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String Inspired Z' Model With Stable Proton and Light Neutrino Masses

Alon E. Faraggi[*1,2] and Marc Thormeier[†1]

[1] *Theoretical Physics Department, University of Oxford, Oxford, OX1 3NP, United Kingdom*

[2] *Theory Division, CERN, CH–1211 Geneva, Switzerland*

Abstract

Grand unification and its incarnation in the form of heterotic–string unification, are the only extensions of the Standard Model that are rooted in the structure of the Standard Model itself. In this context it was proposed that the exclusiveness of proton stability and suppression of neutrino masses necessitates the existence of an additional $U(1)_{Z'}$ symmetry, which is of non–GUT origin and remains unbroken down to intermediate or low energies. Realistic string models frequently give rise to non–GUT $U(1)$ symmetries, which arise from the flavor symmetries in the models. In this paper we demonstrate in a string–inspired toy model that such a stringy Z' can indeed guarantee proton longevity and viable phenomenology in the neutrino sector as well as in the quark and charged lepton sectors.

*faraggi@thphys.ox.ac.uk
†thor@thphys.ox.ac.uk

1 Introduction

Following the astounding experimental achievements of the previous decade, the Standard Model of particle physics is firmly established as the correct effective description of nature in the accessible energy range. The experimentally observed multiplet structure of the Standard Model, as well as other qualitative indications, strongly favor the embedding of the Standard Model in larger unifying groups. This embedding can be realized either at the level of an effective field theory below the Planck scale, or directly in a heterotic–string realization, formulated as a model of perturbative quantum gravity at the Planck scale.

A general consequence of the Standard Model multiplet unification is that the unification scale is separated from the electroweak scale by fifteen orders of magnitude. This arises from the fact that the multiplet unification of the Standard model spectrum results in proton decay mediating operators. Compatibility of this hypothesis with the measured gauge parameters at the electroweak scale and the observed suppression of the left–handed neutrino masses then provide further strong support for the validity of the grand unification scenario. The unification hypothesis substantially reduces the number of free parameters of the Standard Model and is hence also well motivated from the perspective of reducing some of the arbitrariness found in the Standard Model. String theory then provides a consistent framework for quantum gravity unification, in the context of which all the Standard Model parameters should, in principle, be calculable from a minimal set of fundamental parameters.

An important augmentation to the unification program is that of space–time supersymmetry. While not yet substantiated experimentally, supersymmetry is supported by several observations, including the consistency of a large top quark mass with the supersymmetric radiative electroweak symmetry breaking. However, proton longevity is more problematic in supersymmetric extensions of the Standard Model [1], which admit dimension four and five baryon and lepton number violating operators [2]. In the MSSM one imposes a global symmetry, $R-$parity (R_p), which forbids the dangerous dimension four operators, while the difficulty with the dimension five operators can only be circumvented if one further assumes that the relevant Yukawa couplings are sufficiently suppressed. The proton lifetime problem becomes especially acute in the context of heterotic–string unification, in which all operators that are compatible with the local gauge symmetries are expected to arise from nonrenormalizable terms. Indeed this issue has been examined in the past by a number of authors [3, 4, 5, 6, 7]. The avenues explored range from the existence of matter parity at special points in the moduli space of particular models to the emergence of non–Abelian custodial symmetries in specific compactifications [5].

The most realistic heterotic–string models constructed to date are the models constructed in the free fermionic formulation [8]. This has given rise to a large set of semi–realistic models [9, 10, 11, 12, 13, 5, 14, 15], which differ in their detailed phenomenological characteristics and share an underlying $Z_2 \times Z_2$ orbifold structure [16].

Past investigations examined several possibilities to explain the proton longevity. For example, ref. [17] stipulated the possibility that the $U(1)_{Z'}$, which is embedded in $SO(10)$, remains unbroken down to the TeV scale and suppresses the problematic dimension 4 operators. In ref. [5] it was shown that the free fermionic string models occasionally give rise to non–Abelian custodial symmetries, which forbid proton decay mediating operators to all orders of non–renormalizable terms. These proposals, however, fall short of providing a satisfactory solution, as they are, in general, exclusive to the generation of light neutrino masses through a see–saw mechanism. For example, the absence of the $SO(10)$ 126 representation in string models [18] necessitates that the $SO(10)$ $U(1)_{Z'}$ be broken at a high scale, rather than at a low scale. Similarly, to date, the existence of the custodial non–Abelian symmetries seems to be exclusive to the generation of a see–saw mass matrix.

Refs. [6, 7] proposed that heterotic–string unification necessitates the existence of an additional $U(1)$ symmetry, beyond the Standard Model, which remains unbroken down to low or intermediate energy. Invariance under the extra $U(1)$ forbids the proton decay mediating operators, which can be generated only after $U(1)_{Z'}$ breaking. The magnitude of the proton decay mediating operators is therefore proportional to the $U(1)_{Z'}$ breaking scale, $\Lambda_{Z'}$, which is in turn constrained by the proton lifetime limit, and other phenomenological constraints. On the other hand the additional $U(1)$ should not forbid quark, lepton and seesaw mass terms. It was argued [6, 7] that the required $U(1)$ symmetry is not of the type that arises in $SO(10)$ or E_6 GUTs. Rather, it should arise from the $U(1)$ symmetries in the string models that are external to the GUT symmetries.

By studying the spectrum and symmetries of the string model of ref. [12] Pati showed [6] that $U(1)$ symmetries with the required properties do indeed exist in the string models, and arise from combinations of the flavor symmetries. In ref. [7] it was shown that the required symmetries can in fact remain unbroken by the choices of supersymmetric flat directions in the string vacuum. In the present paper we continue to explore these ideas. The basic approach that we pursue is to develop field theory models that are inspired from the string models. By this we aim to confront the string features with the experimental data. Our main goal in the present paper is to examine whether the string symmetries that are used to guarantee the proton longevity can indeed at the same time allow for adequate suppression of the neutrino masses. That this is a nontrivial requirement can be seen from the fact that if we insisted on gauged $B-L$ as the proton protector, it would imply a neutrino in the MeV range [20].

We note that the possibility of utilizing $U(1)$ gauge symmetry to insure proton longevity has also been discussed in the context of purely field theory models [21]. The novelty here is the consideration of $U(1)$ symmetries that arise in the context the realistic free fermionic string models. From the string models we extract the charges of the Standard Model fields under the various $U(1)$ symmetries. We examine which combinations of the $U(1)$ symmetries forbid the dangerous baryon and lepton number violating operators while still allowing for the see–saw mass terms. We then assume that all but two of the additional $U(1)$s are broken in the string vacuum by the choices of supersymmetric flat directions. The first one being the $U(1)_{Z'}$ which is embedded in $SO(10)$, and the second being the additional unbroken $U(1)$ that arises from the flavor symmetries of the string model. We proceed to construct the see–saw mass matrix by postulating additional Higgs spectrum and comment how it may arise from the string models. We then demonstrate that such additional stringy $U(1)$ symmetries can indeed guarantee proton longevity and viable phenomenology in the neutrino sector, as well as in the quark and charged lepton sectors.

2 Gauge symmetries in free fermionic models

In this section we discuss the general structure of the realistic free fermionic models, and of the additional $U(1)$ symmetries that arise in these models. The free fermionic heterotic–string formulation yields a large number of three generation models, which possess an underlying $Z_2 \times Z_2$ orbifold structure [16], and differ in their detailed phenomenological characteristics. We emphasize the features of the models that are common to this large class of realistic models.

The free fermionic models are constructed by specifying a set of boundary conditions basis vectors and the one–loop GSO projection coefficients [8]. The physical massless states are obtained by acting on the vacuum with bosonic and fermionic operators and by applying the generalized GSO projections. The $U(1)$ charges, $Q(f)$, with respect to the unbroken Cartan generators of the four dimensional gauge group, which are in one to one correspondence with the $U(1)$ currents f^*f for each complex fermion f, are given by:

$$Q(f) = \frac{1}{2}\alpha(f) + F(f), \tag{2.1}$$

where $\alpha(f)$ is the boundary condition of the world–sheet fermion f in the sector α, and $F_\alpha(f)$ is a fermion number operator.

The four dimensional gauge group in the three generation free fermionic models arises as follows. The models can in general be regarded as constructed in two stages. The first stage consists of the NAHE set, $\{\mathbf{1}, S, b_1, b_2, b_3\}$ [13]. The gauge group after imposing the GSO projections induced by the NAHE set basis vectors is $SO(10)\times SO(6)^3\times E_8$ with $N = 1$ supersymmetry. The space–time vector bosons that generate the gauge group arise from the Neveu–Schwarz sector and from the sector $\mathbf{1} + b_1 + b_2 + b_3$. The Neveu–Schwarz sector produces the generators of $SO(10) \times SO(6)^3 \times SO(16)$. The sector $\zeta \equiv \mathbf{1} + b_1 + b_2 + b_3$ produces the spinorial $\mathbf{128}$ of $SO(16)$ and completes the hidden gauge group to E_8. At the level of the NAHE set the sectors b_1, b_2 and b_3 produce 48 multiplets, 16 from each, in the 16 representation of $SO(10)$. The states from the sectors b_j are singlets of the hidden E_8 gauge group and transform under the horizontal $SO(6)_j$ $(j = 1, 2, 3)$ symmetries. This structure is common to all the realistic free fermionic models. At this stage we anticipate that the $SO(10)$ group gives rise to the Standard Model group factors, whereas the $SO(6)^3$ groups may produce additional symmetries that can play a role in safeguarding the proton lifetime.

The second stage of the free fermionic basis construction consists of adding to the NAHE set three (or four) additional boundary condition basis vectors. These additional basis vectors reduce the number of generations to three chiral generations, one from each of the sectors b_1, b_2 and b_3, and simultaneously break the four dimensional gauge group. The $SO(10)$ symmetry is broken to one of its subgroups $SU(5) \times U(1)$ [9], $SO(6) \times SO(4)$ [11], $SU(3) \times SU(2)^2 \times U(1)$ [15] or $SU(3) \times SU(2) \times U(1)^2$ [10, 12, 14]. Similarly, the hidden E_8 symmetry is broken to one of its subgroups by the basis vectors which extend the NAHE set. This hidden E_8 subgroup may, or may not, contain $U(1)$ factors which are not enhanced to a non–Abelian symmetry. As the Standard Model states are not charged with respect to these $U(1)$ symmetries, they cannot play a role in suppressing the proton decay mediating operators, and are therefore not discussed further here. On the other hand, the flavor $SO(6)^3$ symmetries in the NAHE–based models are always broken to flavor $U(1)$ symmetries, as the breaking of these symmetries is correlated with the number of chiral generations. Three such $U(1)_j$ symmetries are always obtained in the NAHE based free fermionic models, from the subgroup of the observable E_8, which is orthogonal to $SO(10)$. These are produced by the world–sheet currents $\bar{\eta}\bar{\eta}^*$ $(j = 1, 2, 3)$, which are part of the Cartan sub–algebra of the observable E_8. Additional unbroken $U(1)$ symmetries, denoted typically by $U(1)_j$ $(j = 4, 5, ...)$, arise by pairing two real fermions from the sets $\{\bar{y}^{3,\cdots,6}\}$, $\{\bar{y}^{1,2}, \bar{\omega}^{5,6}\}$ and $\{\bar{\omega}^{1,\cdots,4}\}$. The final observable gauge group depends on the number of such pairings.

Subsequent to constructing the basis vectors and extracting the massless spectrum the analysis of the free fermionic models proceeds by calculating the superpotential. The cubic and higher-order terms in the superpotential are obtained by evaluating the correlators

$$A_N \sim \langle V_1^f V_2^f V_3^b \cdots V_N^b \rangle, \tag{2.2}$$

where V_i^f (V_i^b) are the fermionic (scalar) components of the vertex operators, using the rules given in [19]. Typically, one of the $U(1)$ factors in the free-fermion models is anomalous, and generates a Fayet–Iliopoulos term which breaks supersymmetry at the Planck scale. The anomalous $U(1)$ is broken, and supersymmetry is restored, by a non–trivial VEV for some scalar field that is charged under the anomalous $U(1)$. Since this field is in general also charged with respect to the other anomaly-free $U(1)$ factors, some non-trivial set of other fields must also get non–vanishing VEVs $\mathcal{V}$ to ensure a supersymmetric vacuum. Some of these fields will appear in the nonrenormalizable terms (2.2), leading to effective operators of lower dimension. Their coefficients contain factors of order $\mathcal{V}/M \sim 1/10$. Typically the solution of the D– and F–flatness constraints break most or all of the horizontal $U(1)$ symmetries.

The question is whether there exist string symmetries, which are beyond the GUT symmetries and can provide an appealing explanation for the proton lifetime. In a beautifully insightful paper [6] Pati studied this question in the model of ref [12], and showed that such symmetries indeed exist in the string models. In ref. [7] it was shown that the required symmetries can in fact remain unbroken below the string scale, and hence provide the needed suppression. However, as emphasized above, an important question is whether the additional symmetry that suppresses the baryon and lepton number violating operators, still allows at the same time the generation of a neutrino seesaw mass matrix and hence an acceptable neutrino mass spectrum. In this paper we address this question.

3 Proton decay and superstring Z'

For concreteness we focus on the string model of ref. [12], giving rise to the observable gauge symmetry

$$SU(3) \times SU(2) \times U(1)_C \times U(1)_L \times U(1)_1 \times \ldots \times U(1)_6. \tag{3.1}$$

$U(1)_C$ and $U(1)_L$ will, apart from the SM's hypercharge, produce one of the additional gauge bosons which appears in the string models, and constrain the proton decay mediating operators. The charges of the Standard Model states under $U(1)_L$ and $U(1)_C$ are fixed in the string model, as are the charges of $U(1)_{1,\cdots,6}$. We further assume that the low energy spectrum contains two light Higgs representations, which are the $\bar{h}_1$ and h_{45} of [12], with

$$\begin{aligned} H^U &= \bar{h}_1 = [(\bar{1},0);(\bar{2},1)]_{0,-1,0,0,0,0}, \\ H^D &= h_{45} = [(1,0);(2,-1)]_{-\frac{1}{2},-\frac{1}{2},0,0,0,0}. \end{aligned} \tag{3.2}$$

Prior to rotating the anomaly into a single $U(1)_{6'}$, all of the $U(1)_{1,\cdots,6}$ are anomalous: $\mathrm{Tr}U_1 = \mathrm{Tr}U_2 = \mathrm{Tr}U_3 = 24$, $\mathrm{Tr}U_4 = \mathrm{Tr}U_5 = \mathrm{Tr}U_6 = -12$. These can be expressed by one anomalous combination which is unique and five non–anomalous ones:*

$$\begin{pmatrix} U(1)_{1'} \\ U(1)_{2'} \\ U(1)_{3'} \\ U(1)_{4'} \\ U(1)_{5'} \\ U(1)_{6'} \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -2 \\ 1 & 1 & 1 & 2 & 2 & 2 \\ 2 & 2 & 2 & -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} U(1)_1 \\ U(1)_2 \\ U(1)_3 \\ U(1)_4 \\ U(1)_5 \\ U(1)_6 \end{pmatrix}; \tag{3.3}$$

The Standard Model weak hypercharge $U(1)_Y$ and the orthogonal $U(1)_{Z'}$ combinations are given by

$$\begin{pmatrix} U(1)_Y \\ U(1)_{Z'} \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{2} \\ 1 & -1 \end{pmatrix} \begin{pmatrix} U(1)_C \\ U(1)_L \end{pmatrix}. \tag{3.4}$$

*The normalization of the different $U(1)$ combinations is fixed by the requirement that the conformal dimension of the massless states gives $\bar{h}_1 = 1$ in the new basis. Here we neglect the normalization factors as we are only interested in the invariant superpotential terms.

The complete massless spectrum and the charges under the four dimensional gauge group are given in ref. [12]. After the rotation (3.4) one gets the charges

Field	$\overline{E}$	$\overline{U}$	Q	$\overline{N}$	$\overline{D}$	L	H^U	H^D
$U(1)_Y$	1	$-\frac{2}{3}$	$\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
$U(1)_{Z'}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$-\frac{3}{2}$	$-\frac{3}{2}$	-1	1

(3.5)

in agreement with the hypercharges of the Standard Model. Furthermore (3.3) leads to the charges

Field	$U(1)_{1'}$	$U(1)_{2'}$	$U(1)_{3'}$	$U(1)_{4'}$	$U(1)_{5'}$	$U(1)_{6'}$
$Q_1,\ \overline{D}_1,\ \overline{N}_1$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$
$L_1,\ \overline{E}_1,\ \overline{U}_1$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$
$Q_2,\ \overline{D}_2,\ \overline{N}_2$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$
$L_2,\ \overline{E}_2,\ \overline{U}_2$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$
$Q_3,\ \overline{D}_3,\ \overline{N}_3$	0	-1	0	1	$-\frac{1}{2}$	$\frac{3}{2}$
$L_3,\ \overline{E}_3,\ \overline{U}_3$	0	-1	0	-1	$\frac{3}{2}$	$\frac{1}{2}$
H^U	1	-1	0	0	-1	-2
H^D	0	-1	0	0	-1	-2

. (3.6)

We note that $U(1)_{1'}, ..., U(1)_{4'}$ are generation-dependent.

The superpotential of the MSSM+N+$\not{R}_p$ is given by

$$
\begin{aligned}
W \;=\;& \varepsilon^{ab}\, \delta^{xy}\, G^{(U)}_{ij}\, Q^i_{ax}\, H^U_b\, \overline{U}^j_y \\
&+ \varepsilon^{ab}\, \delta^{xy}\, G^{(D)}_{ij}\, Q^i_{ax}\, H^D_b\, \overline{D}^j_y \\
&+ \varepsilon^{ab}\, G^{(E)}_{ij}\, L^i_a\, H^D_b\, \overline{E}^j \qquad + \varepsilon^{ab}\, G^{(N)}_{ij}\, L^i_a\, H^U_b\, \overline{N}^j \\
&+ \varepsilon^{ab}\, \mu\, H^D_a\, H^U_b \qquad + \Gamma_{ij}\, \overline{N}^i\, \overline{N}^j \\
&+ \frac{1}{2}\, \varepsilon^{ab}\, \Lambda_{ijk}\, L^i_a\, L^j_b\, \overline{E}^k \\
&+ \varepsilon^{ab}\, \delta^{xy}\, \Lambda'_{ijk}\, Q^i_{ax}\, L^j_b\, \overline{D}^k_y \qquad + \Xi_i\, \overline{N}^i \\
&+ \frac{1}{2}\, \varepsilon^{xyz}\, \Lambda''_{ijk}\, \overline{U}^i_x\, \overline{D}^j_y\, \overline{D}^k_z \qquad + \varepsilon^{ab}\, \Upsilon_i\, \overline{N}^i\, H^D_a\, H^U_b \\
&+ \varepsilon^{ab}\, K_i\, L^i_a\, H^U_b \qquad + \Lambda'''_{ijk}\, \overline{N}^i\, \overline{N}^j\, \overline{N}^k .
\end{aligned}
\tag{3.7}
$$

Here the particle content is given by the MSSM–superfields plus three generations of right–handed neutrinos. Q, L, U, D, N, E represent the two left–handed $SU(2)$–doublets of the quarks and leptons, and the right–handed up–quarks, down–quarks, neutrinos and electrons, respectively; H^D, H^U are the two left–handed $SU(2)$ Higgs doublets; an overbar denotes charge conjugation. The gauge group is $SU(3)\times SU(2)\times$

$U(1)_Y$. a,b are $SU(2)$–indices (taking on the values $1,2$), x,y,z are $SU(3)$–indices (taking on the values $1,2,3$), i,j,k are generational indices, taking on the values $1,2,3$; *the heaviest generation is labeled by* 1*, which is* <u>*not*</u> *the same convention as e.g. in some papers on* $\not{R}_p$*; concerning* Λ'_{ijk}*, note that we work with the operator* $QL\overline{D}$ *rather than* $LQ\overline{D}$*.* Summation over repeated indices is implied; $\delta^{\cdots}$ is the Kronecker symbol, $\varepsilon^{\cdots}$ symbolizes any tensor that is totally antisymmetric with respect to the exchange of two indices, with $\varepsilon^{12\cdots}=1$. All other symbols are coupling constants. The right blocks of (3.7) contain right–handed neutrinos, the left ones do not; the upper blocks contain R_p–conserving terms, the lower ones do not. Thus the superpotential of the MSSM is given by the upper left block.

The unbroken new symmetries $U(1)_{1',\cdots,6'}$ strongly reduce the number of renormalizable operators in the low-energy superpotential of the MSSM+N+$\not{R}_p$: Imposing $U(1)_{Z'}\times U(1)_{\ldots'}$ on (3.7) gives that

$$\begin{array}{lll}
U(1)_{Z'}\times U(1)_{1'} & \text{allows} & G^{(U)}_{22}, G^{(N)}_{22}, G^{(D)}_{12}, G^{(D)}_{21}, G^{(D)}_{33}, G^{(E)}_{12}, G^{(E)}_{21}, G^{(E)}_{33},\\
U(1)_{Z'}\times U(1)_{2'} & \text{allows} & G^{(U)}_{ij}, G^{(N)}_{ij}, G^{(D)}_{ij}, G^{(E)}_{ij} \text{ for } i,j=1,2,\\
U(1)_{Z'}\times U(1)_{3'} & \text{allows} & G^{(U)}_{ii}, G^{(N)}_{ii} \text{ for } i=1,2,3;\\
 & & G^{(D)}_{12}, G^{(D)}_{21}, G^{(D)}_{33}, G^{(E)}_{12}, G^{(E)}_{21}, G^{(E)}_{33}, \mu,\\
U(1)_{Z'}\times U(1)_{4'} & \text{allows} & G^{(U)}_{ii}, G^{(N)}_{ii} \text{ for } i,j=1,2,3;\\
 & & G^{(U)}_{21}, G^{(U)}_{12}, G^{(N)}_{21}, G^{(N)}_{12};\ \mu.\\
U(1)_{Z'}\times U(1)_{5'} & \text{allows} & G^{(U)}_{ij}, G^{(N)}_{ij} \text{ for } i,j=1,2,3,\\
U(1)_{Z'}\times U(1)_{6'} & \text{allows} & G^{(U)}_{ij}, G^{(N)}_{ij} \text{ for } i,j=1,2,3.
\end{array} \tag{3.8}$$

If $U(1)_{Z'}$ and *all* $U(1)_{\ldots'}$ were exact, only the two terms $Q^2H^U\overline{U}^2$ and $L^2H^U\overline{N}^2$ in (3.7) would be allowed. Thus we assume that all but one, labeled $U(1)_{?'}$, get broken near the heterotic–string scale $M_S\sim 10^{18}$GeV. Hence the gauge group at high energies below the Planck–scale is given by

$$SU(3)\times SU(2)\times U(1)_Y\times U(1)_{Z'}\times U(1)_{?'}. \tag{3.9}$$

We assume that the $\overline{N}^i$ do not get a VEV. Thus to break the $U(1)_{Z'}\times U(1)_{?'}$ gauge symmetries we add the left–handed SM–singlet fields A,B and R,S,T to the fields of the MSSM+N. The underlying $SO(10)$ structure of the model demands A,B to be a vector–like couple with respect to $U(1)_{Z'}$, with A having the charge of the right–handed neutrino $\overline{N}$. We take A,B to be uncharged with respect to $U(1)_{?'}$, as we require that the VEVs of A,B do not break $U(1)_{?'}$ because we assume the breaking scale of $U(1)_{Z'}$ to be much higher than the breaking scale of $U(1)_{?'}$. The phenomenological constraints that we discuss below imply that $\Lambda_{?'}$ has to be considerably above the electroweak scale. The stringy background of the model requires R,S,T to be uncharged under $U(1)_{Z'}$. Fixing their $U(1)_{?'}$–charges basically is the only freedom in our string–inspired model. The choices we make here aim at generating a simple see–saw matrix. In the string models the additional Higgs spectrum may arise from fundamental states in the massless string spectrum, or from product of fields that produce these effective quantum numbers [22].

From (3.8) we see that $U(1)_{4'}$, $U(1)_{5'}$, $U(1)_{6'}$ are too restrictive, and $U(1)_{1'}$ still allows too few entries in $G^{(U)}, G^{(N)}$ to be $U(1)_{?'}$. $U(1)_{2'}$ and $U(1)_{3'}$ look more promising, and we study both cases. $U(1)_{2'}$ leads to a model with no mass–scale, as it forbids the μ–term. If $U(1)_{2'}$ suppresses, or forbids, the proton decay mediating operators, while still allowing the see–saw mass terms, and setting

the order of the μ–parameter by its breaking scale, then $U(1)_{2'}$ yields an example of a $U(1)$ symmetry that, provided that it remains unbroken down to a sufficiently low scale, can address the proton lifetime and neutrino mass issues, as well as provide a solution to the μ–problem. However, $U(1)_{2'}$ also forbids the mass terms for one generation, which are therefore suppressed by $\Lambda_{2'}/M_S$. One could contemplate the possibility that this deficit is cured by radiative corrections. $U(1)_{3'}$ allows both masses for all generations and also a μ–term. In this case therefore we have to ascertain that the additional Higgs VEVs do not generate a large μ–term. However, unlike $U(1)_{2'}$, $U(1)_{3'}$ does not forbid the proton–destabilizing dimension five operators, and in particular, the operators $Q^3Q^3Q^3L^3$ and $U^3U^3D^3E^3$. Therefore in the case of $U(1)_{3'}$ additional suppression of the dimension five operators is required. In the context of the string models such suppression is anticipated due to the additional $U(1)$ symmetries that are broken by the choices of supersymmetric flat directions at the string scale.

3.1 The case of $U(1)_{3'}$

We first focus on the case of $U(1)_{Z'} \times U(1)_{3'}$. We work with the following charges:

Field	A	B	R	S	T
$U(1)_{Z'}$	$\frac{5}{2}$	$-\frac{5}{2}$	0	0	0
$U(1)_{3'}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0

(3.10)

As T is completely uncharged it cannot break any symmetry and we therefore impose $\langle T\rangle = 0$. Instead of (3.7) we now obtain

$$\begin{aligned} W &= \varepsilon^{ab}\,\delta^{xy}\,G^{(U)}_{ij}\,Q^i_{ax}\,H^U_b\,\overline{U}^j_y \;+\; \varepsilon^{ab}\,\delta^{xy}\,G^{(D)}_{ij}\,Q^i_{ax}\,H^D_b\,\overline{D}^j_y \\ &+\; \varepsilon^{ab}\,G^{(E)}_{ij}\,L^i_a\,H^D_b\,\overline{E}^j \;+\; \varepsilon^{ab}\,G^{(N)}_{ij}\,L^i_a\,H^U_b\,\overline{N}^j \;+\; \varepsilon^{ab}\,\mu\,H^D_a\,H^U_b \\ &+\; \varepsilon^{ab}\,P\,T\,H^D_a H^U_b \;+\; \varepsilon^{ab}\,\Upsilon'\,A\,L^3_a\,H^U_b \;+\; \Theta\,T \;+\; M_N\,B\,\overline{N}^3 \;+\; M_R\,R\,S \\ &+\; M_A\,A\,B \;+\; M_T\,T\,T \;+\; \Upsilon''\,B\,R\,\overline{N}^1 \;+\; \Upsilon'''\,B\,S\,\overline{N}^2 \;+\; \Upsilon''''\,B\,T\,\overline{N}^3 \\ &+\; P'\,A\,B\,T \;+\; P''\,R\,S\,T \;+\; P'''\,T\,T\,T, \end{aligned} \qquad (3.11)$$

with the $G_{ij}^{(\dots)}$ having only three entries (see (3.8)). Guided by the structure of the string models we impose $\Theta = 0$. From the superpotential above we obtain the following see–saw mass matrix (n_L^3 being the fermionic component of $L^3 = N_L^3$, a being the fermionic component of A, etc.),

$$
\begin{pmatrix} n_L^1 & n_L^2 & n_L^3 & \overline{n_R}^1 & \overline{n_R}^2 & \overline{n_R}^3 & r & s & t & a & b \end{pmatrix}
$$
$$
\begin{pmatrix}
 & & & \langle H^U\rangle & & & & & & & \\
 & & & & \langle H^U\rangle & & & & & & \\
 & & & & & \langle H^U\rangle & & & & \langle H^U\rangle & \\
\langle H^U\rangle & & & & & & \langle B\rangle & & & & \langle R\rangle \\
 & \langle H^U\rangle & & & & & & \langle B\rangle & & & \langle S\rangle \\
 & & \langle H^U\rangle & & & & & & \langle B\rangle & & M_N \\
 & & & \langle B\rangle & & & & M_R & \langle S\rangle & & \\
 & & & & \langle B\rangle & & M_R & & \langle R\rangle & & \\
 & & & & & \langle B\rangle & \langle S\rangle & \langle R\rangle & M_T & \langle B\rangle & \langle A\rangle \\
 & & \langle H^U\rangle & & & & & & \langle B\rangle & & M_A \\
 & & & \langle R\rangle & \langle S\rangle & M_N & & & \langle A\rangle & M_A &
\end{pmatrix}
\begin{pmatrix} n_L^1 \\ n_L^2 \\ n_L^3 \\ \overline{n_R}^1 \\ \overline{n_R}^2 \\ \overline{n_R}^3 \\ r \\ s \\ t \\ a \\ b \end{pmatrix},
\tag{3.12}
$$

where in (3.12) coefficients of order one are not displayed explicitly, and the $n_L^3 a$–entry is discussed further below. The entries in (3.12) are constrained by several phenomenological considerations. In order to avoid D–term SUSY–breaking contributions that are larger than 1 TeV we impose $\left|\langle R\rangle^2 - \langle S\rangle^2\right| \leq 1\ (\text{TeV})^2$ and $\left|\langle A\rangle^2 - \langle B\rangle^2\right| \leq 1\ (\text{TeV})^2$. As the $U(1)_{Z'}$ and $U(1)_{3'}$ breaking scales are assumed to be above the SUSY breaking scale we thus have

$$
\langle A\rangle \sim \langle B\rangle, \quad \langle R\rangle \sim \langle S\rangle. \tag{3.13}
$$

After $U(1)_{Z'}$ breaking the term $\Upsilon'\ A\ L^3\ H^U$ may generate a very large effective K_3, see (3.7). One might argue that it is possible to rotate this term away by a unitary field-redefinition of L^3 and H^D, so that

$$
\begin{pmatrix} \mu & K_3 \end{pmatrix} \begin{pmatrix} H^D \\ L^3 \end{pmatrix} \longrightarrow \begin{pmatrix} \tilde{\mu} & 0 \end{pmatrix} \begin{pmatrix} \widetilde{H}^D \\ \widetilde{L}^3 \end{pmatrix}, \tag{3.14}
$$

but this would induce a very large $\tilde{\mu}$, namely $|\tilde{\mu}|^2 = |\mu|^2 + |\Upsilon'\ \langle A\rangle|^2$, which is not desired, as $\mu \ll \langle A\rangle$. We therefore have to demand that

$$
\Upsilon' \leq \frac{\langle H^U\rangle}{\langle A\rangle}. \tag{3.15}
$$

Thus, in the see–saw mass matrix $\mathbf{M}$ in (3.12) we can neglect the $n_L^3 a$-entries, as $\langle H^U\rangle^2/\langle A\rangle \ll \langle H^U\rangle$.

We impose the following relation on the parameters appearing in (3.12) which will yield appealing neutrino mass and mixing spectrum

$$M_N \sim \frac{\langle A\rangle^{\frac{3}{2}}}{\langle R\rangle^{\frac{1}{2}}}, \tag{3.16}$$

with M_N much larger than $\langle A\rangle$. Equation (3.16) is constrained by requiring no F–term SUSY–breaking contributions that are bigger than 1TeV, i.e. $M_N\langle A\rangle \leq$ 1TeV $\times\, M_S$. Similar constraints on products like $\langle A\rangle\langle R\rangle$ are automatically fulfilled with the condition guaranteeing no large contributions to the μ–term, that we discuss further below.

We must also insure that the dangerous proton decay mediating operators are sufficiently suppressed. Such operators may arise from non–renormalizable terms of the form of[†]

$$\frac{A\ R}{M_S^2}\ Q^3\ L^3\ \overline{D}^k \quad \text{and} \quad \frac{A\ R}{M_S^2}\ \overline{U}^3\ \overline{D}^3\ \overline{D}^k. \tag{3.17}$$

After $U(1)_{Z'}$ and $U(1)_{3'}$ symmetry breaking one gets

$$\Lambda'_{33k} \text{ and } \Lambda''_{33k} \sim \frac{\langle A\rangle\ \langle R\rangle}{M_S^2}. \tag{3.18}$$

Proton decay limits impose that (see for example [23])

$$\Lambda^{'}_{33k}\ \Lambda^{''}_{33k}\ \leq\ 2\times 10^{-27}\left(\frac{m_{\widetilde{d^k_R}}}{100\text{ GeV}}\right)^2; \tag{3.19}$$

taking the masses of the SUSY-particles to be of O(1TeV), one gets that the relation above is fulfilled if

$$\Lambda^{'}_{33k} \leq 5\times 10^{-13} \quad \text{and} \quad \Lambda^{''}_{33k} \leq 5\times 10^{-13}. \tag{3.20}$$

Comparing this with eq. (3.18) gives the condition

$$\langle A\rangle\ \langle R\rangle\ \leq\ 5\times 10^{23}(\text{GeV})^2. \tag{3.21}$$

Similarly, potential terms contributing to the electroweak Higgs mixing parameter should be adequately suppressed or absent altogether. As the VEV of T vanishes, the superpotential term TH^DH^U does not generate a large μ–term. To guarantee that the two gauge–invariant non–renormalizable terms

$$\frac{A\ B}{M_S}\ H^D\ H^U,\quad \frac{R\ S}{M_S}\ H^D\ H^U \tag{3.22}$$

[†] $k=1,2$, because $k=3$ does not have to be considered due to the antisymmetry of Λ''_{ijk} under the exchange of j,k.

do not contribute substantially to the μ–term after $U(1)_{Z'}$ and $U(1)_{3'}$ gauge symmetry breaking, we impose

$$\langle A\rangle \leq 10^{10}\mathrm{GeV}. \tag{3.23}$$

Since we have $\langle A\rangle \gg \langle R\rangle$ eq. (3.23) implies

$$\langle A\rangle\langle R\rangle \leq 10^{20}(\mathrm{GeV})^2, \tag{3.24}$$

so that the operators (3.17) are rendered harmless, and also F–term SUSY–breaking is avoided.

As stated earlier, the $U(1)_{2'}/U(1)_{3'}$–charges of the three generations are not family universal. Consequently, the breaking scale of $U(1)_{2'}/U(1)_{3'}$ has to be sufficiently high to avoid contradiction with Flavor Changing Neutral Currents (FCNC):

$$\langle R\rangle \geq 30\times 10^3\mathrm{GeV}. \tag{3.25}$$

For simplicity we assume‡

$$M_R \sim M_T \sim \langle R\rangle, \quad M_A \sim \langle A\rangle. \tag{3.26}$$

We plug (3.13), (3.15), (3.16) and (3.26) into (3.12) and determine its eigenvalues. The eight large ones are approximately given by

$$\pm M_N, \pm\langle A\rangle, \pm\langle A\rangle, \pm\langle A\rangle, \tag{3.27}$$

all large enough that the corresponding particles cannot have been observed. However, we are of course most interested in the three small eigenvalues. Writing

$$\mathbf{H} = \begin{pmatrix} \langle H^U\rangle & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \langle H^U\rangle & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \langle H^U\rangle & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{3.28}$$

and

$$\mathbf{J} = \begin{pmatrix} & & & \langle A\rangle & & & & \langle R\rangle \\ & & & & \langle A\rangle & & & \langle R\rangle \\ & & & & & \langle A\rangle & & \frac{\langle A\rangle^{\frac{3}{2}}}{\langle R\rangle^{\frac{1}{2}}} \\ \langle A\rangle & & & & \langle R\rangle & \langle R\rangle & & \\ & \langle A\rangle & & \langle R\rangle & & \langle R\rangle & & \\ & & \langle A\rangle & \langle R\rangle & \langle R\rangle & \langle R\rangle & \langle A\rangle & \langle A\rangle \\ & & & & & \langle A\rangle & & \langle A\rangle \\ \langle R\rangle & \langle R\rangle & \frac{\langle A\rangle^{\frac{3}{2}}}{\langle R\rangle^{\frac{1}{2}}} & & & \langle A\rangle & \langle A\rangle & \end{pmatrix}, \tag{3.29}$$

‡Other choices either did not lead to substantially different results or produced contradiction with experiment.

we can express the see–saw mass matrix as

$$\mathbf{M} = \begin{pmatrix} 0 & \mathbf{H} \\ \mathbf{H}^T & \mathbf{J} \end{pmatrix}. \tag{3.30}$$

Thus the three lightest eigenvalues and their corresponding eigenvectors are approximately, and to lowest order in $\langle R\rangle/\langle A\rangle$, those of the 3×3 matrix

$$\mathbf{H}^T\mathbf{J}^{-1}\mathbf{H} \;\approx\; \frac{\langle H^U\rangle^2\langle R\rangle}{\langle A\rangle^2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \tag{3.31}$$

namely [§]

$$\pm\; \frac{\langle H^U\rangle^2\langle R\rangle}{\langle A\rangle^2}, \quad \frac{2\langle H^U\rangle^2\langle R\rangle}{\langle A\rangle^2}. \tag{3.32}$$

The mass–degeneracy of the first two eigenvalues can be lifted by including higher orders, and/or non–universal coefficients, to obtain (3.36). All expressions go to zero for vanishing Higgs VEV, as they should. The diagonalizing matrix of (3.31) is given by

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} n_L^1 \\ n_L^2 \\ n_L^3 \end{pmatrix} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \tag{3.33}$$

Here ν symbolizes the light mass–eigenstates. This result can be interpreted as the (currently disfavored by SNO data [25]) maximal atmospheric neutrino mixing [24] ($\vartheta_{23} \sim 45°$), and small mixing angle MSW solar neutrino oscillation ($\vartheta_{13} \sim \vartheta_{12} \sim 0°$), i.e. one has to fix the values of $\langle R\rangle$ and $\langle A\rangle$ with

$$\left|m_2^2 - m_3^2\right| \;=\; 3.5\times 10^{-3}(\mathrm{eV})^2, \tag{3.34}$$

$$\left|m_3^2 - m_1^2\right| \;=\; 3.5\times 10^{-3}(\mathrm{eV})^2, \tag{3.35}$$

$$\left|m_1^2 - m_2^2\right| \;=\; 6\times 10^{-6}(\mathrm{eV})^2. \tag{3.36}$$

From the eigenvalues (3.32) we know that $m_3 \sim 2m_2, 2m_1$. Therefore,

$$\frac{\sqrt{3}\;\langle H^U\rangle^2\;\langle R\rangle}{\langle A\rangle^2} \sim \sqrt{3.5\times 10^{-3}(\mathrm{eV})^2}, \tag{3.37}$$

and thus

$$\langle R\rangle \sim 3\times 10^{-15}(\mathrm{GeV})^{-1}\langle A\rangle^2. \tag{3.38}$$

[§]This result can be found by an alternative method which for simplicity we demonstrate only for the simple conventional 2×2 see–saw matrix, which gives the characteristic polynomial $x^2 - Mx - m^2 = 0$. Assuming that $|x^2| \ll |Mx|, m^2$ one has $-Mx - m^2 \approx 0$ so that $x \approx -\frac{m^2}{M}$, reproducing the well-known result. Plugging it into the initial assumption above gives $\frac{m^4}{M^2} \ll M\frac{m^2}{M}, m^2$, justifying the assumption in hindsight.

From (3.25) we hence obtain $\langle A\rangle^2 \times \frac{3 \times 10^{-15}}{\rm GeV} \geq 30 \times 10^3$GeV, so that together with (3.23) we have the narrow range

$$3 \times 10^9 \text{GeV} \leq \langle A\rangle \leq 10^{10}\text{GeV}. \tag{3.39}$$

Hence

$$30 \text{ TeV} \ \leq \ \langle R\rangle \leq \ 300 \text{ TeV}, \tag{3.40}$$

which leads to

$$M_N \sim 10^{12}\text{GeV}, \tag{3.41}$$

hence fulfilling the constraints of F–term SUSY breaking with $\langle A\rangle = 3 \times 10^9$GeV.

When A acquires a VEV several renormalizable $\not{R}_p$ couplings are generated from the terms

$$\frac{A}{M_S}\ L\ L\ \overline{E},\ \ \frac{A}{M_S}\ Q\ L\ \overline{D},\ \ \frac{A}{M_S}\ \overline{U}\ \overline{D}\ \overline{D}. \tag{3.42}$$

Assuming their coupling constants to be $O(1)$ one gets

$$\begin{aligned}
&\Lambda_{231}, \Lambda_{132}, \Lambda_{123},\\
&\Lambda'_{231}, \Lambda'_{132}, \Lambda'_{113},\\
&\Lambda'_{311}, \Lambda'_{322}, \Lambda'_{223}, \Lambda'_{333}\\
&\Lambda''_{113}, \Lambda''_{223}, \Lambda''_{312},
\end{aligned} \tag{3.43}$$

all being suppressed by a factor of $\frac{\langle A\rangle}{M_S} \sim 10^{-8}$, which is below the current upper bounds at high energies.¶ We also note that the lepton number violating operators in (3.42) can contribute to the left–handed neutrino masses through fermion–sfermion loops, with $m_\nu \sim 3\Lambda'^2/(16\pi^2) m_b^2/M_{\rm SUSY}$ [27]. However, with the constraint (3.39), this contribution is at most $m_\nu \sim 5 \cdot 10^{-11}$eV and therefore negligible.

3.2 The case of $U(1)_{2'}$

Turning to $U(1)_{Z'} \times U(1)_{2'}$ we work with the following charge assignment:‖

Field	A	B	R	S	T
$U(1)_{Z'}$	$\frac{5}{2}$	$-\frac{5}{2}$	0	0	0
$U(1)_{2'}$	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	1

(3.44)

¶See Table III in [26]. Note that their convention is to label the heaviest generation '3', and that they work with $LQ\overline{D}$ rather than $QL\overline{D}$.

‖Being far from obvious we have not found any other $U(1)_{2'}$–charge assignment for R, S, T that produced a stable proton, a sensible see–saw matrix and the generation of a μ–term. Just as for $U(1)_{3'}$, the $U(1)_{2'}$–charges of R, S, T are opposite to the ones of the neutrinos.

It allows the non–renormalizable term

$$\frac{T\ T\ H^U\ H^D}{M_S}, \tag{3.45}$$

so that we can generate the μ–term with

$$\langle T\rangle^2 \sim 10^2 \text{ GeV} \times M_S \quad \Rightarrow \quad \langle T\rangle \sim 10^{10} \text{ GeV}. \tag{3.46}$$

Thus if $\mathcal{N}, \mathcal{N}', \mathcal{N}''$ (see (3.50)) are ≤ 10 the superpotential terms RST, RRT, SST cannot contribute substantially to F–term SUSY–breaking (see (3.51) and (3.52)). Since the $U(1)_{2'}$–charge of $(H^U H^D)$ is -2, and neither a three–field combination of R, S, T or one of these fields alone has a compensating $U(1)_{2'}$–charge of $+2$, the $U(1)_{2'}$–charges forbid

$$\frac{(R\ or\ S\ or\ T)\ (R\ or\ S\ or\ T)\ (R\ or\ S\ or\ T)\ H^U\ H^D}{M_S^2}$$
$$\text{and}\quad \frac{A\ B\ (R\ or\ S\ or\ T)\ H^U\ H^D}{M_S^2}. \tag{3.47}$$

Furthermore, since

$$Q_R, Q_S, Q_T \neq \frac{3}{2}, \tag{3.48}$$

the terms

$$\frac{A\ (R\ or\ S\ or\ T)\ Q^3\ L^3\ \overline{D}^{1,2}}{M_S^2} \quad \text{and} \quad \frac{A\ (R\ or\ S\ or\ T)\ \overline{U}^3\ \overline{D}^3\ \overline{D}^{1,2}}{M_S^2}, \tag{3.49}$$

are forbidden (unlike the case with $U(1)_{3'}$) and thus the proton is protected. The dimension five operator $\frac{1}{M_S}Q^3Q^3Q^3L^3$ is also forbidden (again unlike $U(1)_{3'}$). It can be generated from $\frac{T\ T\ T\ T}{M_S^5}Q^3Q^3Q^3L^3$, which however is highly suppressed. Similarly, all other dimension five operators are suppressed by at least $\langle T\rangle/M_S$ and are therefore adequately suppressed. The renormalizable superpotential is given by

$$\begin{aligned} W \quad = \quad & \varepsilon^{ab}\ \delta^{xy}\ G^{(U)}_{ij}\ Q^i_{ax}\ H^{\mathcal{U}}_b\ \overline{U}^j_y\ +\ \varepsilon^{ab}\ \delta^{xy}\ G^{(D)}_{ij}\ Q^i_{ax}\ H^{\mathcal{D}}_b\ \overline{D}^j_y\ +\ \varepsilon^{ab}\ G^{(E)}_{ij}\ L^i_a\ H^{\mathcal{D}}_b\ \overline{E}^j \\ & +\ \varepsilon^{ab}\ G^{(N)}_{ij}\ L^i_a\ H^{\mathcal{U}}_b\ \overline{N}^j\ +\ M_A\ A\ B\ +\ \Upsilon'_i\ B\ R\ \overline{N}^i\ +\ \Upsilon''_i\ B\ S\ \overline{N}^i \\ & +\ \Upsilon'''\ B\ T\ \overline{N}^3\ +\ \mathcal{N}\ R\ S\ T\ +\ \mathcal{N}'\ R\ R\ T\ +\ \mathcal{N}''\ S\ S\ T, \end{aligned} \tag{3.50}$$

with $i, j = 1, 2$. Looking at the charges we see that we can avoid D–term SUSY–breaking with

$$\langle R\rangle = \langle S\rangle = \langle T\rangle, \quad \langle A\rangle = \langle B\rangle. \tag{3.51}$$

Thus (3.25) is fulfilled.

As in the case of $U(1)_{3'}$ we have $\langle R\rangle \ll \langle A\rangle$. M_A and $\langle A\rangle$ have to be chosen such that the SUSY–breaking F–terms are adequately suppressed. Thus, we impose

$$\begin{aligned} M_A \, \langle A\rangle \; &\leq \; 1 \text{ TeV} \times M_S; \\ \Big(\Upsilon_i' \text{ and } \Upsilon_i'' \text{ and } \Upsilon'''\Big) \, \langle R\rangle \, \langle A\rangle \; &\leq \; 1 \text{ TeV} \times M_S \\ \Rightarrow \; \langle A\rangle \; &\leq \; \frac{10^{11} \text{ GeV}}{\Big(\Upsilon_i' \text{ and } \Upsilon_i'' \text{ and } \Upsilon'''\Big)}. \end{aligned} \tag{3.52}$$

The see–saw matrix is again of the form (3.30), with

$$\mathbf{H} = \begin{pmatrix} \langle H^U\rangle & \langle H^U\rangle & 0 & 0 & 0 & 0 & 0 & 0 \\ \langle H^U\rangle & \langle H^U\rangle & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{3.53}$$

and

$$\mathbf{J} = \begin{pmatrix} & & & \langle A\rangle & \langle A\rangle & & & \langle R\rangle \\ & & & \langle A\rangle & \langle A\rangle & & & \langle R\rangle \\ & & & & & \langle A\rangle & & \langle R\rangle \\ \langle A\rangle & \langle A\rangle & & \langle R\rangle & \langle R\rangle & \langle R\rangle & & \\ \langle A\rangle & \langle A\rangle & & \langle R\rangle & \langle R\rangle & \langle R\rangle & & \\ & & \langle A\rangle & \langle R\rangle & \langle R\rangle & & & \\ & & & & & & & M_A \\ \langle R\rangle & \langle R\rangle & \langle R\rangle & & & & M_A & \end{pmatrix}, \tag{3.54}$$

where we have not displayed explicitly the coupling constants $G_{ij}^{(N)}$, Υ_i', Υ_i'', Υ''', $\mathcal{N}$,$\mathcal{N}'$, $\mathcal{N}''$ [**]. The eight large eigenvalues corresponding to heavy particles are approximately

$$\pm \langle A\rangle, \pm\langle A\rangle, \pm\langle A\rangle, \pm M_A. \tag{3.55}$$

It turns out that to a good approximation the small eigenvalues of the see–saw matrix, corresponding to the very light neutrinos, are independent of M_A, one has

$$\mathbf{H}^T\mathbf{J}^{-1}\mathbf{H} \; \approx \; \frac{\langle H^U\rangle^2\langle R\rangle}{\langle A\rangle^2} \begin{pmatrix} \alpha & \beta & 0 \\ \beta & \gamma & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{3.56}$$

α, β, γ being complicated functions of $G_{ij}^{(N)}, \Upsilon_i', \Upsilon_i'', \Upsilon''', \mathcal{N}, \mathcal{N}', \mathcal{N}''$. From the null–row in the matrix above it follows that one of the neutrinos is massless. It is possible to assign values between ± 0.1 and ± 10 to $G_{ij}^{(N)}, \Upsilon_i', \Upsilon_i'', \Upsilon''', \mathcal{N}, \mathcal{N}', \mathcal{N}''$ such that one gets nearly degenerate masses for the other two neutrinos, and maximal mixing

[**] These couplings however cannot be completely random, otherwise the matrix could be singular, e.g. all couplings exactly equal to unity does not work, unlike the case for $U(1)_{3'}$.

between them, obeying (3.58). In order for this to reproduce the experimental data (3.34) with $m_3 \sim 0$ one thus needs

$$m_{1,2} = \pm\frac{\langle H^U\rangle^2\langle R\rangle}{\langle A\rangle^2} = \pm 6\times 10^{-11}\text{GeV} \quad \Rightarrow \quad \langle A\rangle \sim 10^{12}\text{GeV}. \tag{3.57}$$

In order to avoid substantial contributions to SUSY–breaking F–terms we thus have to impose (see (3.52))

$$\Upsilon_i', \Upsilon_i'', \Upsilon''' \ \leq 10^{-1}, \tag{3.58}$$

and $M_A \leq 10^9$GeV[††]. From the terms (3.42) one gets

$$\begin{aligned} &\Lambda_{131}, \Lambda_{231}, \Lambda_{132}, \Lambda_{232}, \Lambda_{123};\\ &\Lambda'_{131}, \Lambda'_{231}, \Lambda'_{132}, \Lambda'_{232}, \Lambda'_{123}, \Lambda'_{113},\\ &\Lambda'_{311}, \Lambda'_{321}, \Lambda'_{312}, \Lambda'_{322}, \Lambda'_{213}, \Lambda'_{223};\\ &\Lambda''_{131}, \Lambda''_{123}, \Lambda''_{213}, \Lambda''_{223}, \Lambda''_{312}, \end{aligned} \tag{3.59}$$

this time all of them are suppressed by $\frac{\langle A\rangle}{M_S} = O(10^{-6})$, again not contradicting the bounds of Table III in [26][‡‡]. Similarly, we note that the fermion–sfermion contribution to the left–handed neutrino masses [27] are adequately suppressed.

4 Discussion and Conclusions

The structure of the Standard Model spectrum indicates the realization of grand unification structures in nature. On the other hand the proton longevity severely constrains the possible extensions of the Standard Model and serves as a useful guide in attempts to understand the origin of the Standard Model gauge and matter spectrum. The realistic free fermionic heterotic–string models reproduce the grand unification structures that are suggested by the Standard Model and represent the most realistic string models constructed to date. As such the realistic free fermionic models serve as a useful probe to the fundamental characteristics of the possibly true string vacuum, as well as to various properties that the string vacuum should possess in order to satisfy various phenomenological constraints. It was proposed previously that proton stability necessitates the existence of an additional $U(1)$ symmetry, which remains unbroken down to intermediate or low energies. Furthermore, the required symmetry is not of the type that arises in Grand Unified Theories, but is of intrinsic string origin. The realistic free fermionic models do indeed give rise to $U(1)$ symmetries, which are external to the GUT symmetries, and forbid the proton decay mediating operators.

[††] However, M_A should be sufficiently large so that the two particles with mass $\sim M_A$ would not have been detected

[‡‡] We note again that in our convention '1' labels the heaviest generation; we also work with $QL\overline{D}$ instead of $LQ\overline{D}$.

One of the vital issues in heterotic–string Grand Unification is whether it is possible to adequately suppress the proton decay mediating operators, and at the same time insure that the left–handed neutrino masses are sufficiently small. That this is a non–trivial task is seen, for example, from the fact that a VEV of the neutral component in the 16 and $\overline{16}$ of $SO(10)$, which is used in the string models to generate the required large see–saw mass scale, at the same time can induce the dangerous proton decay mediating operators from non–renormalizable terms. The solution pursued here therefore advocates the existence of an additional $U(1)$ symmetry, which forbids the proton decay mediating operators, while it permits the neutrino seesaw–matrix mass terms, as well as those of the quarks and charged–leptons. Thus, provided that the additional $U(1)$ is broken at a sufficiently low scale, the proton decay mediating operators will be adequately suppressed. In this paper we demonstrated that indeed the symmetries that appear in the string models can guarantee proton longevity and simultaneously suppress the left–handed neutrino masses. We achieved this by assuming additional Higgs spectrum that is needed to break the $U(1)_{Z'} \times U(1)_{?'}$ and by detailed construction of the neutrino see–saw mass matrix. Such additional Higgs spectrum arises in the string models from fundamental states or from product of states that produce the effective quantum number of the states that we assumed here. We then analyzed the neutrino see–saw mass matrix and showed that the resulting neutrino mass spectrum is in qualitative agreement with the experimentally observed values.

It is interesting to note that among the semi–realistic orbifold models constructed to date the free fermionic models are unique in the sense that they are the only ones that have been shown to admit the $SO(10)$ embedding of the Standard Model spectrum. This is true of the Z_3 heterotic string models [28] as well as the type I string models, which do not admit the chiral 16 of $SO(10)$ in the perturbative massless spectrum. This in particular means that these models do not admit the canonical $SO(10)$ embedding of the weak–hypercharge. In these models the weak–hypercharge arises from a non–minimal combination of the world–sheet $U(1)$ currents, which correctly reproduces the Standard Model hypercharge assignments. Therefore, the models do contain a number of $U(1)$ symmetries that may suppress the proton decay mediating operators, and the suppression depends on the $U(1)_{Z'}$ symmetry breaking scale. However, the non–canonical embedding of the weak hypercharge typically results in exotic states which cannot be decoupled from the massless spectrum. As regard the issue of neutrino masses, the non–standard embedding of the weak–hypercharge implies that this issue can only be studied on a case by case basis, and general patterns are more difficult to decipher. The case of type I constructions presents additional novelties. In this case the Standard Model gauge group typically arises from a product of $U(n)$ groups. Therefore, embedding of the Standard Model gauge group in an enlarged gauge structure is not obtained in type I string models. Consequently, one then often finds a gauged $U(1)_B$ in these models, which guarantees proton longevity. The suppression of neutrino masses in this context still remains an open issue that can only be addressed on a case by case basis. However, due to the non–standard embedding of the weak–hypercharge, these models have typically been considered in the context of low scale gravity models [29]. In which case the suppression of of left–handed neutrino masses may arise from the large volume of the extra dimensions [30].

To conclude, the Standard Model multiplet structure, augmented with the right–handed neutrinos, strongly indicates the realization of an underlying $SO(10)$ structure in nature. Preservations of the $SO(10)$ embedding in the context of string unification has proven to be a highly non–trivial task. In fact, among the semi–realistic orbifold models, only the free fermionic models are known to reproduce this desired structure. However, the problems of proton longevity and the simultaneous suppression of the left–handed neutrino masses, still pose an extremely severe challenge to Grand Unified, and heterotic–string, models. It has been proposed that these challenges necessitate the existence of an extra non–GUT $U(1)$ symmetry, broken at low energies, and that the required symmetries do arise in the free fermionic heterotic–string models. In this paper by constructing detailed string inspired models, we showed that the additional stringy $U(1)$ symmetries can indeed perform their designated task. The relative simplicity of the additional Higgs spectrum that we assumed here, suggests that further more detailed models might be able to fully reproduce the observed quark and lepton mass spectrum, as well as safeguarding the proton lifetime.

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