Effects of jet algorithms from higher order QCD in $W^{\pm}$ mass determinations at LEP2[1]

E. Maina*

Dipartimento di Fisica Teorica – Università di Torino,
Istituto Nazionale di Fisica Nucleare – Sezione di Torino,
Via Pietro Giuria 1, 10125 Torino, Italy.

S. Moretti*

CERN – Theory Division
CH-1211 Geneva 23, Switzerland

Abstract

We analyse the impact of systematic effects due to the scale dependence of QCD corrections in combination with the use of different jet clustering algorithms in the measurement of the $W^{\pm}$ mass in the fully hadronic decay mode of W^+W^- pairs produced at LEP2. We consider the CC03 and CC11 channels through $\mathcal{O}(\alpha_s)$ at the parton level. We find that the associated uncertainties can be of order 100 MeV, thus competitive with those possibly arising in the non-perturbative regime and indeed above the current experimental estimates.

* E-mails: maina@to.infn.it, stefano.moretti@cern.ch.

1 Introduction

Over the past years, LEP2 has been producing and studying $W^\pm$ bosons. One of the main goals of such a collider was the determination of $M_{W^\pm}$ with a target accuracy of 40–50 MeV. This has apparently been achieved. By combining their data in all possible $W^\pm$ decay channels, the four LEP experiments quoted the following result:

$$M_{W^\pm} = 80.427 \pm 0.046 \text{ GeV}, \tag{1}$$

that compares rather favourably with the estimates obtained at $p\bar{p}$ colliders [1]. This measurement is extremely important: if combined with an improved determination of the top mass, m_t (soon to be performed at the Tevatron, during Run 2), it can lead to a rather stringent prediction of the Higgs mass, from a fit to high precision electroweak (EW) data.

One of the experimental strategies adopted to measure the $W^\pm$ mass at LEP2 has been the kinematic reconstruction of the $W^\pm$ resonance through the momenta of its decay products[2], e.g., in the fully hadronic channel: $e^+e^- \to W^+W^- \to 4$ jets. A 'cleaner' measurement is certainly performed in the semi-leptonic channel, i.e., $e^+e^- \to W^+W^- \to 2$ jets $\ell^\pm$ plus an undetected neutrino (with $\ell = e, \mu$). However, the contemporaneous presence in this case of missing energy in the final state and of photon radiation in the initial state (ISR), loosens the kinematic constraints that can be applied in the $W^\pm$ mass reconstruction procedure. Besides, the fully hadronic decay rate has a somewhat higher statistics than the semi-leptonic one. Therefore, although the event reconstruction is made harder in multi-jet final states by the larger number of tracks in the detector and by the usual uncertainties related to measuring jet energies and directions (a task much less complicated in the case of leptons), the $W^+W^- \to 4$ jets mode represented an accurate means of determining $M_{W^\pm}$ at LEP2 [2]. In fact, as shown in Ref. [2], the separate results obtained from the $W^+W^- \to q\bar{q}'Q\bar{Q}'$ and $W^+W^- \to q\bar{q}'\ell\bar{\nu}_\ell$ channels are consistent, with a difference in mass which is very small:

$$\Delta M_{W^\pm}(q\bar{q}'Q\bar{Q}' - q\bar{q}'\ell\bar{\nu}_\ell) = +9 \pm 44 \text{ MeV}. \tag{2}$$

Nonetheless, systematics errors on $M_{W^\pm}$ are somewhat larger in the $q\bar{q}'Q\bar{Q}'$ than in the $q\bar{q}'\ell\bar{\nu}_\ell$ channels: see Table 2 of Ref. [2].

Let us examine then more closely the kind of problems associated with the $W^+W^- \to 4$ jets signature. One of the issues is the problem of estimating theoretical biases due to the relatively unknown 'colour-reconnection' (CR) [3] and 'Bose-Einstein correlation' (BEC) [4] effects (see [5] for a theoretical review and [6] for an experimental one). Things go as follows. In $e^+e^- \to W^+W^- \to 4$ jets, one should expect some interference effects between the two hadronic $W^\pm$ decays, simply because the decay products from the two different gauge bosons can overlap considerably in space-time. In fact, at LEP2 energies, the separation between the two $W^\pm$ decay vertices is ~ 0.1 fm, that is, much smaller than the typical hadronisation scale, ~ 1 fm. Hence, the two hadronic $W^\pm$ decays can no longer be considered as separate, since final-state interactions (CR) and/or identical-particle symmetrisation (BEC) can play a non-negligible

---

[2]An alternative method is the so-called 'threshold scan', wherein a value for $M_{W^\pm}$ is fitted to the shape of the $e^+e^- \to W^+W^-$ total cross section for $\sqrt{s}$ in the vicinity of $2M_{W^\pm}$ (the result quoted in eq. (1) does also include measurements obtained in this way).

role, possibly leading to an apparent 'shift' in the reconstructed $W^{\pm}$ mass resonance [5, 6]. Unfortunately, because of our current lack of understanding of non-perturbative QCD[3], such interference effects can only be estimated theoretically in the context of different 'models'. Most of the latter can be constrained by looking at experimental observables which are sensitive to either phenomenon. For example, CR effects would be manifest in the central region, particularly in the single-particle distributions at low momenta, as produced in the fully hadronic versus the semi-leptonic channel, whereas BECs would lead to an increase in the correlation function for W^+W^-, as compared to that for a single $W^{\pm}$ [11]. While the jury is still out in the case of CR effects, there is an increasing evidence that BEC effects are very small if at all present [12]. In practice, the values that the LEP experimental collaborations assign to the systematic errors on $M_{W^{\pm}}$ due to CR and BEC effects range from 30 to 66 MeV and from 20 to 67 MeV, respectively (40 and 25 MeV are adopted in the combined results) [2].

Other problems in the fully hadronic decay channel of W^+W^- pairs are associated with the definition of 'jets'. The problematic here is twofold. Firstly, because two identical decays take place in the same event, one has the phenomenon of mis-pairing of jets. That is, even in the ideal case in which all tracks are correctly ascribed to the parton from which they originate, one has to cope with the ambiguity that it is in practice impossible to uniquely assign any pair among the four reconstructed jets to the parent $W^{\pm}$ on the sole basis of the event topology. Of all possible combinations of di-jet systems, only one is correct. Thus, an 'intrinsic' background exists in $W^+W^- \to 4$ jets events, in terms of simple combinatorics. Secondly, because of the large hadronic multiplicity, one also has the phenomenon of mis-assignment of tracks. In this case, the ambiguity stems from the fact that a track assigned to a jet, the latter eventually identified as a parton originating from one of the $W^{\pm}$'s, might have actually been produced in the fragmentation of another parton coming from the second $W^{\mp}$ decay.

Both these phenomenological aspects are clearly dependent upon the 'jet clustering algorithm' (see Ref. [13] for a review), wherein the number of hadronic tracks is reduced one at a time by combining the two most (in some sense) nearby ones (hereafter, we will quantify the 'distance' between two particles i and j by means of a variable denoted by y_{ij}). This (binary) joining procedure is stopped by means of a resolution parameter, y_{cut}, and the final 'clusters' yielding y_{ij} values all above y_{cut} are called jets[4]. In Ref. [13], it was precisely this dependence that was investigated, by using standard Monte Carlo (MC) simulation programs based on a parton shower (PS) approach (see [14]), such as HERWIG [15], JETSET/PYTHIA [16] and ARIADNE [17]. The results presented there did show a rather dramatic effect in the reconstructed $M_{W^{\pm}}$ values, due to the choice of the jet finder, and of its resolution parameter as well. However, any shift on $M_{W^{\pm}}$ of this sort can be estimated accurately, as it is simply due to kinematic effects induced by the jet clustering algorithm itself in reconstructing the quark momenta starting from the PS (or after hadronisation). In practice, it can be treated as a well quantifiable correction to be applied to the reconstructed $M_{W^{\pm}}$ value, in order to reproduce the true one. Using the same MC programs, one can also determine the typical size of the

---

[3]In fact, the perturbative effects of CR are expected to be small, because of order $\sim (C_F\alpha_s)^2/N_C^2 \times \Gamma_{W^{\pm}}/M_{W^{\pm}}$ [7] – see also Refs. [8, 9] – so are partonic 'Fermi-Dirac correlations' (as opposed to hadronic BECs) [10].

[4]Here and in the following, the word 'cluster' refers to hadrons or calorimeter cells in the real experimental case, to partons in the theoretical perturbative calculations, and also to intermediate jets during the clustering procedure.

systematic errors due to the hadronisation process, by comparing the outputs of the various programs. Finally, background effects can be accounted for by exploiting the numerous event generators available on the market for $e^+e^- \to 4$ jets, both in EW [18] and QCD [15]–[17], [19] processes (see Ref. [20] for a dedicated study of the impact of such QCD background effects in $e^+e^- \to W^+W^- \to 4$ jets).

Other systematic effects remain instead quite beyond control. These are intimately related to the way predictions are made within standard perturbation theory. That is, to the fact that only a finite number of terms of a perturbative series are generally computed over all the available phase space. Or alternatively, that only over a restricted region of it, all terms of a series can be summed to all orders. Whereas the availability of the latter is in general more crucial to the estimation of a total cross section, that of the former can be decisive for the study of more exclusive observables. Given the relative size of the EW and QCD coupling 'constants' at LEP2, it is clear that the dominant higher order effects will be due to the emission and/or absorption of gluons.

Several QCD effects entering $e^+e^- \to W^+W^- \to 4$ jets events have been studied so far. For a start, it should be mentioned that the amplitude for $e^+e^- \to W^+W^- \to q\bar{q}'Q\bar{Q}'$ (the so-called CC03 channel) is quite trivial to derive, in fact, more of a textbook example. It represents the lowest-order (LO) contribution to the $e^+e^- \to W^+W^- \to 4$ jets signal. Higher-order QCD contributions involving gluons are, for example, the real ones (i.e., tree-level processes): $e^+e^- \to W^+W^- \to q\bar{q}'Q\bar{Q}'g$ and $e^+e^- \to W^+W^- \to q\bar{q}'Q\bar{Q}'gg$ events, which have been calculated in Refs. [21] and [8], respectively, as well as $e^+e^- \to W^+W^- \to q\bar{q}'Q\bar{Q}'g^*$, with the gluon splitting in a quark-antiquark pair, which was considered in Ref. [10] (see also [22]). One-loop QCD corrections to $e^+e^- \to W^+W^- \to q\bar{q}'Q\bar{Q}'$ are also known to date [23], and they have been interfered with the LO amplitudes and eventually combined with the single real gluon emission contribution of Ref. [21] into the complete $\mathcal{O}(\alpha_s)$ result [23]. In Ref. [24], the full $\mathcal{O}(\alpha_s)$ corrections were computed for the case of the so-called CC11 channel [18], also including irreducible background effects in addition to $e^+e^- \to W^+W^-$ production and decay. Finally, two-loop effects due to the virtual exchange of two gluons between the two quark pairs in hadronic W^+W^- decays were estimated in Ref. [9] in the 'soft limit' and found to be either small (colour-singlet exchange) or large (colour-octet exchange) but symmetric around $M_{W^\pm}$, hence unobservable in general.

We make use here of the calculations of Refs. [23, 24] in order to assess the size of the typical theoretical error due to the truncation of the perturbative series at order α_s and the systematic effects that it introduces in observable quantities, primarily, in the 'line-shape' of the $W^\pm$ resonance, as determined by using different jet finders to select the hadronic sample. Our motivations to carry out such a study are dictated by the following considerations. For a start, NLO corrections to both CC03 and CC11 have been found to be rather large in general [23, 24], with their actual size clearly depending upon the algorithm used. Furthermore, it is well known that differential distributions are typically more sensitive (particularly in presence of cuts over the phase space available to gluon emission) to higher order effects than fully inclusive quantities, such as total cross sections, where virtual and real contributions tend to cancel to a larger extent. Besides, in the case of the $W^\pm$-mass line-shape, one would expect the distortion effects to be mainly induced by relatively hard and non-collinear gluons, which should be better modelled by an exact NLO calculation than by the PS models exploited in Ref. [13].

3

All QCD predictions have an intrinsic dependence on an arbitrary scale, hereafter denoted by μ, entering at any order in α_s. This scale is not fixed a priori. On the one hand, although the structure of the QCD perturbative expansion does not prescribe which value should be adopted for μ, an obvious requirement is that it should be of the order of the energy scale involved in the problem: i.e., the CM energy $\sqrt{s}$ (see Ref. [25] for detailed discussions). On the other hand, the physical scales of gluon emissions that actually give rise to multi-jet configurations are to be found down to the energy scale $\sqrt{y_{\mathrm{cut}}s}$. In practice, one should avoid building up large logarithmic terms related to the (unphysical) 'mismatch' between the process scale $\mu \approx \sqrt{s}$ and the emission scale $\sqrt{y_{\mathrm{cut}}s}$ and it is well known that it may be necessary to adopt a different scale for each observable in order to best describe experimental data taken at fixed $\sqrt{s}$ [26]. It is precisely the μ-dependence of the truncated perturbative series that is treated very differently by each jet-clustering algorithm [13, 27, 28] and the corresponding effects on observable quantities are what we aim to study. In our calculation, QCD effects appear through $\mathcal{O}(\alpha_s)$ only, so that the μ-dependence is merely the one affecting the strong coupling constant at lowest order. Whereas its impact is trivial to assess (and account for) in the case of total inclusive rates, this is no longer true for differential quantities (such as mass distributions), because of the different kinematics of lowest and higher order contributions, respectively. Notice, however, that a problem arises when studying the scale dependence of α_s results for algorithms based on different measures, as for the same y_{cut} the total cross section at NLO can be significantly different. A more consistent procedure was outlined in Ref. [28]: that is, to compare the NLO scale dependence of the various schemes not at the same y_{cut} value, rather at the same LO rate. This is our approach.

In order to make more manifest the effects of the interplay between the $\mathcal{O}(\alpha_s)$ corrections to $W^+W^- \to 4$ partons events and the jet clustering schemes tested, we have not considered here Coulomb corrections to CC03 [29] (their relevance is anyway modest beyond the W^+W^- threshold). We also have neglected the implementation of the mentioned BEC and CR phenomena, as these mainly arise in the non-perturbative domain. Similarly, hadronisation and detector effects were not investigated, nor those due to ISR. We refer the reader to Refs. [13] and [24], respectively, where their impact was studied in detail.

As centre-of-mass (CM) energy representative of LEP2 we have used the value $\sqrt{s} = 175$ GeV. As for the parameters of the theory, we have adopted (in the fixed-width approach) $M_{Z^0} = 91.189$ GeV, $\Gamma_{Z^0} = 2.497$ GeV, $M_{W^\pm} = 80.430$ GeV, $\Gamma_{W^\pm} = 2.087$ GeV, $\sin^2\theta_W = 0.231$, $\alpha_{em} = 1/128.07$ and the one-loop expression for α_s (for consistency), with $\Lambda_{\mathrm{QCD}}^{N_F=4} = 0.283$ GeV (yielding $\alpha_s(\sqrt{s}) = 0.123$)[5]. Furthermore, we have kept all quarks massless as a default, in order to speed up the numerical evaluations. Electron and positron have mass zero too, so has the neutrino. Also, we have neglected Cabibbo-Kobayashi-Maskawa (CKM) mixing terms (i.e., we have taken the CKM rotation matrix to be diagonal).

As jet clustering schemes[6], we have used a selection of the binary ones, in which only two objects are clustered together at any step. These are the following. The JADE (J) one [30],

---

[5]The one-loop values of α_s adopted here are consistent with the two-loop one extracted from experimental fits to shape variables at the Z^0 peak, $\alpha_s(M_{Z^0}) = 0.116$, for the same choice of $\Lambda_{\mathrm{QCD}}^{N_F=4}$.

[6]We acknowledge here the well admitted abuse in referring to the various jet 'finders' both as algorithms and as schemes, since the last term was originally intended to identify the composition law of four-momenta when pairing two clusters: in our case, the so-called E-scheme, i.e., $p_{ij}^\mu = p_i^\mu + p_j^\mu$ (other choices have negligible impact on our conclusions).
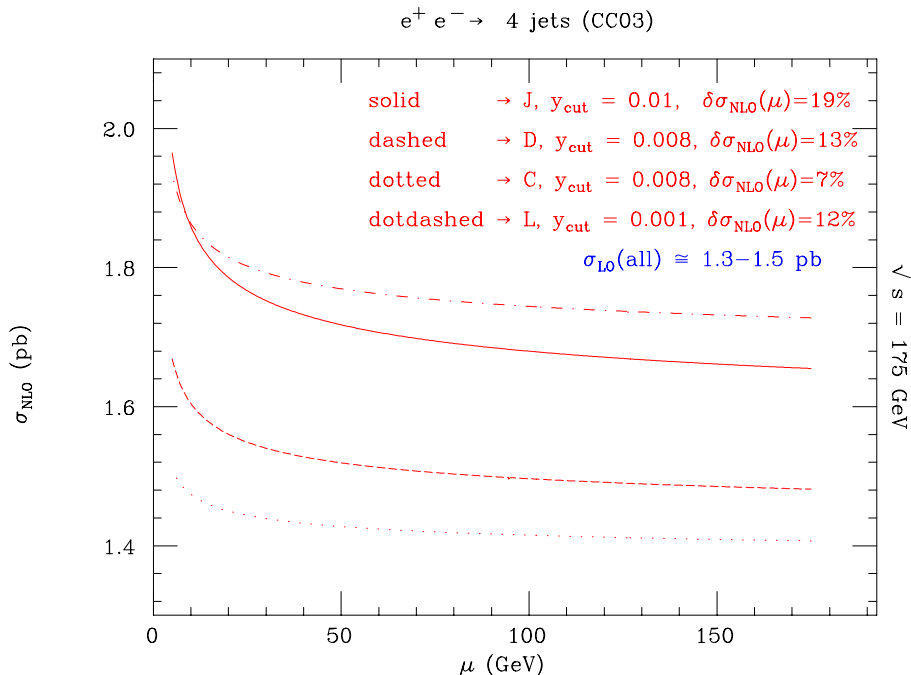
Figure 1: NLO cross sections for $e^+e^- \to W^+W^- \to u\bar{d}s\bar{c}$ (CC03), as a function of the QCD scale μ, as given by four different jet clustering algorithms, at $\sqrt{s} = 175$ GeV. (The values of y_{cut} are chosen such that the LO rates are approximately equal for all schemes.) Total hadronic rates are obtained by multiplying those above times four.

which uses as a measure of separation (or 'metric') the quantity

$$y_{ij}^J = \frac{2E_iE_j(1 - \cos\theta_{ij})}{s}. \tag{3}$$

The Durham (D) [31] and the Cambridge (C) [27] ones, both using

$$y_{ij}^D \equiv y_{ij}^C = \frac{2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})}{s}. \tag{4}$$

(The Cambridge algorithm in fact only modifies the clustering procedure of the Durham jet finder.) We also have adopted the LUCLUS or LUND (L) jet finder [32], for which one has

$$y_{ij}^L = \frac{2|p_i|^2|p_j|^2(1 - \cos\theta_{ij})}{(|p_i| + |p_j|)^2 s}, \tag{5}$$

however, with the same clustering procedure of the Cambridge scheme and without 'preclustering' and 'reassignment' (see Ref. [32]), i.e., as done in Ref. [13] (where it was labelled as CL). In eqs. (3)–(5), $E_i(|p_i|)$ and $E_j(|p_j|)$ are the energies(moduli of the tree-momenta) and θ_{ij} the

5

angular separation of any pair ij of particles in the final state, to be compared against the resolution parameter y_{cut}. The choice of these particular schemes has a simple motivation. The D, C and L ones are different versions of 'transverse-momentum' based algorithms, whereas the J one uses an 'invariant-mass' measure (the numerator of eq. (3) coincides with the invariant mass of the partons ij, when the latter are massless, as is the case here). In fact, these two categories are those that have so far been employed most in phenomenological studies of jet physics in electron-positron collisions, with the former gradually overshadowing the latter, thanks to their reduced scale dependence in higher order QCD, e.g., in the case of the $\mathcal{O}(\alpha_s^2)$ three- [13, 28, 33] and $\mathcal{O}(\alpha_s^3)$ four-jet rates [34], and to smaller hadronisation effects in the same contexts [13, 28].

Our multi-jet sample is selected at the parton level, by requiring a final state with *at least* four resolved objects (i.e., with all their y_{ij}'s above a given y_{cut}). When five objects survive, the two yielding the smallest y_{ij} value (according to the metric used) are joined together, so to always produce a four-particle final state. In doing so, we conform to typical experimental approaches: see, e.g., Ref. [35]. The impact of a different treatment of five-jet contributions was assessed in Ref. [23].

Fig. 1 illustrates the dependence of the CC03 NLO rates upon the unknown μ scale, for our four default jet clustering algorithms, for representative choices of y_{cut} such that the LO rates in the various schemes are approximately equal. (In fact, differences at LO are typically within 15%; the actual numbers being: $\sigma_{LO} = 1.38(1.32)[1.32]\{1.54\}$ pb for J(D)[C]{L}.) The variation of the NLO rates with μ, for values of the latter ranging between 5 GeV and $\sqrt{s}$[7], denoted by $\delta\sigma_{\text{NLO}}(\mu)$, depends upon the jet algorithm, varying significantly, between 7% (C scheme) and 19% (J scheme). The K-factors, for $\mu = \sqrt{s}$, are also very different, from one algorithm to another, again with the minimum corresponding to the C scheme ($K = 1.07$) and the maximum to the J one ($K = 1.20$). All these values are however lower limits. In fact, as μ is decreased α_s increases, hence the relative size of the NLO effects grows larger too, in each case.

In Fig. 2, we compare the differential distributions in the 'average' mass, M_{ave}[8], defined as follows. Out of the three possible combinations of pairs of jet-jet systems, we choose the one for which the two reconstructed $W^{\pm}$ masses, M_{R_1} and M_{R_2}, minimise

$$\Delta M = |M_{R_1} - M_{W^{\pm}}| + |M_{R_2} - M_{W^{\pm}}| \tag{6}$$

and then define

$$M_{\text{ave}} = \frac{1}{2}(M_{R_1} + M_{R_2}). \tag{7}$$

This variable has been extensively used as an estimator of the true $M_{W^{\pm}}$ value since, at tree level, the difference between M_{ave} and the average between the two $W^{\pm}$ masses that one would reconstruct if the quarks could always be paired correctly is very small[9]. We present the differential spectra in the average mass as obtained at LO and NLO by using the four jet clustering algorithms with y_{cut} values as before. The scale is here set to $\mu = \sqrt{s}$. A clear tendency is visible,

---

[7]Note that by restraining μ to values higher than the hadronisation scale Q_0, which is of order 1 GeV, a perturbative analysis is in principle always justified. However, too low a value of μ would imply a very large α_s, in turn rendering the fixed order predictions unreliable. As a compromise, we will be considering μ-values well above Q_0 in the reminder of our study (35 GeV and above).

[8]Actually, in the difference between this and the true $W^{\pm}$ mass, $M_{W^{\pm}}$.

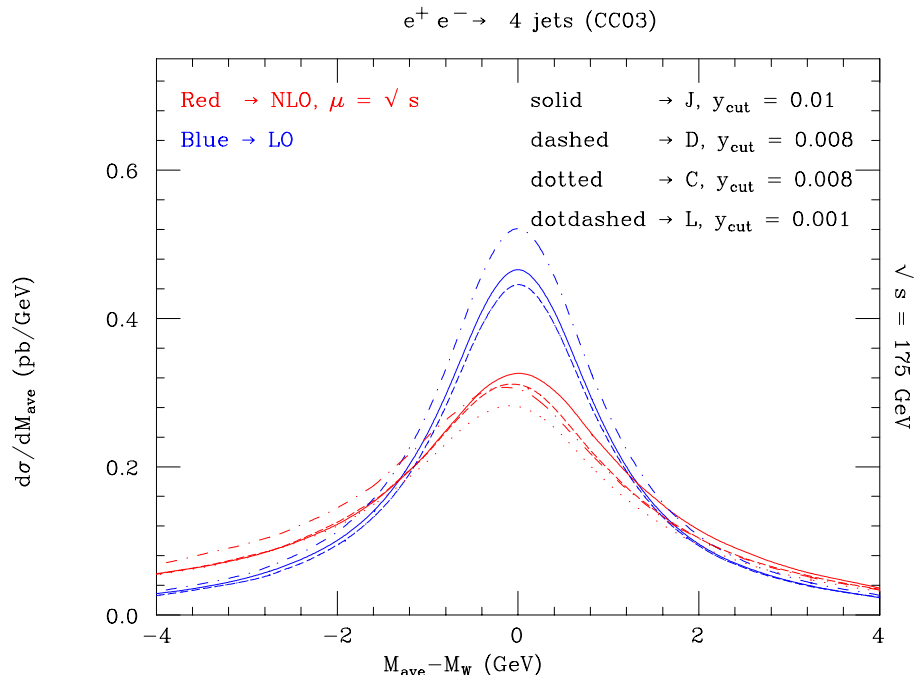[9]One could consider more sophisticated approaches, but this is beyond the scope of this paper.

Figure 2: LO and NLO differential distribution in the 'average' mass (as defined in the text) for $e^+e^- \to W^+W^- \to u\bar{d}s\bar{c}$ (CC03), the latter with QCD scale $\mu = \sqrt{s}$, as given by four different jet clustering algorithms, at $\sqrt{s} = 175$ GeV. (The values of y_{cut} are chosen such that the LO rates are approximately equal for all schemes.) Notice that the dashed and dotted blue-lines are visually indistinguishable. Total hadronic rates are obtained by multiplying those above times four.

a shift towards low mass values in the M_{ave} distribution at NLO, with respect to the LO case, and this depends upon the jet algorithm being used and its y_{cut} value. The generation of this low mass tail at NLO was already observed and discussed in Ref. [23], where its consequences for a determination of $M_{W^\pm}$ were described in details.

Instead, we are concerned here with the fact that the actual shape of the $M_{\mathrm{ave}} - M_{W^\pm}$ distribution at NLO depends upon μ (this was set to $\sqrt{s}$ as default in [23]), through the choice of both the jet finder and its resolution parameter. Ultimately then, so will do the value of $M_{W^\pm}$ extracted from that distribution. To study this effect, we plot in Fig. 3 the differential distribution of the quantity defined in eqs. (6)–(7), now for three different choices of μ, e.g., $35, 100$ GeV and $\sqrt{s}$, for our default choice of jet algorithms and y_{cut}'s. There exists a visible variation with μ; besides, the previously observed dependence on the choice of the algorithm and y_{cut} persists at different μ's.

The fact that the μ-dependence of the $W^+W^- \to 4$ jet rates could have a strong impact in measurements of the $W^\pm$ mass is already reflected in the results reported in Tab. 1. There,
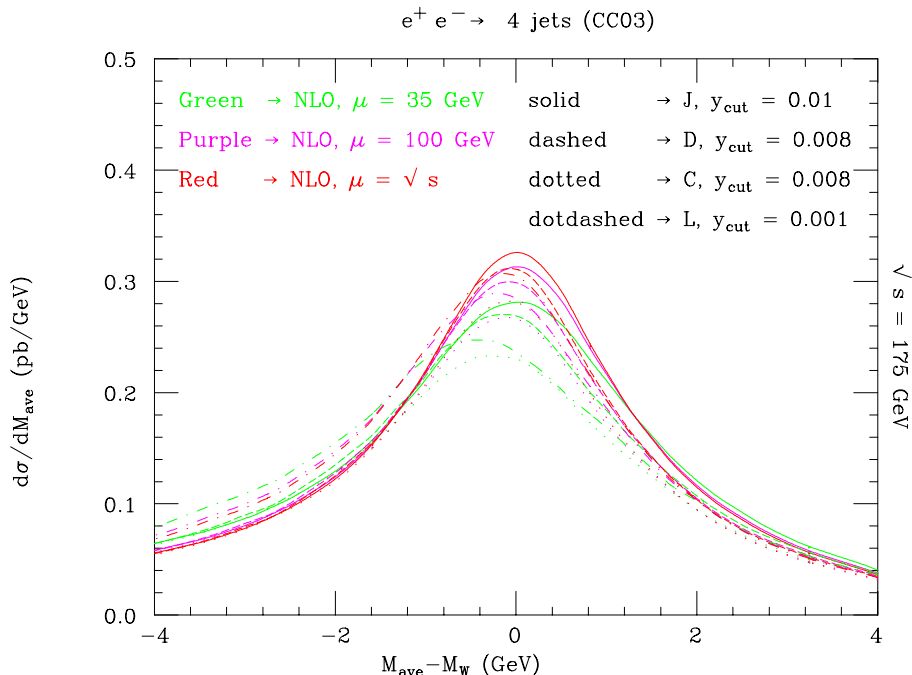
Figure 3: NLO differential distribution in the 'average' mass (as defined in the text) for $e^+e^- \to W^+W^- \to u\bar{d}s\bar{c}$ (CC03), for three choices of the QCD scale μ, as given by four different jet clustering algorithms, at $\sqrt{s} = 175$ GeV. (The values of y_{cut} are chosen such that the LO rates are approximately equal for all schemes.) Total hadronic rates are obtained by multiplying those above times four.

we have collected the values obtained for the mean deviation from $M_{W^\pm}$ of the reconstructed average mass M_{ave}, as predicted by our default jet clustering algorithms and separations. Both at LO and NLO, the difference $< M_{\text{ave}} - M_{W^\pm} >$ is negative, as already observed in [23]. Besides, by comparing the higher order predictions obtained with μ varying from 35 to 175 GeV, one may notice that systematic uncertainties on $M_{W^\pm}$ could turn out be very large in the end, since $< M_{\text{ave}} - M_{W^\pm} >$ can be as large as 500 MeV (in the L scheme).

In order to quantify more realistically the systematic uncertainty on the determination of the $W^\pm$ mass induced by the unknown scale μ, we perform a `MINUIT` [36] fit on the LO and NLO distributions, with a fitting function of the form

$$f(m) = c_1 \frac{c_2^2 c_3^2}{(m^2 - c_2^2)^2 + c_2^2 c_3^2} + g(m) \tag{8}$$

where the term $g(m)$ is meant to simulate a smooth background due to mis-assigned jets induced

	< $M_{\mathrm{ave}} - M_{W^\pm}$ > (GeV)		
Algorithm	$\mu = \sqrt{s}/5 = 35$ GeV	$\mu = \sqrt{s} = 100$ GeV	$\mu = \sqrt{s} = 175$ GeV
J	−0.34	−0.34	−0.34
	−1.91	−1.67	−1.57
D	−0.28	−0.28	−0.28
	−1.71	−1.49	−1.40
C	−0.28	−0.28	−0.28
	−1.94	−1.66	−1.55
L	−0.35	−0.35	−0.35
	−2.49	−2.13	−1.99

Table 1: Mean difference between M_{ave} and $M_{W^\pm}$ as obtained from (some of) the spectra in Figs. 2–3. First line is for LO results (these do not depend upon μ), second line is for the NLO ones. Recall that the input value for the $W^\pm$ mass is 80.430 GeV.

by the clustering algorithm. For the latter, we adopt two possible choices

$$
g(m) = \begin{cases} c_4 + c_5\ (m - c_2) + c_6\ (m - c_2)^2, \\ \\ c_4 \frac{1}{1+\exp((m-c_5)/c_6)}, \end{cases} \tag{9}
$$

that is, a three-term polynomial and a smeared step function (motivated by the kinematical-limit shoulder at large masses and on the same footing as in Ref. [13]). Notice that in eq. (8) we have implicitly assumed a Breit-Wigner shape characterised by a peak height c_1, a position c_2 and a width c_3, corresponding to the normalisation h[10], $M_{W^\pm}$ and $\Gamma_{W^\pm}$, respectively, of the distributions in Figs. 2–3. To first approximation, the difference between the values of the coefficient c_2 as obtained from fitting the above curves is then a measure of the typical size of the systematic error that we are investigating.

Tab. 2 reproduces the results of one of our fits. Whereas the values of some of the parameters (such as the height h and the width $\Gamma_{W^\pm}$ of the distributions) depend sensibly on the choice of the mass interval used for the fit and/or the form of the background, the values extracted for $M_{W^\pm}$ (at a given perturbative order) are remarkably more stable[11]. The parameters in the table should be taken as representative of the qualitative features of all fits that we performed. From there, one notices a strong dependence of the fitted $M_{W^\pm}$ value upon the jet scheme already at lowest order, with variations of up to 160 MeV. Besides, a common features at LO to both fits and all jet clustering algorithms is a smaller reconstructed $W^\pm$ mass, respect to the true one (with a maximum −200 MeV shift for the J scheme, irrespectively of the shape of the background). At NLO, the variation (for a 'fixed' μ) of the fitted $M_{W^\pm}$ value upon the jet algorithm is even larger, up to almost 350 MeV (for the L scheme, when $\mu = 35$ GeV and

---

[10]This is related to the input normalisation for `MINUIT` and is irrelevant to our purposes.

[11]For reason of space, we do not reproduce here the values of the coefficients c_4, c_5 and c_6 which characterise the background.

Algorithm	$\mu = \sqrt{s}/5 = 35$ GeV			$\mu = \sqrt{s} = 175$ GeV		
	h	$M_{W^\pm}$ (GeV)	$\Gamma_{W^\pm}$ (GeV)	h	$M_{W^\pm}$ (GeV)	$\Gamma_{W^\pm}$ (GeV)
	Polynomial background					
J	243.028	80.237	6.835	243.028	80.237	6.835
	517.757	80.464	3.237	575.412	80.454	2.828
D	316.973	80.336	4.953	316.973	80.336	4.953
	480.344	80.298	3.163	539.973	80.343	2.788
C	288.930	80.389	5.520	288.930	80.389	5.520
	422.196	80.211	3.450	486.111	80.292	2.930
L	288.913	80.399	5.547	288.913	80.399	5.547
	453.062	80.085	3.801	529.413	80.217	3.142
	Smeared step function background					
J	239.120	80.237	6.704	239.120	80.237	6.704
	516.169	80.451	3.213	601.842	80.472	3.008
D	319.045	80.336	5.014	319.045	80.336	5.014
	521.475	80.286	3.450	549.312	80.332	2.890
C	285.231	80.389	5.410	285.231	80.389	5.410
	454.114	80.226	3.619	517.671	80.320	3.163
L	284.960	80.400	5.428	284.960	80.400	5.428
	456.933	80.154	3.791	515.275	80.221	3.127

Table 2: Fits to (some of) the M_{ave} spectra in Figs. 2–3. First three columns are for $\mu = \sqrt{s}/5$, last three are for $\mu = \sqrt{s}$, with $\sqrt{s} = 175$ GeV. A Breit-Wigner shape is always assumed, supplemented by a three-term polynomial (upper section) or a smeared step function (lower section) to emulate the intrinsic background. First line is for LO results (these do not depend upon μ), second line is for the NLO ones. Here, we have fitted the M_{ave} distributions over the mass interval 75 to 85 GeV. Recall that the input value for the $W^\pm$ mass is 80.430 GeV.

assuming a polynomial background). Besides, at this order, the shift in mass can have either sign: positive (for the J scheme) or negative (for all others).

Clearly, the NLO results are those most representative of the theory. Besides, all effects discussed above are well quantifiable, since they are merely of kinematical origin (the different handling of gluon radiation by the various jet clustering algorithms). As already stressed repeatedly, it is the systematics associated with the choice of μ that is beyond theoretical control. It turns out that such an intrinsic uncertainty of the fixed-order QCD prediction can be rather large in the determination of the $W^\pm$ mass: compare the values for $M_{W^\pm}$ as obtained in the NLO fits and given on the left of Tab. 2 to those on the right. The differences between the reconstructed $W^\pm$ values for $\mu = 35$ and 175 GeV (which, hereafter, we denote by $\delta_{\mathrm{NLO}} M_{W^\pm}$) can become as large as 130 MeV (for the L scheme, in presence of a polynomial background). The J scheme seems to be here the least sensitive to μ-variations (a 10 to 20 MeV effect), with the D and C ones falling in between: however, this pattern was not always respected in the various fits that we carried out. Notice that, in this exercise, we have restrained ourselves to values of μ not smaller than $\sqrt{s}/5$, precisely in order to avoid the mentioned logarithmic effects induced by a choice of μ too close to the jet-scheme-dependent emission scale $\sqrt{y_{cut}s}$ (see also Footnote 7). Indeed, for μ in the above interval, we have found that to change the values of y_{cut} (still maintaining the typical four-jet separations used in experimental analyses) does not affect our conclusions. In fact, the latter do not change either, if one adopts a different recombination procedure of the cluster momenta.

In summary, we have verified that theoretical errors arising from the use of different jet clustering algorithms in treating the fixed-order $\mathcal{O}(\alpha_s)$ corrections from perturbative QCD, in the prediction of experimental observables used for the extraction of the $W^\pm$ boson mass in the hadronic channel $e^+e^- \to W^+W^- \to 4$ jets (CC03) at LEP2, can be competitive with similar systematic effects that could be induced by non-perturbative dynamics, such as CR and BECs, e.g., as predicted in the hadronisation model of Ref. [37]. In particular, using various jet definitions, we have shown that these uncertainties on $M_{W^\pm}$ can be of order 100 MeV, hence larger than current experimental assumptions on the size of the theoretical error. Unfortunately, from our analysis, a preferred choice of jet clustering algorithm, which would minimise such effects, did not emerge clearly.

We also have verified that the inclusion of irreducible background via CC11 diagrams has little impact on our main conclusions, so has the incorporation of ISR effects. Similarly, a different treatment of the widths in the resonant propagators (the fixed-width scheme was adopted here) has negligible consequences for both CC03 and CC11. A different choice of $\sqrt{s}$ (at fixed y_{cut}'s, or vice versa) yields similar estimates of $\delta_{\mathrm{NLO}} M_{W^\pm}$ to those given here. Also, if one enforces typical $W^\pm$ mass reconstruction cuts, say, $|M_{R_i} - M_{W^\pm}| < \delta$, for $i = 1, 2$ and 10 GeV $< \delta < 30$ GeV, see eqs. (6)–(7), typical values of $\delta_{\mathrm{NLO}} M_{W^\pm}$ remain in the above range, despite the effects on the actual event rates can be dramatic [23].

Finally, effects due to the kinematic interplay between jet clustering algorithms and PS (including hadronisation) were not in the original intentions of this study, as they have already been addressed in Ref. [13]. Whereas the latter can be estimated in the context of an event level MC analysis, those considered here are intrinsic uncertainties of the theory. The results of our present analysis point to the fact that such perturbative QCD effects may not yet be under control, at least in the context of $W^\pm$ mass determinations from the hadronic data samples

11

collected at LEP2. Taking also into account the results of Ref. [13] in the same context, one may conclude that a thorough reassessment of the theoretical systematics entering the $e^+e^- \to W^+W^- \to$ hadrons channel is in order, given the importance that the precise knowledge of the $W^\pm$ mass has in constraining the properties of yet undiscovered particles, such as the Higgs boson mass.

Acknowledgements

References

[1] See, e.g.:
LEP Electroweak Working Group, `http://lepewwg.web.cern.ch/LEPEWWG/lepww/mw/`.

[2] The LEP Collaborations ALEPH, DELPHI, L3, OPAL and the LEP W Working Group, preprint LEPEWWG/MASS/2001-02; ALEPH 2001-044 PHYSICS 2001-017, DELPHI 2001-122 PHYS 899, L3 Note 2695, OPAL TN-697, July 2001 (to be found in [1]).

[3] G. Gustafson, U. Pettersson and P.M. Zerwas, *Phys. Lett.* **B209** (1988) 90;
T. Sjöstrand and V.A. Khoze, *Phys. Rev. Lett.* **72** (1994) 28; *Z. Phys.* **C62** (1994) 281;
G. Gustafson and J. Häkkinen, *Z. Phys.* **C64** (1994) 659;
V.A. Khoze, W.J. Stirling, S. Moretti, A. Ballestrero and E. Maina, *Z. Phys.* **C74** (1997) 493.

[4] L. Lönnblad and T. Sjöstrand, *Phys. Lett.* **B351** (1995) 293.

[5] B.R. Webber, *J. Phys.* **G24** (1998) 287.

[6] A. Ballestrero et al., *J. Phys.* **G24** (1998) 365.

[7] T. Sjöstrand and V.A. Khoze, in Ref. [3].

[8] E. Accomando, A. Ballestrero and E. Maina, *Phys. Lett.* **B362** (1995) 141;
M. Smith, *Eur. Phys. J.* **C9** (1999) 95.

[9] A. Donnachie and P.V. Landshoff, *Phys. Lett.* **B421** (1998) 341.

[10] S. Moretti, *Eur. Phys. J.* **C16** (2000) 115.

[11] B.R. Webber, talk given at 19th International Symposium on Lepton and Photon Interactions at High-Energies (LP 99), Stanford, California, 9-14 August 1999, preprint CERN-TH/99-387, Cavendish-HEP-99/16, December 1999, `hep-ph/9912292`.

[12] N.K. Watson, talk given at Moriond Electroweak 2001, `hep-ph/0105230`.

[13] S. Moretti, L. Lönnblad and T. Sjöstrand, *JHEP* **08** (1998) 001.

[14] R.K. Ellis, W.J. Stirling and B.R. Webber, "QCD and Collider Physics" (Cambridge University Press, Cambridge 1996).

[15] G. Corcella, I.G. Knowles, G. Marchesini, S. Moretti, K. Odagiri, P. Richardson, M.H. Seymour and B.R. Webber, preprint Cavendish-HEP-99/17, December 1999, `hep-ph/9912396`; JHEP **01** (2001) 010; preprint CAVENDISH-HEP-01-08, CERN-TH-2001-173, DAMTP-2001-61, July 2001, `hep-ph/0107071`.

[16] T. Sjöstrand, *Comp. Phys. Comm.* **39** (1984) 347;
M. Bengtsson and T. Sjöstrand, *Comp. Phys. Comm.* **43** (1987) 367;
T. Sjöstrand, preprint LU-TP/95-20, December 1998, `hep-ph/9508391`.

[17] L. Lönnblad, *Comp. Phys. Comm.* **71** (1992) 15; preprint December 1997, to be found at `http://www.thep.lu.se/staff/leif/ariadne/`.

[18] D. Bardin and R. Kleiss (conveners), preprint September 1997, `hep-ph/9709270`, in Proceedings of the Workshop 'Physics at LEP2', G. Altarelli, T. Sjöstrand and F. Zwirner (editors), CERN 96-01 (and references therein).

[19] R. Kuhn, F. Krauss, B. Ivanyi and G. Soff, *Comput. Phys. Commun.* **134** (2001) 223;
F. Krauss, R. Kuhn and G. Soff, *Acta Phys. Polon.* **B30** (1999) 3875;
B.R. Webber, talk at the XXXV Rencontres de Moriond, Les Arcs, France, March 2000, preprint Cavendish-HEP-00/05, May 2000, `hep-ph/0005035`.

[20] S. Moretti and W.J. Stirling, *Eur. Phys. J.* **C9** (1999) 81.

[21] N. Brown, *Z. Phys.* **C51** (1991) 107.

[22] S. Moretti, *Z. Phys.* **C75** (1997) 465; *Eur. Phys. J.* **C9** (1999) 229.

[23] E. Maina and M. Pizzio, *Phys. Lett.* **B369** (1996) 341.

[24] E. Maina, R. Pittau and M. Pizzio, *Phys. Lett.* **B429** (1998) 354; preprint DFTT-48-97, September 1997, `hep-ph/9709454`.

[25] G. Grunberg, *Phys. Lett.* **B95** (1980) 70;
S.J. Brodsky, G.P. Lepage and P.B. Mackenzie, *Phys. Rev.* **D28** (1983) 228;
P.M. Stevenson, *Nucl. Phys.* **B231** (1984) 65;
H.D. Politzer, *Nucl. Phys.* **B194** (1982) 493.

[26] DELPHI Collaboration, *Eur. Phys. J.* **C14** (2000) 557.

[27] Yu.L. Dokshitzer, G.D. Leder, S. Moretti and B.R. Webber, *JHEP* **08** (1997) 001.

[28] S. Bethke, Z. Kunszt, D.E. Soper and W.J. Stirling, *Nucl. Phys.* **B370** (1992) 310; Erratum, preprint `hep-ph/9803267`.

[29] V.S. Fadin, V.A. Khoze and A.D. Martin, *Phys. Lett.* **B311** (1993) 311;
D. Bardin, W. Beenakker and A. Denner, *Phys. Lett.* **B317** (1993) 213;
V.S. Fadin, V.A. Khoze, A.D. Martin and A. Chapovsky, *Phys. Rev.* **D52** (1995) 1377;
V.A. Khoze and W.J. Stirling, *Phys. Lett.* **B356** (1995) 373;
V.S. Fadin, V.A. Khoze, A.D. Martin and W.J. Stirling, *Phys. Lett.* **B363** (1995) 112.

[30] JADE Collaboration, *Z. Phys.* **C33** (1986) 23;
S. Bethke, Habilitation thesis, preprint LBL 50-208 (1987).

[31] Yu.L. Dokshitzer, contribution cited in the 'Report of the Hard QCD Working Group', in Proceedings of the Workshop 'Jet Studies at LEP and HERA', Durham, December 1990, *J. Phys.* **G17** (1991) 1537;
S. Catani, Yu.L. Dokshitzer, M. Olsson, G. Turnock and B.R. Webber, *Phys. Lett.* **B269** (1991) 432.

[32] T. Sjöstrand, *Comp. Phys. Comm.* **28** (1983) 227.

[33] M. Bilenky, G. Rodrigo and A. Santamaria, talk given at the IVth International Symposium on Radiative Corrections (RADCOR98), Barcelona, Catalonia, Spain, 8-12 September 1998, preprint FTUV/98-94, IFIC/98-95, December 1998, `hep-ph/9812433`; contribution to the XXIX International Conference on High Energy Physics, Vancouver, Canada, July 1998, preprint FTUV/98-79, IFIC/98-80, November 1998, `hep-ph/9811465`;
M. Bilenky, S. Cabrera, J. Fuster, S. Marti, G. Rodrigo and A. Santamaria, *Phys. Rev.* **D60** (1999) 114006.

[34] Z. Nagy and Z. Trócsányi, *Phys. Rev.* **D59** (1999) 014020; Erratum, *ibidem* **D62** (2000) 099902; *ibidem* **D57** (1998) 5793.

[35] See, e.g.:
ALEPH Collaboration, *Phys. Lett.* **B422** (1998) 384;
DELPHI Collaboration, *Eur. Phys. J.* **C2** (1998) 581.

[36] F. James and M. Roos, *Comp. Phys. Comm.* **10** (1975) 343.

[37] J. Ellis and K. Geiger, *Phys. Lett.* **B404** (1997) 230.