minimum values of m_0 are found when $A_0 \sim 0$, whereas $A_0 < 0$ is preferred for larger $\tan\beta$. The $b \to s\gamma$ constraint also becomes relevant for larger $\tan\beta$, and the minimum value of m_0 is generally found when $A_0 \sim -150$ GeV. In Fig. 2b, we show only the stronger Higgs mass bound, $m_h > 113.5$ GeV. The solid (red) curves ignores the cosmological constraint (6), whereas the dashed (green) curve includes it. We see that when the constraint (6) is ignored, there is a small range in $\tan\beta \sim 8$ where $m_0 = 0$ is allowed.

Also shown in Fig. 2 are upper limits on m_0. These apply to the 'bulk', 'coannihilation' and rapid-annihilation 'funnel' regions allowed by cosmology [15], but not to the 'focus-point' region [17]. The latter is typically a narrow strip in the $(m_{1/2}, m_0)$ plane that appears at much larger m_0 than the range studied here, the precise location being quite uncertain, being rather sensitive to the choices of input parameters, particularly the top quark mass, and higher-order effects in the model [18]. Setting aside the focus-point region, the maximum value of m_0 is always given by the tip of the coannihilation region. The mass of the lighter stau increases with A_0, and the position of the coannihilation tail also scales with A_0. The maximum value we have considered is $A_0 = 3$ TeV. This choice is somewhat arbitrary, but seems to us relatively conservative.

Panels (c) and (d) of Fig. 2 are for $\mu < 0$, with $A_0 = 0$ and $A_0 \neq 0$, respectively. They show similar qualitative features to the corresponding panels (a) and (b) for $\mu > 0$, with the notable exception that $m_0 = 0$ is disallowed, even if one uses only the weaker cosmological bound $m_{\tilde{\tau}_1} > m_\chi$. Because of the correlation between μ and the anomalous magnetic moment of the muon, constraint (4) is not satisfied for $\mu < 0$. Note also the increased importance of the $b \to s\gamma$ constraint is seen by the merging of the curves in panel (c) at lower $\tan\beta$ than in (a).

We also see in Fig. 2 a lower bound on $\tan\beta$, which is ~ 3 for $\mu > 0$ and the weaker Higgs mass bound $m_h > 110.5$ GeV, rising to ~ 4 for the stronger requirement $m_h > 113.5$ GeV. The lower limit would be much stronger, $\tan\beta \sim 8$, if one required constraint (4) coming from the anomalous magnetic moment of the muon.

There are several issues that can be addressed in view of the allowed region for m_0 seen in Fig. 2. Reaches for the discovery of sparticles has been discussed rather extensively recently [19], so we concentrate our attention on other issues. One is the menace of FCNI [2] that was mentioned earlier in this paper. One may ask whether the constraints discussed above allow $m_0/m_{1/2}$ to be sufficiently small for (at least some of) the FCNI constraints to be obeyed in a natural way. Consider, for example, the constraint imposed by the real part

of $K^0 - \overline{K}^0$ mixing [3]. One should consider box diagrams with chargino exchange, which yield [20]

$$|(\delta^u_{LL})_{22} - (\delta^u_{LL})_{11}| < 0.3 \times \frac{m_{1/2}}{200 \text{ GeV}}, \tag{3}$$

where $(\delta^u_{LL})_{ii} \equiv (m_0^2)_{ii}/m_{\tilde{q}}^2$ is related to the difference between the second- and first- generation up-squark soft supersymmetry-breaking masses squared, in the canonical CKM basis. Assuming that $m_{\tilde{q}}^2 \simeq 6m_{1/2}^2$, we see that (3) is satisfied, for $m_{1/2} = 200$ GeV, by $(m_0^2)_{22} - (m_0^2)_{11} \lesssim 7 \times 10^4$ GeV^2. This condition is clearly satisfied for a substantial range of m_0 compatible with our lower limit (2), even for maximal non-universality between $(m_0^2)_{22}$ and $(m_0^2)_{11}$. One should also consider box diagrams with gluino exchange, which yield [21]

$$|(\delta^d)_{12}| < 0.003 \tag{4}$$

for $m_{1/2} = 200$ GeV, where $(\delta^d)_{12} \equiv (m_0^2)_{12}/m_{\tilde{q}}^2$ parameterizes a possible off-diagonal term in the down-squark soft supersymmetry-breaking mass-squared matrix, again in the CKM basis. For $m_{1/2} = 200$ GeV and again assuming $m_{\tilde{q}}^2 \simeq 6m_{1/2}^2$, the upper limit (4) requires $(m_0^2)_{12} \lesssim 700$ GeV^2. Like the chargino constraint, this gluino constraint may be satisfied in a fairly natural way by an off-diagonal entry that is not very much smaller than our lower limit (2). A complete analysis of the FCNI issue goes beyond the scope of this paper, and we refer the reader to [21] for a review. However, we do note that the experimental upper limit on $\mu \to e\gamma$ decay does require the degeneracy between the slepton species to be rather complete [2]. For this reason, in particular, it is desirable to find an explanation why the (necessarily) non-zero soft supersymmetry-breaking scalar masses should be generation-independent.

In this connection, we re-examine whether the no-scale hypothesis that $m_0 = 0$ [3] is really excluded. What we have shown above is that $m_0 \neq 0$ *at the GUT scale*. In a complete quantum theory of gravity, such as string theory, the GUT scale is typically somewhat smaller than the Planck scale. One can therefore imagine a scenario in which $m_0 = 0$ *at the Planck scale*, with GUT interactions then renormalizing this starting value: $m_0 \neq 0$ at the GUT scale. As an example how this might work, we consider the minimal supersymmetric $SU(5)$ GUT. In this case, the soft supersymmetry-breaking scalar masses of the **10** and $\overline{\mathbf{5}}$ representations of $SU(5)$ are renormalized differently above m_{GUT}. In the one-loop approximation:

$$\frac{\partial m_{\mathbf{10}}^2}{\partial t} = \frac{1}{16\pi^2}\left[-\frac{144}{5}g_5^2 m_{1/2}^2\right], \tag{5}$$

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[3]We do not consider the imaginary part of $K^0 - \overline{K}^0$ mixing, regarding CP violation in the MSSM as an independent challenge.

$$\frac{\partial m_{\bar{\mathbf{5}}}^2}{\partial t} \;\; = \;\; \frac{1}{16\pi^2}\left[-\frac{96}{5}g_5^2 m_{1/2}^2\right], \tag{6}$$

where $t \equiv \ln(\mu^2/\mu_0^2)$, g_5 is the $SU(5)$ gauge coupling, and we have neglected renormalization by Yukawa couplings.

To estimate the order of magnitude of the generation-independent value of m_0 that may in this way be generated at the GUT scale, we insert the value $g_5^2/4\pi \simeq 1/20$ into (6), finding $\partial m_0^2/\partial t \sim 0.1 \times m_{1/2}^2$. A complete analysis of the coupled set of GUT renormalization-group equations for the soft supersymmetry-breaking parameters goes beyond the scope of this paper, but this rate of renormalization is clearly sufficient to generate values of m_0 compatible with our lower limit (2), even if the effective Planck scale is only an order of magnitude beyond the GUT scale.

We conclude that, at least within the the CMSSM framework we have discussed, experimental evidence indicates for the first time that soft supersymmetry-breaking scalar masses *must be non-zero*, at least *at the GUT scale*. This follows in general from the LEP lower limit on m_h [5] and the requirement that the LSP not be charged [10]. As we have indicated, the required magnitude of m_0 is not necessarily a disaster for FCNI, at least in the quark sector. Moreover, the lower limit on m_0 is still compatible with vanishing scalar masses *at the Planck scale*, as suggested by no-scale models [3].

Acknowledgments

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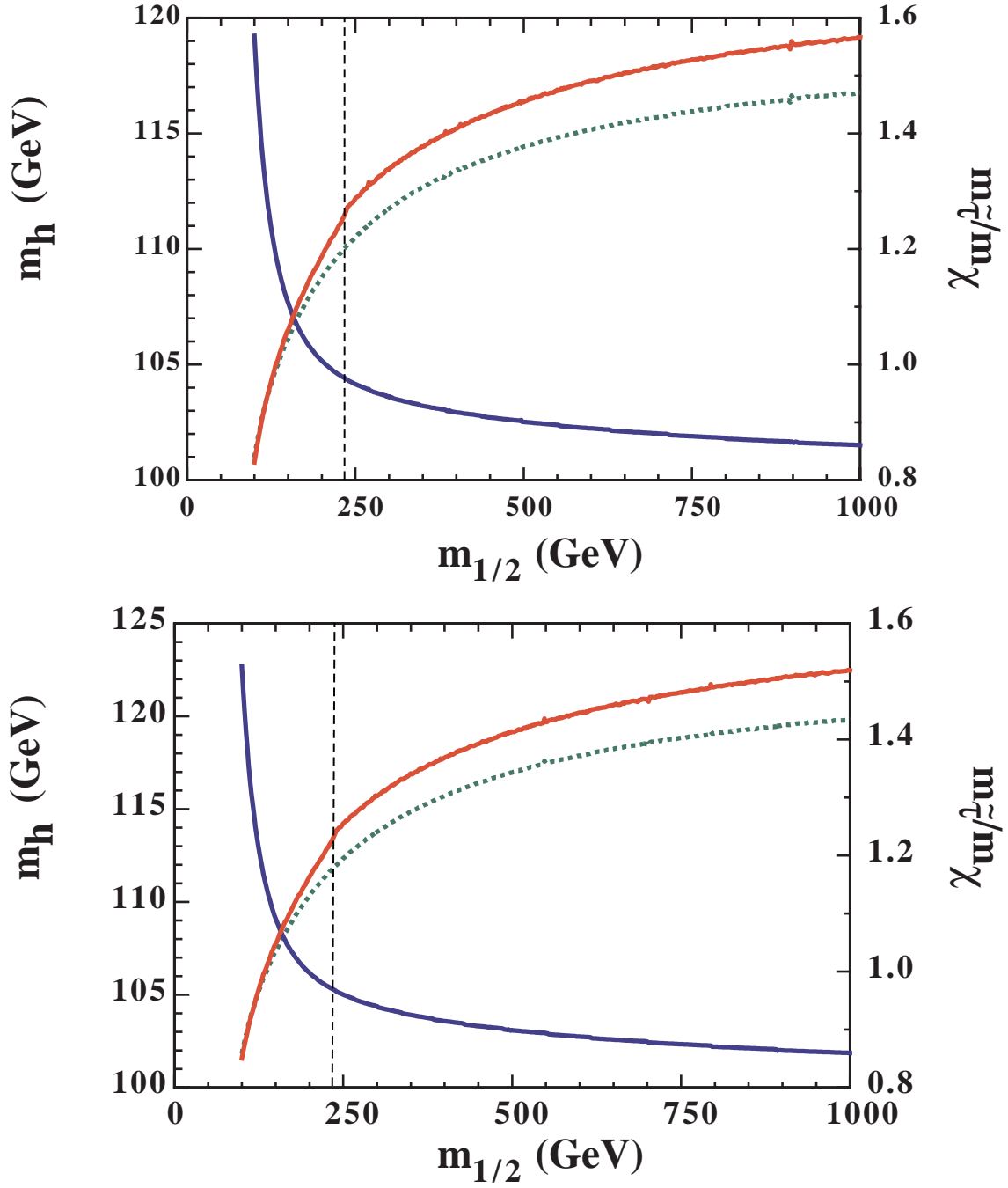[21] M. Misiak, S. Pokorski and J. Rosiek, hep-ph/9703442.

Figure 1: *The price of requiring $m_0 = 0$, showing the interplay of the constraints (i) to (v) for (a) $\tan\beta = 10, \mu > 0$ and $m_t = 175$ GeV, and (b) for $\tan\beta = 10, \mu > 0$ and $m_t = 180$ GeV. The red solid (green dashed) curves with positive slope show the calculated value of m_h using the* FeynHiggs *code [6] (using HHH [14]). The blue solid curve with negative slope shows the ratio $m_{\tilde\tau}/m_\chi$. The vertical thin dashed line shows the lower limit on $m_{1/2}$ due to the selectron mass limit.*
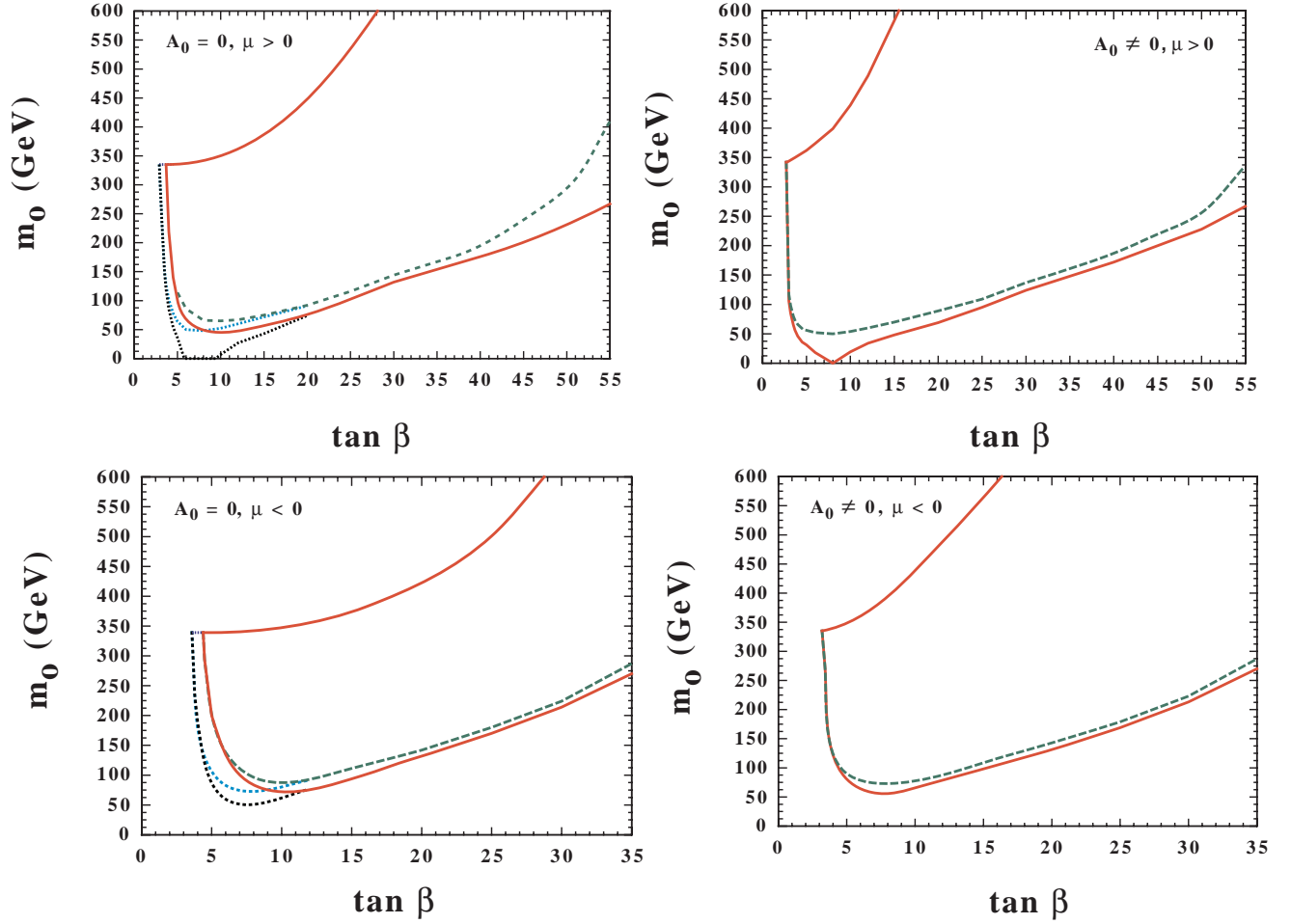
Figure 2: *Lower and upper limits on m_0 due to the different constraints, for (a) $\mu > 0$ and $A_0 = 0$, (b) $\mu > 0$ and $A_0 \neq 0$, (c) $\mu < 0$ and $A_0 = 0$, (b) $\mu < 0$ and $A_0 \neq 0$. The solid (red) line and dashed (green) are for $m_h > 113.5$ GeV, the dotted lines for $m_h > 110.5$ GeV, as calculated using the* `FeynHiggs` *code [6]. The solid (red) and darker dotted (black) lines only require $m_{\tilde{\tau}_1} > m_\chi$, whereas the dashed (green) and lighter dotted (blue) lines require $\Omega_\chi h^2 > 0.1$.*